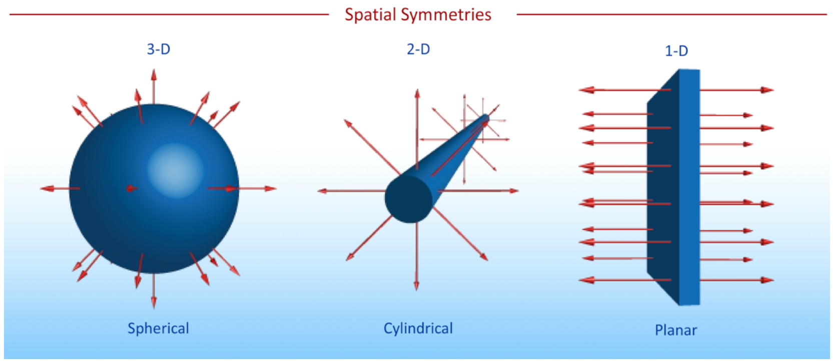
# *Electricity & Magnetism*
# *Lecture 4: Gauss' Law*

## Today's Concepts:

A) Conductors

B) Using Gauss' Law



| |
|---|
| Some iclicker questions are not collected /read (it says no submission even though we should have gotten the right asnwer). |
| . |
| I am super confused |
| Do we not consider Simi-conductors in this class? |
| Are there like any reliable video sources on youtube for this course? |
| I dont understand the "Checkpoint Charged Conducting Sphere and Spherical Shell" question at all. Can you please explain when E and q are 0 outside of a surface (ex. in between surfaces, under a surface) |
| not sure what kind of problems we would face on this topic |
| Everything was really confusing :( |
| This was a very difficult very confusing lecture. Please HELP!!!!!! |
| Please go over the situations where you need to calculate the electric field inside of a spherical insulator and conductor. |
| i have no idea as to what is going on |
| This lecture was the most confusing so far, there are too many things and given too quickly. Especially the infinite line charge. |
| What is this??? It's all Greek to me |
| i found not loving you to be the most difficult concept <3 |
| Could you please explain why the charge on a conductor goes to the surface. |
| Can you go over the charged shell? |
| Do we need to memorize all these equations for different insulators/conductors? |
| could you please go over the third question in the checkpoint? why is the field between the solids equal 0? |
| please go over the pre lecture questions. |
| Help me pls i have no idea what this is |
| Could you please elaborate on the example in the prelecture where a charge is enclosed within a shell with an inner and outer radius? |

# Stuff you asked about:

- if the conductor allows charges to move freely then why $E_{inside} = 0$?
- ¨Will we have do any integrals?”
- Some iclicker questions are not collected /read (it says no submission even though we should have gotten the right asnwer).
- Could you explain why the E field in the conductive sphere example is 0? what does equilibrium mean in this case? Also how does E field not depend on distance from the plane in the case of infinite planes of charge? would a charge halfway across the galaxy experience the same electric force from an infinite plane of charge as a charge right next to the plane?
- What does Gauss's Law describe?
-

# *More videos*

"Are there like any reliable video sources on youtube for this course?"

- Electric Flux (MIT)
- The Mechanical Universe: Electric Fields

# Office Hours

Ali, P9416 Wed 14:30 to 15:20

Cristina, P9416 Thurs 15:30 to 16:20

Neil, P9444, MWF 11:00 to 12:00

Other times by arrangement.

# Conductors = Charges Free to Move

## Claim: $E = 0$ inside any conductor at equilibrium

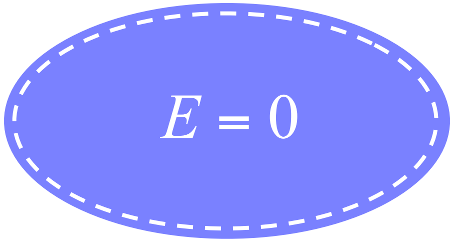
Charges in conductor move to make $E$ field zero inside. (Induced charge distribution).

If $E \neq 0$, then charge feels force and moves!

## Claim: Excess charge on conductor only on surface at equilibrium

Why?

- Apply Gauss' Law
  - Take Gaussian surface to be just inside conductor surface
  - $E = 0$ everywhere inside conductor → $\oint_{\text{surface}} \vec{E} \cdot \vec{A} = 0$
  - Gauss' Law: $\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$ → $Q_{enc} = 0$



# Gauss' Law + Conductors + Induced Charges

$$\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

ALWAYS TRUE!

$$E = \frac{Q_{enc}}{A\varepsilon_o}$$

If choose a Gaussian surface that is entirely in metal, then $E = 0$ so $Q_{enclosed}$ must also be zero!

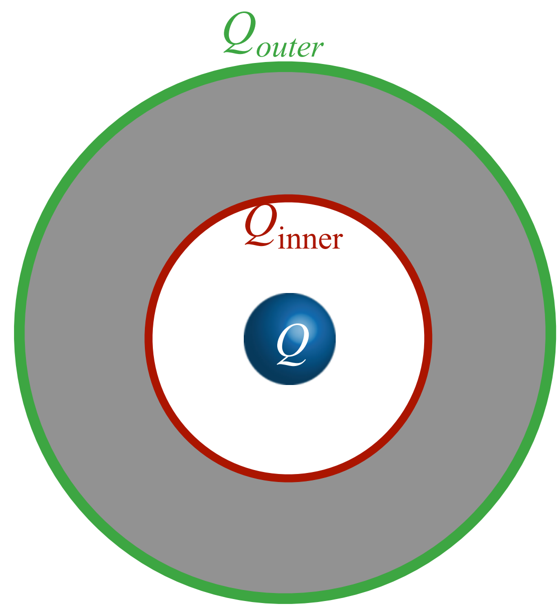
How Does This Work?

Charges in conductor move to surfaces to make $Q_{enclosed} = 0$.

We say charge is induced on the surfaces of conductors

# Clicker Question: Charge in Cavity of Conductor

A particle with charge $+Q$ is placed in the center of an uncharged conducting hollow sphere. How much charge will be induced on the inner and outer surfaces of the sphere?




A) inner = $-Q$, outer = $+Q$

B) inner = $-Q/2$ , outer = $+Q/2$

C) inner = $0$, outer = $0$

D) inner = $+Q/2$, outer = $-Q/2$

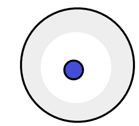
E) inner = $+Q$, outer = $-Q$

Since $E = 0$ in conductor

- Gauss' Law: $\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$ → $Q_{enc} = 0$

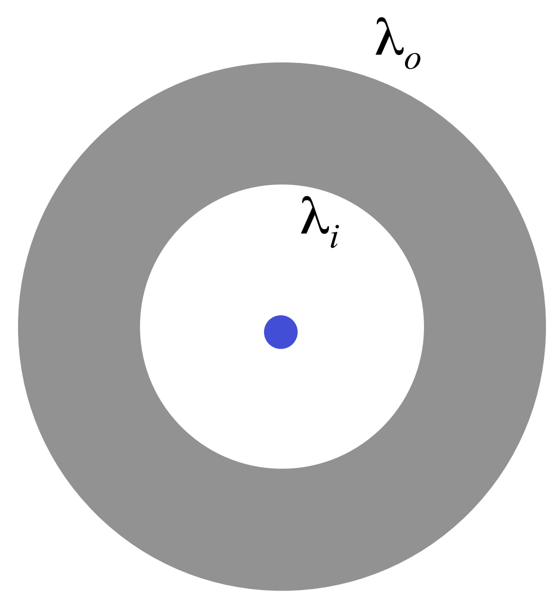
# Clicker Question: Infinite Cylinders

A long thin wire has a uniform positive charge density of 2.5 C/m. Concentric with the wire is a long thick conducting cylinder, with inner radius 3 cm, and outer radius 5 cm. The conducting cylinder has a net linear charge density of –4 C/m.



What is the linear charge density of the induced charge on the inner surface of the conducting cylinder ($\lambda_i$) and on the outer surface ($\lambda_o$)?

| | | | | | |
|---|---|---|---|---|---|
| $\lambda_i$: | +2.5 C/m | –4 C/m | –2.5 C/m | –2.5 C/m | 0 |
| $\lambda_o$: | –6.5 C/m | 0 | +2.5 C/m | –1.5 C/m | –4 C/m |
| | A | B | C | D | E |



# Gauss' Law

$$\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull $E$ outside and get $E = \frac{Q_{enc}}{A\varepsilon_0}$

In General, integral to calculate flux is difficult…. and not useful!

To use Gauss' Law to calculate $E$, need to choose surface carefully!

1) Want $E$ to be constant and equal to value at location of interest

OR

2) Want $E$ dot $A = 0$ so doesn't add to integral

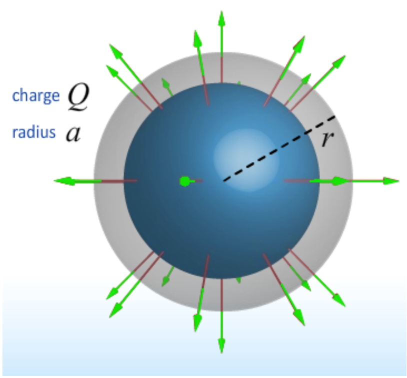
# Gauss' Law Symmetries

$$\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull $E$ outside and get $E = \dfrac{Q_{enc}}{A\varepsilon_0}$

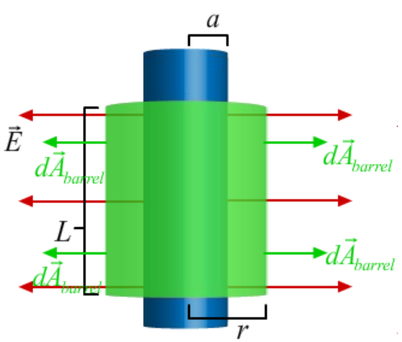
## Spherical




$$A = 4\pi r^2$$

$$E = \frac{Q_{enc}}{4\pi r^2 \varepsilon_0}$$

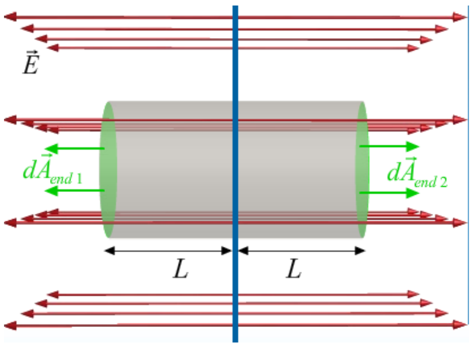
## Cylindrical




$$A = 2\pi r L$$

$$E = \frac{\lambda}{2\pi r \varepsilon_0}$$

## Planar

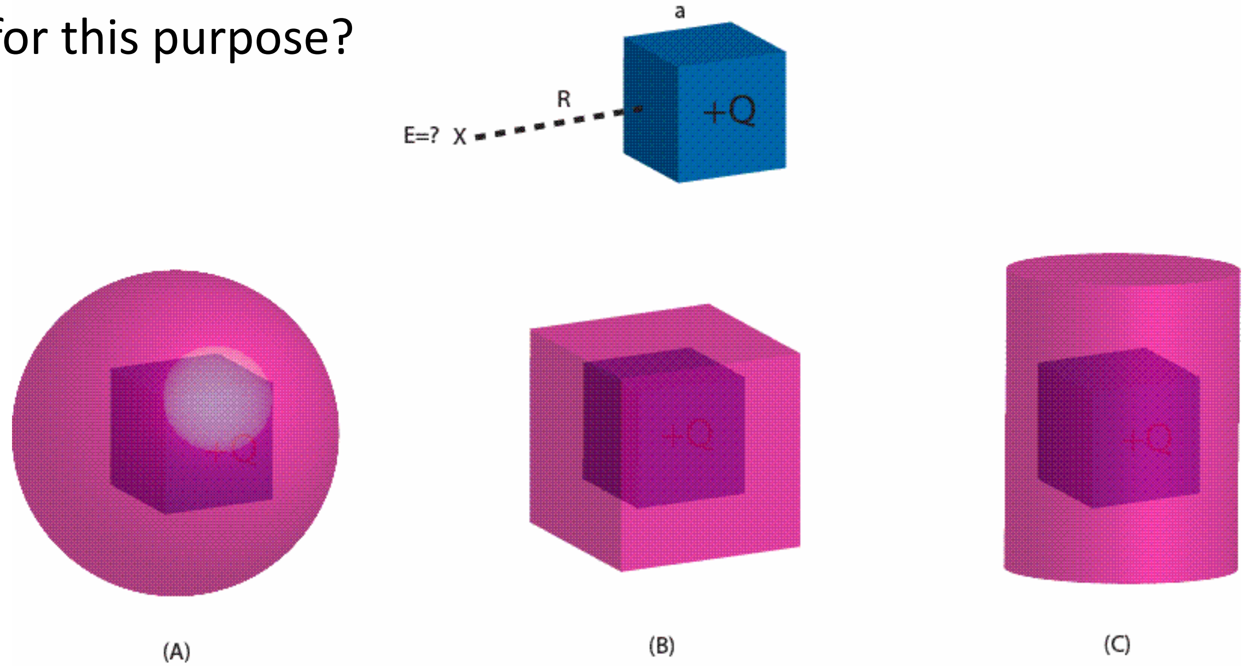



$$A = 2\pi r^2$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

# CheckPoint: Gaussian Surface Choice

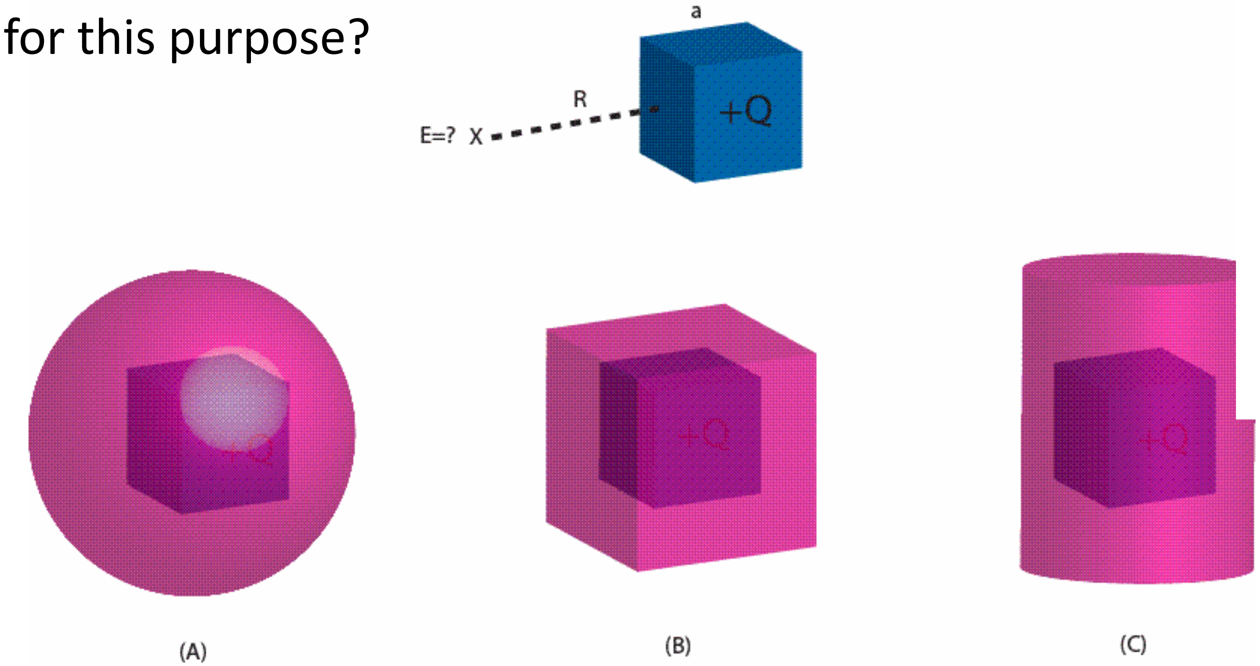
You are told to use Gauss' Law to calculate the electric field at a distance **R** away from a charged cube of dimension **a**. Which of the following Gaussian surfaces is best suited for this purpose?

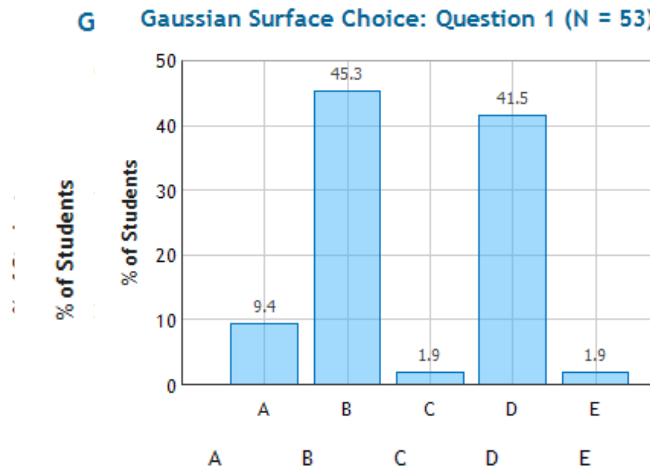



A. a sphere of radius R+½a
B. a cube of dimension R+½a
C. a cylinder with cross sectional radius of R+½a
   and arbitrary length
D. This field cannot be calculated using Gauss' law
E. None of the above

# CheckPoint Results: Gaussian Surface Choice

You are told to use Gauss' Law to calculate the electric field at a distance **R** away from a charged cube of dimension **a**. Which of the following Gau
for this purpose?




A. a sphere of radius R+½a
B. a cube of dimension R+½a
C. a cylinder with cross sectional radius of R+½a
and arbitrary length
D. This field cannot be calculated using Gauss' law
E. None of the above

**THE CUBE HAS NO GLOBAL SYMMETRY!**

THE FIELD AT THE FACE OF THE CUBE
IS NOT
PERPENDICULAR OR PARALLEL

| | | | |
|---|---|---|---|
| 3D | POINT | ® | SPHERICAL |
| 2D | LINE | ® | CYLINDRICAL |
| 1D | PLANE | ® | PLANAR |

“could you please go over the third question in the checkpoint? why is the field between the solids equal 0?”

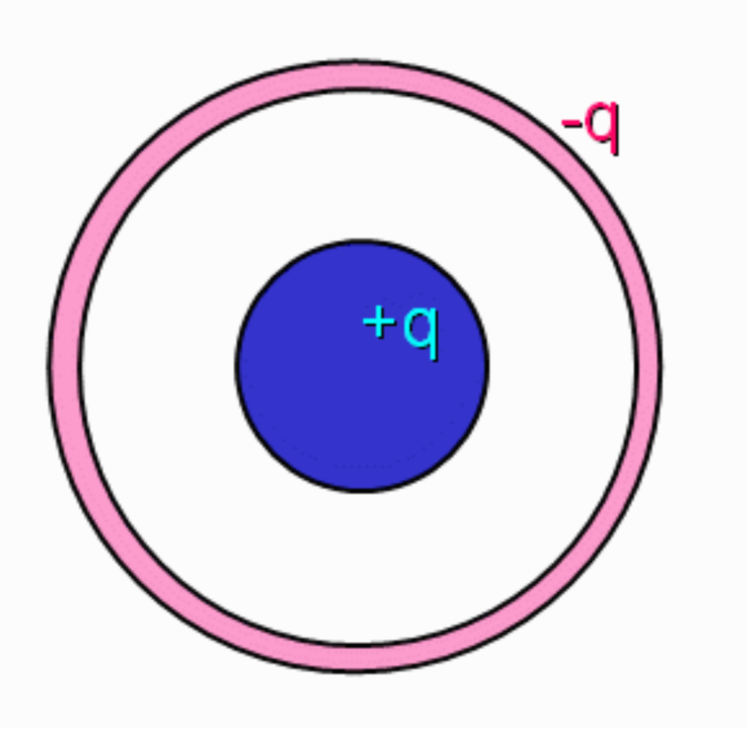
# CheckPoint: Charged Conducting Sphere & Shell 1

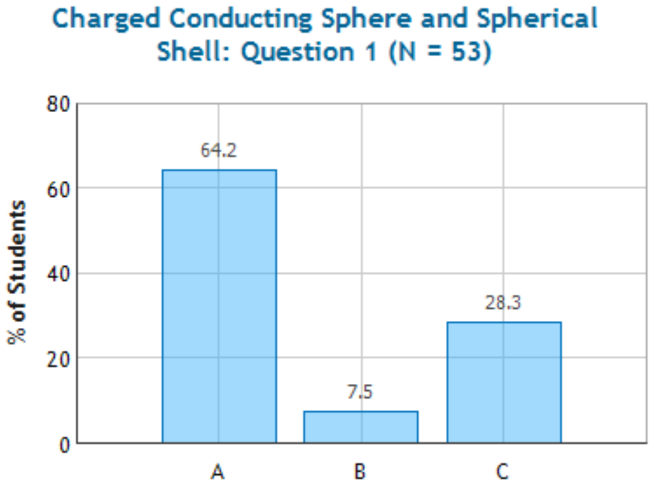
A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same. Which of the following statements best describes the electric field in the region between the spheres?

A. The field points radially outward

B. The field points radially inward

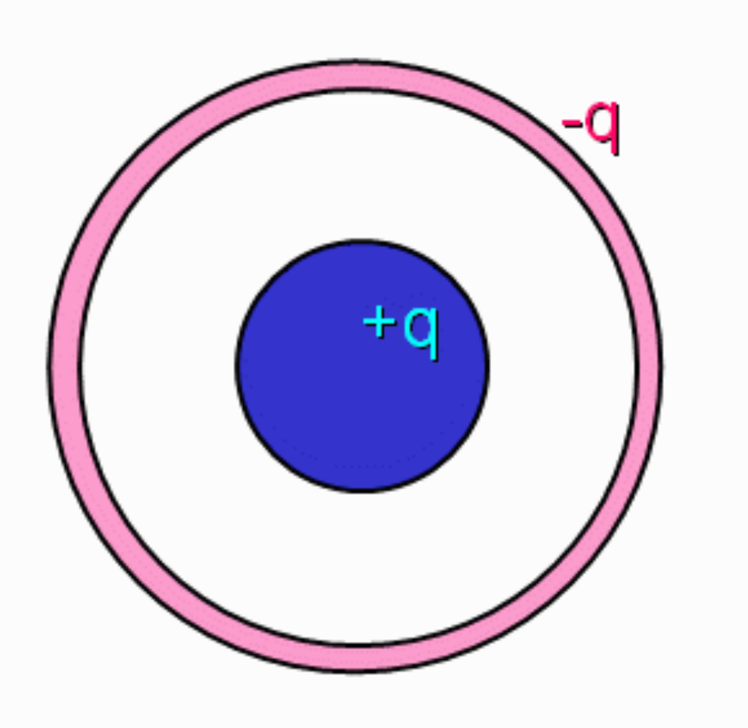
C. The field is zero



# CheckPoint: Charged Conducting Sphere & Shell 2

A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same. Which of the following statements best describes the electric field in the region outside the red sphere?




A. The field points radially outward
B. The field points radially inward
C. The field is zero

"Since they have the same charge, the efield from the red sphere is larger than the efield from the blue sphere. So the red field points inwards, the blue sphere points outwards so the resultant is outward"

" closest influence is inwards"

"if +q = –q, the field is zero, because the enclosed charge inside the Gaussian surface is +q + (-q) = 0, since E = total q / A, and q is 0, E is also 0"

# CheckPoint: Charged Sphericlal Shell

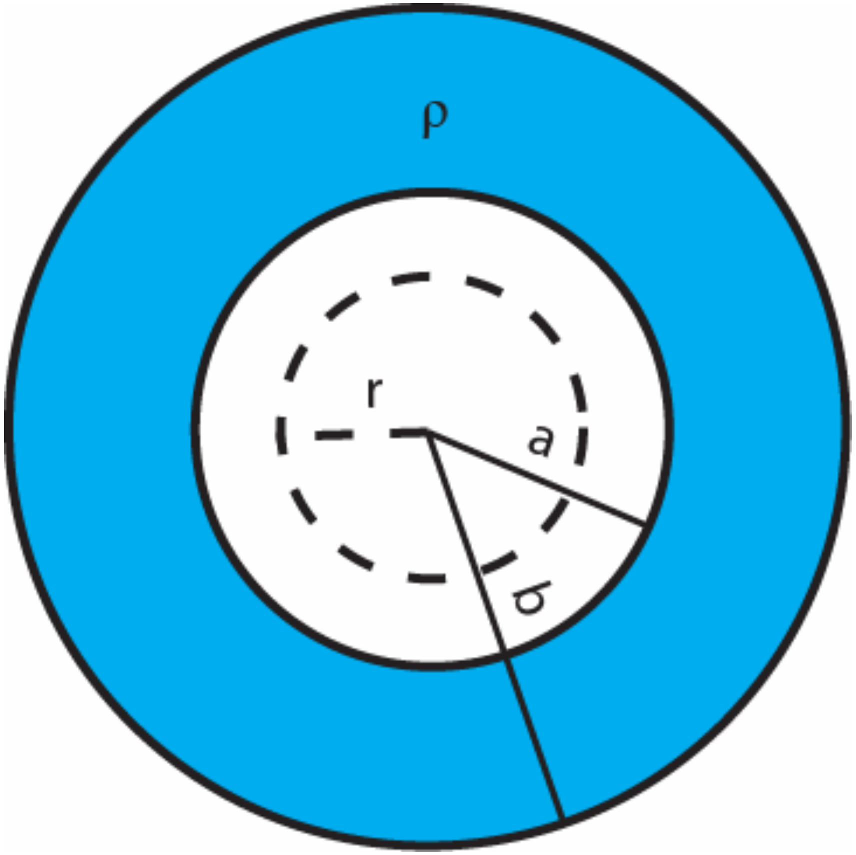
A charged spherical insulating shell has inner radius a and outer radius b. The charge density on the shell is **ρ**. What is the magnitude of the E-field at a distance r away from the center of the shell where r < a?

A. $\rho/\varepsilon_o$

B. zero

C. $\rho(b^3-a^3)/(3\varepsilon_o r^2)$

D. none of the above



# CheckPoint Results: Charged Sphericlal Shell

A charged spherical insulating shell has inner radius a and outer radius b. The charge density on the shell is **ρ**. What is the magnitude of the E-field at a distance r away from the center of the shell where r < a?

A. $\rho/\varepsilon_o$

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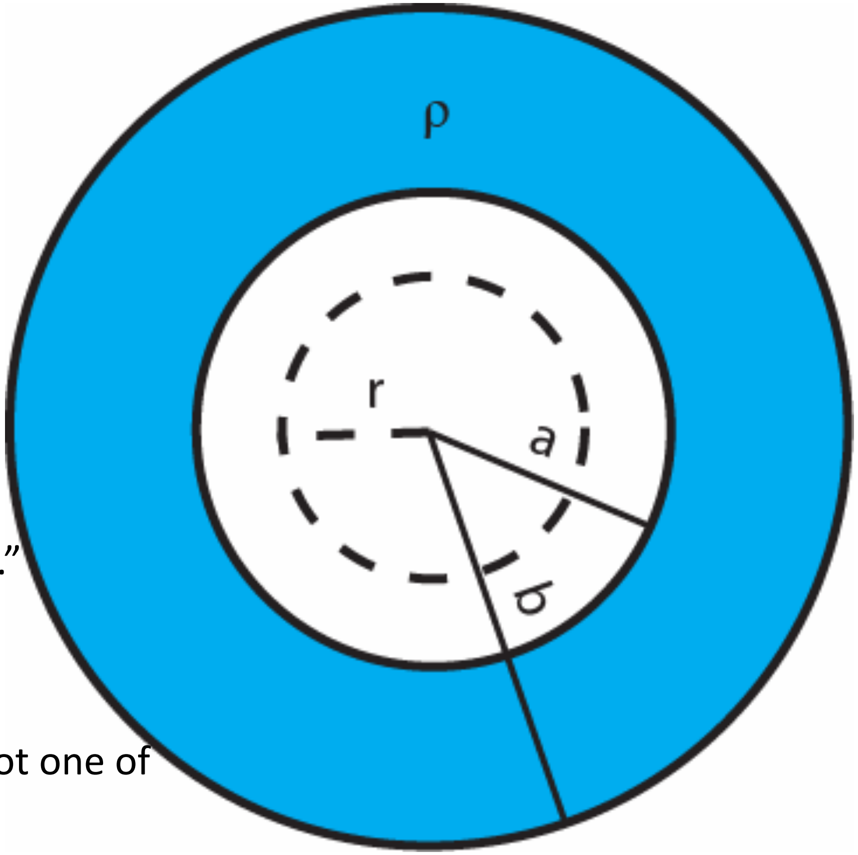



Latest

"I'm not actually sure but it seems like the right answer."

Trick!!!! There is no enclosed charge in r<a HA!

The e-field at r is given by ρ*r/(3*epsilon-0), which is not one of the options.

Applications?

# On the Effectiveness of Aluminium Foil Helmets:

# An Empirical Study

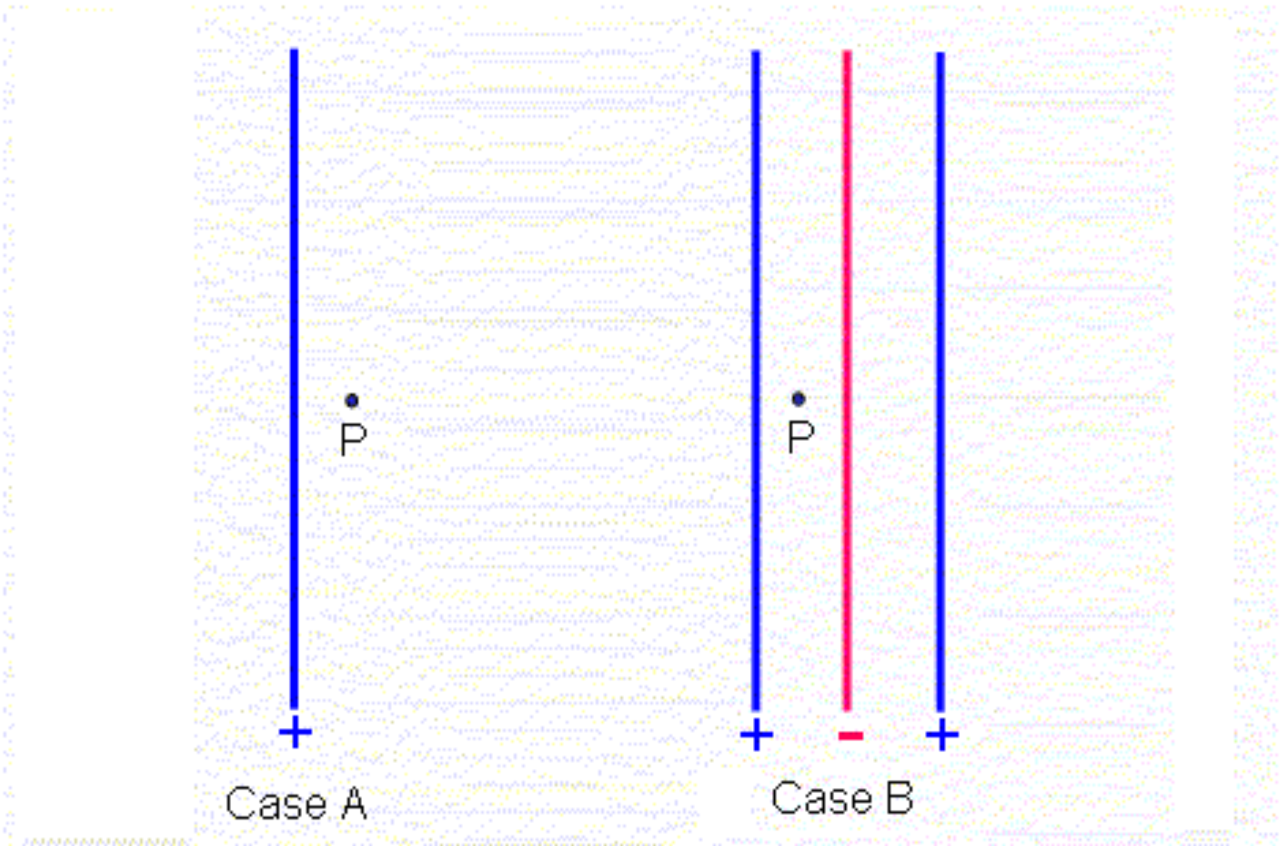
*Ali Rahimi[1], Ben Recht [2], Jason Taylor [2], Noah Vawter [2]*
*17 Feb 2005*

1: Electrical Engineering and Computer Science department, MIT.
2: Media Laboratory, MIT.

# CheckPoint: Infinite Sheets of Charge

In both cases shown below, the colored lines represent positive (blue) and negative (red) charged planes. The magnitudes of the charge per unit area on each plane is the same. In which case is the magnitude of the electric field at point P bigger?

A. Case A
B. Case B
C. They are the same



# CheckPoint Results: Infinite Sheets of Charge

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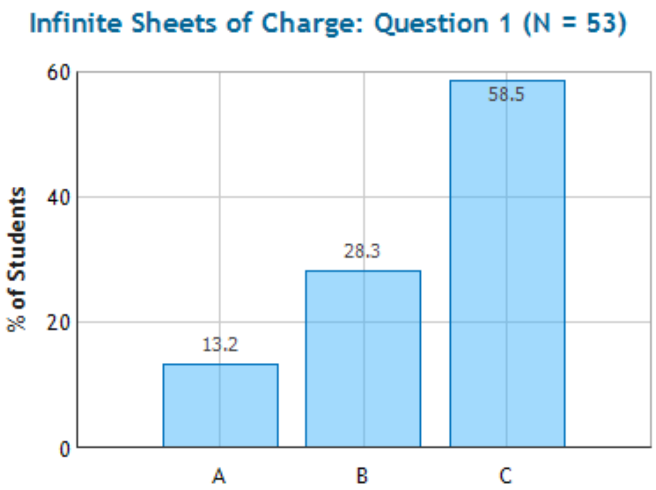
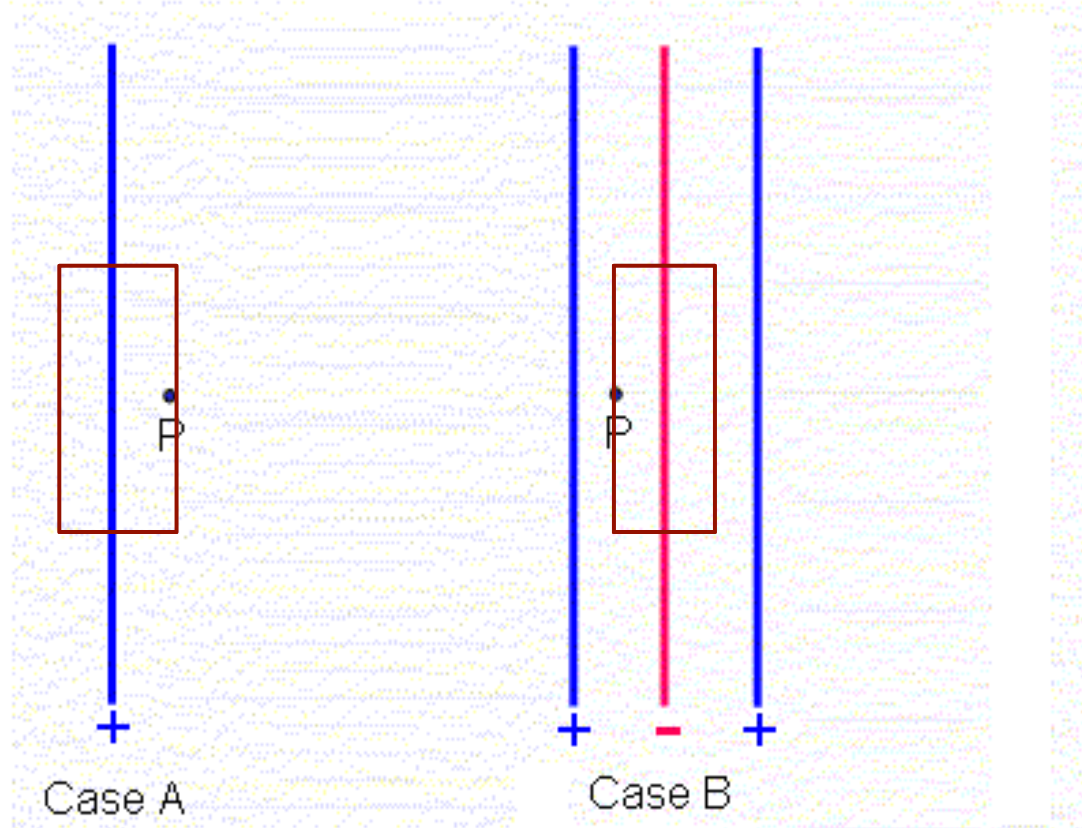
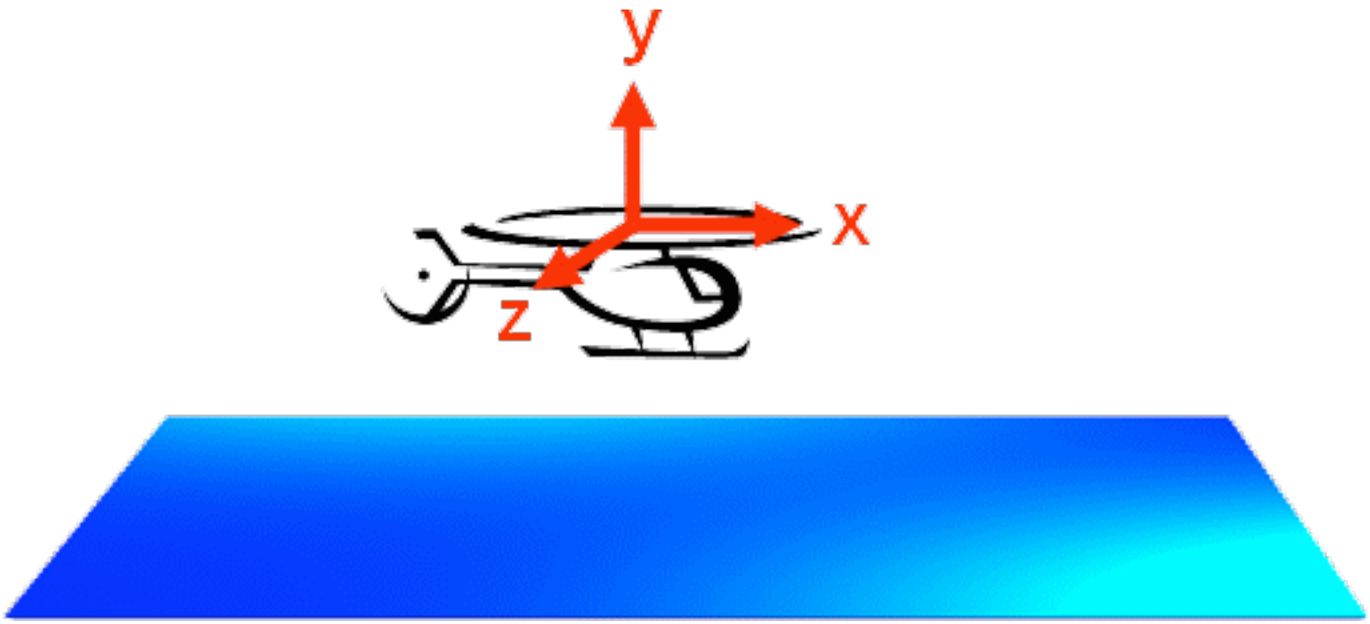
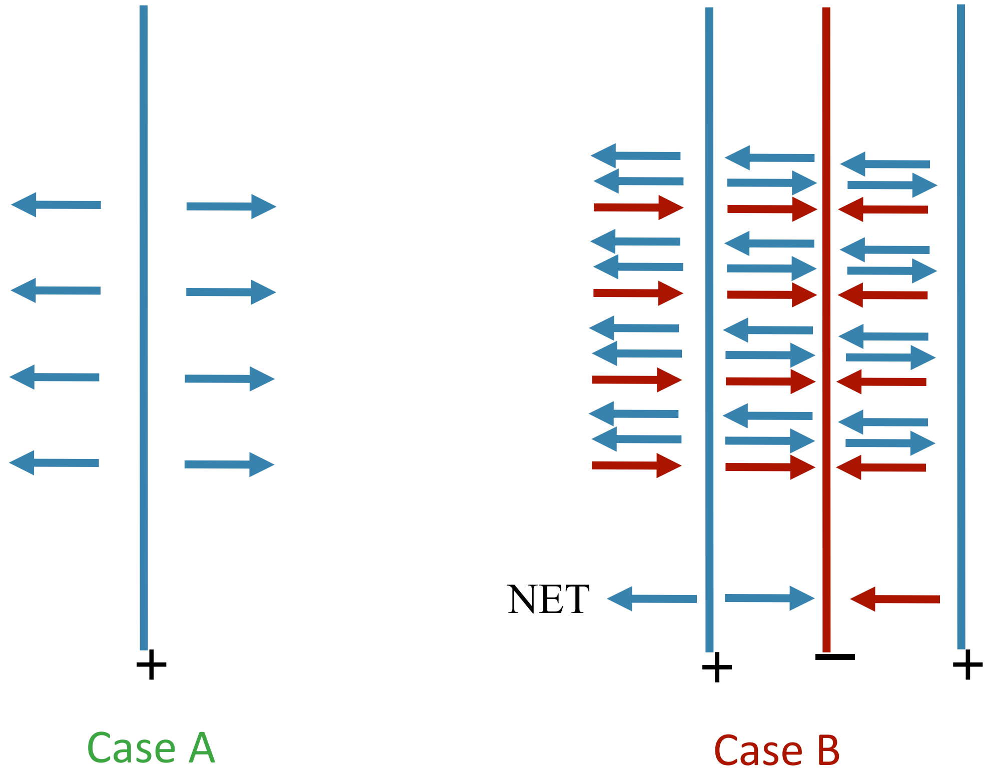


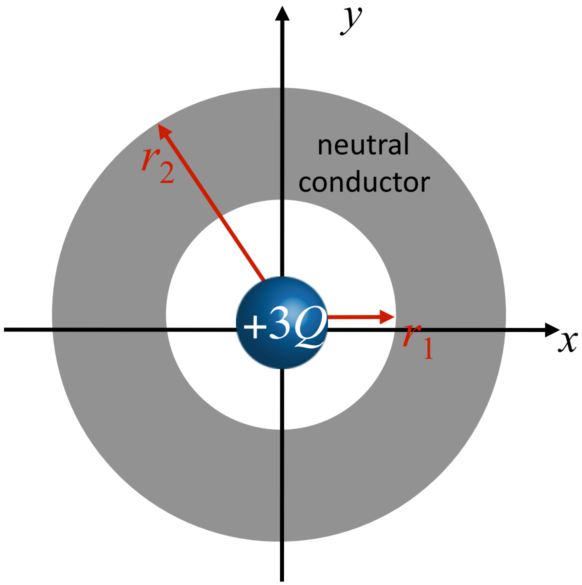




y
x
z

# Superposition:



# Calculation




Point charge $+3Q$ at center of neutral conducting shell of inner radius $r_1$ and outer radius $r_2$.

a) What is $E$ everywhere?

First question: Do we have enough symmetry to use Gauss' Law to determine $E$?

Yes, Spherical Symmetry (what does this mean???)

A) Magnitude of $E$ is $fcn$ of $r$

B) Magnitude of $E$ is $fcn$ of $(r\text{-}r_1)$

C) Magnitude of $E$ is $fcn$ of $(r\text{-}r_2)$

D) None of the above

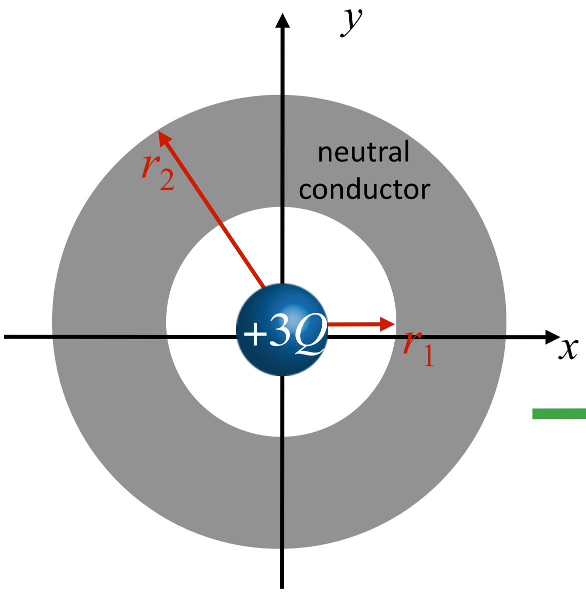
A) Direction of $E$ is along $\hat{x}$

B) Direction of $E$ is along $\hat{y}$

C) Direction of $E$ is along $\hat{r}$

D) None of the above

# Calculation




Point charge $+3Q$ at center of neutral conducting shell of inner radius $r_1$ and outer radius $r_2$.

A) What is $E$ everywhere?

We know:

magnitude of $E$ is *fcn* of $r$

direction of $E$ is along $\hat{r}$

We can use Gauss' Law to determine $E$

Use Gaussian surface = sphere centered on origin

$$\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$r < r_1$

$$\oint_{\text{surface}} \vec{E} \cdot \vec{A} = E 4\pi r^2$$

$$Q_{enc} = +3Q$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{3Q}{r^2}$$

$r_1 < r < r_2$

A) $E = \frac{1}{4\pi\varepsilon_0} \frac{3Q}{r^2}$

B) $E = \frac{1}{4\pi\varepsilon_0} \frac{3Q}{r_1^2}$

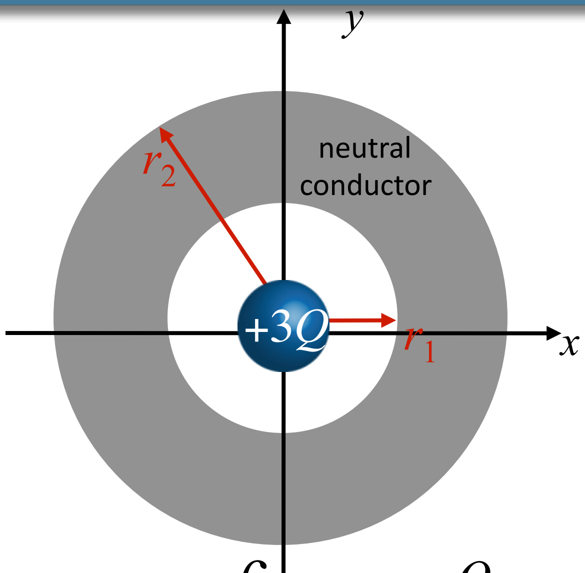
C) $E = 0$

$r > r_2$

A) $E = \frac{1}{4\pi\varepsilon_0} \frac{3Q}{r^2}$

B) $E = \frac{1}{4\pi\varepsilon_0} \frac{3Q}{(r - r_2)^2}$

C) $E = 0$




Point charge $+3Q$ at center of neutral conducting shell of inner radius $r_1$ and outer radius $r_2$.

A) What is $E$ everywhere?

We know:

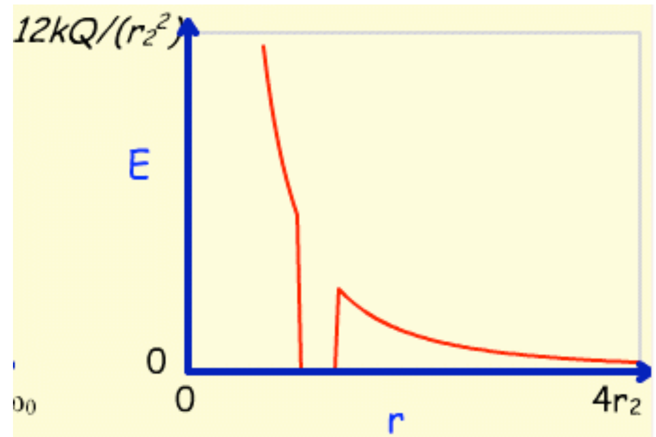
$r < r_1$

$r > r_2$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{3Q}{r^2}$$

$r_1 < r < r_2$ $\quad E = 0$

$$\oint_{\text{surface}} \vec{E}\cdot\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

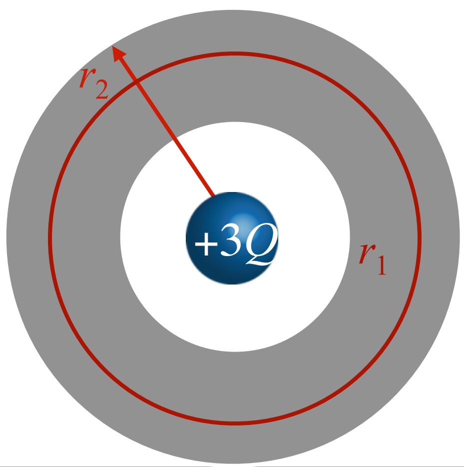



B) What is charge distribution at $r_1$?

A) $\sigma < 0$

B) $\sigma = 0$

C) $\sigma > 0$



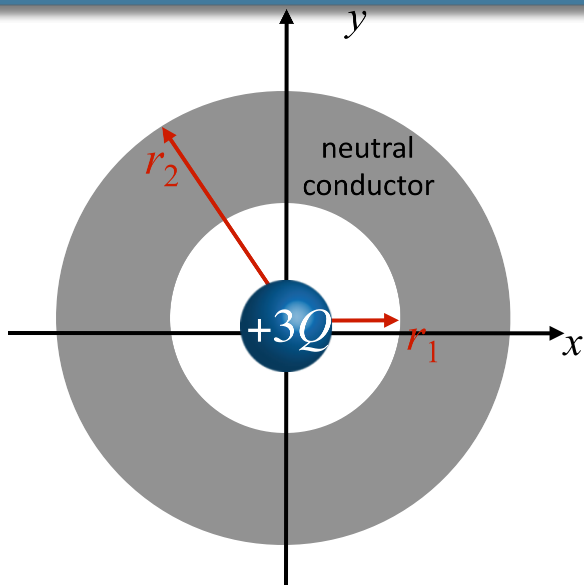

Gauss' Law:

$$E = 0 \rightarrow Q_{enc} = 0 \rightarrow \sigma_1 = \frac{-3Q}{4\pi r_1^2}$$

Similarly:

$$\sigma_2 = \frac{+3Q}{4\pi r_2^2}$$

# Calculation

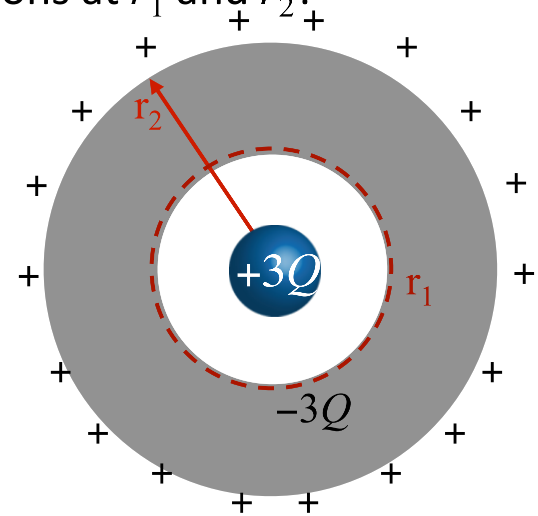




Suppose the conductor has a charge of $-Q$

A) What is $E$ everywhere?

B) What are charge distributions at $r_1$ and $r_2$?

$$\oint_{\text{surface}} \vec{E} \cdot \vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$




$r < r_1$

A) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{r^2}$

B) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{r^2}$

C) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$

$r > r_2$

A) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3Q}{r^2}$

B) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Q}{r^2}$

C) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$

$r_1 < r < r_2$

$$E = 0$$