Electricity & Magnetism Lecture 8: Capacitors

Today's Concept:

Capacitors

(Capacitors in a circuits, Dielectrics, Energy in capacitors)

- Yesterday’s homework credit 100% until next week.
  - Because of the maintenance outage lasted longer
- There are practise problems on flipit that you can work.

– Review Problems

31. Coulomb's Law
32. Electric Fields
33. Electric Flux and Field Lines
34. Gauss' Law
35. Electric Potential Energy
36. Electric Potential
37. Conductors and Capacitors

Stuff you asked about:

- “the formula for calculating capacitance in parallel and series are the opposite of the formula for resistance “
- “Please go over the Capactitors with and without a dielectric from checkpoint.”
- “this stuff is a lot harder than what we’ve been doing before, one thing i don’t understand is when you disconnect a battery from a capacitor what exactly happens, how come it still has a charge?? ”
- “can we add our own formulas to the sheet like how we did for phys 120 last term?”
  - Within reason.

Capacitors, connected and unconnected

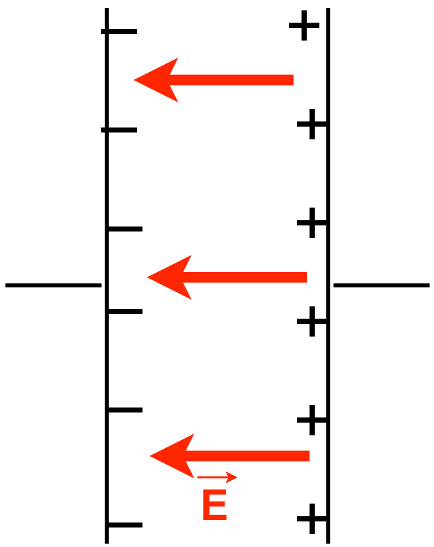
- A capacitor that is unconnected to a battery has constant charge: $V = Q/C$ (V is determined by Q)
- A capacitor connected to a battery has a constant voltage. $Q = CV$ (Q is determined by V)
- Capacitors in parallel have the same voltage. Charge may redistribute among them: $Q_1/C_1 = Q_2/C_2 = Q_2/C_2 \ldots$
- Capacitors in series have the same charge.

Dielectric

- Charged capacitor, not connected to battery
- Dielectric makes ΔV smaller

air capacitor

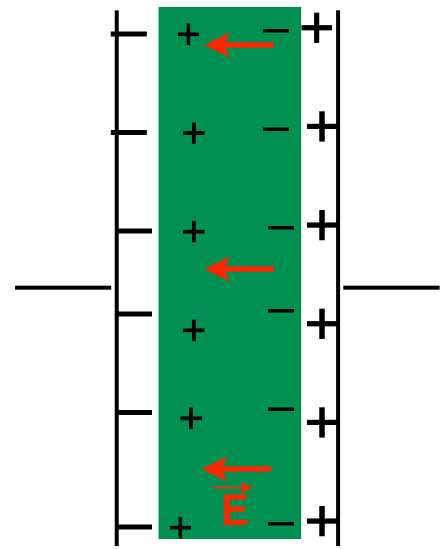
$\kappa = 1$



$C = \varepsilon_0 A/L$

dielectric capacitor

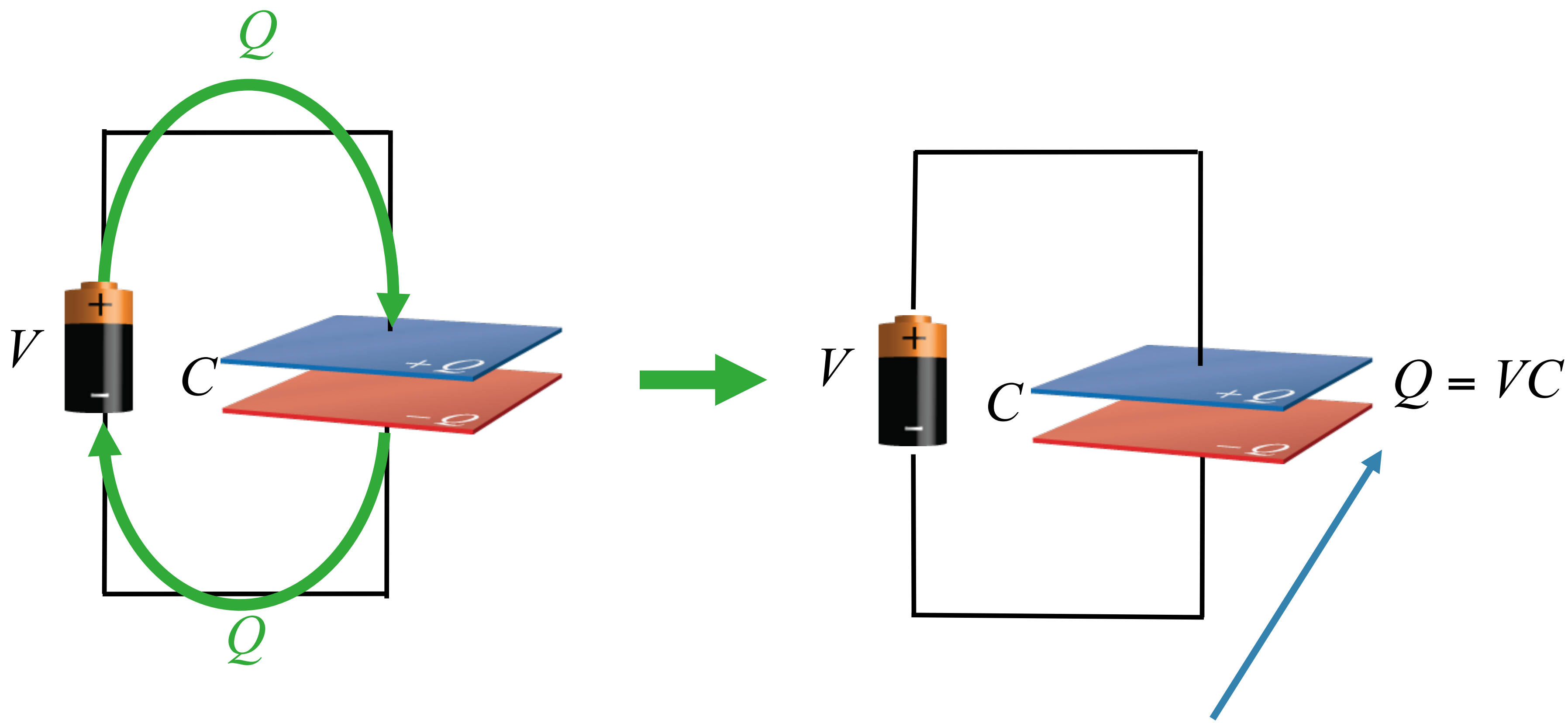
$\kappa > 1$



$C = \kappa\varepsilon_0 A/L$

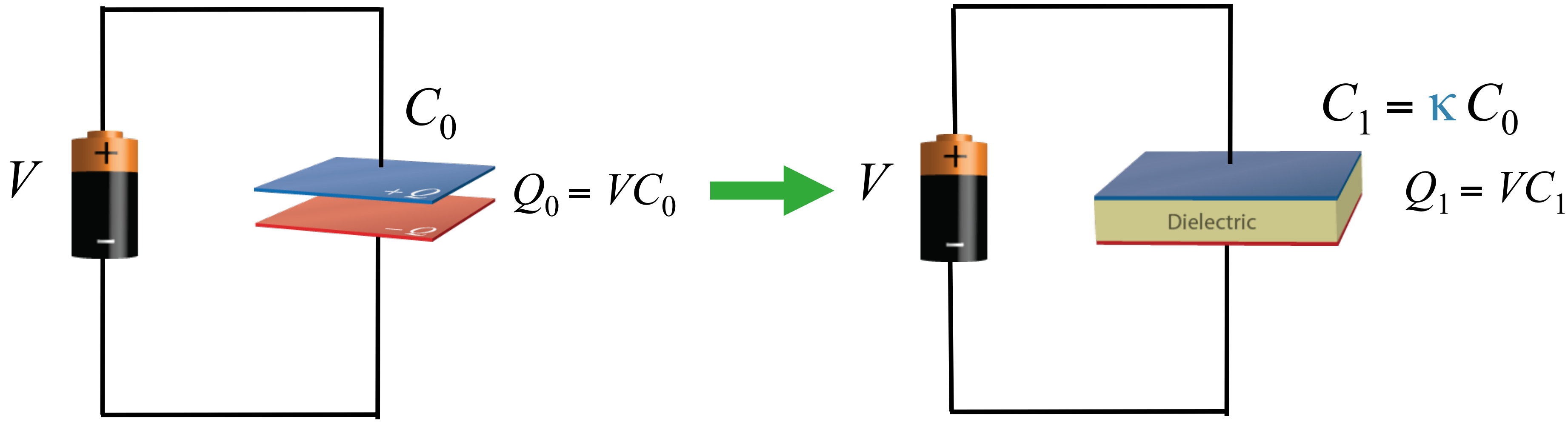
E inside dielectric is smaller than E outside.
The charge on the surface of the dielectric partially cancels the E field from the charge on the plates.
Therefore the ΔV between the plates is less.

Simple Capacitor Circuit



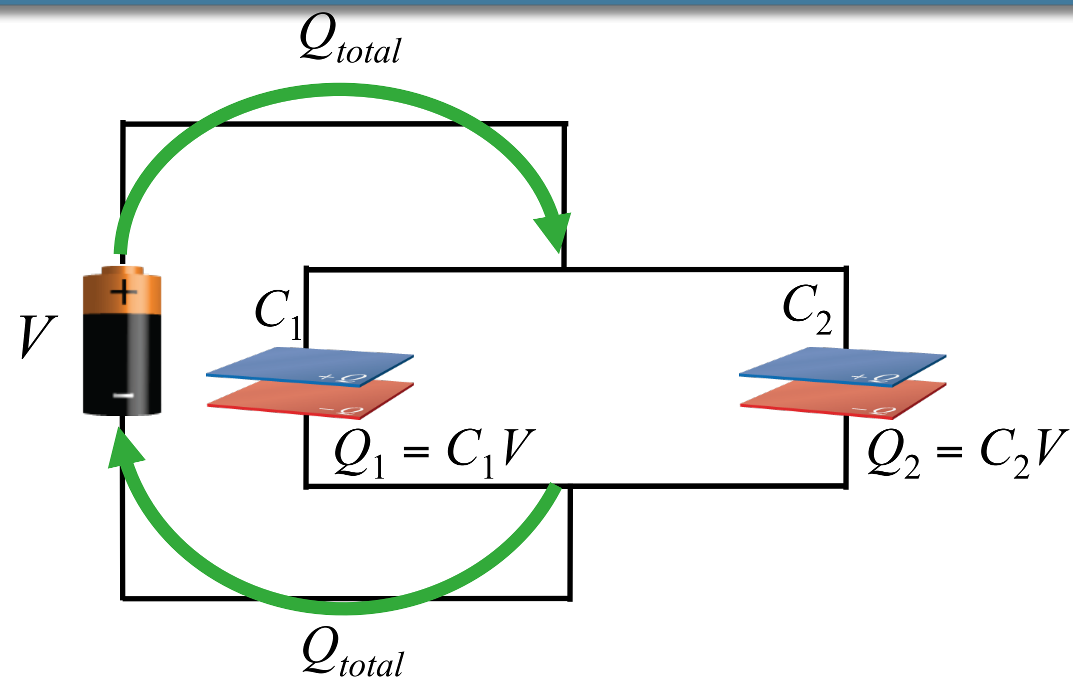
This "Q" really means that the battery has moved charge Q from one plate to the other, so that one plate holds $+Q$ and the other $-Q$.

Dielectrics



By adding a dielectric you are just making a new capacitor with larger capacitance (factor of κ)

Parallel Capacitor Circuit

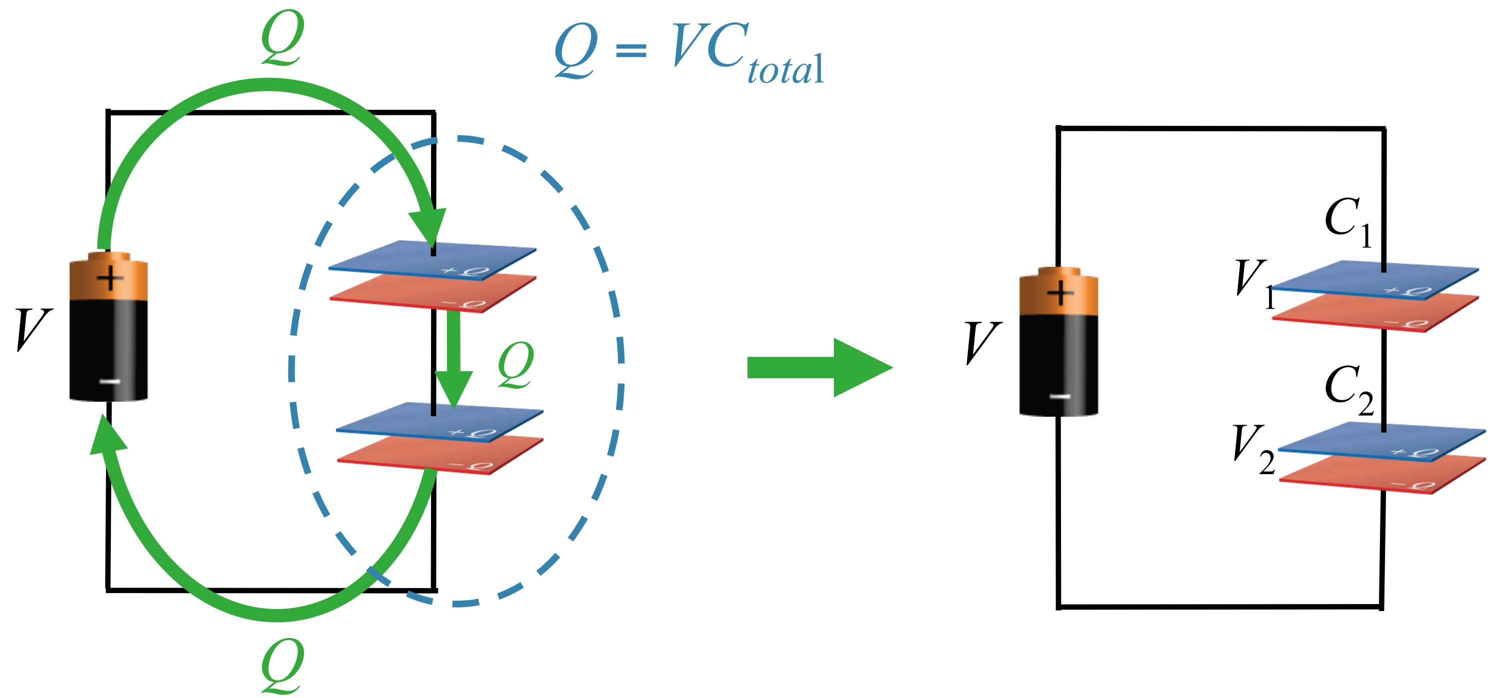


Key point: V is the same for both capacitors

Key Point: $Q_{total} = Q_1 + Q_2 = VC_1 + VC_2 = V(C_1 + C_2)$

$$C_{total} = C_1 + C_2$$

Series Capacitor Circuit



Key point: Q is the same for both capacitors

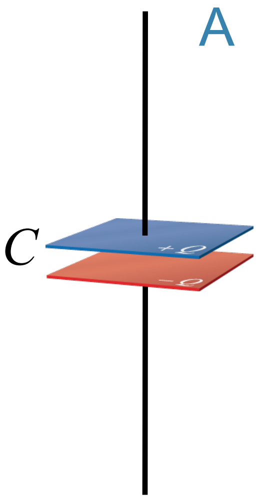
Key point: $Q = VC_{total} = V_1C_1 = V_2C_2$

Also: $V = V_1 + V_2$ → $Q/C_{total} = Q/C_1 + Q/C_2$

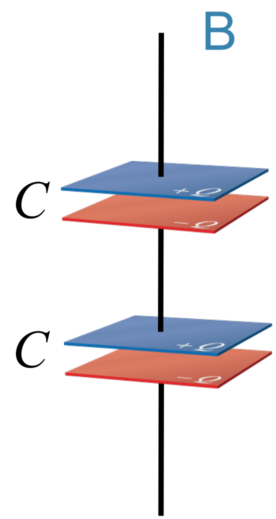
$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$$

CheckPoint: Three Capacitor Configurations

The three configurations shown below are constructed using identical capacitors. Which of these configurations has lowest total capacitance?

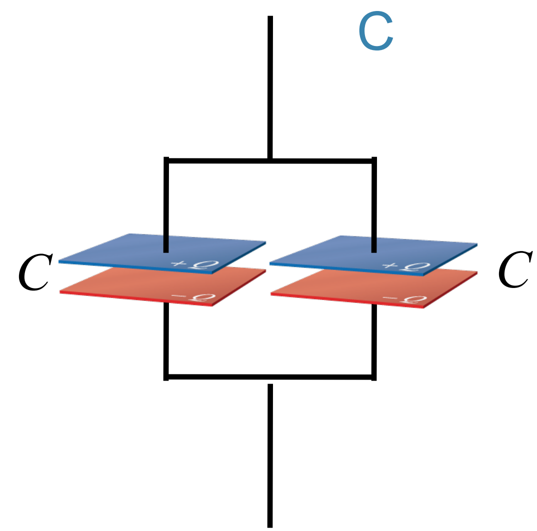


$$C_{total} = C$$



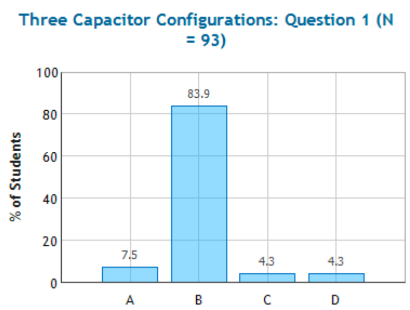
$$1/C_{total} = 1/C + 1/C$$
$$= 2/C$$

$$C_{total} = C/2$$

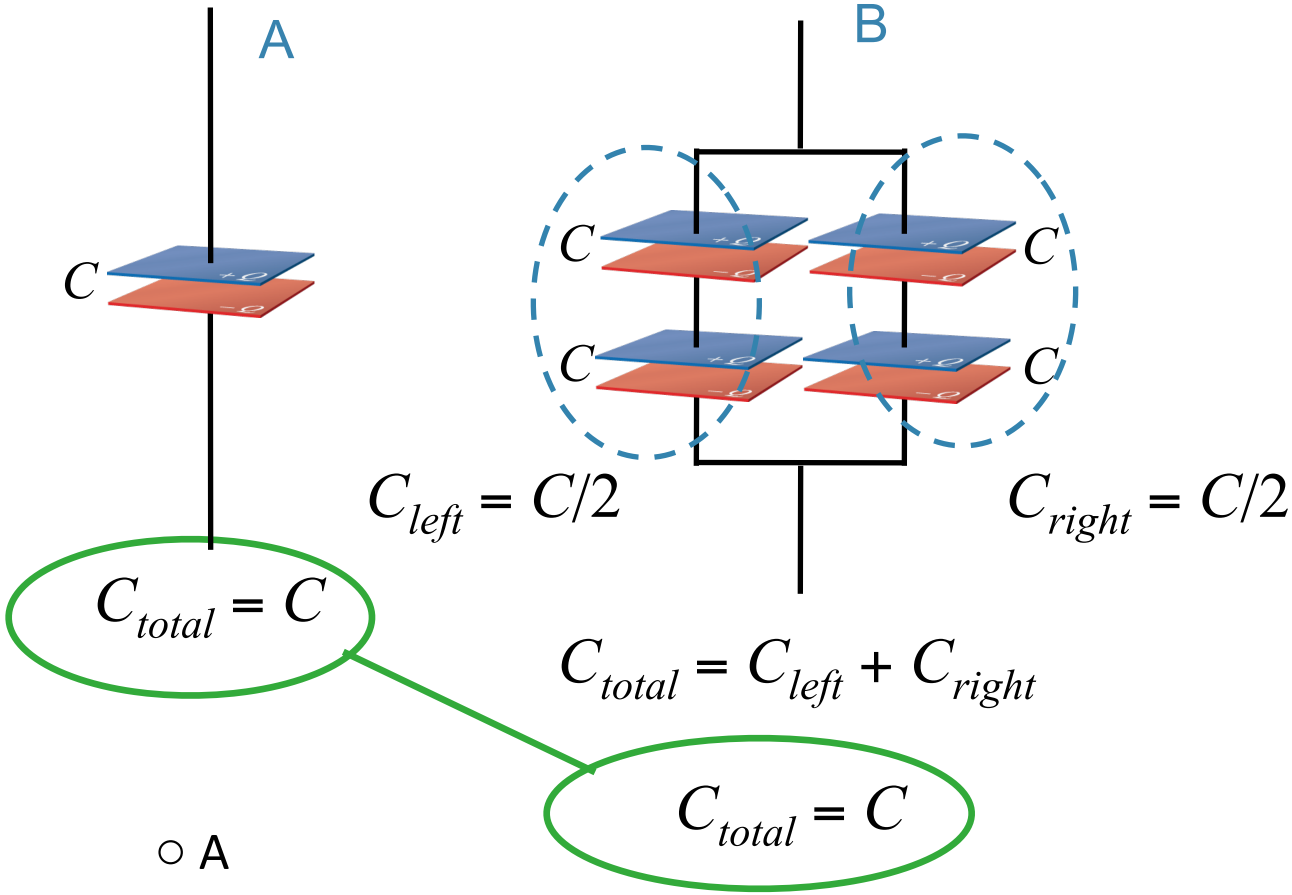


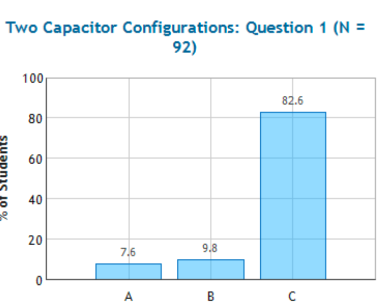
$$C_{total} = 2C$$

D: All 3 are the same



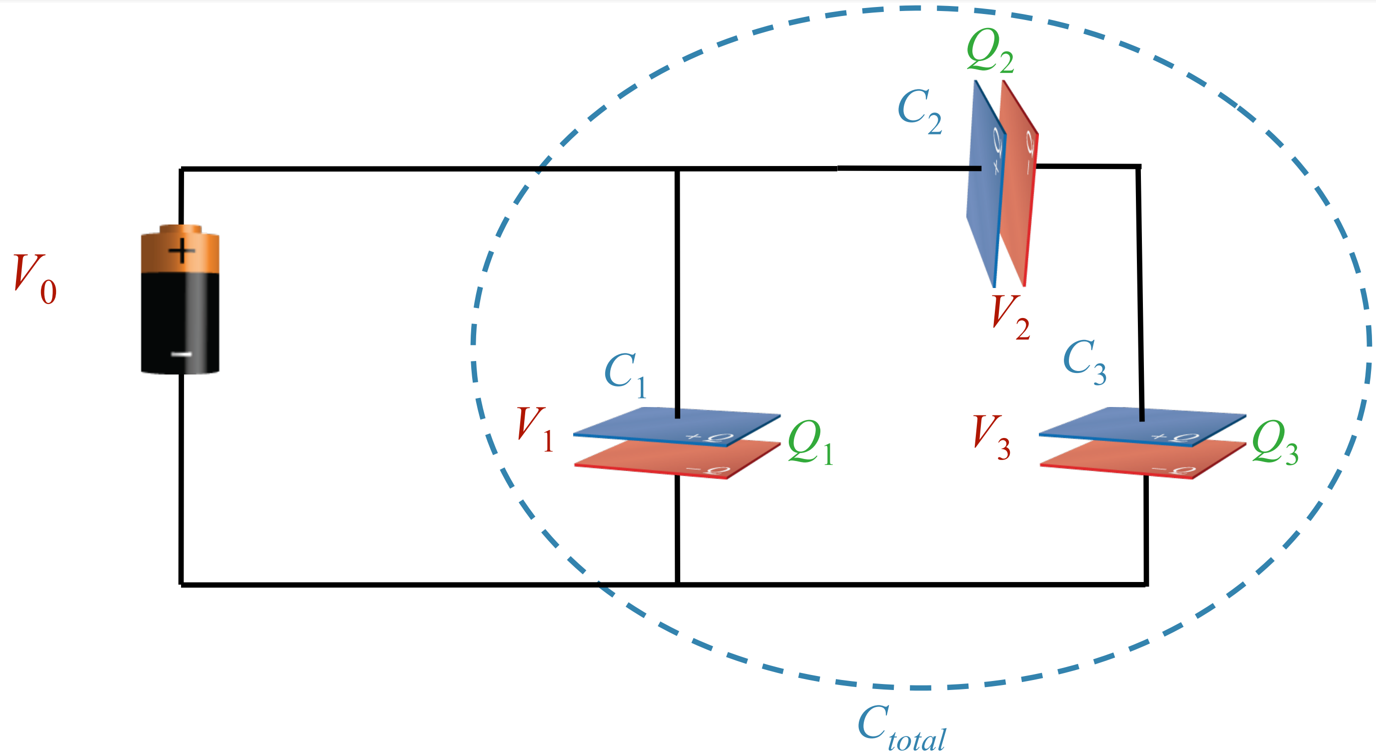
The two configurations shown below are constructed using identical capacitors. Which of these configurations has the lowest overall capacitance?





- ○ A
- ○ B
- ○ Both configurations have the same capacitance

Clicker Question: Capacitor Network



Which of the following is NOT necessarily true:

A) $V_0 = V_1$

B) $C_{total} > C_1$

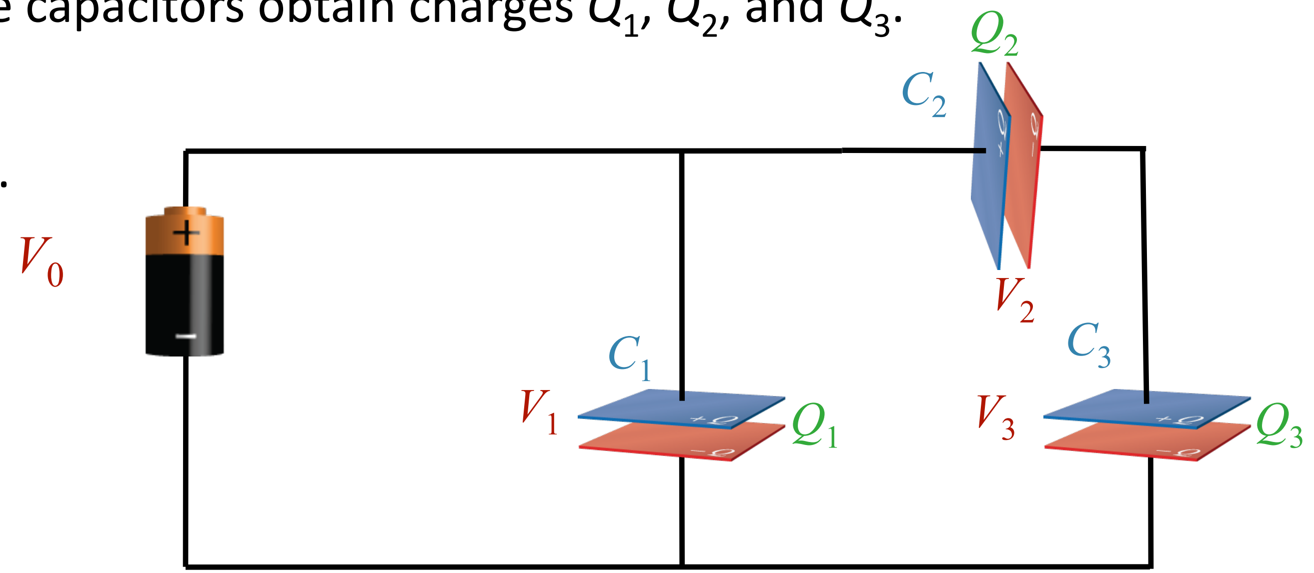
C) $V_2 = V_3$

D) $Q_2 = Q_3$

E) $V_1 = V_2 + V_3$

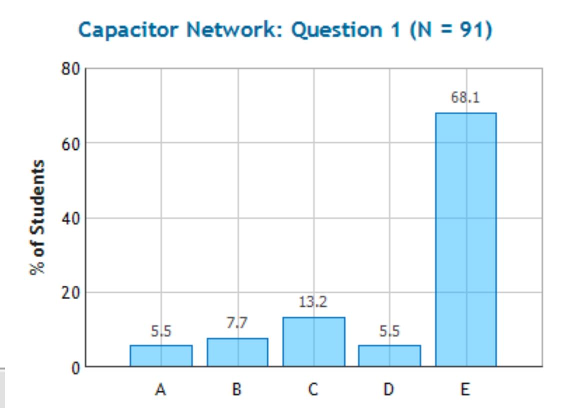
CheckPoint: Capacitor Network

A circuit consists of three unequal capacitors C_1, C_2, and C_3 which are connected to a battery of voltage V_0. The capacitance of C_2 is twice that of C_1. The capacitance of C_3 is three times that of C_1. The capacitors obtain charges Q_1, Q_2, and Q_3.



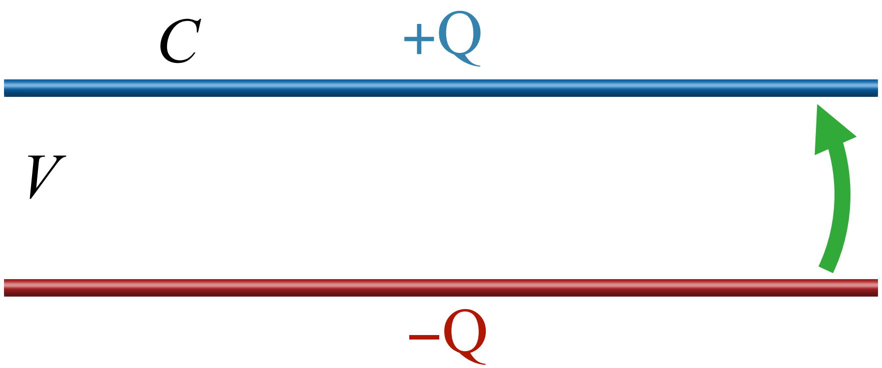
Compare Q_1, Q_2, and Q_3.

- A. $Q_1 > Q_3 > Q_2$
- B. $Q_1 > Q_2 > Q_3$
- C. $Q_1 > Q_2 = Q_3$
- D. $Q_1 = Q_2 = Q_3$
- E. $Q_1 < Q_2 = Q_3$



Energy in a Capacitor

In Prelecture 7 we calculated the work done to move charge Q from one plate to another:



$U = {}^1\!/_2 QV$

$U = {}^1\!/_2 CV^2$ — Most useful

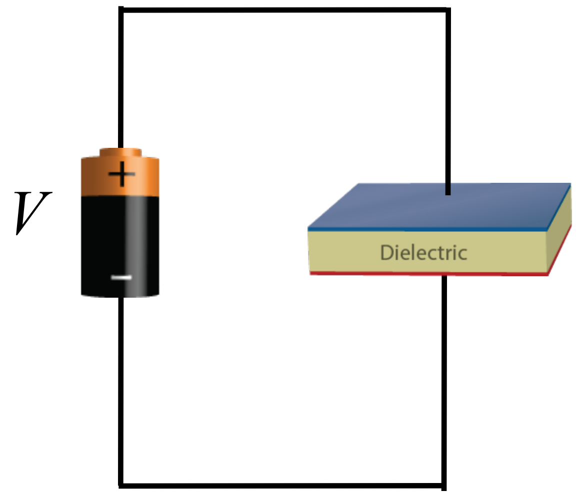
$U = {}^1\!/_2 Q^2/C$

Since $Q = VC$

This is potential energy waiting to be used…

Messing with Capacitors

If connected to a battery V stays constant



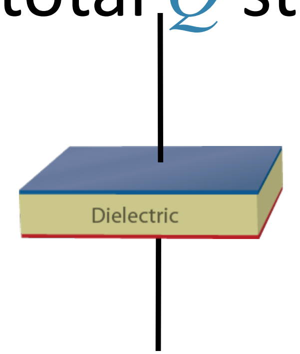
$$V_1 = V$$

$$C_1 = \kappa\, C$$

$$\rightarrow Q_1 = C_1 V_1$$

$$= \kappa\, CV = \kappa\, Q$$

If isolated then total Q stays constant



$$\mathrm{Q}_1 = \mathrm{Q}$$

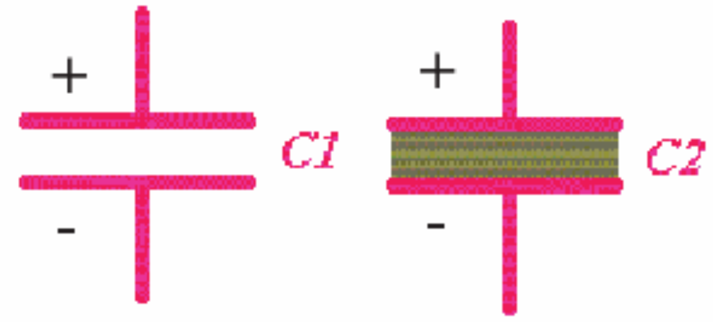
$$\mathrm{C}_1 = \kappa\, \mathrm{C}$$

$$\rightarrow V_1 = Q_1/C_1$$

$$= Q/\kappa\, C = V/\kappa$$

CheckPoint: Capacitors and Dielectrics 1

Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. After C_2 has been charged and disconnected, it is filled with a dielectric.



Compare the voltages of the two capacitors.

A. $V_1 > V_2$

B. $V_1 = V_2$

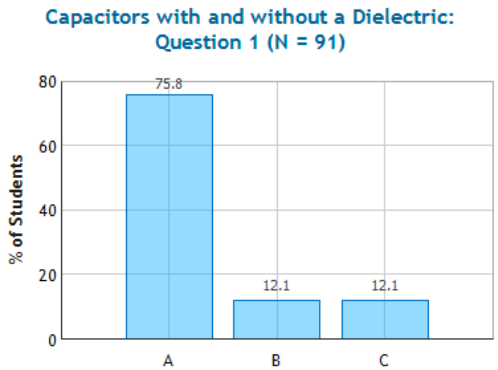
C. $V_1 < V_2$

"The electric field decreases on the second due to the dielectric, so its voltage must decrease."

"The little brown thing is stopping the pikachus from reaching the other side."

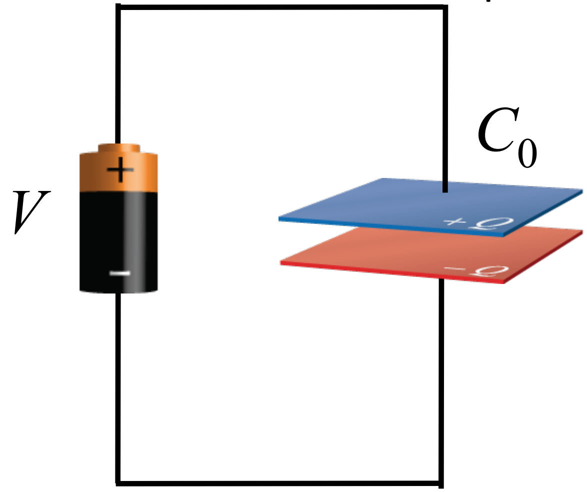
" Dielectrics don't change voltages. "

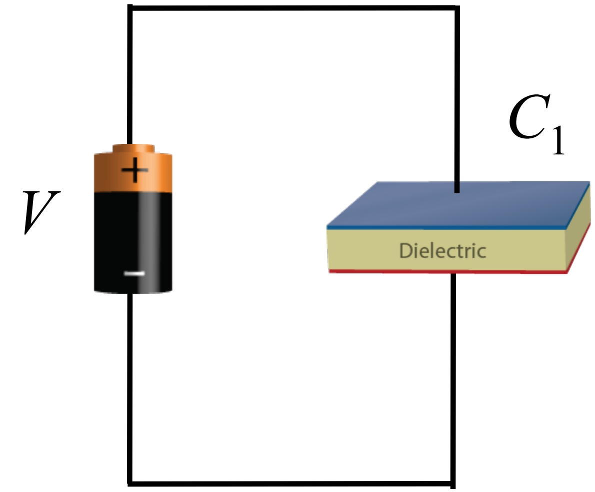
"Capacitance lowers in case 2, meaning V increases in case 2. "



Clicker Question: Messing with Capacitors

Two identical parallel plate capacitors are connected to identical batteries. Then a dielectric is inserted between the plates of capacitor C_1. Compare the energy stored in the two capacitors.





A) $U_1 < U_0$

B) $U_0 = U_1$

C) $U_1 > U_0$

Compare using $U = {}^1/_2 CV^2$

$$U_1/U_0 = \kappa$$

→ Potential Energy goes UP

CheckPoint: Capacitors and Dielectrics 2

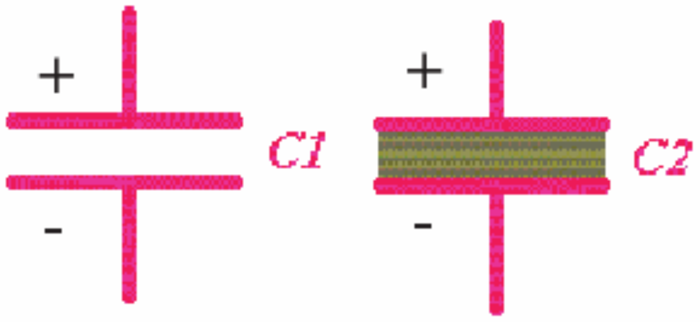
Two identical parallel plate capacitors are given the same charge Q, after which they are disconnected from the battery. After C_2 has been charged and disconnected, it is filled with a dielectric.

Compare the potential energy stored by the two capacitors.

A. $U_1 > U_2$

B. $U_1 = U_2$

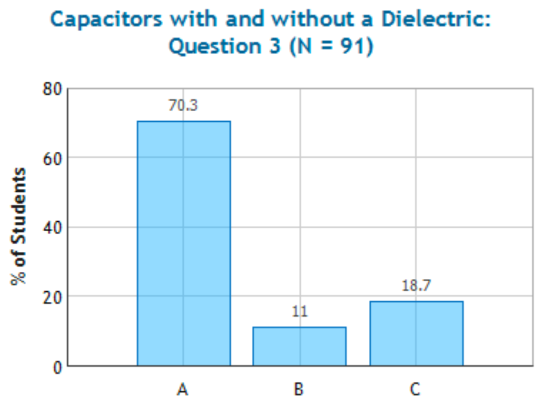
C. $U_1 < U_2$



" If the electric field goes down, then so does the potential energy of the second system.."

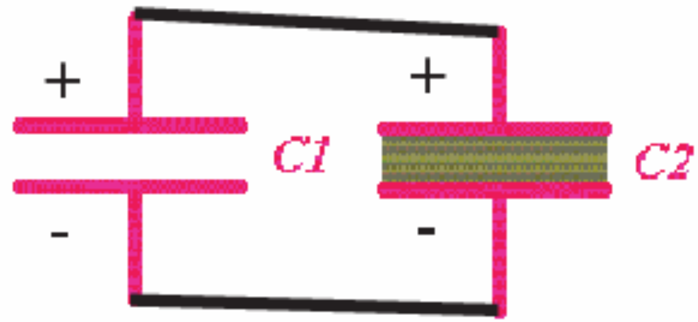
"Dielectrics don't affect potential."

"U = QV. Since voltage remains constant and charge increases, potential energy must also increase."



CheckPoint: Capacitors and Dielectrics 3

The two capacitors are now connected to each other by wires as shown. How will the charge redistribute itself, if at all?

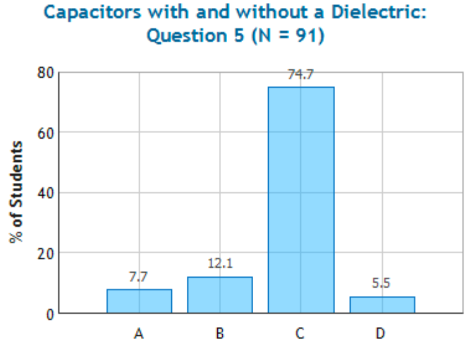


V must be the same !

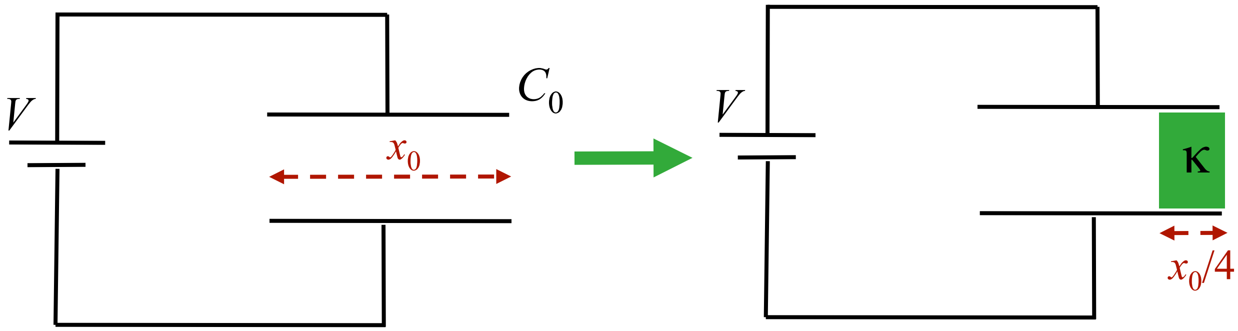
A) The charges will flow so that the charge on C1 will become equal to the charge on C2.
B) The charges will flow so that the energy stored in C1 will become equal to the energy stored in C2
C) The charges will flow so that the potential difference across C1 will become the same as the potential difference across C2.
D) No charges will flow. The charge on the capacitors will remain what it was before they were connected.

Q: $$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \rightarrow Q_1 = \frac{C_1}{C_2} Q_2$$

U: $$U_1 = \tfrac{1}{2} C_1 V^2 \quad U_2 = \tfrac{1}{2} C_2 V^2 \rightarrow U_1 = \frac{C_1}{C_2} U_2$$



Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

Conceptual Analysis:

$$C \equiv \frac{Q}{V}$$

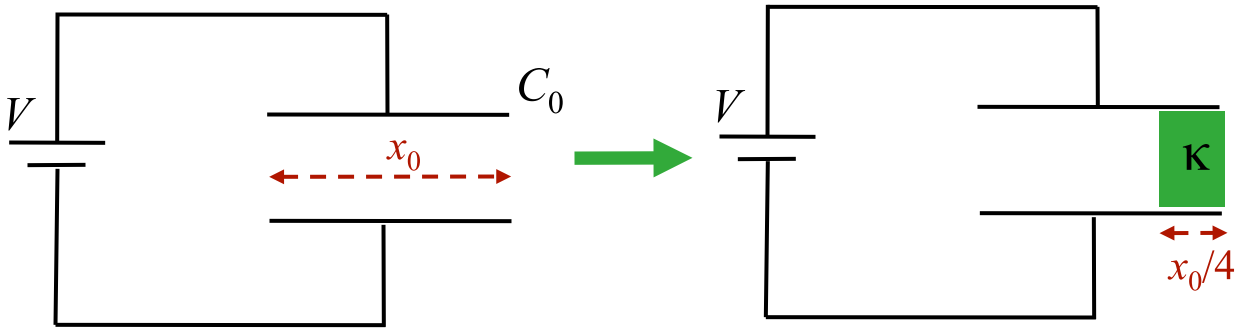
What changes when the dielectric added?

A) Only C B) only Q C) only V **D) C and Q** E) V and Q

Adding dielectric changes the physical capacitor → C changes

V does not change and C changes → Q changes

Calculation



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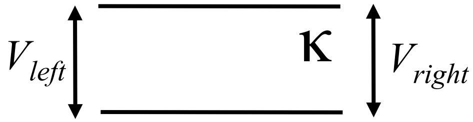
A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

Strategic Analysis:

- Calculate new capacitance C
- Apply definition of capacitance to determine Q

To calculate C, let's first look at:



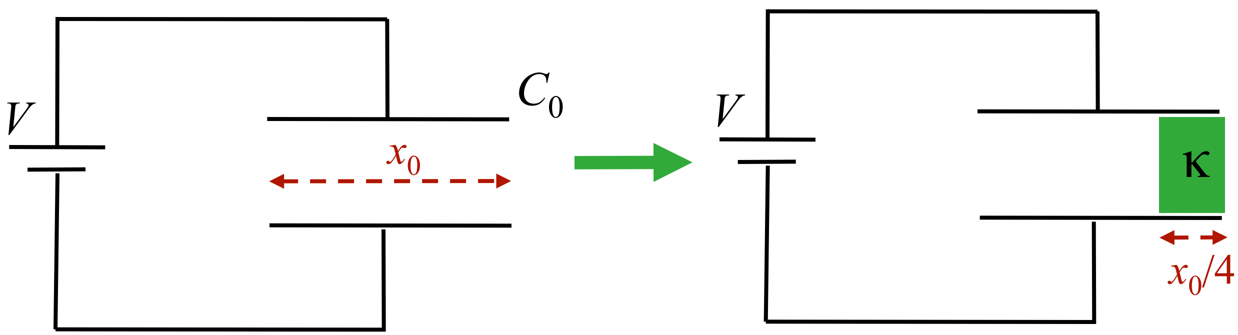
A) $V_{left} < V_{right}$

B) $V_{left} = V_{right}$

C) $V_{left} > V_{right}$

The conducting plate is an equipotential !

Calculation

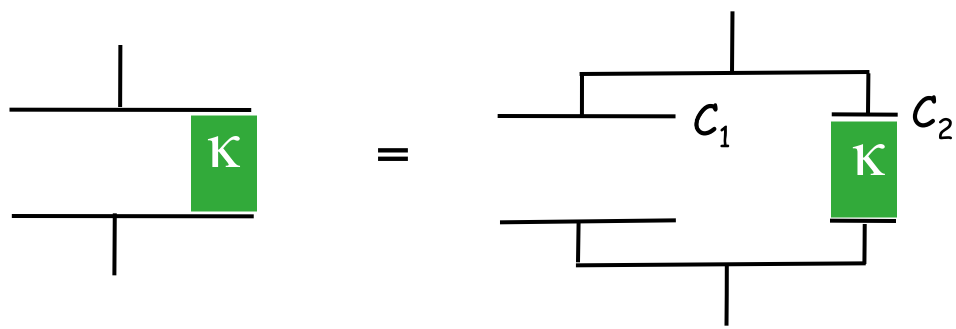


An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.

What is Q_f, the final charge on the capacitor?

Can consider capacitor to be two capacitances, C_1 and C_2, in parallel



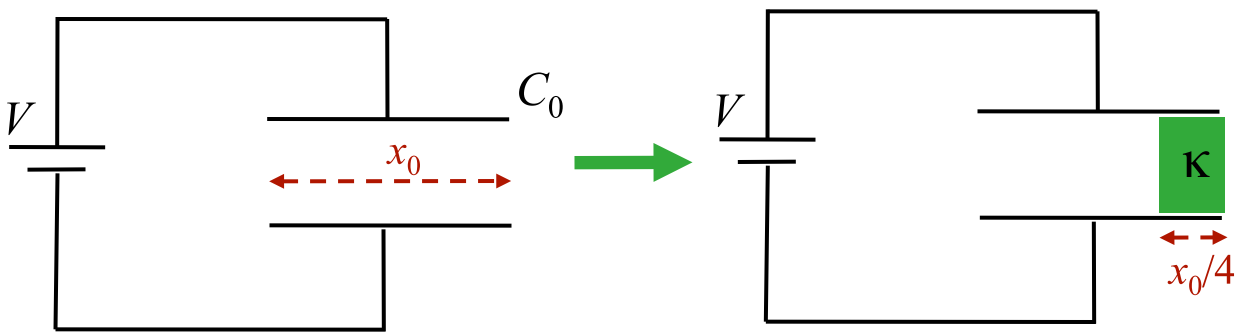
What is C_1 ?

A) $C_1 = C_0$ B) $C_1 = {}^3/_4 C_0$ C) $C_1 = {}^4/_3 C_0$ D) $C_1 = {}^1/_4 C_0$

In general. For parallel plate capacitor: $C = \varepsilon_0 A/d$

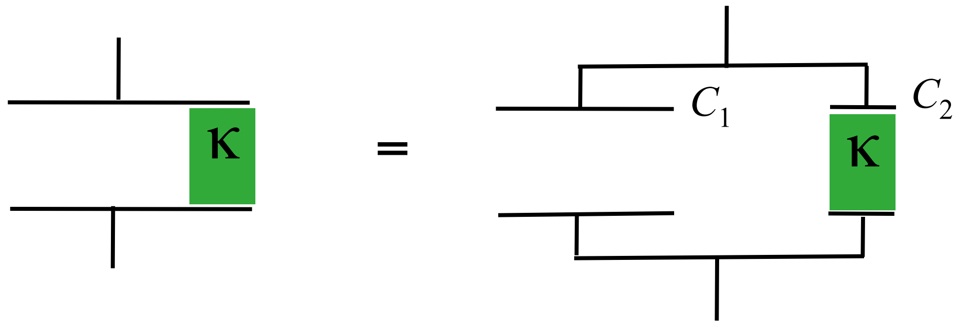
$$A = {}^3/_4 A_0, \quad d = d_0 \rightarrow C_1 = {}^3/_4 (\varepsilon_0 A_0/d_0) \rightarrow C_1 = {}^3/_4 C_0$$

Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.



What is Q_f, the final charge on the capacitor?

$C_1 = {}^3/_4 C_0$

What is C_2 ?

A) $C_2 = \kappa C_0$

B) $C_2 = {}^3/_4 \kappa C_0$

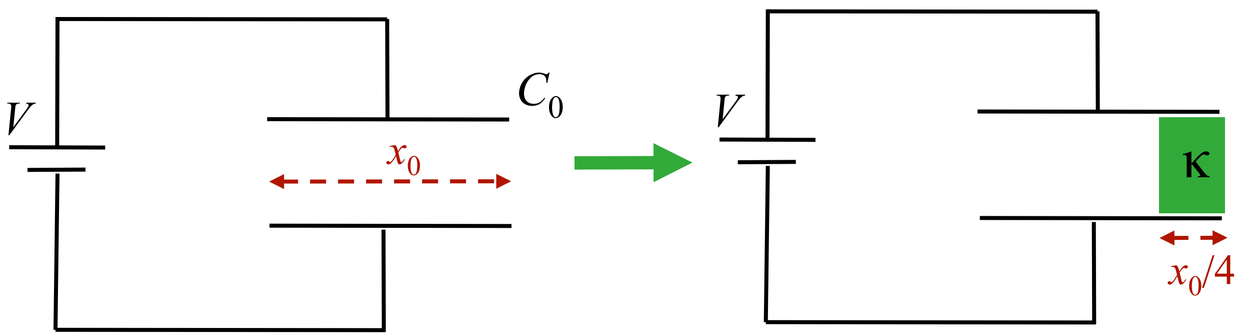
C) $C_2 = {}^4/_3 \kappa C_0$

D) $C_2 = {}^1/_4 \kappa C_0$

In general. For parallel plate capacitor filled with dielectric: $C = \kappa\varepsilon_0 A/d$

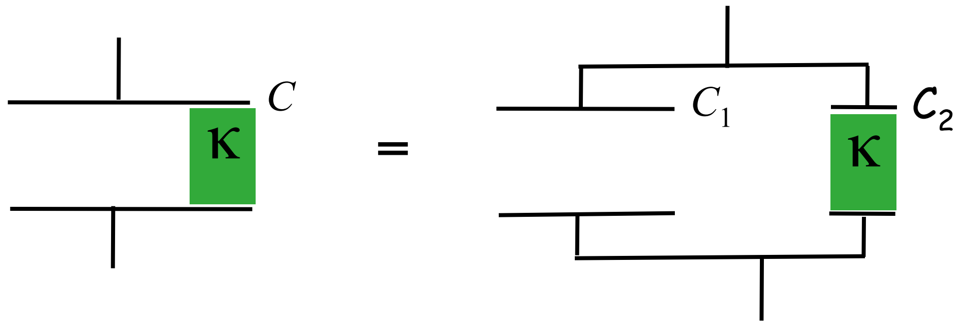
$A = {}^1/_4 A_0$, $d = d_0$ → $C = \frac{1}{4}(\kappa\varepsilon_0 A_0/d_0)$ → $C_2 = {}^1/_4 \kappa C_0$

Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.



What is Q_f, the final charge on the capacitor?

$C_1 = {}^3/_4 C_0$ $\qquad$ $C_2 = {}^1/_4\, \kappa\, C_0$

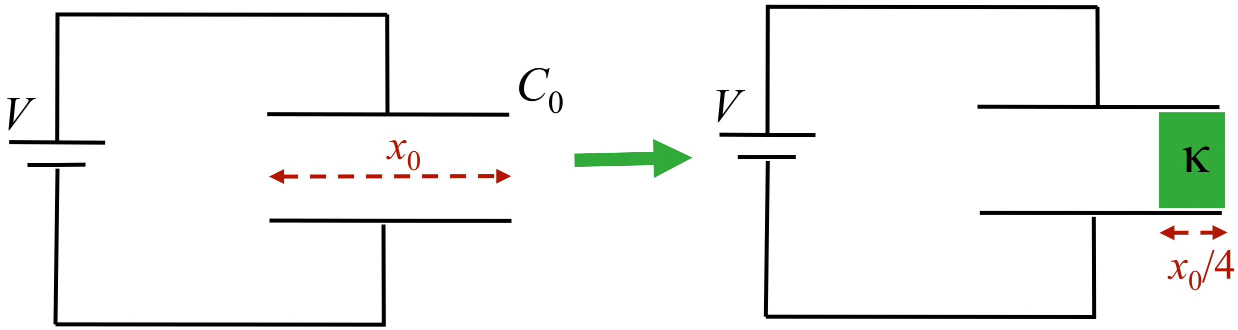
What is C?

A) $C = C_1 + C_2$ $\qquad$ B) $C = C_1 + \kappa C_2$ $\qquad$ C) $C = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$

C = parallel combination of C_1 and C_2: $\quad C = C_1 + C_2$

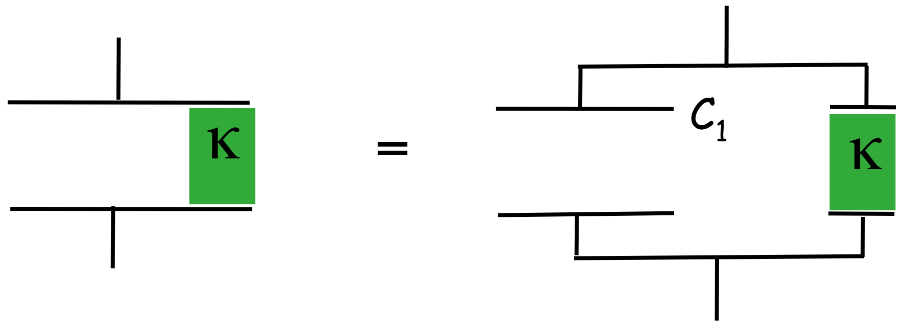
$\rightarrow$ $C = C_0\,({}^3/_4 + {}^1/_4\,\kappa)$

Calculation



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V.

A dielectric (κ) of width $x_0/4$ is inserted into the gap as shown.



What is Q_f, the final charge on the capacitor?

$C_1 = {}^3/_4 C_0$ $\qquad$ $C_2 = {}^1/_4\, \kappa\, C_0$

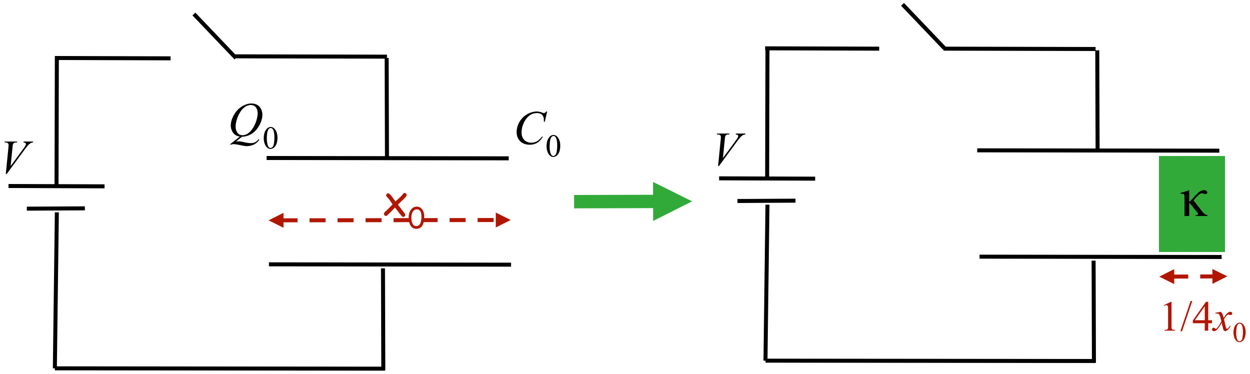
$\rightarrow$ $C = C_0\,({}^3/_4 + {}^1/_4\,\kappa)$

What is Q?

$$C \equiv \frac{Q}{V} \quad \rightarrow \quad Q = VC$$

$$Q = VC_0\left(\frac{3}{4} + \frac{1}{4}\kappa\right)$$

Different Problem



An air-gap capacitor, having capacitance C_0 and width x_0 is connected to a battery of voltage V and then battery is disconnected.

A dielectric (κ) of width $1/4x_0$ is inserted into the gap as shown.

What is V_f, the final voltage on the capacitor?

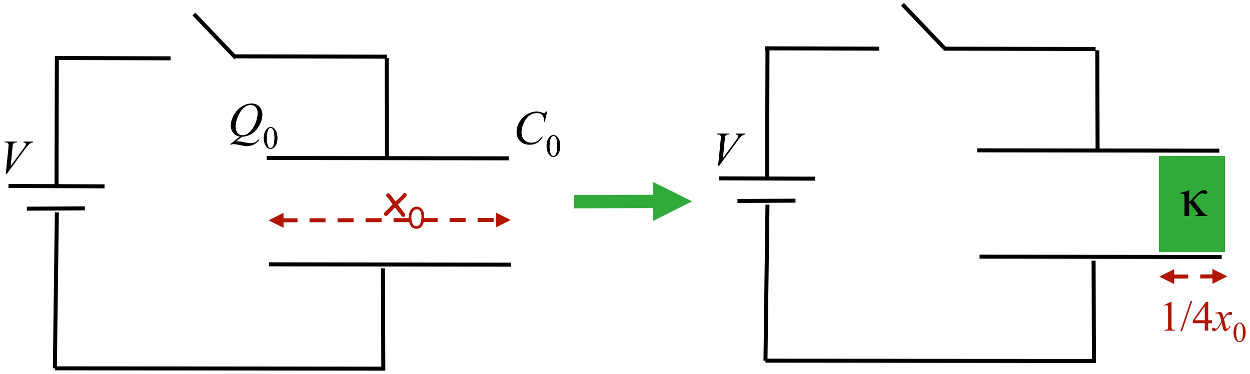
A) $V_f < V$ B) $V_f = V$ C) $V_f > V$

Q stays same: no way to add or subtract → $Q = Q_0 = C_0V$

We know C: (property of capacitor) → $C = C_0\,(^3/_4 + {}^1/_4\,\kappa)$

→ $V_f = Q/C = V/(^3/_4 + {}^1/_4\,\kappa)$

Different Problem



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A dielectric (κ) of width $1/4x_0$ is inserted into the gap as shown.

What is V_f, the final voltage on the capacitor?

$$V_f = Q/C = V/(^3/_4 + {}^1/_4\,\kappa)$$

How did energy stored in capacitor change when dielectric inserted?

A) U increased B) U stayed same C) U decreased

$U = \frac{1}{2} Q^2/C$

Q remained same → U decreased

C increased

Capacitors, connected and unconnected

- A capacitor that is unconnected to a battery has constant charge: $V = Q/C$ (V is determined by Q)
- A capacitor connected to a battery has a constant voltage. $Q = CV$ (Q is determined by V)
- Capacitors in parallel have the same voltage. Charge may redistribute among them: $Q_1/C_1 = Q_2/C_2 = Q_2/C_2$ …
- Capacitors in series have the same charge.