Electricity & Magnetism
Lecture 9: Electric Current

Today's Concept:

Electric Current

Midterm

- B9201: 6:30 pm to 8:00 pm
- Covers up to today's lecture material (Unit 9) and next week's tutorials

Meeting Information

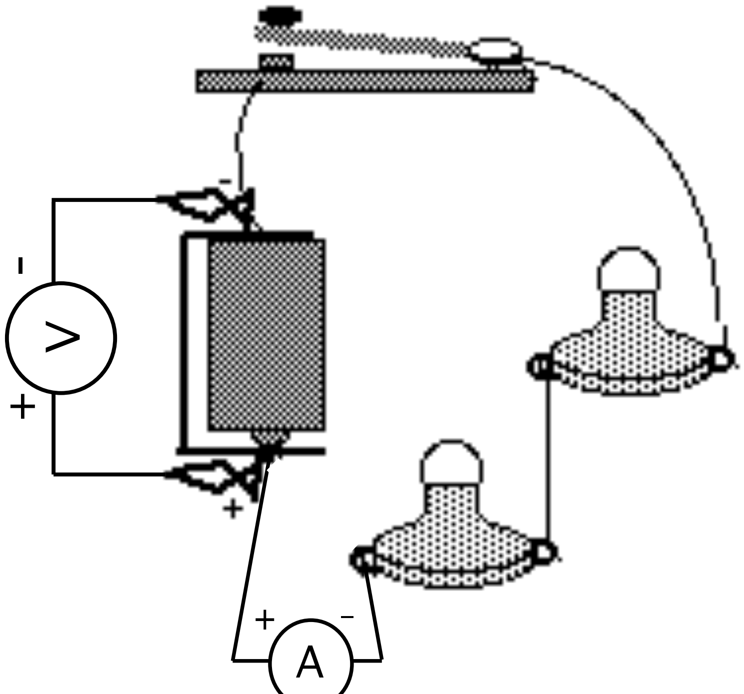
Days & Times	Room	Instructor	Meeting Dates
MoWeFr 09:30 - 10:20	RCBIMAGTH	Neil Alberding	06/05/2019 - 02/08/2019
Fr 18:30 - 20:20	B9201	Midterm	07/06/2019
Fr 18:30 - 20:20	B9201	Midterm	05/07/2019
Th 08:30 - 11:30	TBA	Exam	15/08/2019

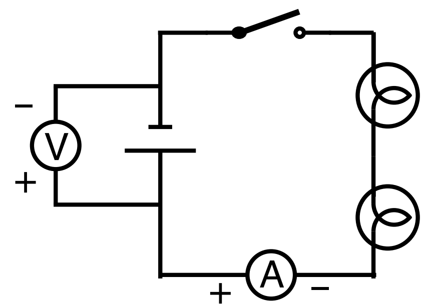
Enrollment Information

How do you feel about circuits

A. I completely understand them from high school
B. Need Review
C. Still hopeless
D. Circuit? What's that?

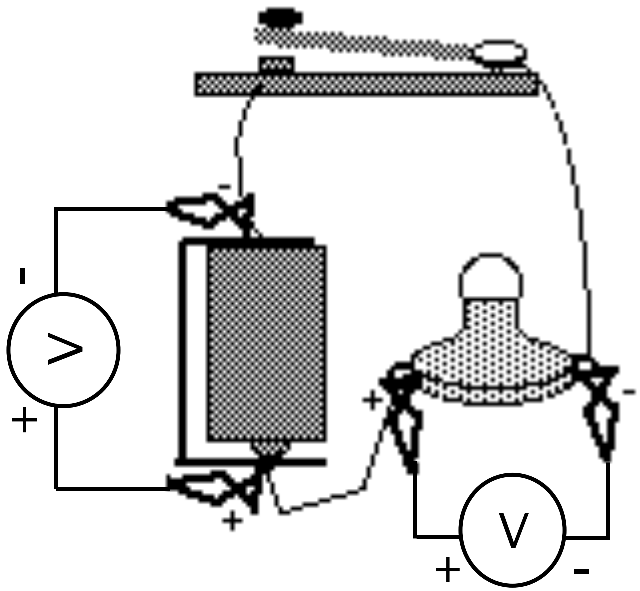
Joke of the day: I could not resist coming to class today

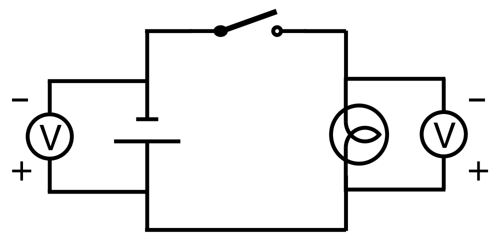




When the switch is closed, and bulbs are identical

A. Top bulb is brighter
B. Bottom bulb is brighter
C. Both are equally bright



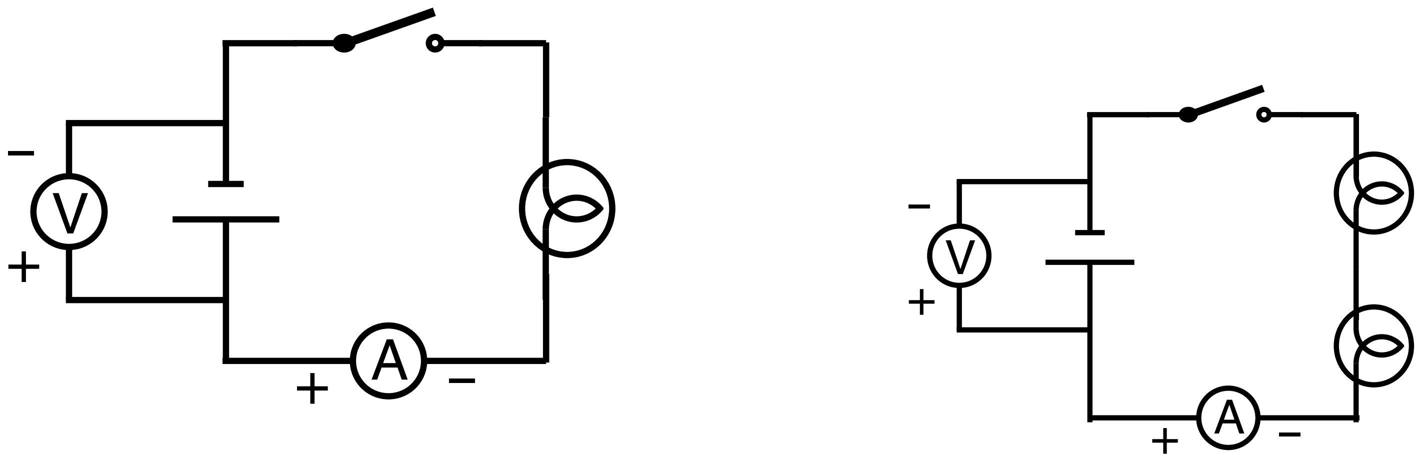


When the switch is closed

A. $V_{battery} = V_{bulb}$

B. $V_{battery} < V_{bulb}$

C. $V_{battery} > V_{bulb}$

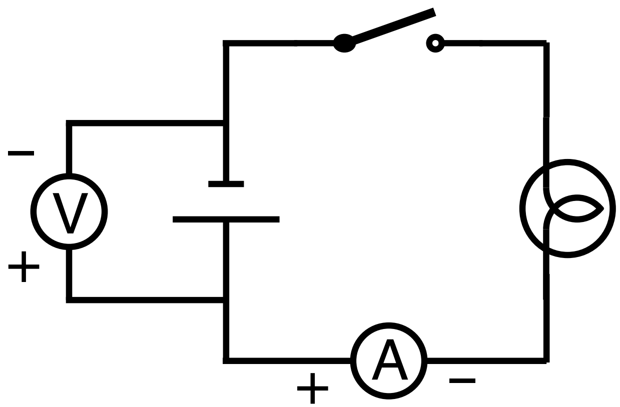


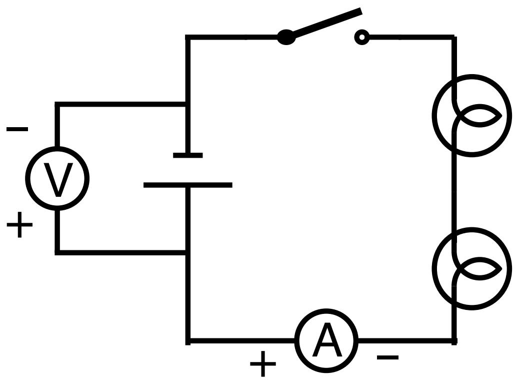
How do the currents measured compare?

A. $I_{left} < I_{right}$

B. $I_{left} = I_{right}$

C. $I_{left} > I_{right}$





How dow the Voltages measured compare?

A. $V_{left} < V_{right}$

B. $V_{left} = V_{right}$

C. $V_{left} > V_{right}$

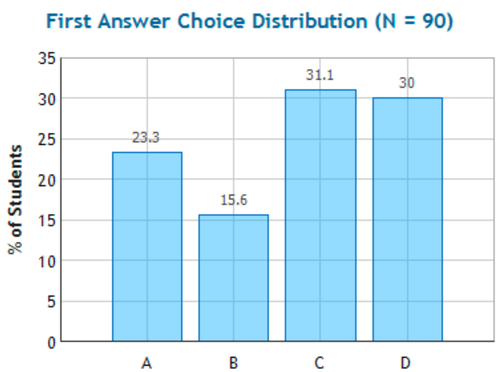
Two resistors, one having half the resistance of the other, are connected to a battery as shown. What is the voltage across the bigger resistor?

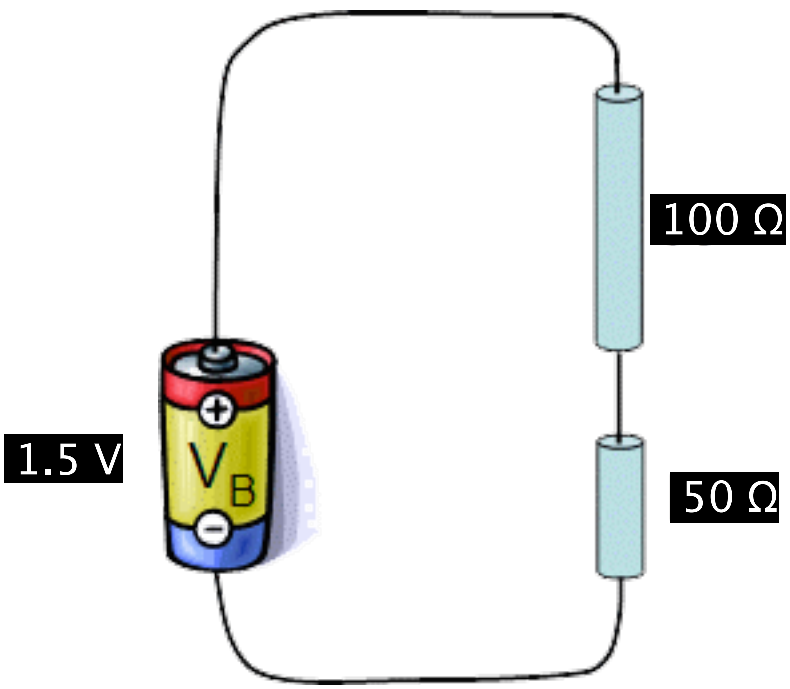
A) $V_B/2 = 0.75$ V

B) $V_B/3 = 0.50$ V

C) $3V_B/2 = 2.25$ V

D) $2V_B/3 = 1.00$ V





Your stuff

- so which way does DC current flow? -_-
- “Please explain how electric field is calculated inside the copper wire.”
- “Since $R = \rho L/A$, the greater the cross sectional area, the smaller the resistance, but the greater the length the higher the resistance. Is that why long cables have to be very thick?”
- “What if I put ammeter right between + and -?”
- “the part relating to the ohm's law and current density stuff makes no sense to me.”

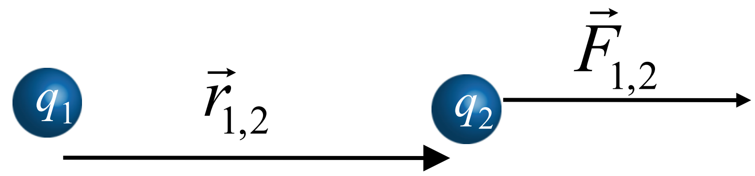
New Symbols

“How many different things will ω symbolize???”

A Big Idea Review

Coulomb's Law
Force law between point charges

$$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2}$$



Electric Field
Force per unit charge

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric Field
Property of Space
Created by Charges
Superposition

Gauss' Law
Flux through closed surface is always proportional to charge enclosed

$$\oint_{\text{surface}} \vec{E}\cdot\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

Gauss' Law
Can be used to determine E field → Spheres, Cylinders, Infinite Planes

Electric Potential
Potential energy per unit charge

$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Capacitance
Relates charge and potential for two conductor system

$$C \equiv \frac{Q}{V}$$

Electric Potential
Scalar Function that can be used to determine E

$$\vec{E} = -\vec{\nabla}V$$

A Note on Units

- Force is newtons: $N = kg \bullet m/s^2$
- Electric Field: newton/coulomb (N/C = V/m)
- Electric potential: newton-meter/coulomb = volt
  - $kg \bullet m^2/s^2C = V$
- Capacitance: farad = coulomb/volt
  - farad is **big**, we usually use
    - $\mu F = 10^{-6}$ F
    - $pF = 10^{-12}$ F (μμF in olden days, "puffs" now)
    - ($nF = 10^{-9}$ not customary in N. America)

Applications of Big Ideas

Conductors
Charges free to move → What Determines How They Move? → They move until $E = 0$!

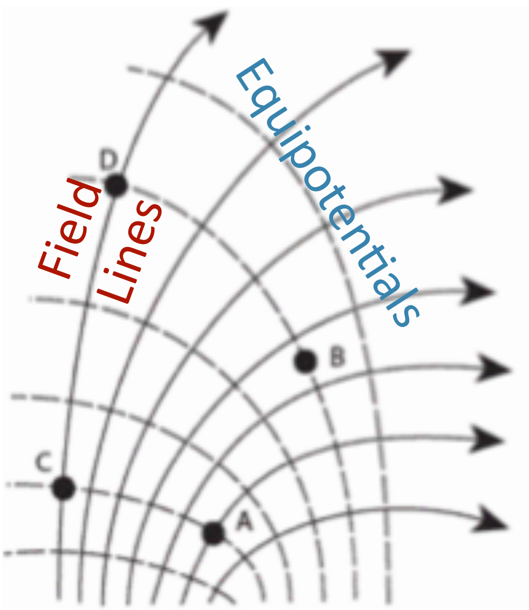
↓

$E = 0$ in conductor determines charge densities on surfaces

← Gauss' Law

Spheres
Cylinders
Infinite Planes

Field Lines & Equipotentials



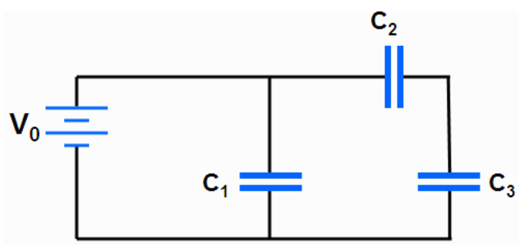
Work Done By E Field

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q\vec{E}\cdot d\vec{l}$$

Change in Potential Energy

$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q} = -\int_a^b \vec{E}\cdot d\vec{l}$$

Capacitor Networks



Series:
$(1/C_{23}) = (1/C_2) + (1/C_3)$
Parallel
$C_{123} = C_1 + C_{23}$

Key Concepts:

1) How resistance depends on A, L, σ, ρ

σ is **conductivity** here (not surface charge density)
ρ is **resistivity** here (not volume charge density).

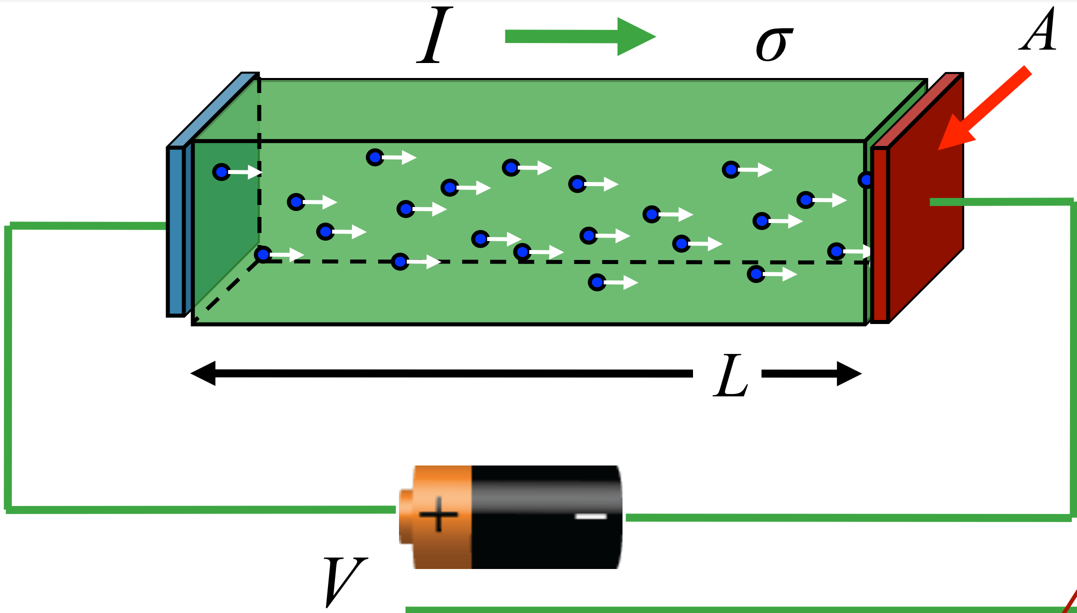
2) How to combine resistors in series and parallel

3) Understanding resistors in circuits

Today's Plan:

1) Review of resistance & preflights

2) Work out a circuit problem in detail



Conductivity – high for good conductors.

Ohm's Law: $J = \sigma E$

Observables:

$$V = EL$$
$$I = JA$$

$$I/A = \sigma V/L$$

$$I = V/(L/\sigma A)$$

R = Resistance

$$\rho = 1/\sigma$$

$$I = V/R$$

$$R = \frac{L}{\sigma A}$$

This is just like Plumbing!

I is like flow rate of water

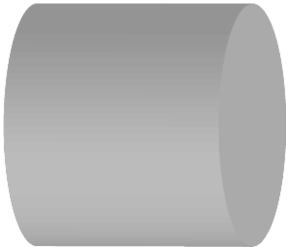
V is like pressure

R is how hard it is for water to flow in a pipe

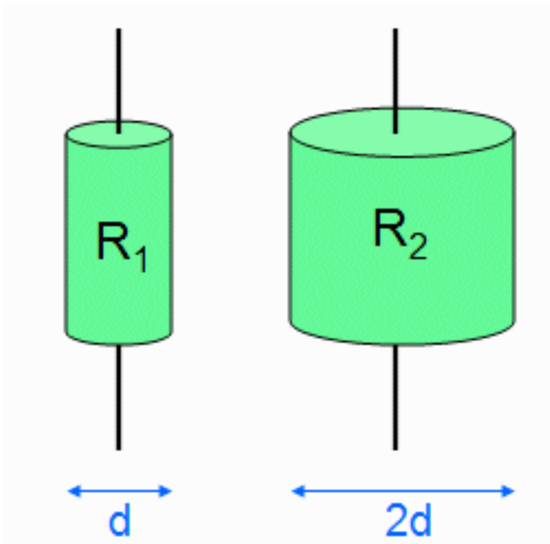
$$R = \frac{L}{\sigma A}$$

To make *R* big, make *L* long or *A* small

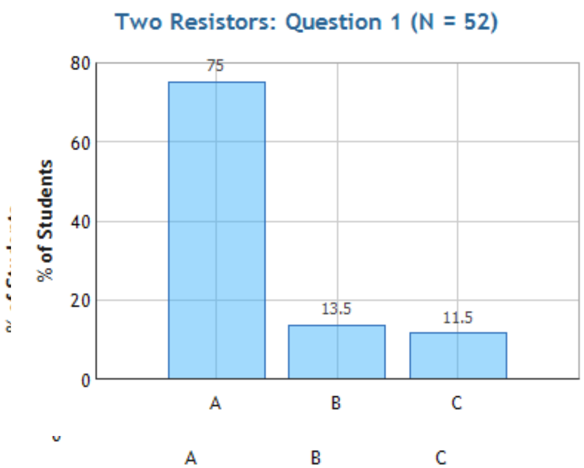
To make *R* small, make *L* short or *A* big



1 CheckPoint: Two Resistors 2



○ $V_1 > V_2$ ○ $V_1 = V_2$ ○ $V_1 < V_2$



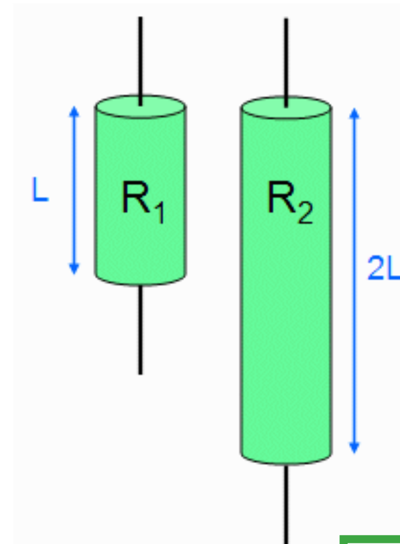
$$A_2 = 4A_1 \Rightarrow V_2 = \tfrac{1}{4}V_1$$

Same current through both resistors

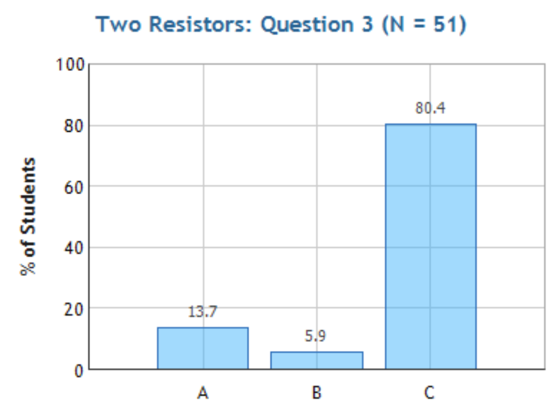
Compare voltages across resistors

$$R \propto \frac{L}{A}$$

$$V = IR \propto \frac{L}{A}$$



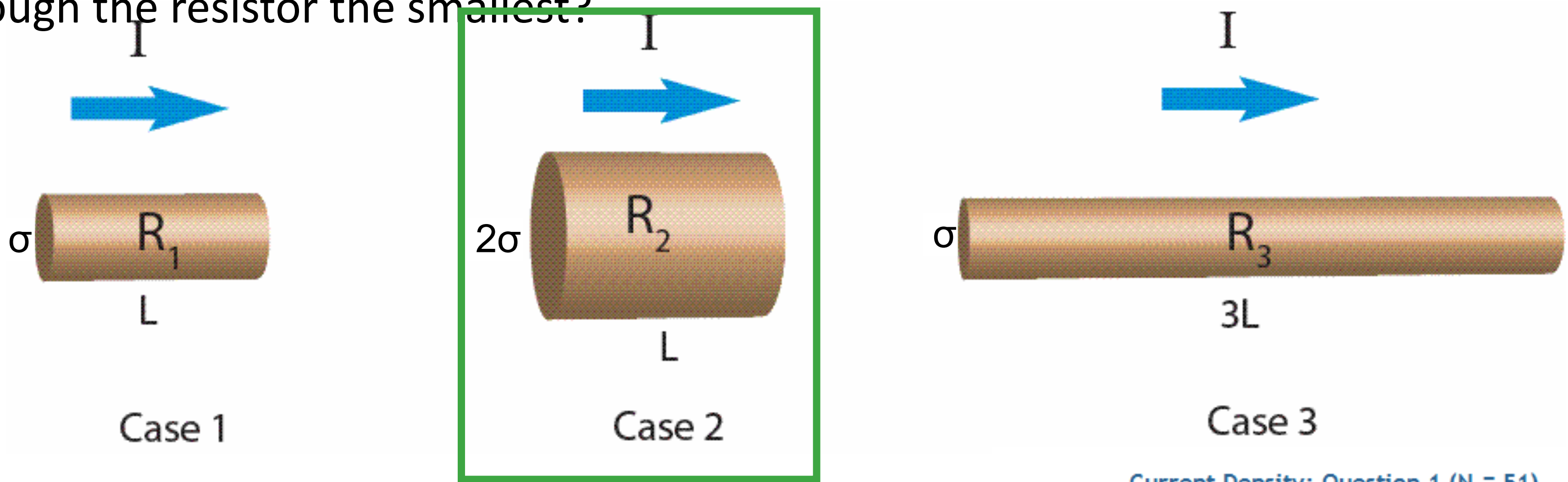
○ $V_1 > V_2$ ○ $V_1 = V_2$ ○ $V_1 < V_2$



$$L_2 = 2L_1 \Rightarrow V_2 = 2V_1$$

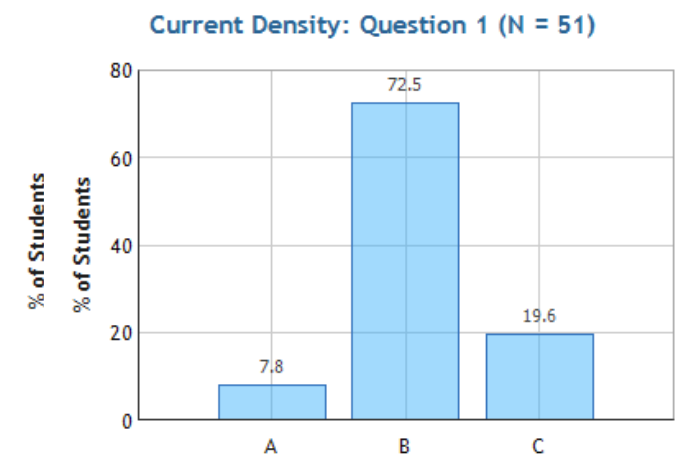
CheckPoint: Current Density

The SAME amount of current I passes through three different resistors. R_2 has twice the cross-sectional area and the same length as R_1, and R_3 is three times as long as R_1 but has the same cross-sectional area as R_1. In which case is the CURRENT DENSITY through the resistor the smallest?

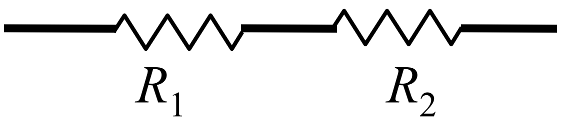
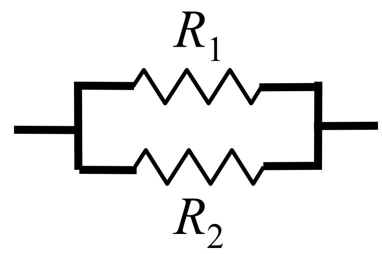


$$J \equiv \frac{I}{A} \longrightarrow J_1 = J_3 = 2J_2$$

Same Current $\longrightarrow$ $J \propto \frac{1}{A}$



Resistor Summary

	Series	Parallel
		
Wiring	Each resistor on the <u>same</u> wire.	Each resistor on a <u>different</u> wire.
Voltage	<u>Different</u> for each resistor. $V_{total} = V_1 + V_2$ Voltage Divider	<u>Same</u> for each resistor. $V_{total} = V_1 = V_2$
Current	<u>Same</u> for each resistor $I_{total} = I_1 = I_2$	<u>Different</u> for each resistor $I_{total} = I_1 + I_2$ Current Divider
Resistance	<u>Increases</u> $R_{eq} = R_1 + R_2$	<u>Decreases</u> $1/R_{eq} = 1/R_1 + 1/R_2$

Symbols

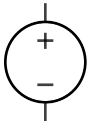
- Resistor symbol (ANSI)
  - N. America, Japan, China(?)

4.7 k = 4700 ohm
1.8 Ω = 1.8 ohm

- Alternate resistor symbol (DIN)
  - Europe, Middle East, Aus/NZ, Africa(?)

4k7 =4700 ohm
1R8= 1.8 ohm

- Voltage Source

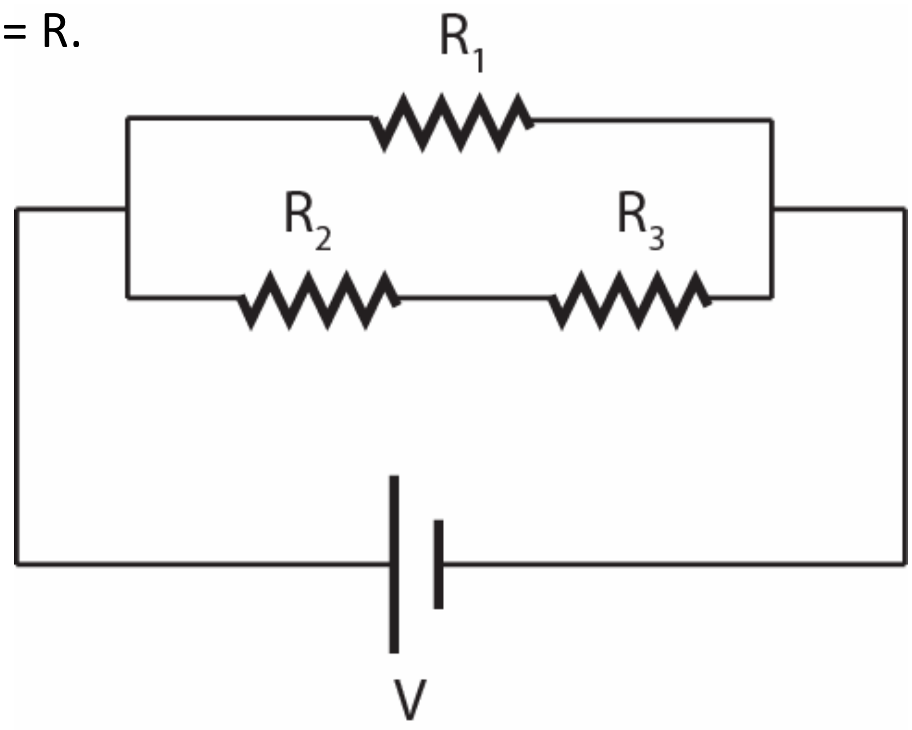


- Electrochemical Cell ("battery")

sometimes used for voltage source

CheckPoint: Resistor Network 1

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.

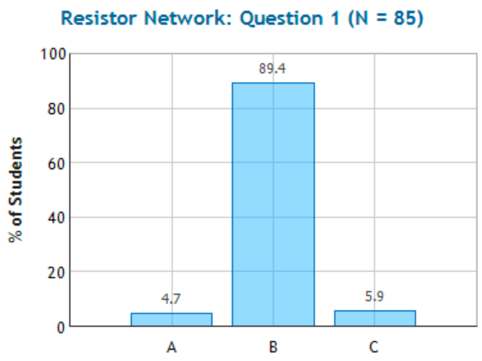


Compare the current through R_2 with the current through R_3:

A. $I_2 > I_3$

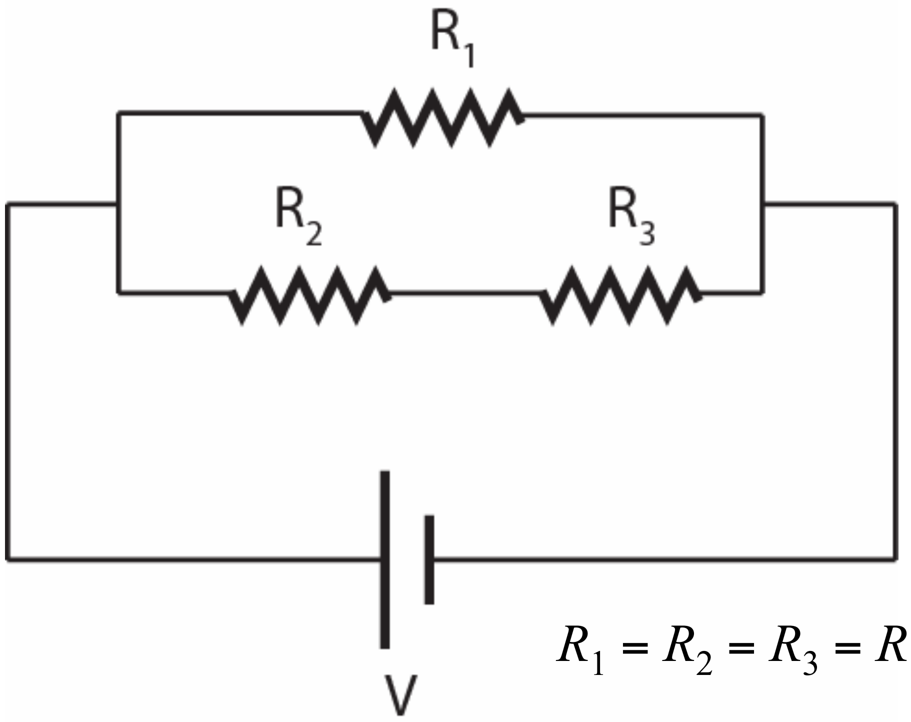
B. $I_2 = I_3$

C. $I_2 < I_3$



R_2 in series with R_3 → Current through R_2 and R_3 is the same

$$I_{23} = \frac{V}{R_2 + R_3}$$



$R_1 = R_2 = R_3 = R$

CheckPoint 2

Compare the current through R_1 with the current through R_2

$I_1 \leftrightarrow I_2$

CheckPoint 3

Compare the voltage across R_2 with the voltage across R_3

$V_2 \leftrightarrow V_3$

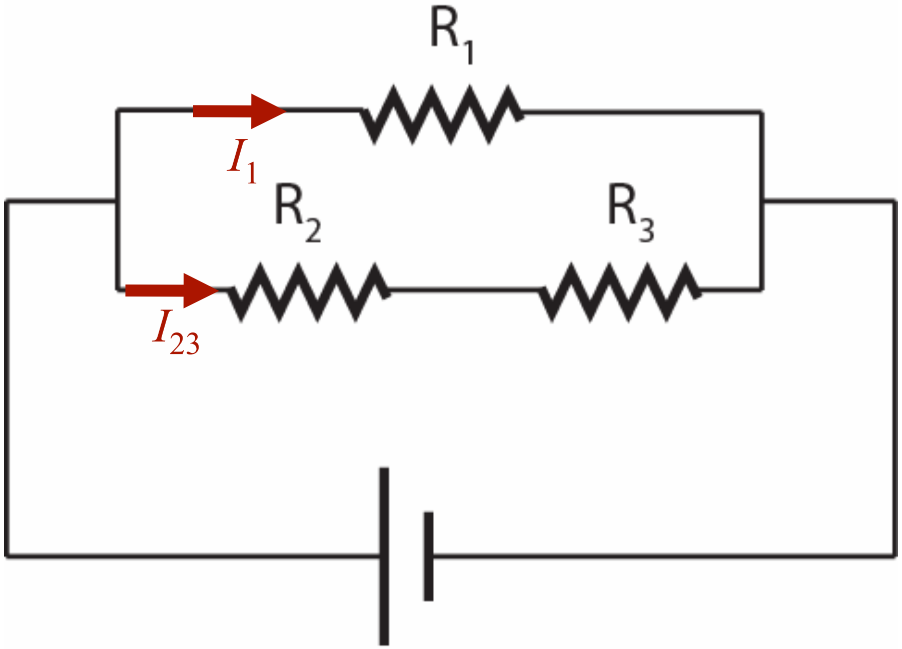
CheckPoint 4

Compare the voltage across R_1 with the voltage across R_2

$V_1 \leftrightarrow V_2$

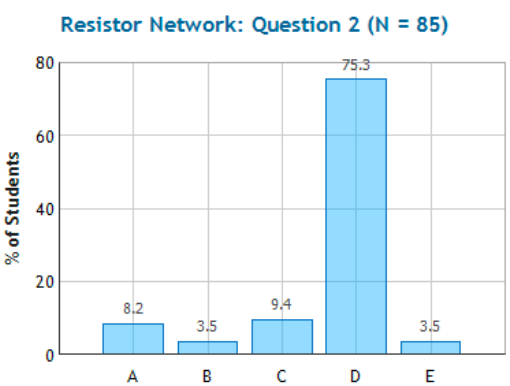
CheckPoint: Resistor Network 2

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.



Compare the current through R_1 with the current through R_2:

A. $I_1/I_{23}=1/2$
B. $I_1/I_{23}=1/3$
C. $I_1 = I_{23}$
D. $I_1/I_{23}=2$
E. $I_1/I_{23}=3$



We know: $$I_{23} = \frac{V}{R_2 + R_3}$$

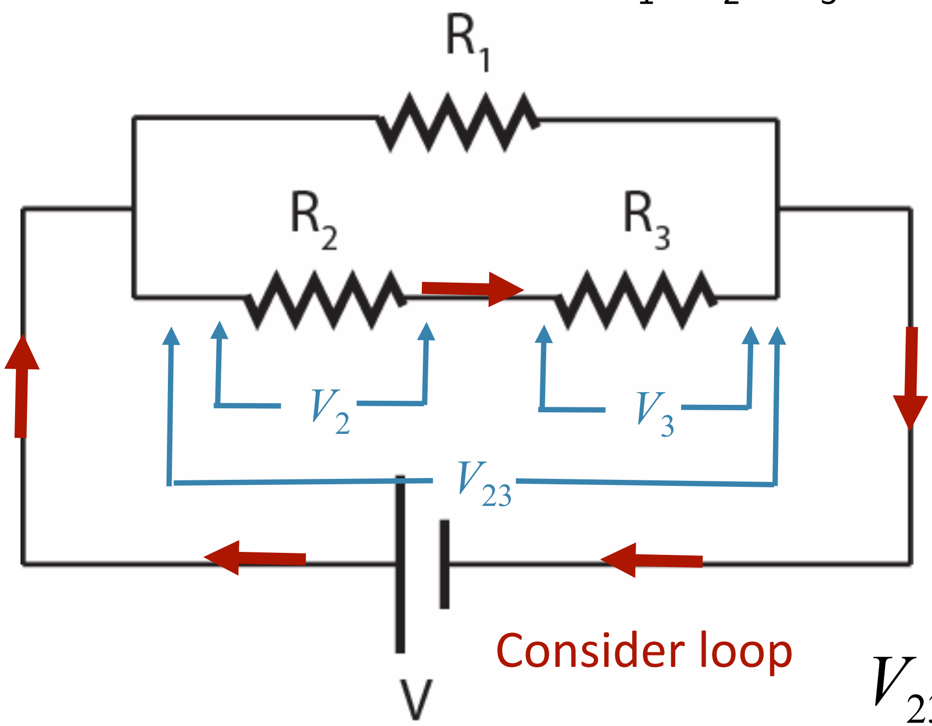
Similarly: $$I_1 = \frac{V}{R_1}$$

$$I_1 = I_{23}\frac{R_2 + R_3}{R_1}$$

$$\frac{I_1}{I_{23}} = \frac{R_2 + R_3}{R_1} = 2$$

CheckPoint: Resistor Network 3

Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.



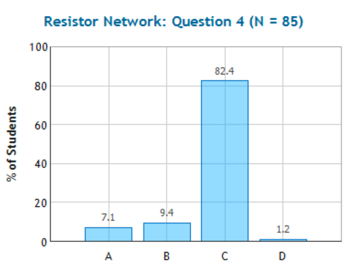
Compare the voltage across R_2 with the voltage across R_3:

A. $V_2 > V_3$

B. $V_2 = V_3 = V$

C. $V_2 = V_3 < V$

D. $V_2 < V_3$



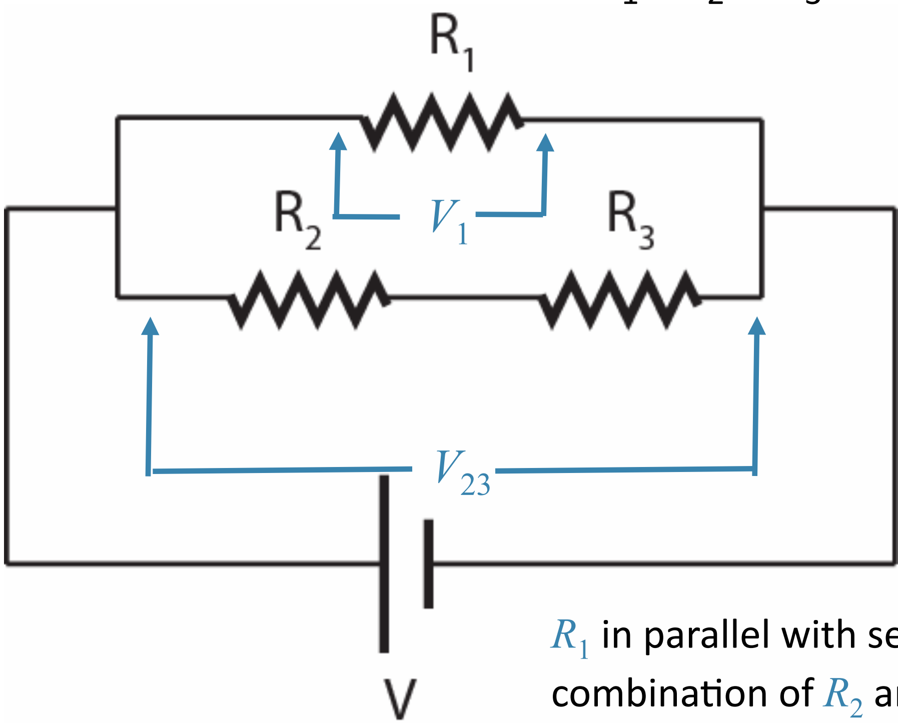
$$V_{23} = V$$

$$V_{23} = V_2 + V_3 \longrightarrow V_2 = V_3 = \frac{V}{2}$$

$$R_2 = R_3 \Rightarrow V_2 = V_3$$

CheckPoint: Resistor Network 4

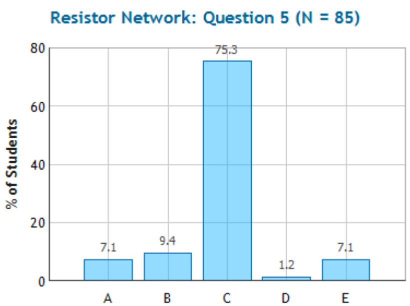
Three resistors are connected to a battery with emf V as shown. The resistances of the resistors are all the same, i.e. $R_1 = R_2 = R_3 = R$.



Compare the voltage across R_1 with the voltage across R_2.

A. $V_1 = V_2 = V$
B. $V_1 = 1/2\ V_2 = V$
C. $V_1 = 2V_2 = V$
D. $V_1 = 1/2\ V_2 = 1/5\ V$
E. $V_1 = 1/2\ V_2 = 1/2\ V$

R_1 in parallel with series combination of R_2 and R_3

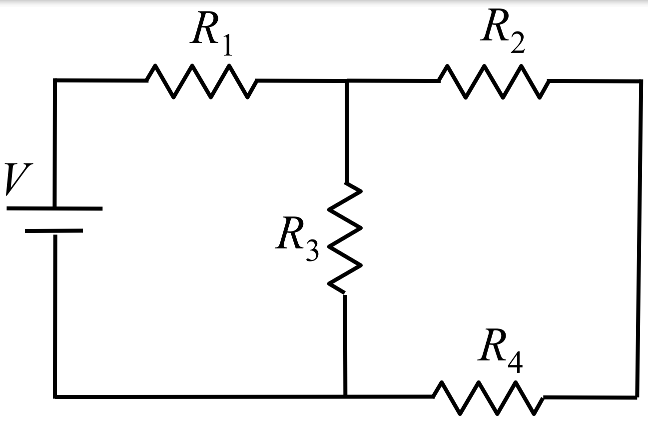


$$V_1 = V_{23}$$

$$R_2 = R_3 \Rightarrow V_2 = V_3 \longrightarrow V_1 = 2V_2 = V$$

$$V_{23} = V_2 + V_3 = 2V_2$$

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Conceptual Analysis:

Ohm's Law: when current *I* flows through resistance R, the potential drop V is given by:

$V = IR$.

Resistances are combined in series and parallel combinations

$$R_{series} = R_a + R_b$$

$$(1/R_{parallel}) = (1/R_a) + (1/R_b)$$

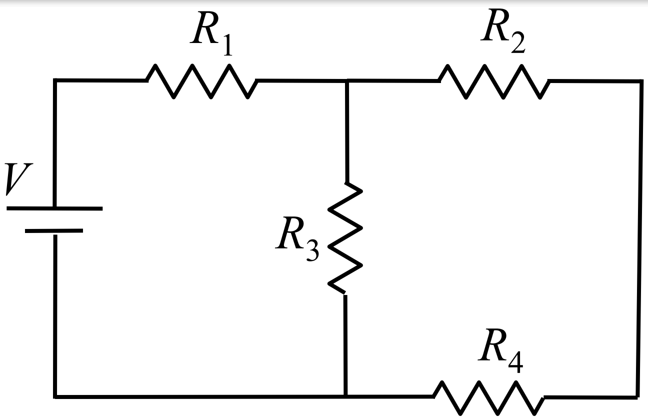
Strategic Analysis:

Combine resistances to form equivalent resistances

Evaluate voltages or currents from Ohm's Law

Expand circuit back using knowledge of voltages and currents

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Combine Resistances:

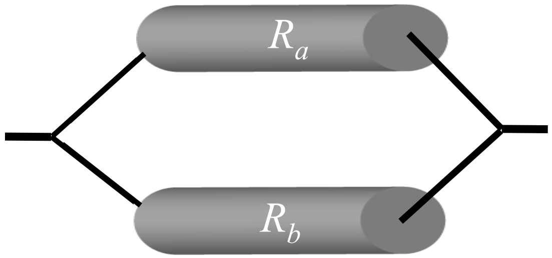
R_1 and R_2 are connected:

A) in series

B) in parallel

C) neither in series nor in parallel

Parallel Combination

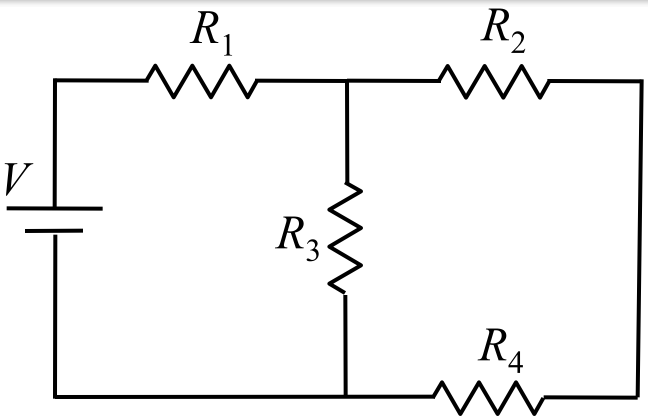


Parallel: Can make a loop that contains only those two resistors

Series Combination

Series : Every loop with resistor 1 also has resistor 2.

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

We first will combine resistances $R_2 + R_3 + R_4$:

Which of the following is true?

A) R_2, R_3 and R_4 are connected in series

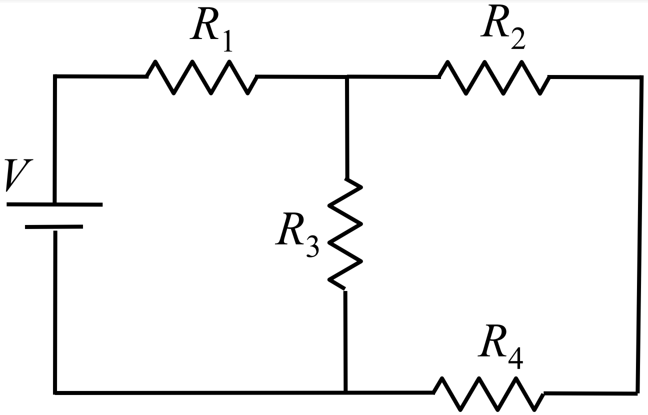
B) R_2, R_3, and R_4 are connected in parallel

C) R_3 and R_4 are connected in series (R_{34}) which is connected in parallel with R_2

D) R_2 and R_4 are connected in series (R_{24}) which is connected in parallel with R_3

E) R_2 and R_4 are connected in parallel (R_{24}) which is connected in parallel with R_3

Calculation



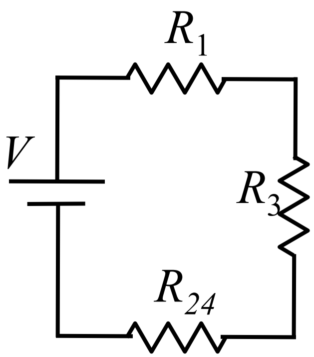
In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

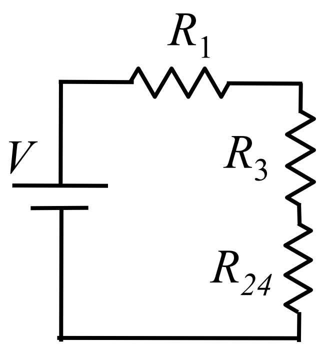
What is V_2, the voltage across R_2?

R_2 and R_4 are connected in series (R_{24}) which is connected in parallel with R_3

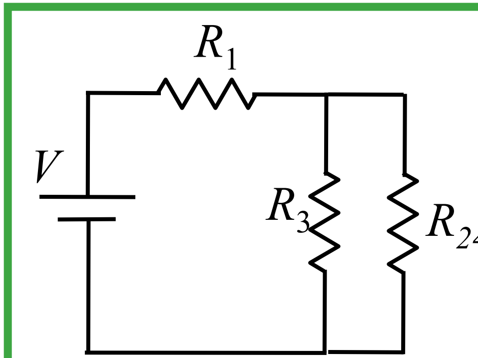
Redraw the circuit using the equivalent resistor R_{24} = series combination of R_2 and R_4.



(A)

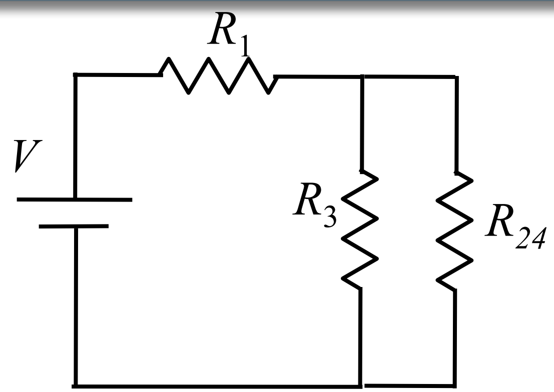


(B)



(C)

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

What is V_2, the voltage across R_2?

Combine Resistances:

R_2 and R_4 are connected in series = R_{24}

R_3 and R_{24} are connected in parallel = R_{234}

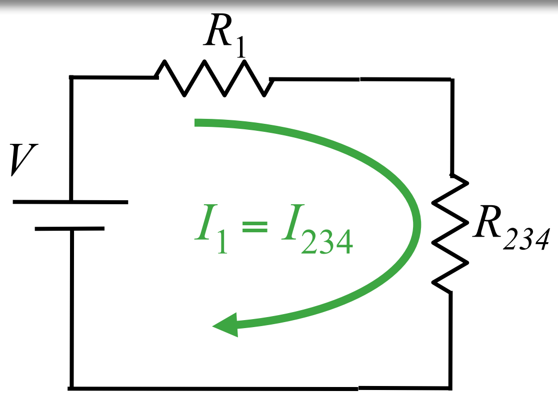
What is the value of R_{234}?

A) $R_{234} = 1\ \Omega$ **B) $R_{234} = 2\ \Omega$** C) $R_{234} = 4\ \Omega$ D) $R_{234} = 6\ \Omega$

R_2 and R_4 in series → $R_{24} = R_2 + R_4 = 2\Omega + 4\Omega = 6\Omega$

$(1/R_{parallel}) = (1/R_a) + (1/R_b)$ → $1/R_{234} = (1/3) + (1/6) = (3/6)\ \Omega^{-1}$ → $R_{234} = 2\ \Omega$

Calculation



In the circuit shown: $V = 18V$,

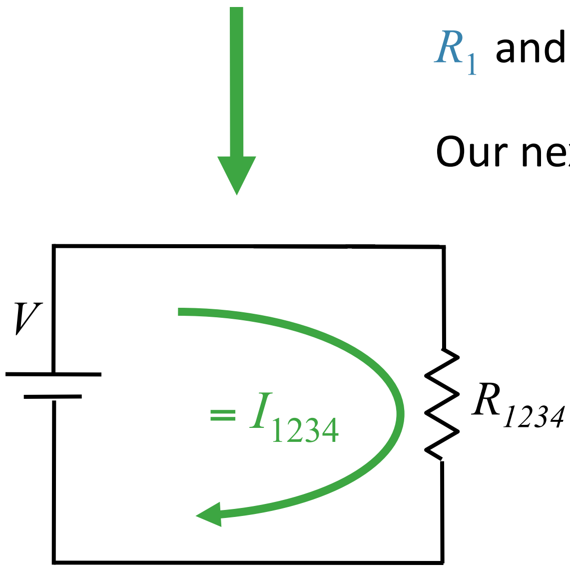
$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

$R_{24} = 6\Omega$ $R_{234} = 2\Omega$

What is V_2, the voltage across R_2?

R_1 and R_{234} are in series. $R_{1234} = 1 + 2 = 3\ \Omega$

Our next task is to calculate the total current in the circuit

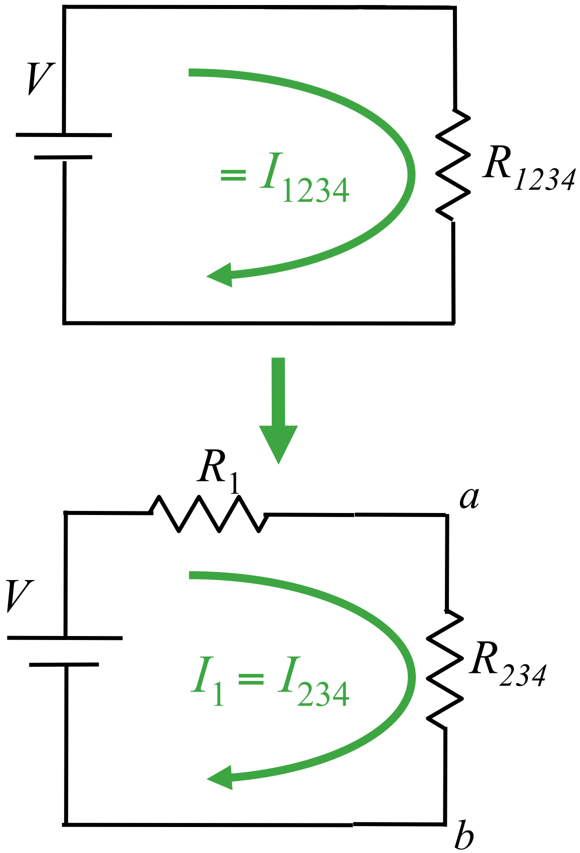


Ohm's Law tells us: $I_{1234} = V/R_{1234}$

$= 18 / 3$

$= 6$ Amps

Calculation



In the circuit shown: $V = 18V$,

$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 3\Omega$, and $R_4 = 4\Omega$.

$R_{24} = 6\Omega$ $\quad R_{234} = 2\Omega$ $\quad I_{1234} = 6\ A$

What is V_2, the voltage across R_2?

$I_{234} = I_{1234}$ Since R_1 in series with R_{234}

$$V_{234} = I_{234} R_{234}$$

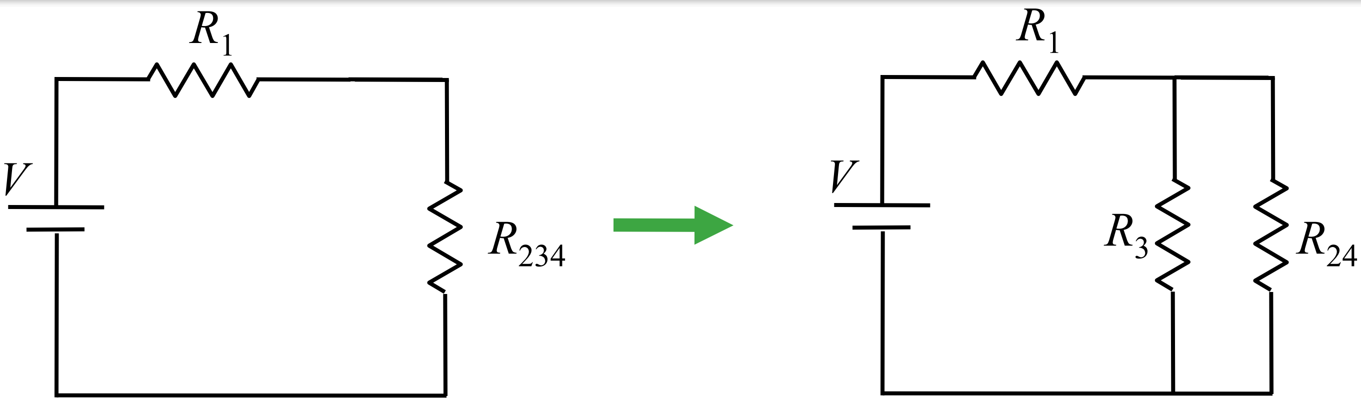
$$= 6 \text{ x } 2$$

$$= 12 \text{ Volts}$$

What is V_{ab}, the voltage across R_{234} ?

A) $V_{ab} = 1\ V$ B) $V_{ab} = 2\ V$ C) $V_{ab} = 9\ V$ D) $V_{ab} = 12\ V$ E) $V_{ab} = 16\ V$

Calculation



$V = 18V$

$R_1 = 1\Omega$

$R_2 = 2\Omega$

$R_3 = 3\Omega$

$R_4 = 4\Omega$

$R_{24} = 6\Omega$

$R_{234} = 2\Omega$

$I_{1234} = 6$ Amps

$I_{234} = 6$ Amps

$V_{234} = 12V$

Which of the following are true?

A) $V_{234} = V_{24}$ B) $I_{234} = I_{24}$ C) Both A+B D) None

R_3 and R_{24} were combined in parallel to get R_{234} → Voltages are same!

What is V_2?

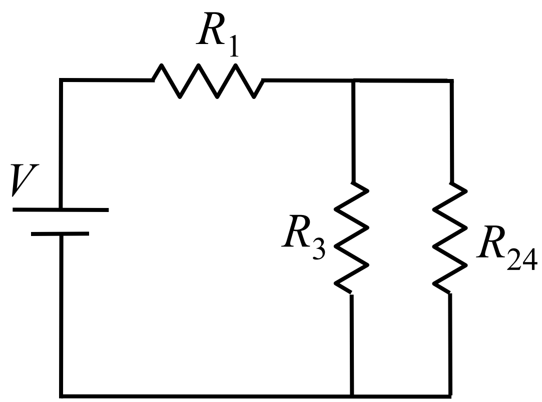
Ohm's Law

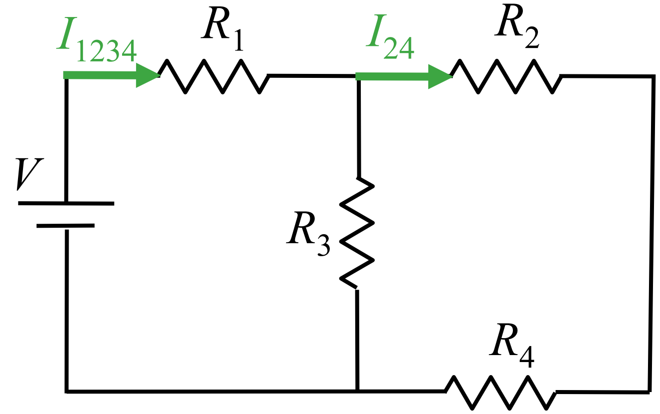
$$I_{24} = V_{24} / R_{24}$$

$$= 12 / 6$$

$$= 2 \text{ Amps}$$

Calculation





$V = 18V$

$R_1 = 1\Omega$

$R_2 = 2\Omega$

$R_3 = 3\Omega$

$R_4 = 4\Omega$.

$R_{24} = 6\Omega$

$R_{234} = 2\Omega$

$I_{1234} = 6$ Amps

$I_{234} = 6$ Amps

$V_{234} = 12V$

$V_{24} = 12V$

$I_{24} = 2$ Amps

What is V_2?

Which of the following are true?

A) $V_{24} = V_2$ B) $I_{24} = I_2$ C) Both A+B D) None

R_2 and R_4 where combined in series to get R_{24} → Currents are same!

The Problem Can Now Be Solved!

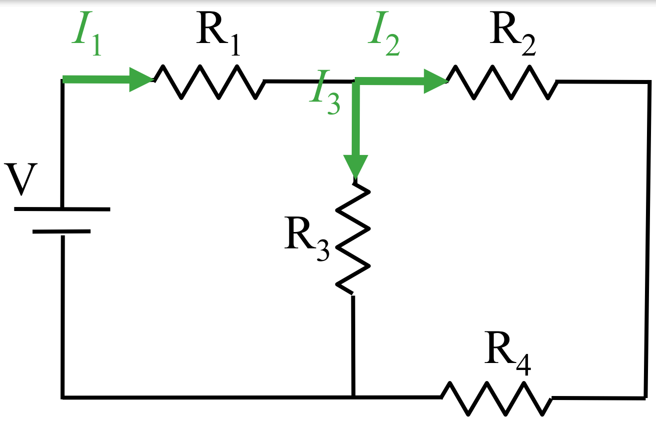
Ohm's Law

$V_2 = I_2 \ R_2$

$= \ 2 \text{ x } 2$

$= 4$ Volts!

Quick Follow-Ups



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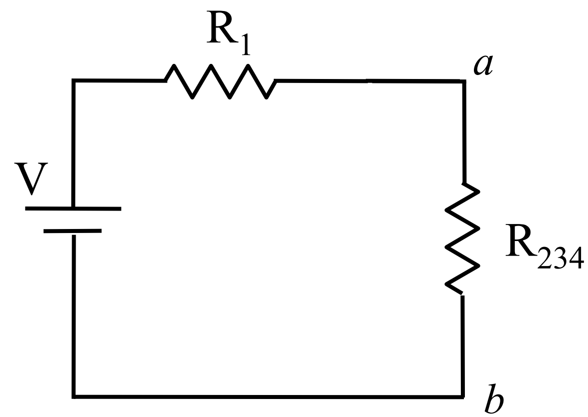


$V = 18V$
$R_1 = 1\Omega$
$R_2 = 2\Omega$
$R_3 = 3\Omega$
$R_4 = 4\Omega$
$R_{24} = 6\Omega$
$R_{234} = 2\Omega$
$V_{234} = 12V$
$V_2 = 4V$
$I_{1234} = 6$ Amps

What is I_3 ?

A) $I_3 = 2\ A$ B) $I_3 = 3\ A$ C) $I_3 = 4\ A$

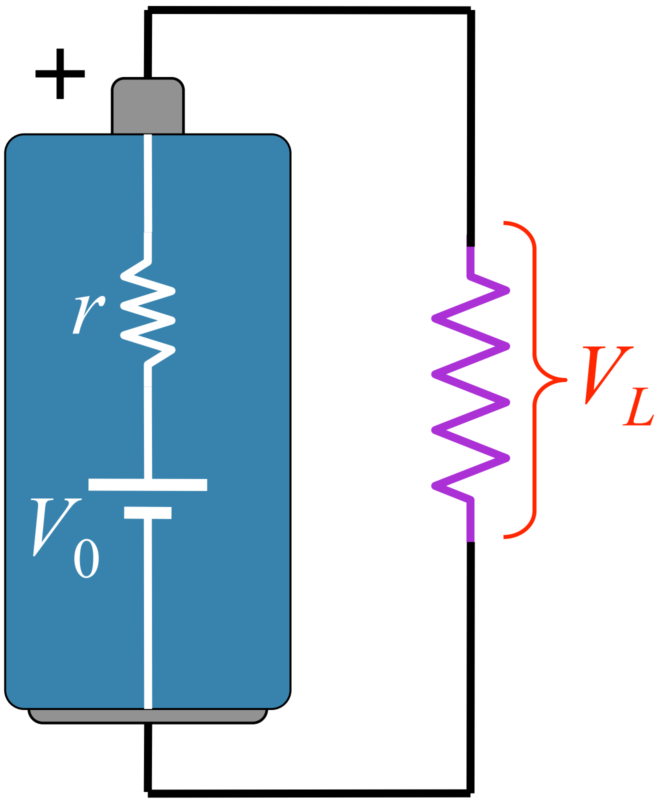
$V_3 = V_{234} = 12V$ → $I_3 = V_3/R_3 = 12V/3\Omega = 4A$

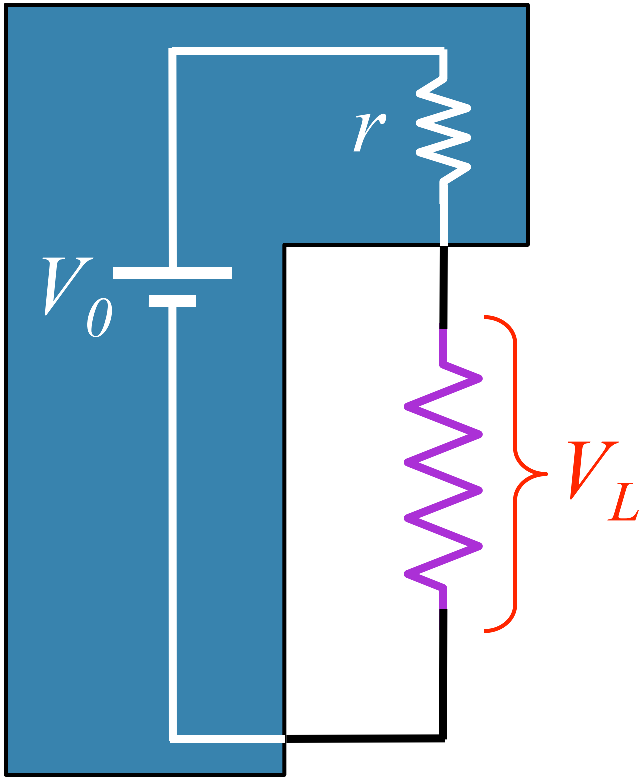
What is I_1 ?

We know $I_1 = I_{1234} = 6\ A$

NOTE: $I_2 = V_2/R_2 = 4/2 = 2\ A$ → $I_1 = I_2 + I_3$ Make Sense?

Model for Real Battery: Internal Resistance





Usually can't supply too much current to the load without voltage "sagging"