Electricity & Magnetism
Lecture 8: Kirchhoff's Rules

Today's Concept:

Kirchhoff's Rules

This week

- Midterm on Friday 6:30 pm to 8:00 pm in B9201
- No lecture on Friday morning
- Today's topic "Kirchhoff's Rules not on this midterm.
- Office hours this week are in a different room:
  - Wednesday: 2:30 – 3:20 P8445.1 (small seminar room)
  - Thursday 3:30 – 4:20 P9416

Comments

- "Please explain Kirchhoff in human language.”

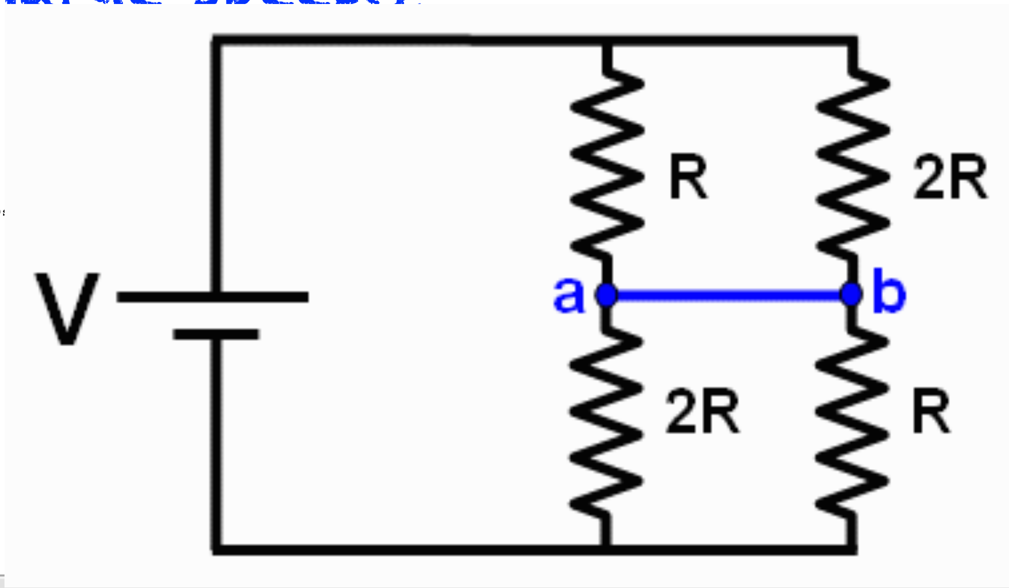
  water, pipes, pumps, tanks ...

- "How can you have a voltage drop across a battery and a voltage gain across a resistor” will talk about these

- "What way does current flow from A to B?". Also, if charges flow through a resistance, then why does I(before) = I(after)? Does the resistance not slow down the current (charge speed)?”

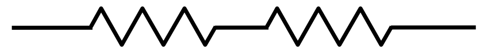
- "The Blue Wire

- "The whole concept o ... ts



Last Time

Resistors in series:

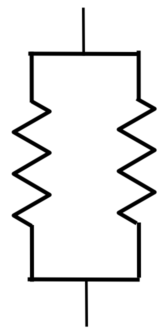


Current through is same.

Voltage drop across is IR_i

$$R_{effective} = R_1 + R_2 + R_3 + \ldots$$

Resistors in parallel:

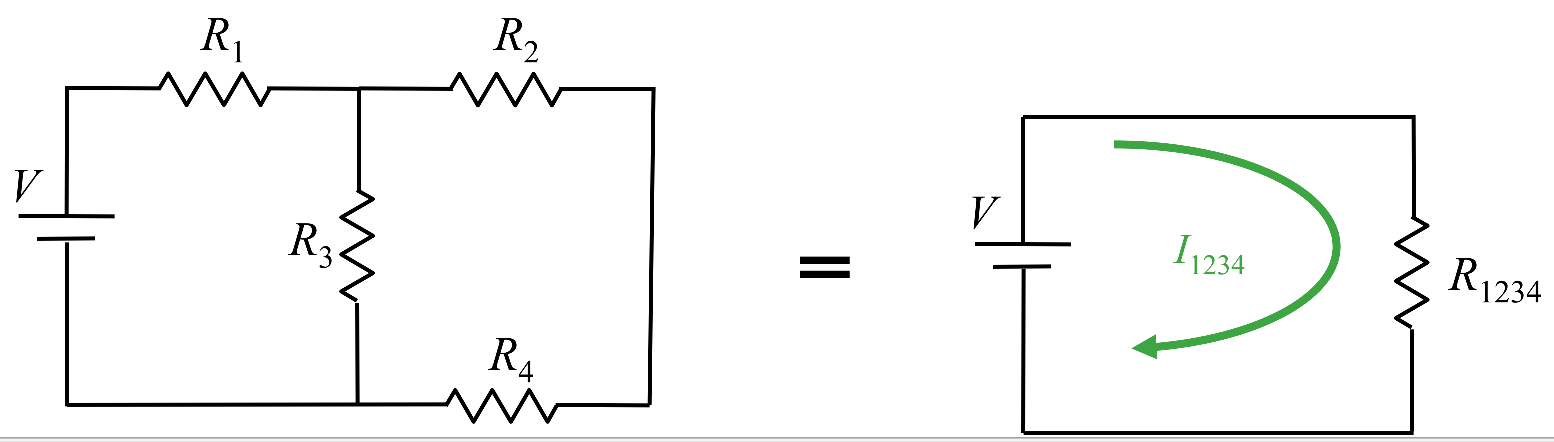


Voltage drop across is same.

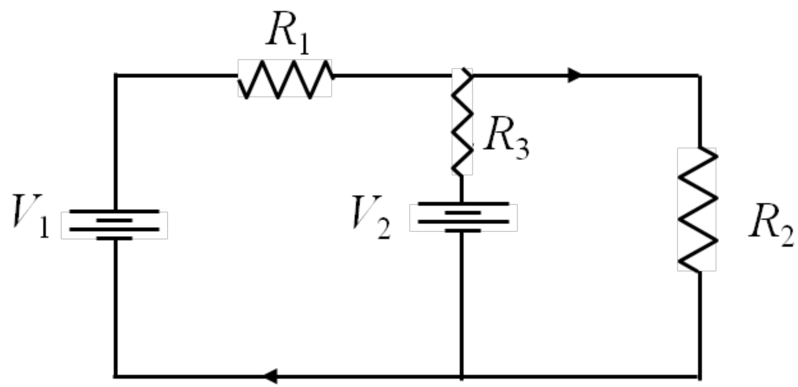
Current through is V/R_i

$$\frac{1}{R_{effective}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

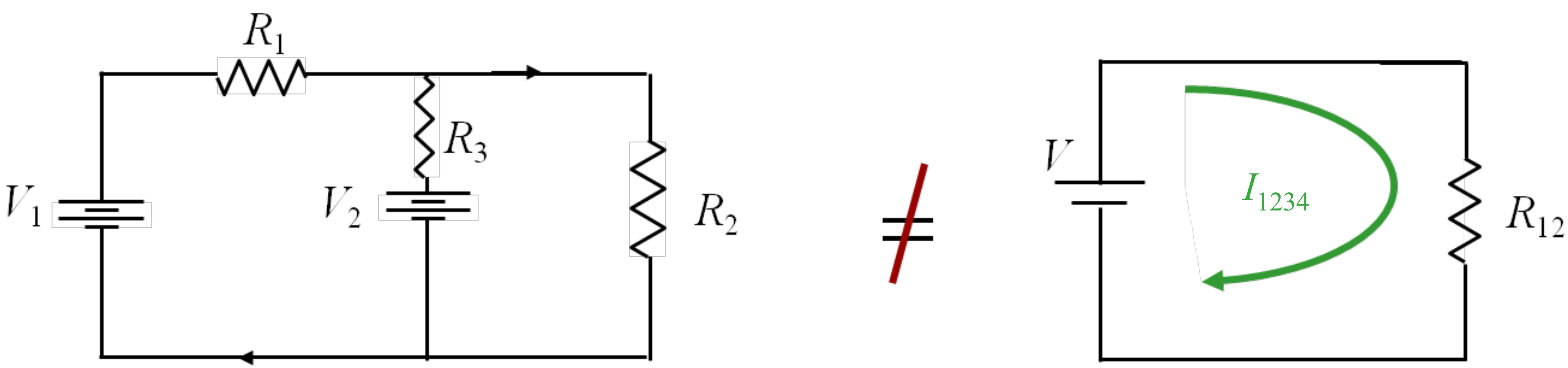
Solved Circuits



New Circuit



How Can We Solve This One?



THE ANSWER: Kirchhoff's Rules

Kirchhoff's Voltage Rule

$$\sum \Delta V_i = 0$$

Kirchhoff's Voltage Rule states that the sum of the voltage changes caused by any elements (like wires, batteries, and resistors) around a circuit must be zero.

WHY?

The potential difference between a point and itself is zero!

Kirchhoff's Current Rule

$$\sum I_{in} = \sum I_{out}$$

Kirchhoff's Current Rule states that the sum of all currents entering any given point in a circuit must equal the sum of all currents leaving the same point.

WHY?

Electric Charge is Conserved

Kirchhoff's Laws

1) Label all currents

Choose any direction

2) Label +/– for all elements

Current goes + ⇒ – (for resistors)

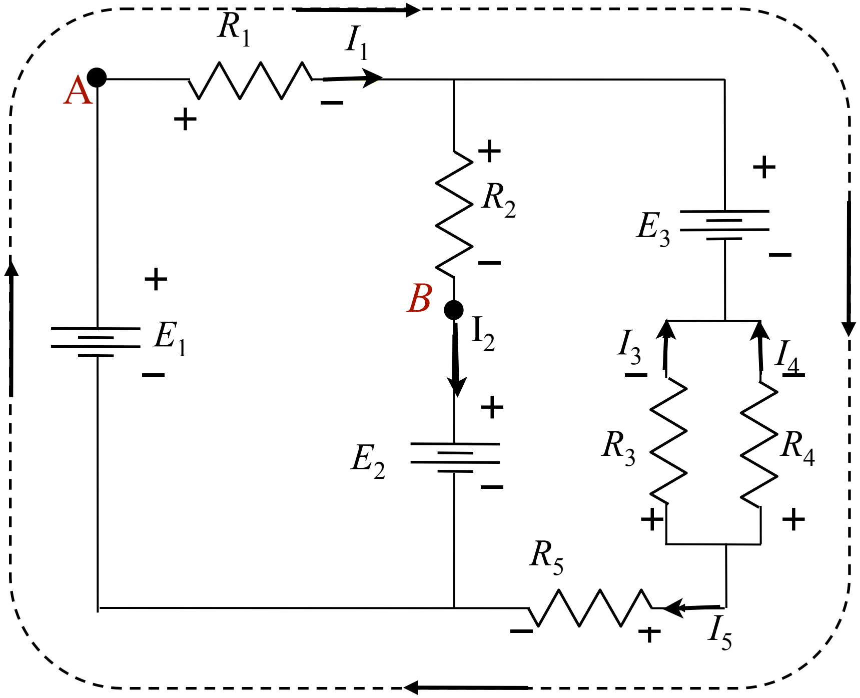
3) Choose loop and direction

Must start on wire, not element.

4) Write down voltage drops

First sign you hit is sign to use.

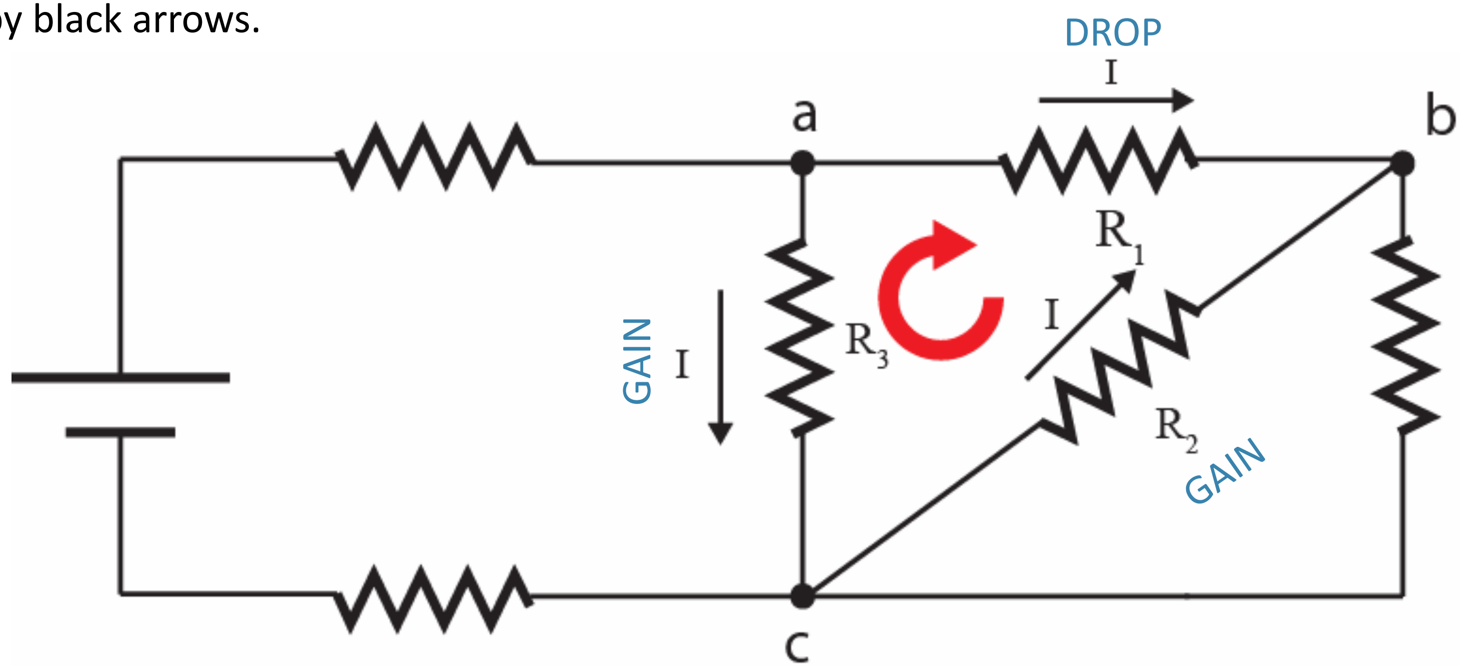
5) Write down node equation $I_{in} = I_{out}$

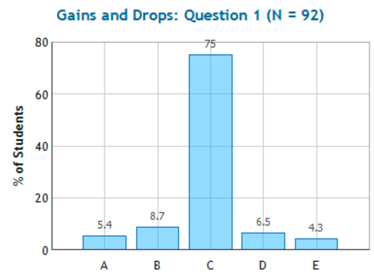


We'll do calculation first today
It's actually the easiest thing to do!

CheckPoint: Gains and Drops

In the following circuit, consider the loop abc. The direction of the current through each resistor is indicated by black arrows.





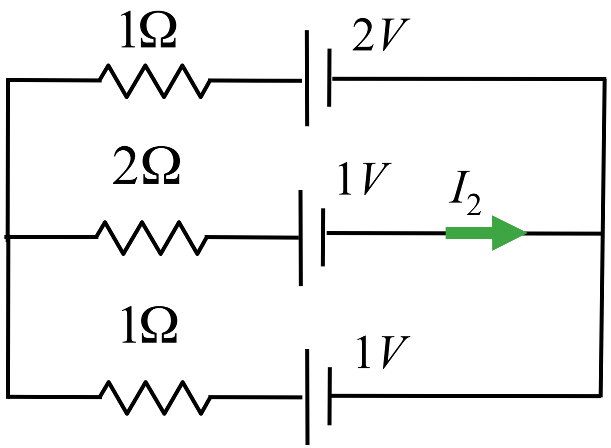
If we are to write Kirchoff's voltage equation for this loop in the clockwise direction starting from point a, what is the correct order of voltage gains/drops that we will encounter for resistors R1, R2 and R3?

A. drop, drop, drop
B. gain, gain, gain
C. drop, gain, gain
D. gain, drop, drop
E. drop, drop, gain

With the current → VOLTAGE DROP

Against the current → VOLTAGE GAIN

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

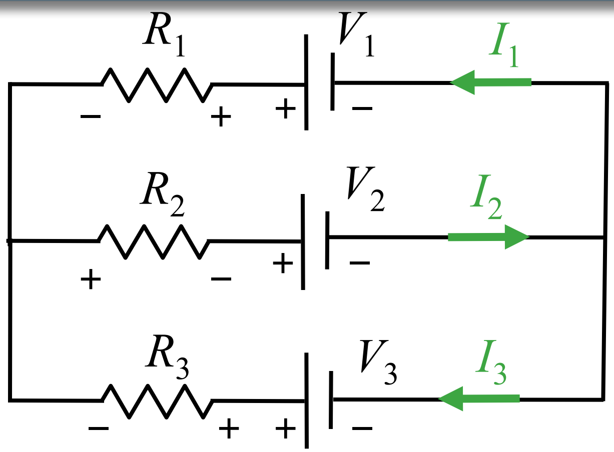
Conceptual Analysis:

- Circuit behavior described by Kirchhoff's Rules:
  - KVR: $\Sigma\, V_{drops} = 0$
  - KCR: $\Sigma\, I_{in} = \Sigma\, I_{out}$

Strategic Analysis

- Write down Loop Equations (KVR)
- Write down Node Equations (KCR)
- Solve

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

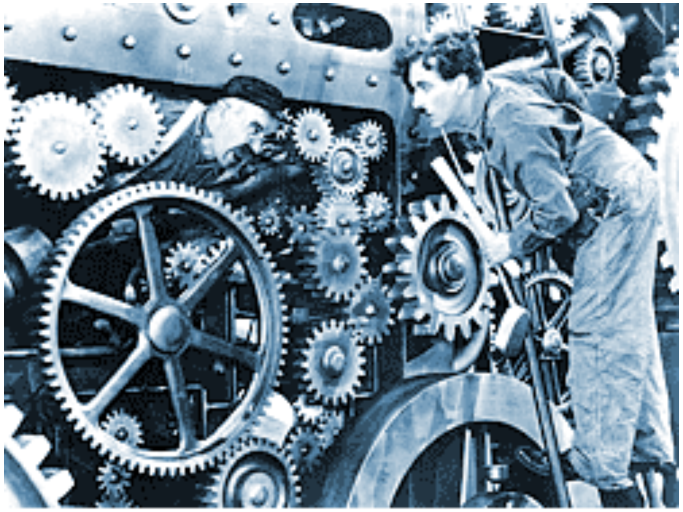
Label and pick directions for each current

Label the + and – side of each element

This is easy for batteries

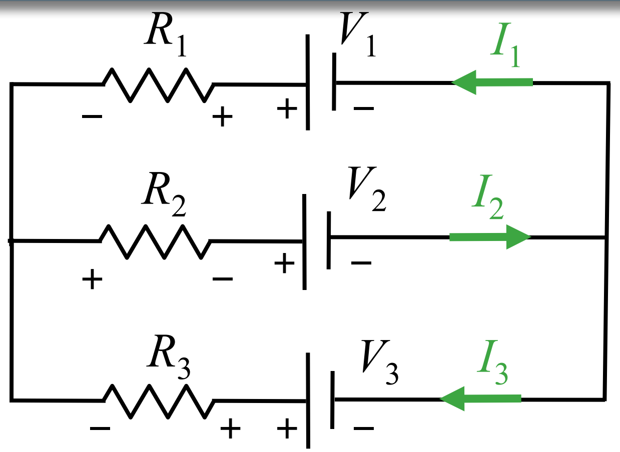
For resistors, the "upstream" side is +

Now write down loop and node equations



Just turn the crank.

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

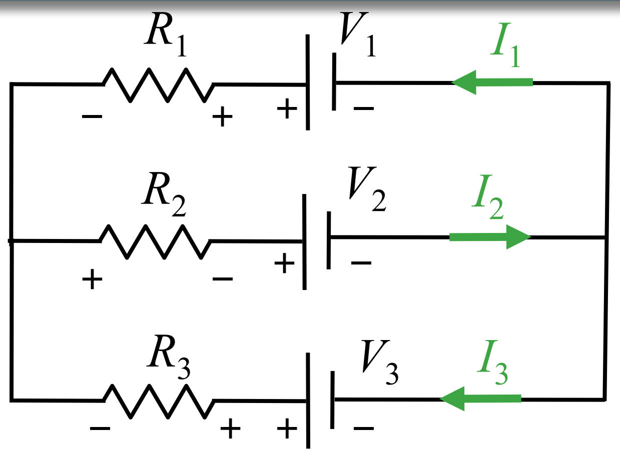
How many equations do we need to write down in order to solve for I_2?

A) 1   B) 2   C) 3   D) 4   E) 5

Why?

- We have 3 unknowns: I_1, I_2, and I_3
- We need 3 independent equations to solve for these unknowns

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

Which of the following equations is NOT correct?

A) $I_2 = I_1 + I_3$

B) $-V_1 + I_1R_1 - I_3R_3 + V_3 = 0$

C) $-V_3 + I_3R_3 + I_2R_2 + V_2 = 0$

D) $-V_2 - I_2R_2 + I_1R_1 + V_1 = 0$

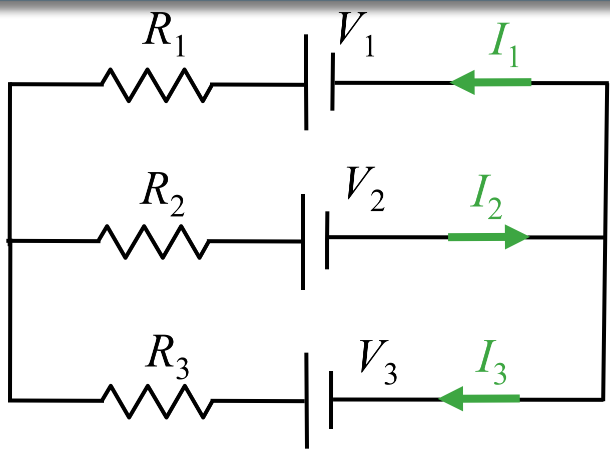
Why?

– (D) is an attempt to write down KVR for the top loop

– Start at negative terminal of V_2 and go clockwise

$V_{gain}\ (-V_2)$ then $V_{gain}\ (-I_2R_2)$ then $V_{gain}\ (-I_1R_1)$ then $V_{drop}\ (+V_1)$

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

We have the following 4 equations:

1. $I_2 = I_1 + I_3$
2. $-V_1 + I_1R_1 - I_3R_3 + V_3 = 0$
3. $-V_3 + I_3R_3 + I_2R_2 + V_2 = 0$
4. $-V_2 - I_2R_2 - I_1R_1 + V_1 = 0$

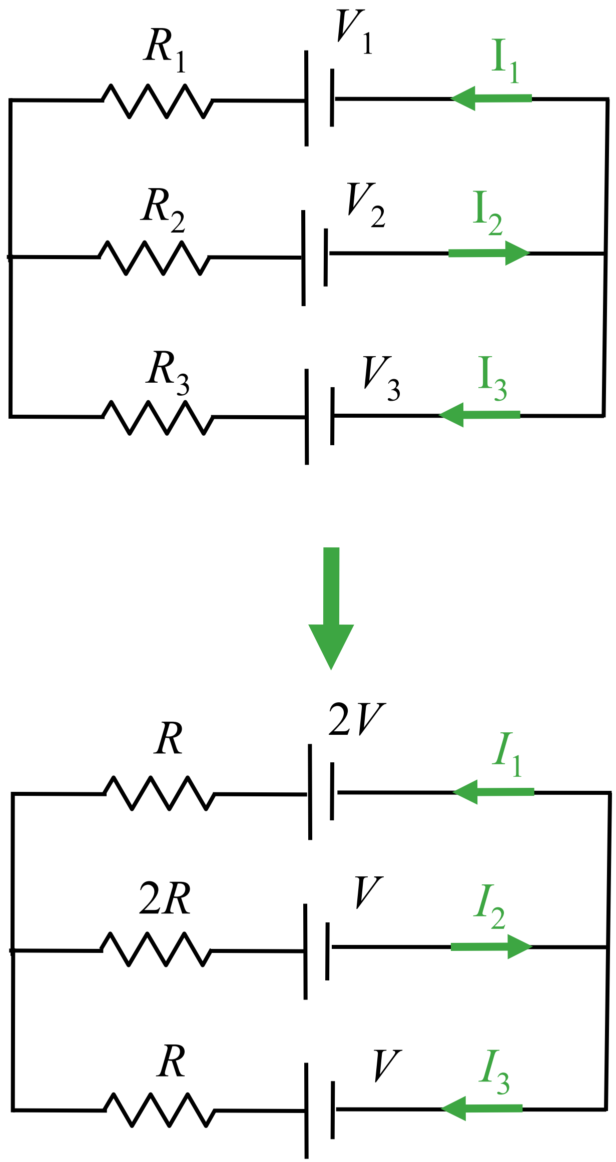
We need 3 equations:
Which 3 should we use?

A) Any 3 will do
B) 1, 2, and 4
C) 2, 3, and 4

Why?

- We need 3 INDEPENDENT equations
- Equations 2, 3, and 4 are NOT INDEPENDENT
  Eqn 2 + Eqn 3 = – Eqn 4
- We must choose Equation 1 and any two of the remaining (2, 3, and 4)

Calculation



In this circuit, assume V_i and R_i are known.

What is I_2 ?

We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$
$$V_1 + I_1R_1 - I_3R_3 + V_3 = 0$$
$$V_2 - I_2R_2 - I_1R_1 + V_1 = 0$$

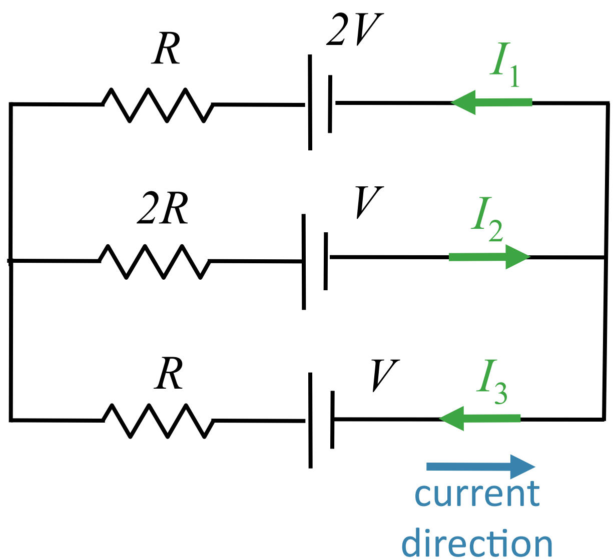
The solution will get very messy!

Simplify: assume $V_2 = V_3 = V$

$$V_1 = 2V$$
$$R_1 = R_3 = R$$
$$R_2 = 2R$$

Calculation: Simplify

In this circuit, assume V and R are known. What is I_2 ?



We have 3 equations and 3 unknowns.

$$I_2 = I_1 + I_3$$

$$-2V + I_1 R - I_3 R + V = 0 \quad \text{(outside)}$$

$$-V - I_2(2R) - I_1 R + 2V = 0 \quad \text{(top)}$$

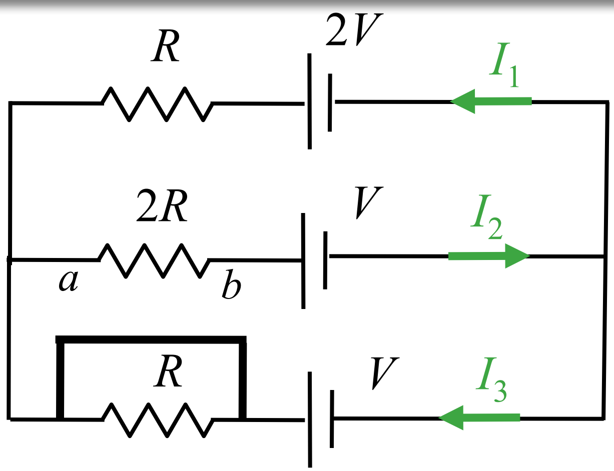
With this simplification, you can verify:

$$I_2 = (\ 1/5)\ V/R$$

$$I_1 = (\ 3/5)\ V/R$$

$$I_3 = (-2/5)\ V/R$$

Follow Up



We know:

$I_2 = (\,1/5)\; V/R$

$I_1 = \;(\,3/5)\; V/R$

$I_3 = (-2/5)\; V/R$

Suppose we short R_3: What happens to V_{ab} (voltage across R_2?)

A) V_{ab} remains the same

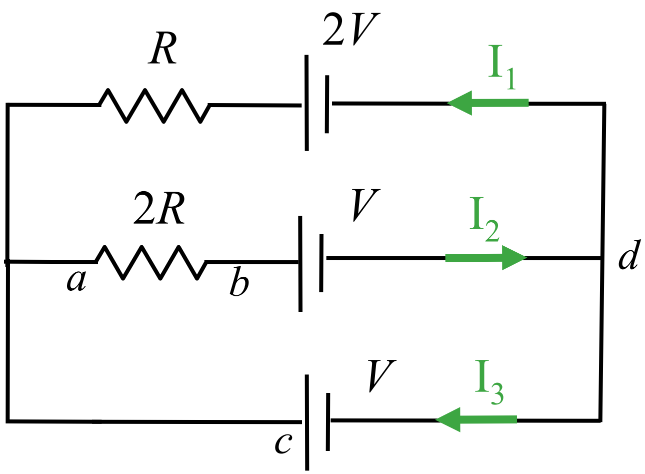
B) V_{ab} changes sign

C) V_{ab} increases

D) V_{ab} goes to zero

Why?

Redraw:

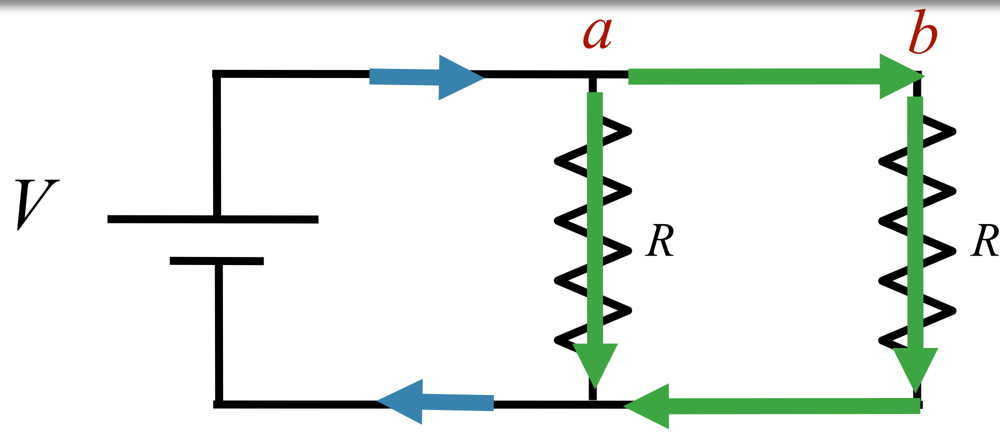


Bottom Loop Equation:

$$V_{ab} + V - V = 0$$

$$V_{ab} = 0$$

Clicker Question



Is there a current flowing between a and b ?

A) Yes
B) No

a & b have the same potential $\not\Rightarrow$ No current flows between a & b

Current flows from battery and splits at a

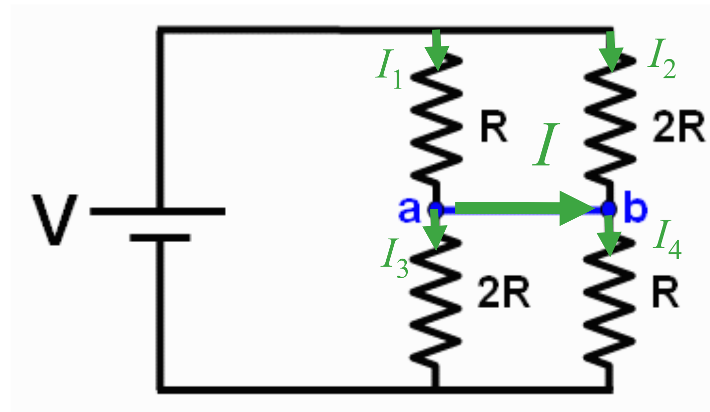
Some current flows down

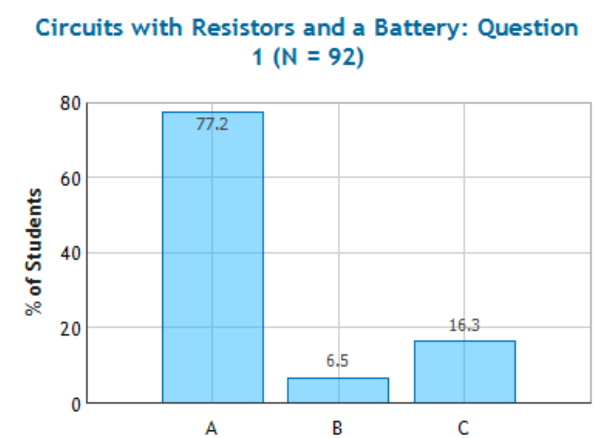
Some current flows right

CheckPoint: Circuits w/ Resistors and a Battery 1

Consider the circuit shown below. Which of the following statements best describes the current flowing in the blue wire connecting points **a** and **b**?

A. Positive current flows from **a** to **b**
B. Positive current flows from **b** to **a**
C. No current flows between **a** and **b**





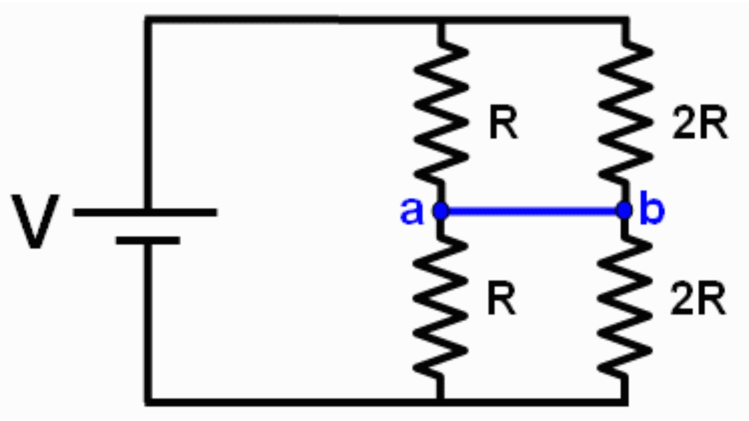
$$I_1 R - I_2\,(2R) = 0 \quad \rightarrow \quad I_2 = ½\ I_1$$

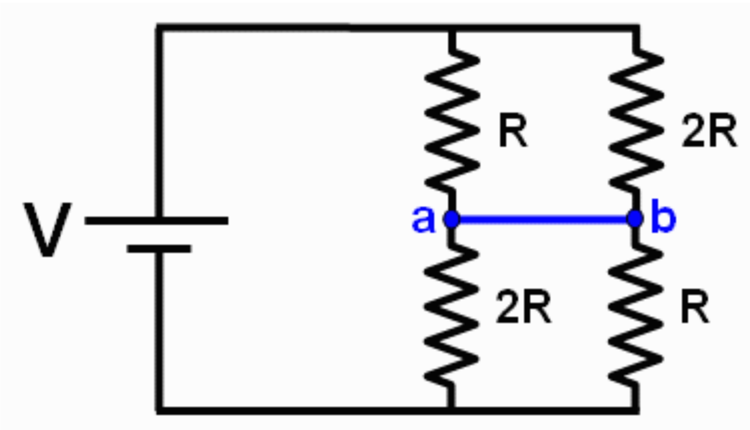
$$I_4 R - I_3\,(2R) = 0 \quad \rightarrow \quad I_4 = 2\,I_3$$

$$I = I_1 - I_3$$

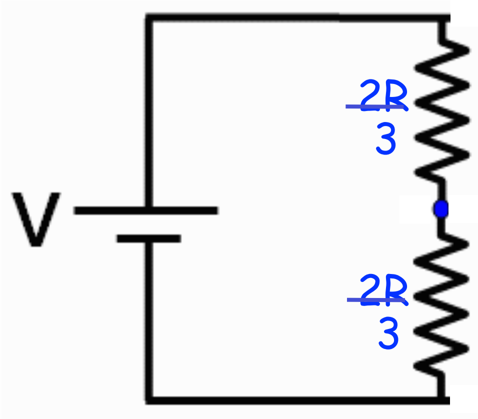
$$I + I_2 = I_4 \quad \rightarrow \quad I_1 - I_3 + ½\,I_1 = 2I_3 \quad \rightarrow \quad I_1 = 2I_3 \quad \rightarrow \quad I = I_3$$

Prelecture *CheckPoint*

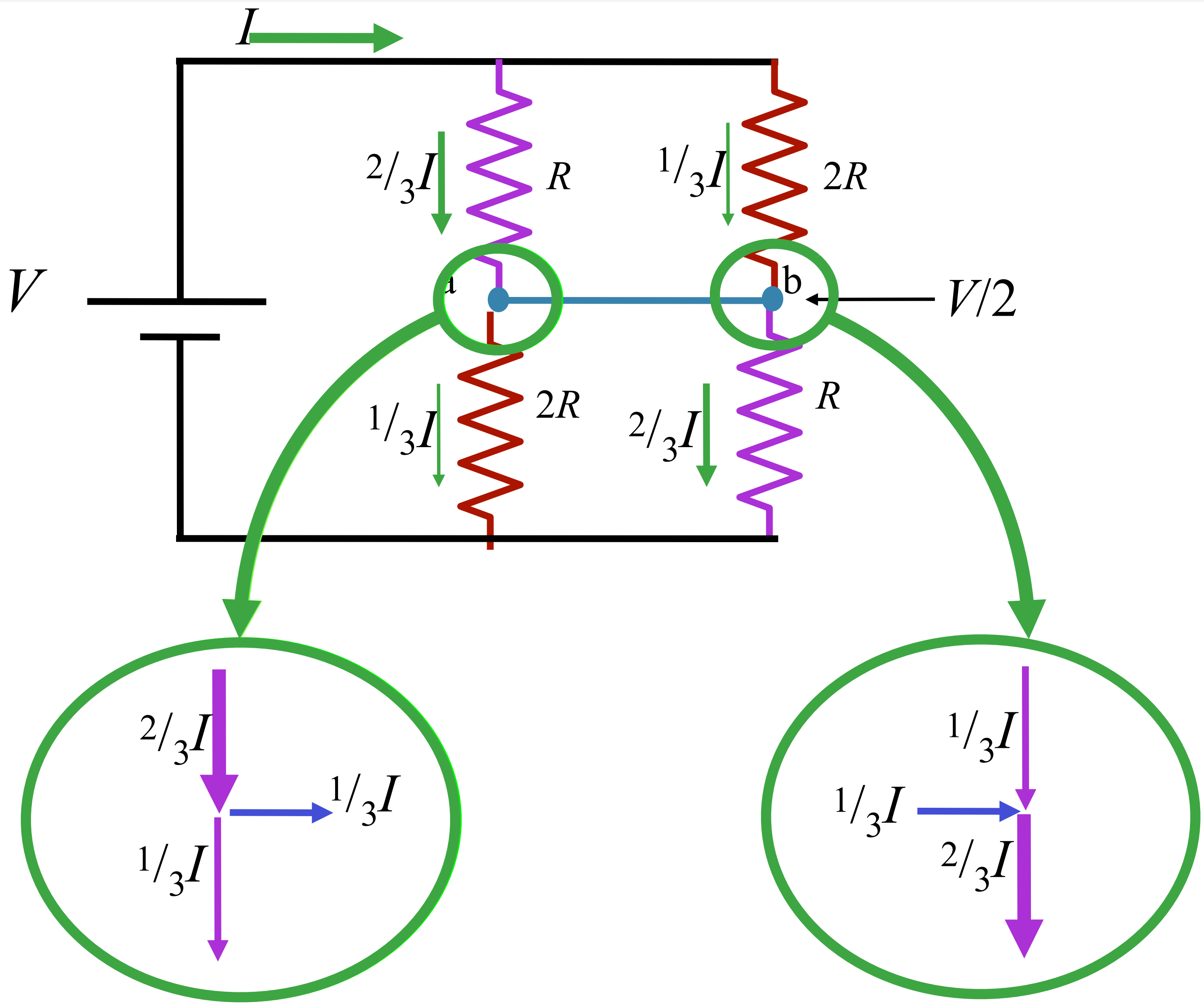




What is the same? Current flowing in and out of the battery.

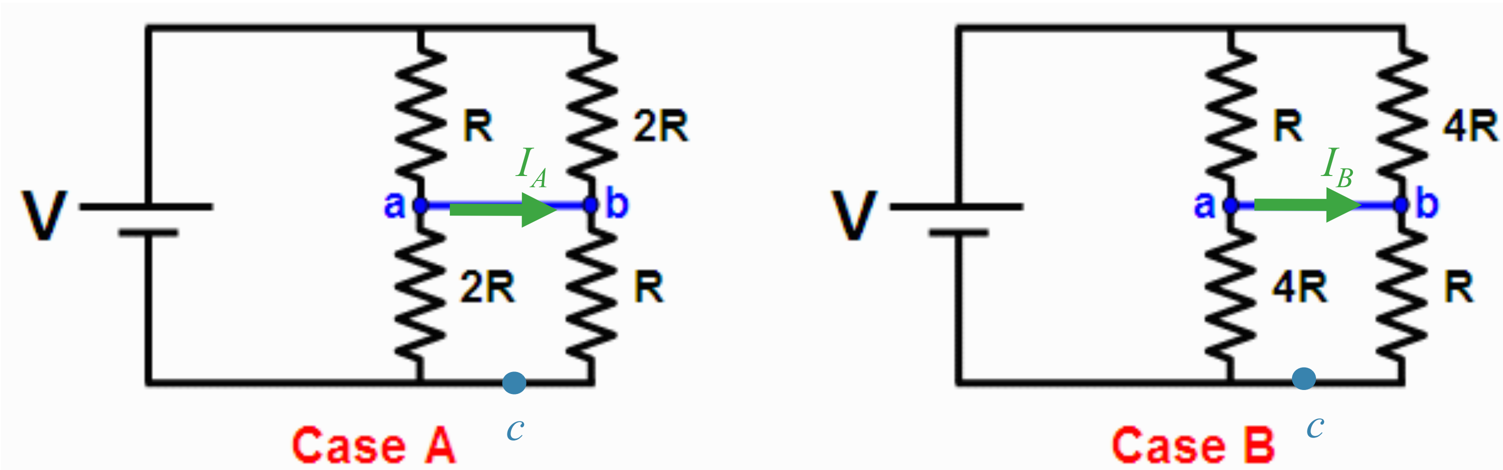


What is different? Current flowing from a to b.


I
V
2/3 I
R
1/3 I
2R
b
V/2
1/3 I
2R
2/3 I
R
2/3 I
1/3 I
1/3 I
1/3 I
1/3 I
2/3 I

CheckPoint: Circuits w/ Resistors and a Battery 2

Consider the circuit shown below. In which case is the current flowing in the blue wire connecting points **a** and **b** bigger?



A Case **A**

B Case **B**

C They are the same

Current will flow from left to right in both cases.

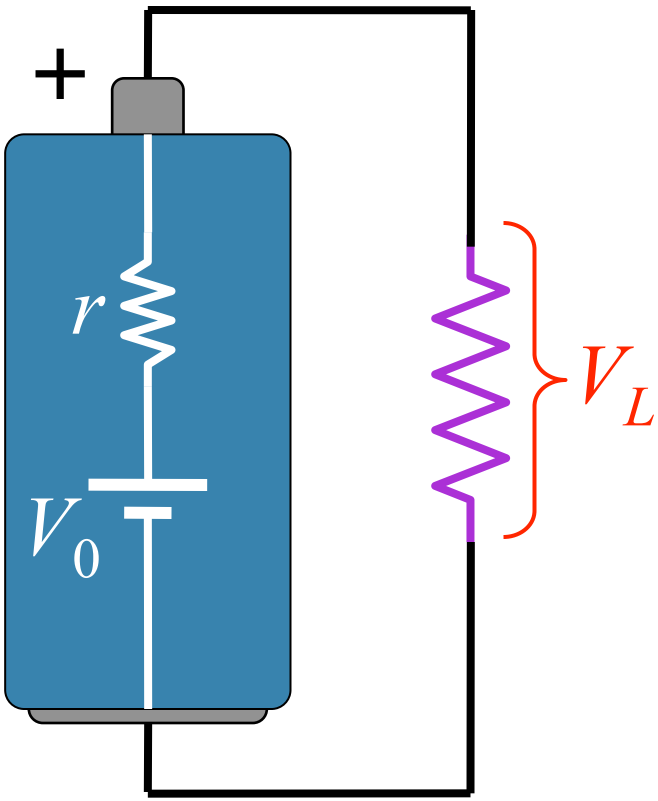
In both cases, $V_{ac} = V/2$

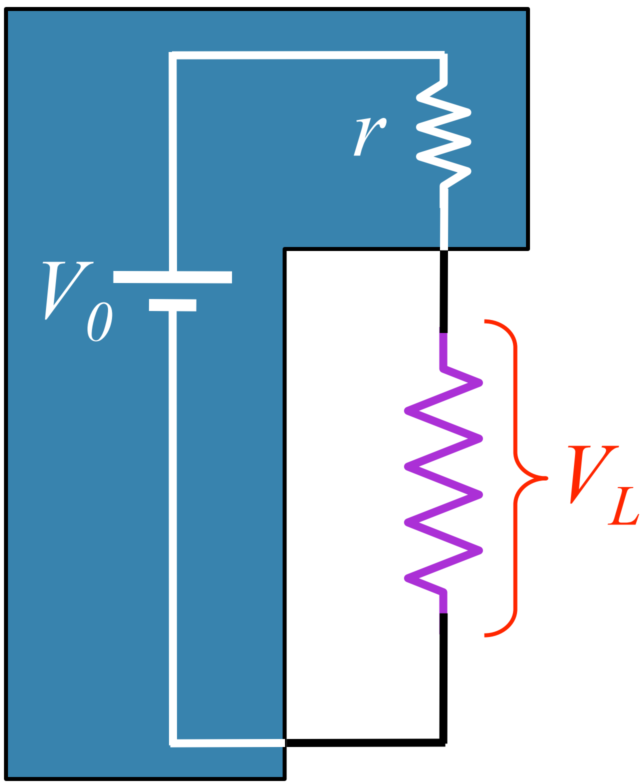
$$I_{2R} = 2I_{4R}$$

$$I_A = I_R - I_{2R}$$
$$= I_R - 2I_{4R}$$

$$I_B = I_R - I_{4R}$$

Model for Real Battery: Internal Resistance





Usually can't supply too much current to the load without voltage "sagging"