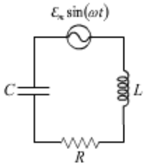
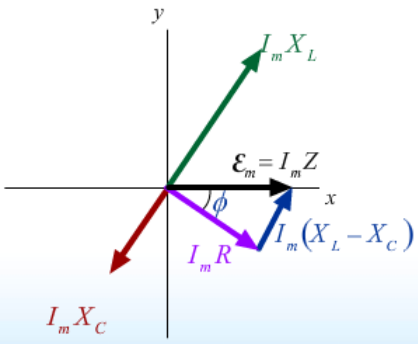
Electricity & Magnetism
Lecture 21



Phase Relation

$$\tan\phi = \frac{X_L - X_C}{R}$$

Voltage Phasor Diagram



Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Maximum Current

$$I_m = \frac{\mathcal{E}_m}{Z}$$

Your Comments

Does the average power change when the circuit goes to resonance?

"help make sense of all the equations!!!"

"do we use phase diagrams to figure all of this stuff out?"

ABSOLUTELY
We will review again today..

Do we care about Q in a practical sense? Is a circuit with low Q desirable?

The concept is important

"Where did all this weird math come from? Root mean squares? The brackets?"

It's all about average values of oscillating quantities

"fro noe of hte hw porblmes yuo spelt evrything as if yuo wree drunk.

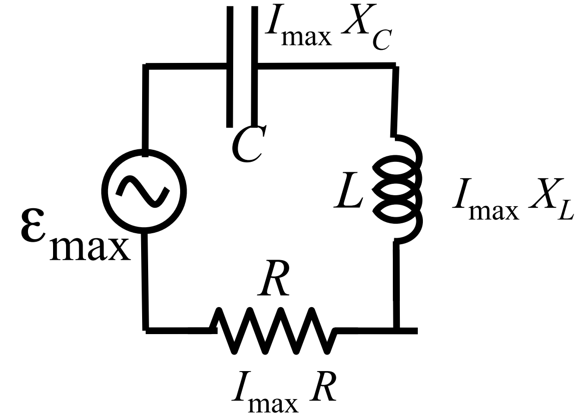
hmmm

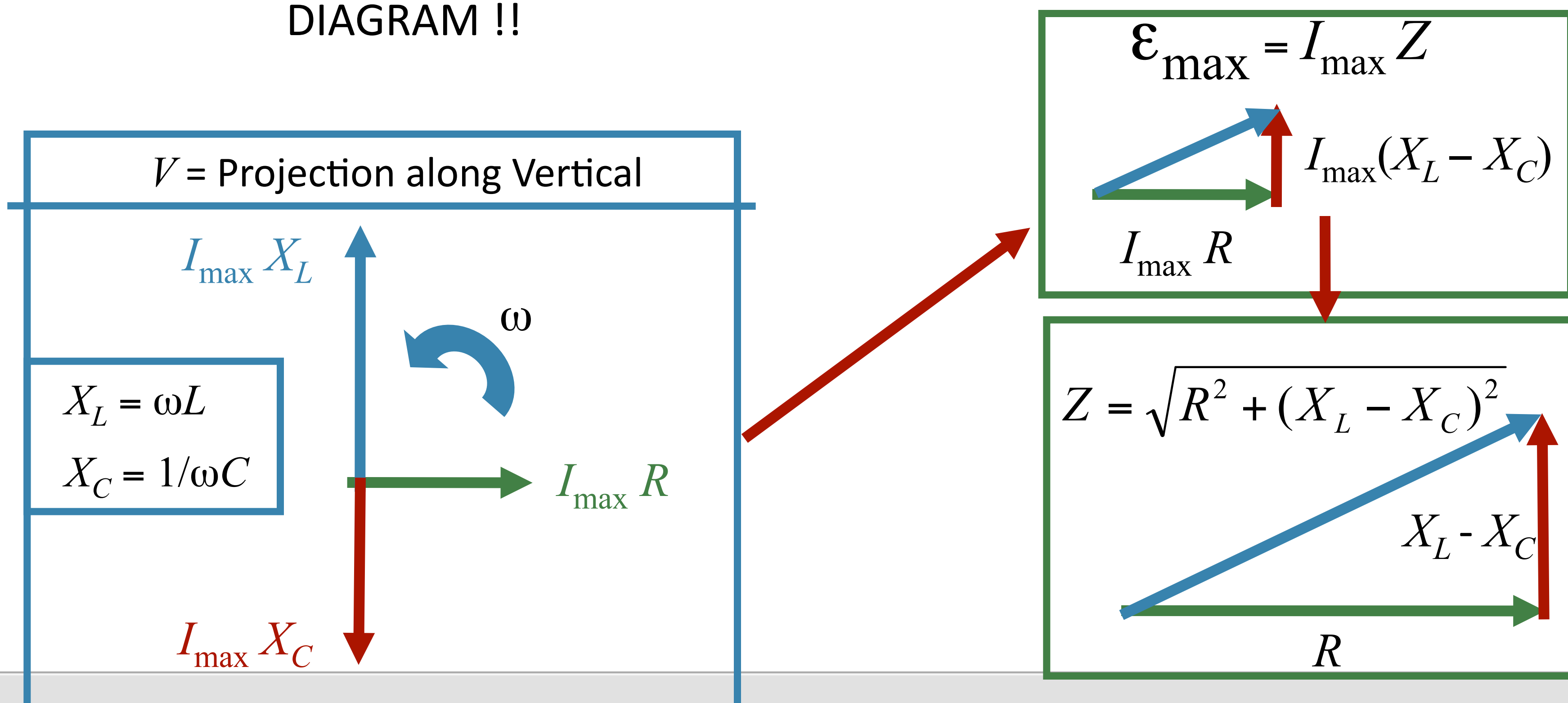
“help make sense of all the equations!!!”

PHASORS ARE THE KEY !
FORMULAS ARE NOT !

START WITH PHASOR DIAGRAM

DEVELOP FORMULAS FROM THE DIAGRAM !!





Peak AC Problems

"Ohms" Law for each element

NOTE: Good for PEAK values only)

$$V_{gen} = I_{max} Z$$

$$V_{Resistor} = I_{max} R$$

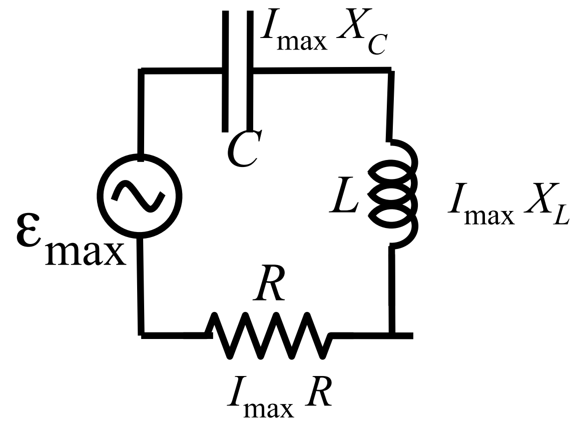
$$V_{inductor} = I_{max} X_L$$

$$V_{Capacitor} = I_{max} X_C$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

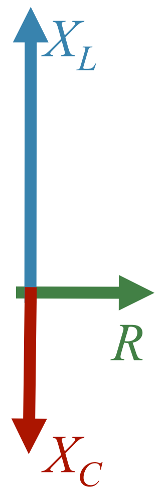


Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

$$X_L = \omega L = 200\,\Omega \qquad Z = \sqrt{R^2 + (X_L - X_C)^2} = 122\,\Omega$$

$$X_C = \frac{1}{\omega C} = 100\,\Omega \qquad I_{max} = \frac{V_{gen}}{Z} = 0.13\,A$$



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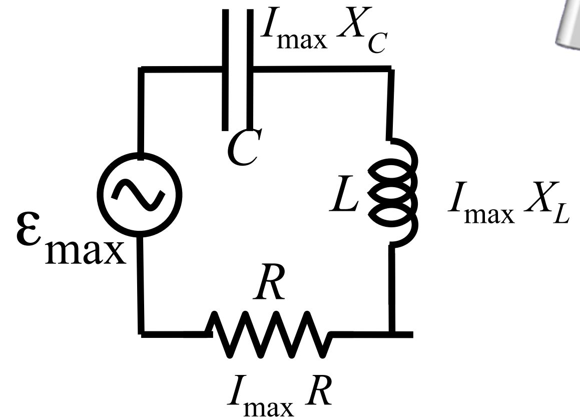


Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

Which element has the largest peak voltage across it?

A) Generator

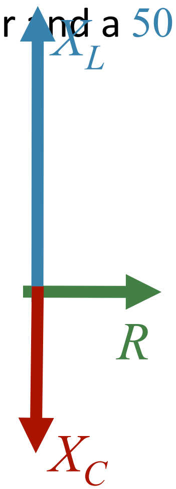
B) Inductor

C) Resistor

D) Capacitor

E) All the same.

$$V_{\max} = I_{\max} X$$



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Peak AC Problems

"Ohms" Law for each element

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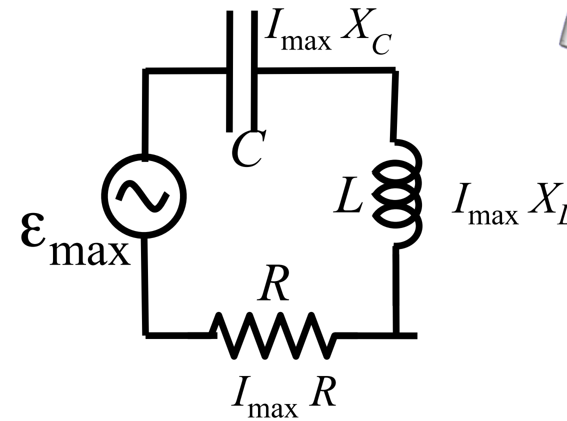


Typical Problem

A generator with peak voltage 15 volts and angular frequency 25 rad/sec is connected in series with an 8 Henry inductor, a 0.4 mF capacitor and a 50 ohm resistor. What is the peak current through the circuit?

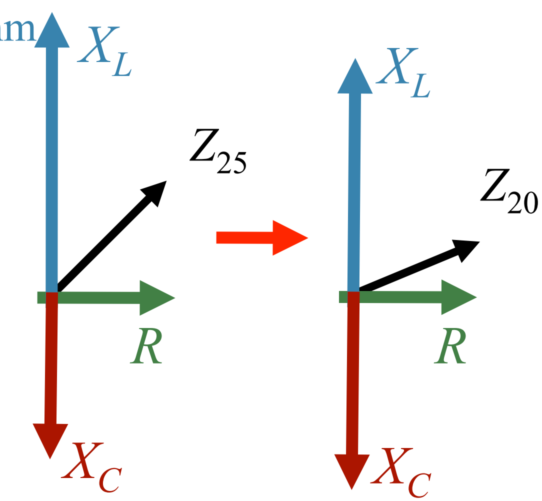
What happens to the impedance if we decrease the angular frequency to 20 rad/sec?

A) Z increases

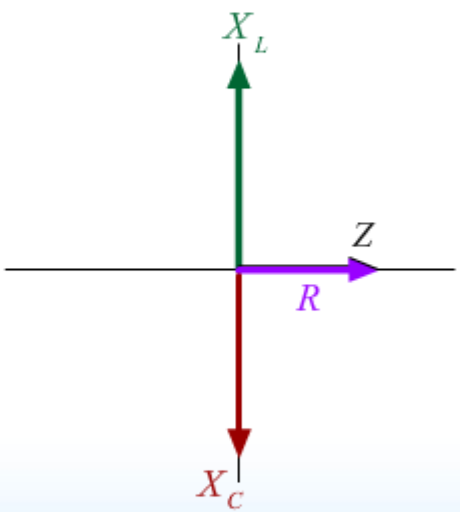
B) Z remains the same

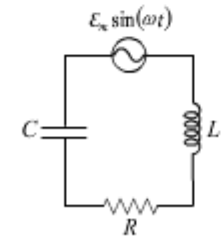
C) Z decreases

$(X_L - X_C)$: $(200 - 100) \rightarrow (160 - 125)$



Resonance





Resonance

I_m is a maximum $\longrightarrow$ $I_m = \dfrac{\mathcal{E}_m}{R}$

$\omega = \omega_o$

Z minimized $\longrightarrow$ $X_L = X_C$

$\phi = 0^o$

$$\boxed{\omega_o = \frac{1}{\sqrt{LC}}}$$

Resonance in AC circuits

ω_0: Frequency at which voltage across inductor and capacitor cancel

R is independent of ω

X_L increases with ω

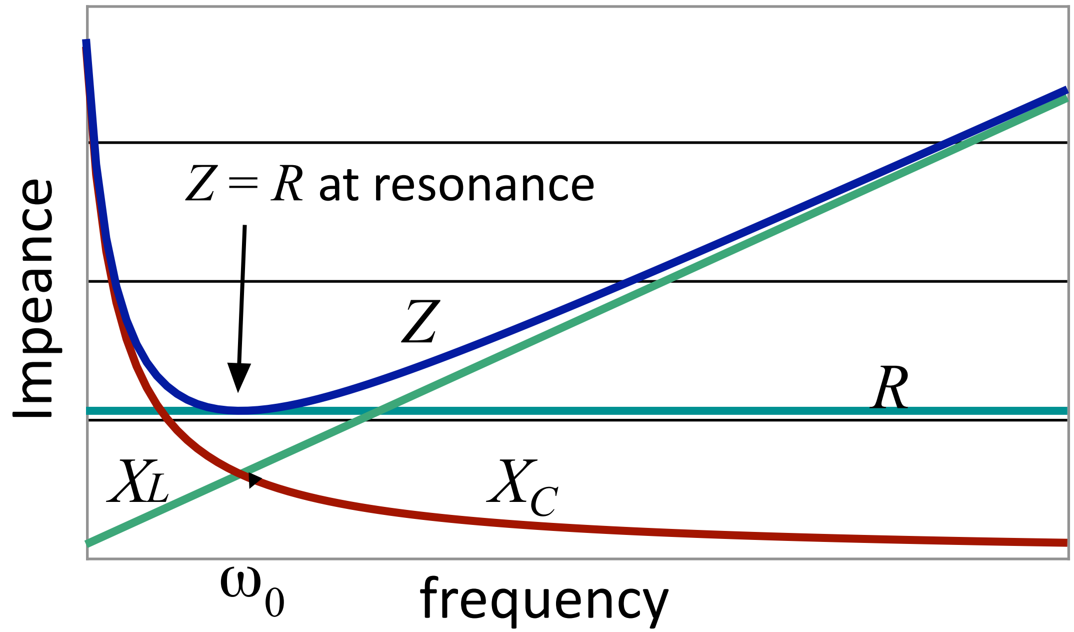
$$X_L = \omega L$$

X_C increases with $1/\omega$

$$X_C = \frac{1}{\omega C}$$

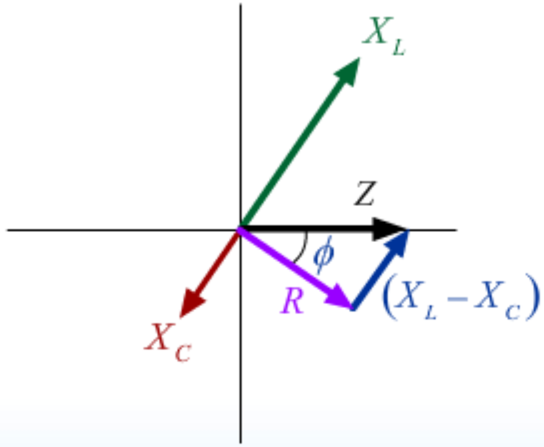
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

is minimum at resonance



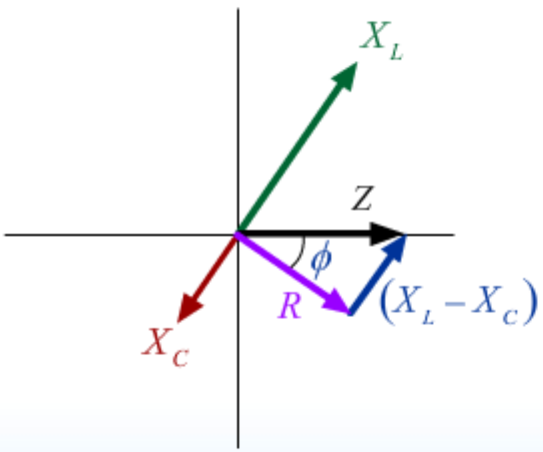
Resonance: $X_L = X_C$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$

Off Resonance



$$I_m = \frac{\mathcal{E}_m}{Z}$$

$$I_m = \frac{\mathcal{E}_m}{R}\frac{R}{\underbrace{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}_{Z}}$$

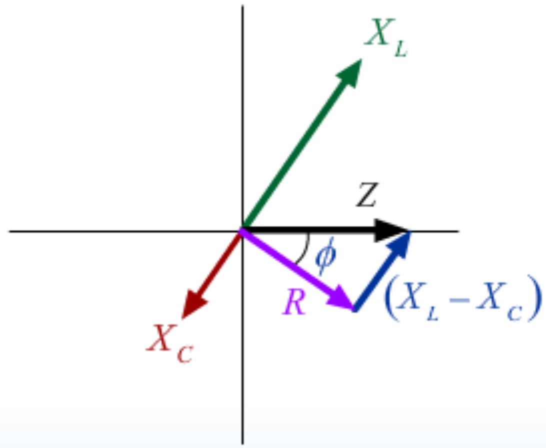


$$x \equiv \frac{\omega}{\omega_o} \qquad Q^2 \equiv \frac{L}{R^2 C} \longrightarrow Q \equiv 2\pi \frac{U_{max}}{\Delta U}$$

$$\boxed{I_m = \frac{\mathcal{E}_m}{R}\frac{1}{\sqrt{1 + Q^2 \frac{(x^2 - 1)^2}{x^2}}}}$$

U_{max} = max energy stored
ΔU = energy dissipated
in one cycle at resonance

Off Resonance



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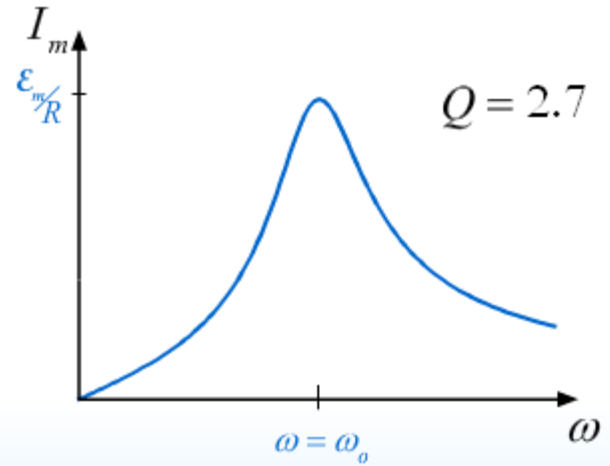
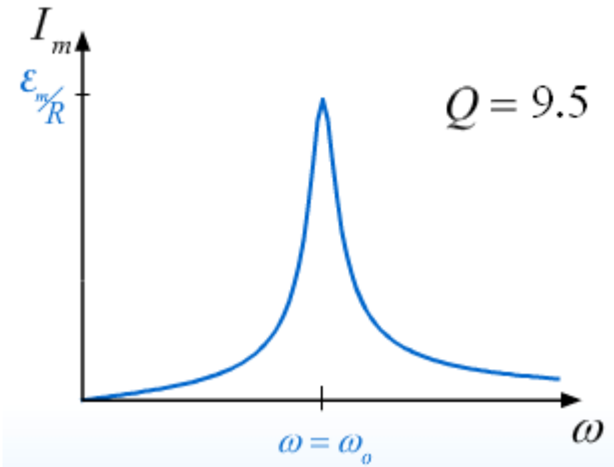
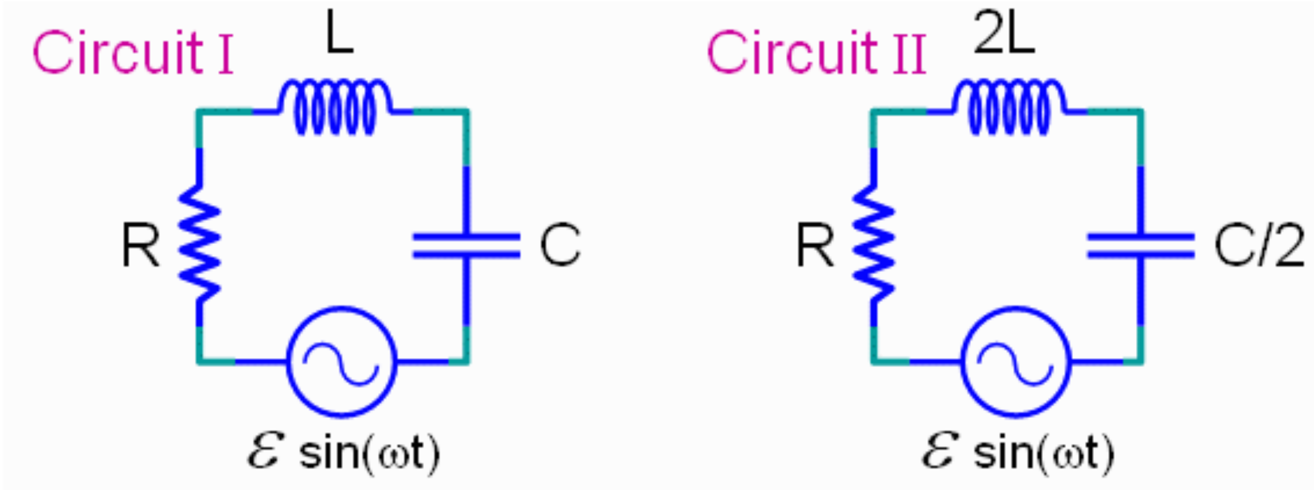
$$\boxed{I_m = \frac{\mathcal{E}_m}{R} \frac{1}{\sqrt{1 + Q^2 \frac{\left(x^2 - 1\right)^2}{x^2}}}}$$





CheckPoint 2



Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

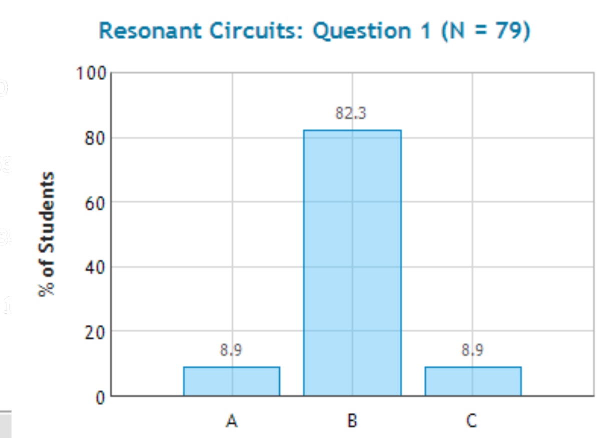
Compare the peak voltage across the resistor in the two circuits

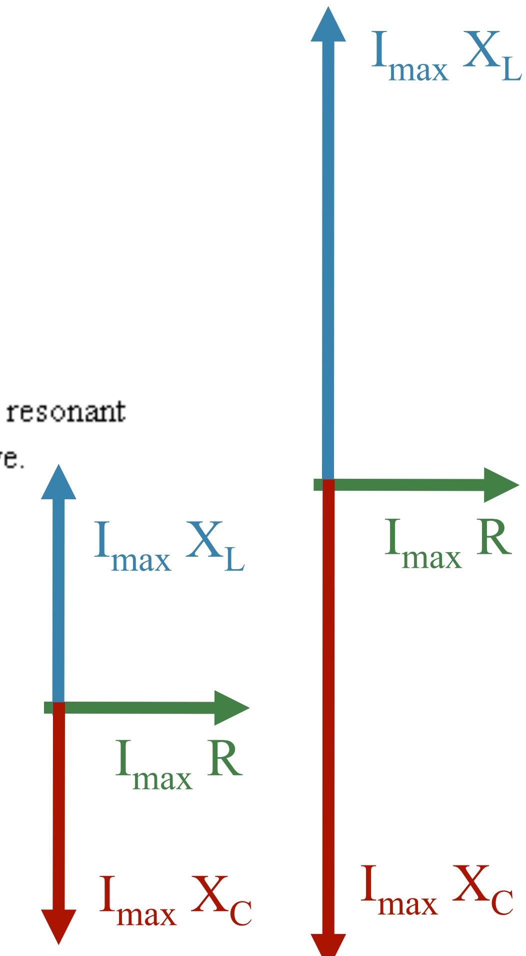
- $V_I > V_{II}$
- $V_I = V_{II}$
- $V_I < V_{II}$

Resonance: $X_L = X_C$

$$Z = R$$

Same since R doesn't change

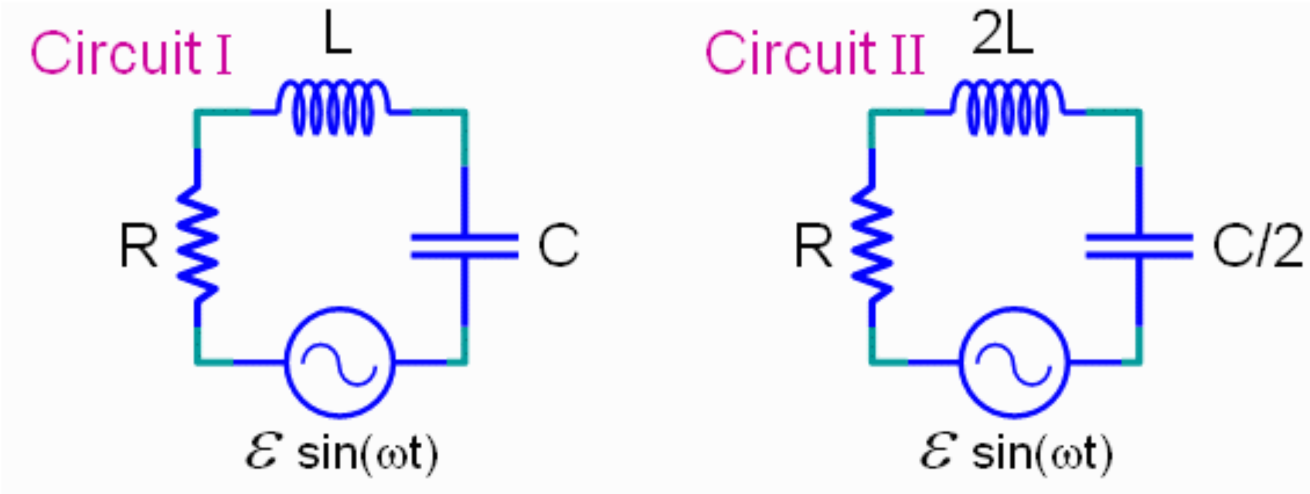




Case 1

Case 2

CheckPoint 4

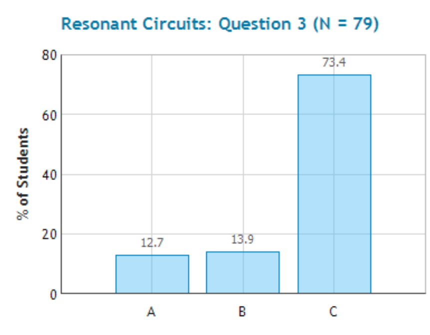


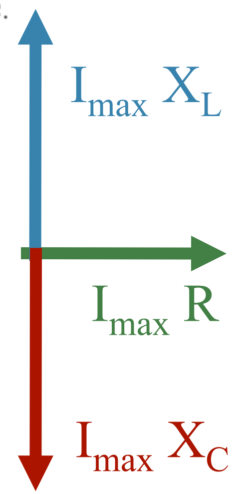
Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

Compare the peak voltage across the inductor in the two circuits

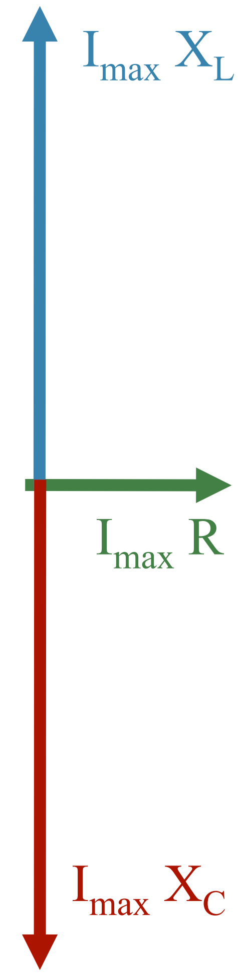
- $V_I > V_{II}$
- $V_I = V_{II}$
- $\boxed{V_I < V_{II}}$

Voltage in second circuit will be twice that of the first because of the $2L$ compared to L.



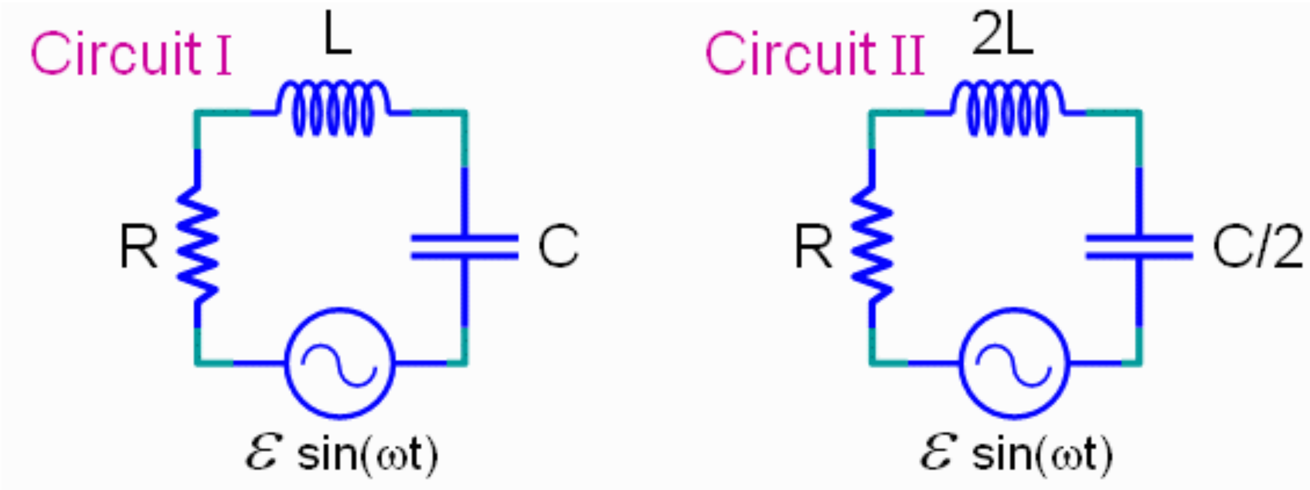


Case 1



Case 2

CheckPoint 6

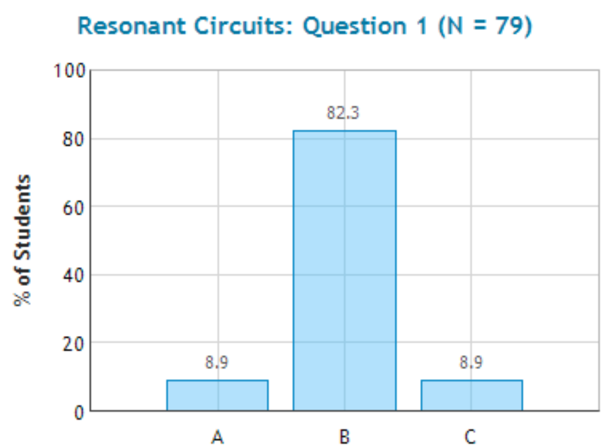


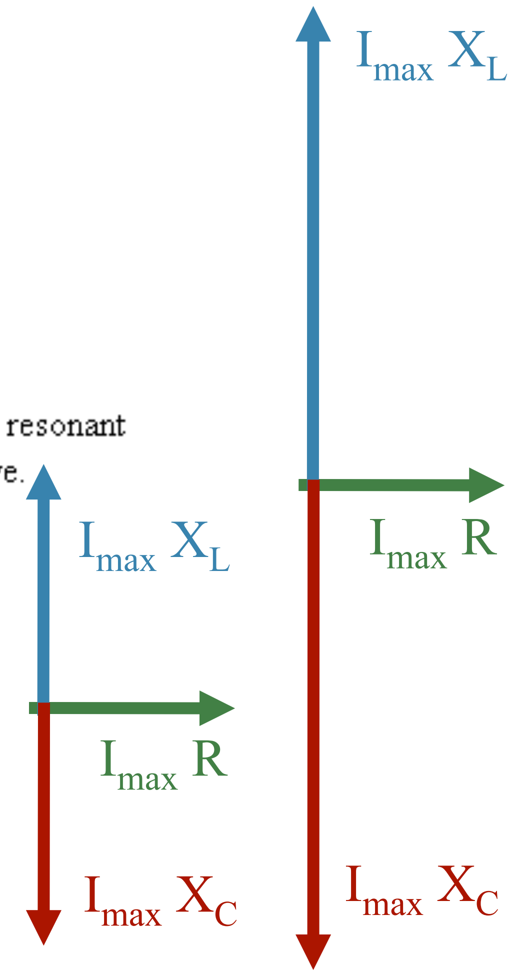
Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

Compare the peak voltage across the capacitor in the two circuits

- ○ $V_I > V_{II}$
- ○ $V_I = V_{II}$
- ○ $\boxed{V_I < V_{II}}$

The peak voltage will be greater in circuit 2 because the value of X_C doubles.

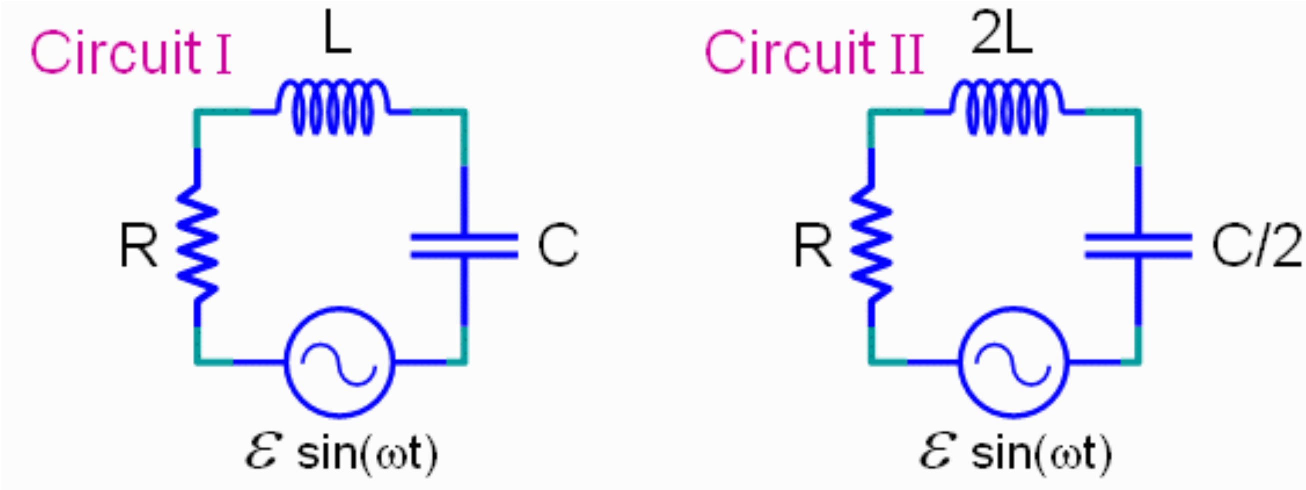




Case 1

Case 2

CheckPoint 8

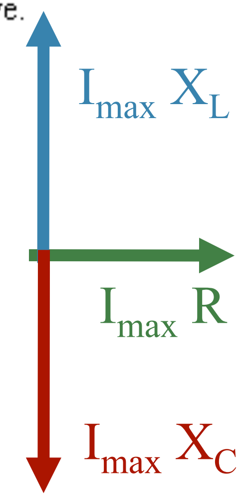


Consider two RLC circuits with identical generators and resistors. Both circuits are driven at the resonant frequency. Circuit II has twice the inductance and 1/2 the capacitance of circuit I as shown above.

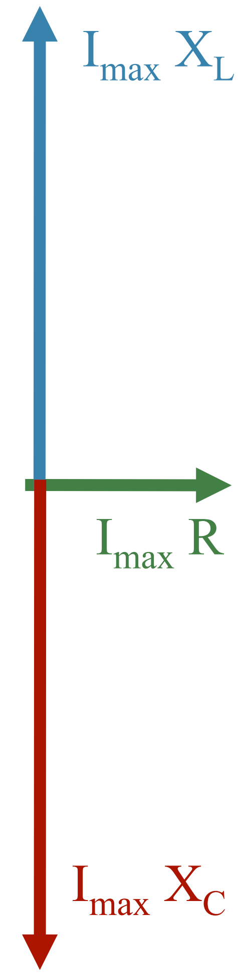
At the resonant frequency, which of the following is true?

- current leads voltage across the generator
- current lags voltage across the generator
- $\boxed{\text{current is in phase with the voltage across the generator}}$

The voltage across the inductor and the capacitor are equal when at resonant frequency, so there is no lag or lead.

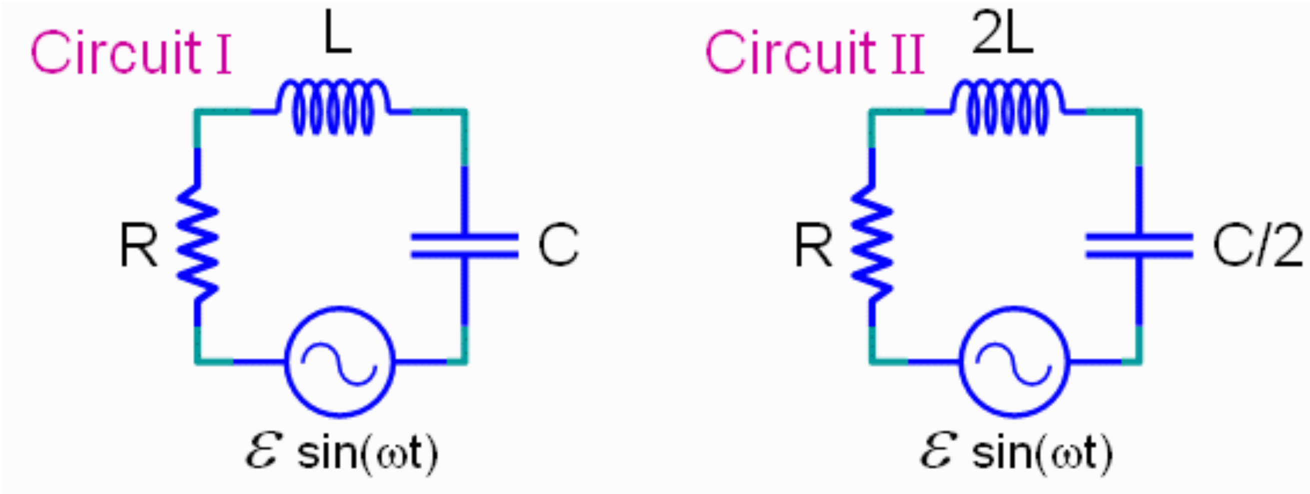


Case 1



Case 2

CheckPoint 8



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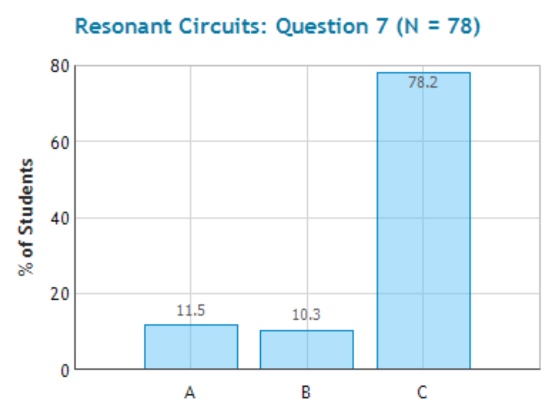
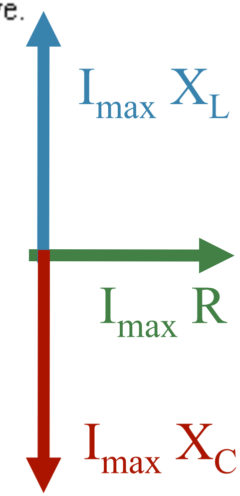
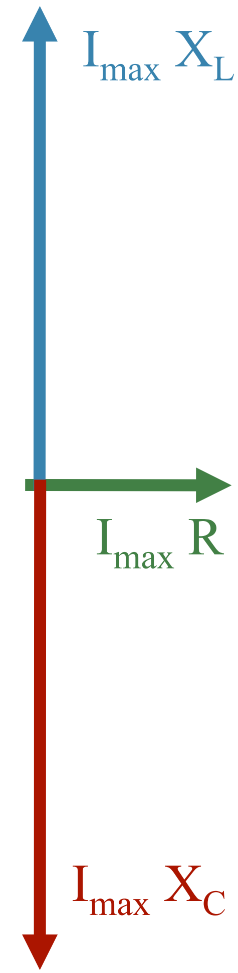




Case 1

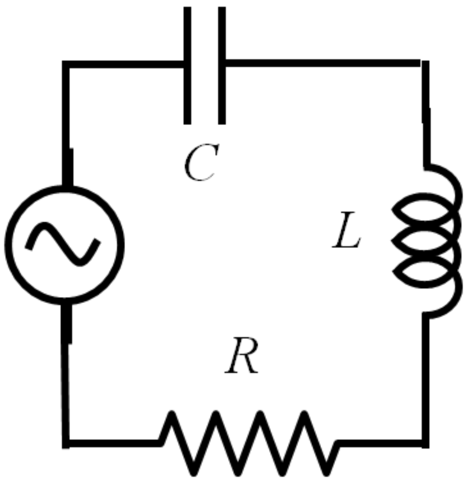


Case 2

Power

$P = IV$ instantaneous always true

- Difficult for Generator, Inductor and Capacitor because of phase
- Resistor I, V are always in phase!



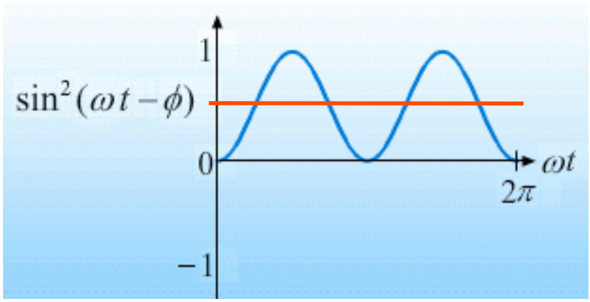
$$P = IV$$
$$= I^2 R$$

Average Power

Inductor and Capacitor = 0   ($\langle \sin\omega t \bullet \cos\omega t \rangle = 0$)

Resistor

$\langle I^2 R \rangle = \langle I^2 \rangle R = \frac{1}{2} I^2_{\text{peak}} R$



RMS = Root Mean Square

$I_{\text{peak}} = I_{\text{rms}} \sqrt{2}$

$\longrightarrow$

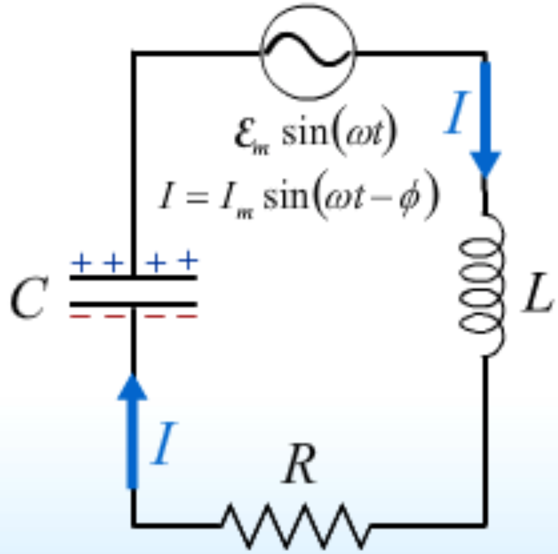
$\boxed{\langle I^2 R \rangle = I_{\text{rms}}^2 R}$

Power Factor

Average Power per Cycle

$$\langle P_{Generator} \rangle = \langle P_{Resistor} \rangle = \frac{1}{2} I_m \, \mathcal{E}_m \boxed{\cos\phi}$$

Power Factor



Power Factor

Note that the power used in terms of rms values is

$$\langle P \rangle = I_{\mathrm{rms}} \mathcal{E}_{\mathrm{rms}} \cos\phi$$

The power factor $\cos\phi < 1$ occurs when the there is inductance in the circuit such as motors.

$$\mathrm{VA} \equiv I_{\mathrm{rms}} \mathcal{E}_{\mathrm{rms}}$$

A power factor < 1 means that excess unneeded current is being delivered which causes waste in the delivery lines.

Therefore in industrial sites, extra capacitance may be installed to make the power factor about 1.

Power Line Calculation

If you want to deliver 1500 Watts at 100 Volts over transmission lines w/ resistance of 5 Ohms. How much power is lost in the lines?

- Current Delivered: $I = P/V = 15$ Amps
- Loss = IV (on line) = $I^2 R = 15 \times 15 \times 5 = 1125$ Watts!

If you deliver 1,500 Watts at 10,000 Volts over the same transmission lines. How much power is lost?

- Current Delivered: $I = P/V = 0.15$ Amps
- Loss = IV (on line) = $I^2 R = 0.125$ Watts

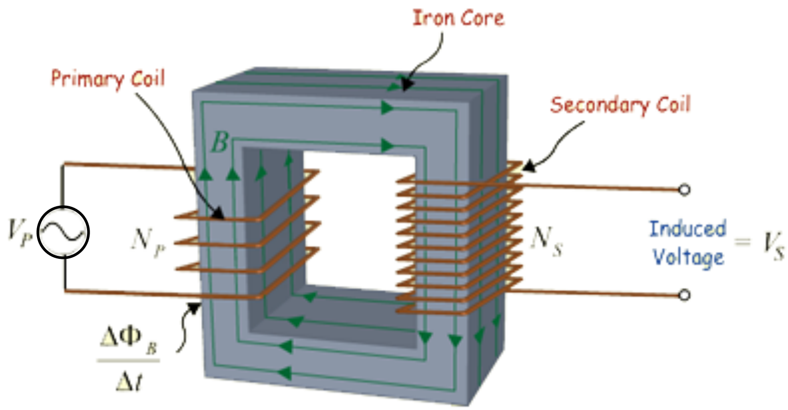
https://en.wikipedia.org/wiki/Electric_power_transmission

Transformers

Application of Faraday's Law

- Changing EMF in Primary creates changing flux
- Changing flux, creates EMF in secondary

$$\frac{V_p}{N_p} = \frac{V_s}{N_s}$$



Efficient method to change voltage for AC.

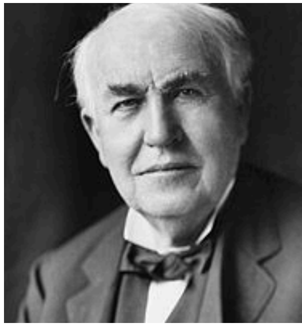
Power Transmission   Loss = I^2R

Power electronics

<u>The Current War (movie)</u>

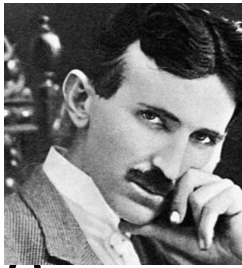
History

DC was used in the early days of electrical power transmission. (Edison, GE)



This meant that the voltage could not be easily changed

DC Power generators needed to be close to the user to avoid transmission line loss.



AC was introduced by a rival company Westinghouse (Tesla).

There was a bitter competition between these systems for a while: The Current War, Edison vs Tesla

Follow-Up from Last Lecture

Consider the harmonically driven series LCR circuit shown.

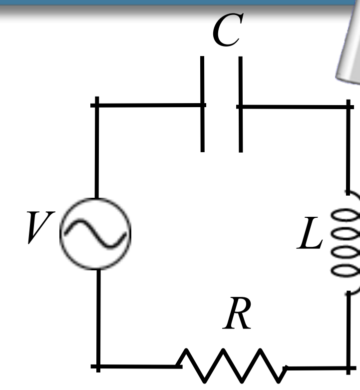
$V_{max} = 100\ V$

$I_{max} = 2\ mA$

$V_{Cmax} = 113\ V$ (= 80 sqrt(2))

The current leads generator voltage by 45^o ($\cos = \sin = 1/\sqrt{2}$)

L and R are unknown.



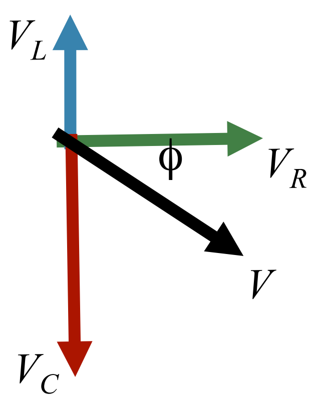
How should we change ω to bring circuit to resonance?

A) decrease ω

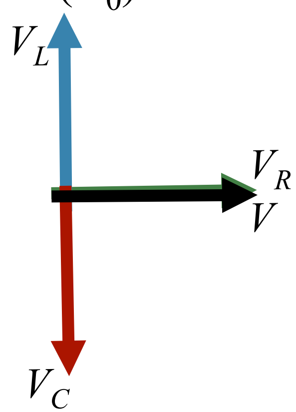
$\boxed{\text{B) increase } \omega}$

C) Not enough info

Original ω



At resonance (ω_0)



At resonance

$X_L = X_C$

$\downarrow$

X_L increases

X_C decreases

$\rightarrow$ ω increases

Current Follow-Up

Consider the harmonically driven series LCR circuit shown.

$V_{max} = 100\text{ V}$

$I_{max} = 2\text{ mA}$

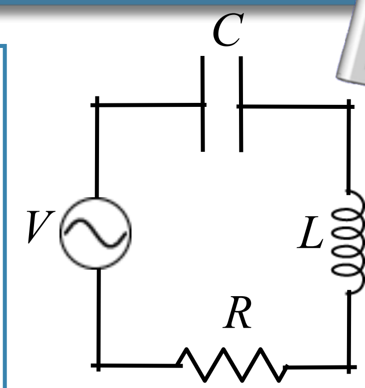
$V_{Cmax} = 113\text{ V } (= 80\sqrt{2})$

$\longrightarrow \quad X_C = 40\sqrt{2}\ k\Omega$

The current leads generator voltage by 45^{o} $(\cos = \sin = 1/\sqrt{2})$

L and R are unknown.

What is the maximum current at resonance?



$$R = 25\sqrt{2}\ k\Omega$$

$$X_L = 15\sqrt{2}\ k\Omega$$

$$\omega_0 = \sqrt{\frac{8}{3}}\,\omega$$

A) $I_{\max}(\omega_0) = \sqrt{2}\,mA$

B) $\boxed{I_{\max}(\omega_0) = 2\sqrt{2}\,mA}$

C) $I_{\max}(\omega_0) = \sqrt{\frac{8}{3}}\,mA$

At resonance
$X_L = X_C$ $\longrightarrow$ $Z = R$ $\longrightarrow$ $I_{\max}(\omega_0) = \dfrac{V_{\max}}{R} = \dfrac{100}{25\sqrt{2}} = 2\sqrt{2}\,mA$

Phasor Follow-Up

Consider the harmonically driven series LCR circuit shown.

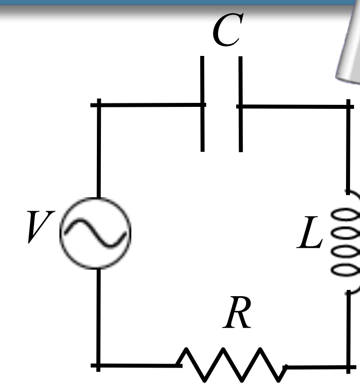
$V_{max} = 100$ V

$I_{max} = 2$ mA

$V_{Cmax} = 113$ V $(= 80\sqrt{2})$

The current leads generator voltage by 45^o $(\cos = \sin = 1/\sqrt{2})$

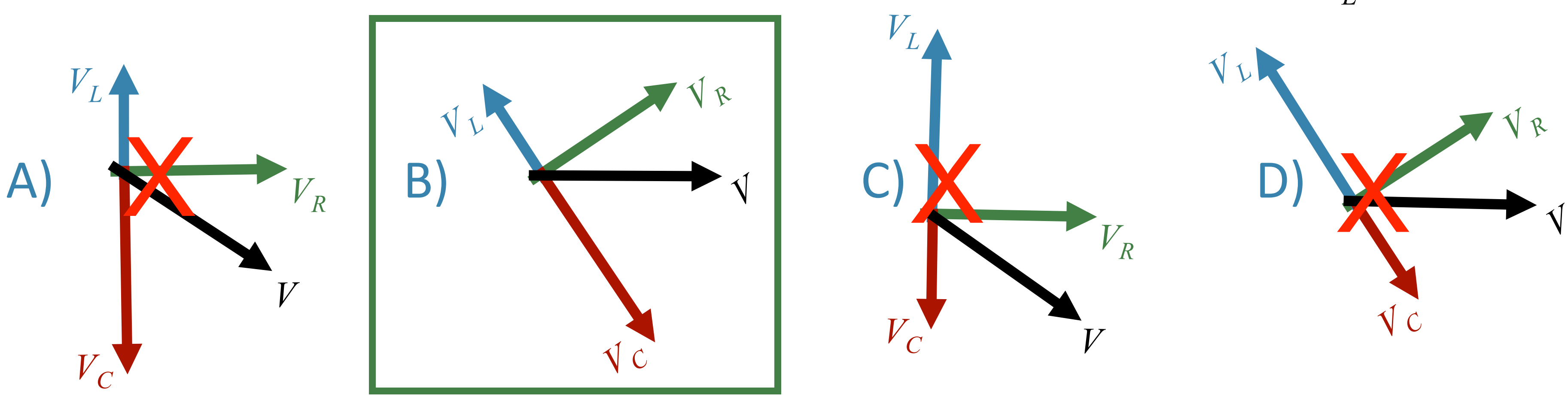
L and R are unknown.



$$R = 25\sqrt{2}\ k\Omega$$

$$X_L = 15\sqrt{2}\ k\Omega$$

What does the phasor diagram look like at t = 0? (assume $V = V_{max}\sin(\omega t)$)



$V = V_{max}\sin(\omega t) \Rightarrow V$ is horizontal at $t = 0$ $(V = 0)$

$\vec{V} = \vec{V}_L + \vec{V}_C + \vec{V}_R$ → $V_L < V_C$ if current leads generator voltage