Electricity & Magnetism
Lecture 22

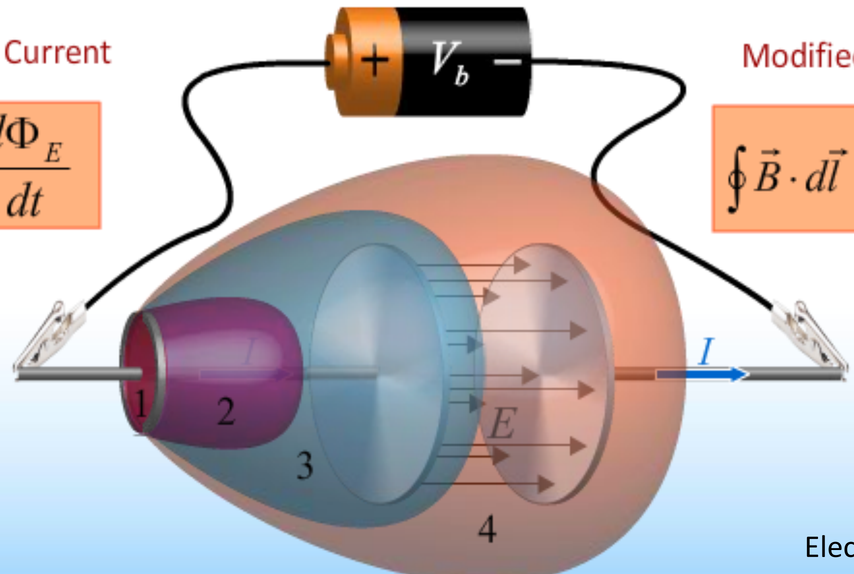
DISPLACEMENT CURRENT and EM WAVES

Displacement Current

$$I_D = \varepsilon_o \frac{d\Phi_E}{dt}$$

Modified Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o (I + I_D)$$



Your Comments

"You LIED about electric potential being the hardest part of the course! Please go through everything in detail because this is a confusing prelecture and I don't understand most of it at all.

ALL TRUE
We will try to make clear, at least the BIG IDEAS

Can you go over the two loops with the Faraday's law and modified Ampere's law? That diagram confused the heck out of me.

Must the electric and magnetic fields lie in perpendicular planes, and if so, why?

Induced B is perpendicular to changing E and vice versa

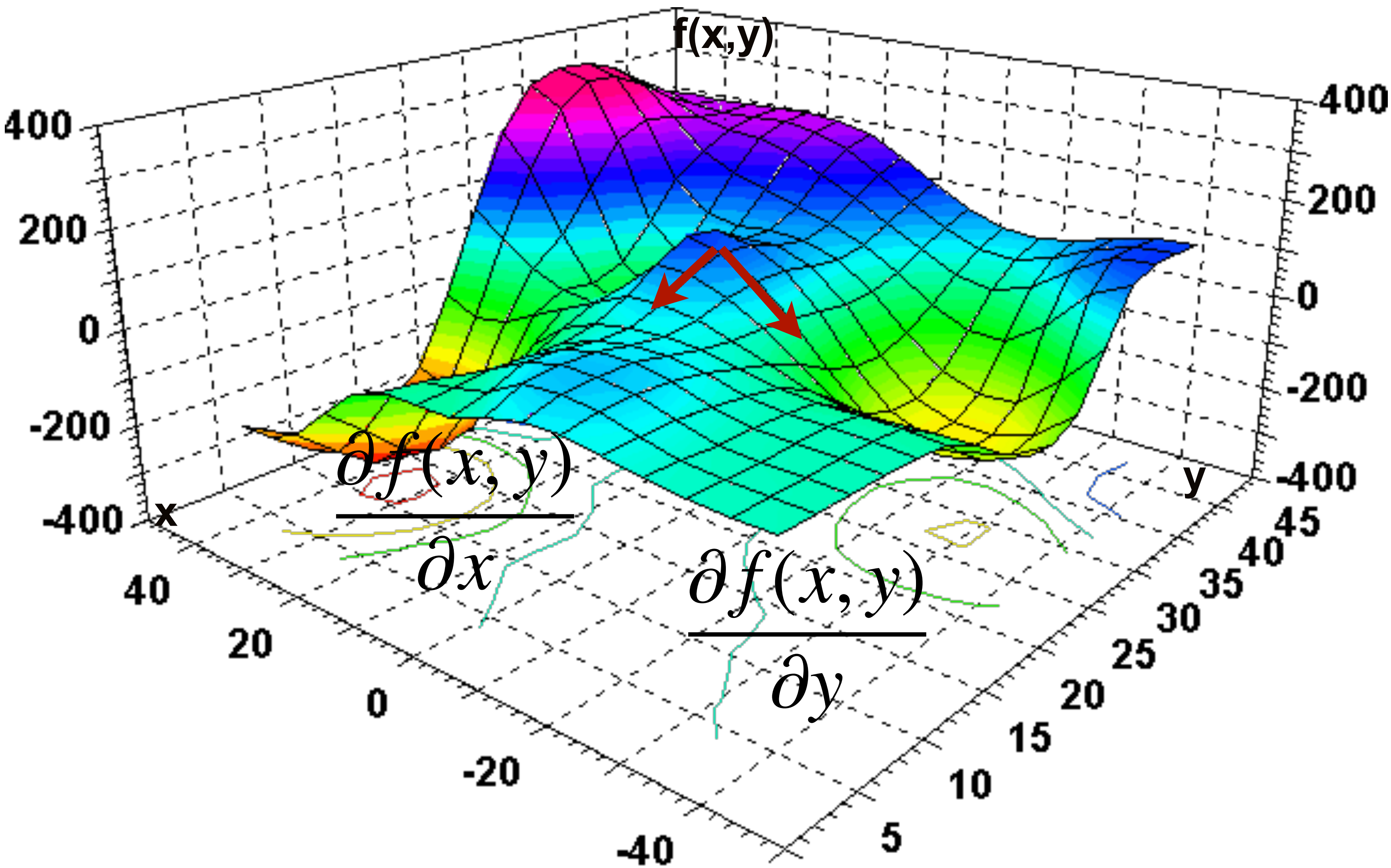
"B field looks like the shadow of E field, or the other way around"

Will we be expected to do the math shown in the prelecture, or will exam questions be more focused on the given wave equation

We will discuss waves
You will not have to solve new diff eqns

What is partial differentiation?

When considering the slope along one axis, imagine that the other axis variable is constant


f(x,y)
400
200
0
-200
-400
x
40
20
0
-20
-40
y
5
10
15
20
25
30
35
40
45
$\frac{\partial f(x,y)}{\partial x}$
$\frac{\partial f(x,y)}{\partial y}$

Mechanical Universe and Beyond

Episode on Maxwell's Equations

- Historical context
- Visual animations

What We Knew Before Prelecture 22

Maxwell's Equations

Gauss' Law for E Fields

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$$

Gauss' Law for B Fields

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

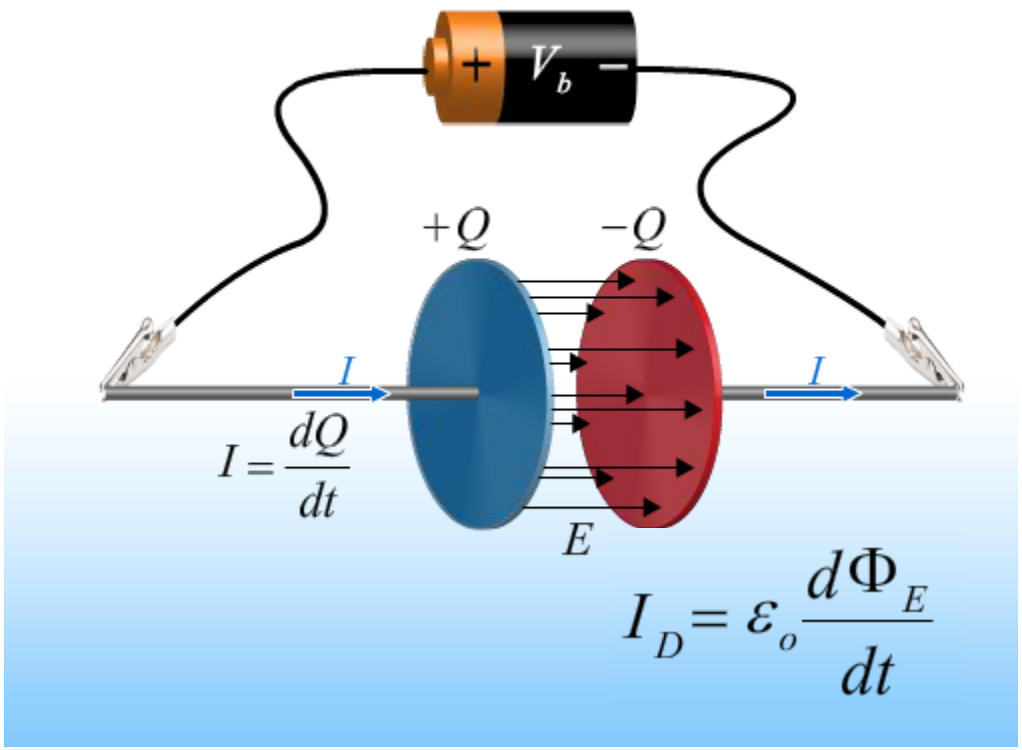
Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed}$$

After Prelecture 21: Modify Ampere's Law

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enclosed} = \mu_o (I + I_D)$$



$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$\Phi_E = EA = \frac{Q}{\varepsilon_0}$$

$$Q = \varepsilon_0 \Phi_E$$

$$\frac{dQ}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} \equiv I_D$$

Displacement Current

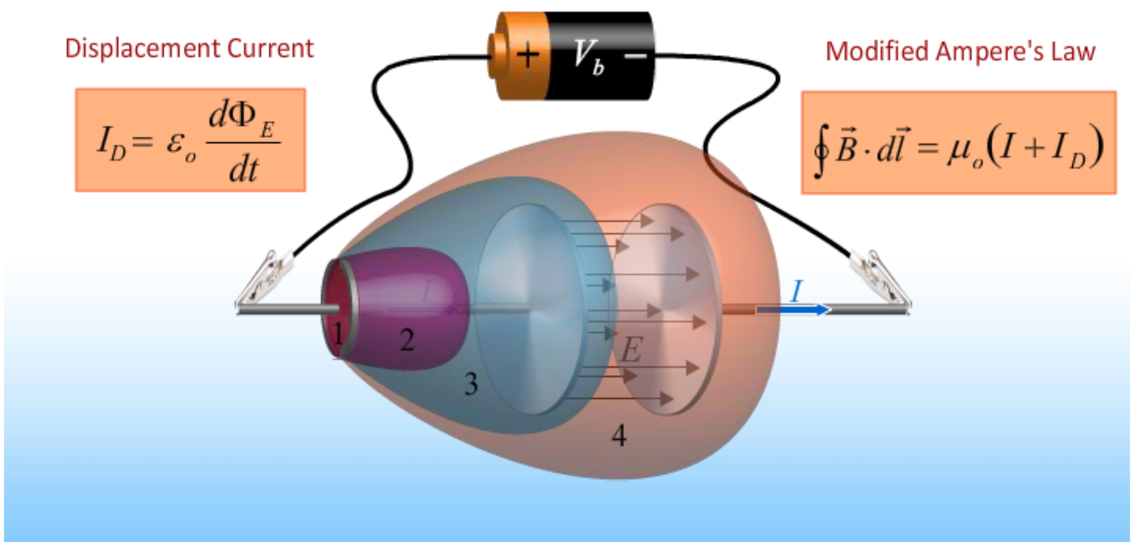
Real Current: Charge Q passes through area A in time t:

$$I = \frac{dQ}{dt}$$

Displacement Current: Electric flux through area A changes in time

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

DISPLACEMENT CURRENT and EM WAVES



Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$$

↕

Modified Ampere's Law

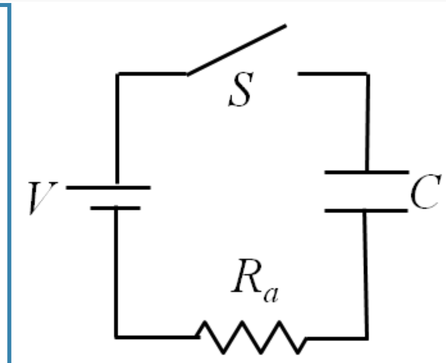
$$\oint \vec{B} \cdot d\vec{l} = \mu_o \varepsilon_o \frac{d}{dt}\int \vec{E} \cdot d\vec{A}$$

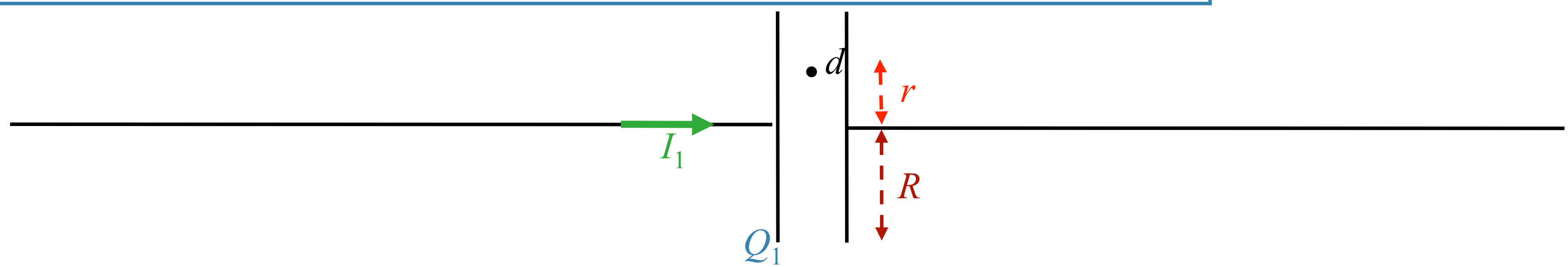
Free space

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.

At time t_1, what is the magnetic field B_1 at a radius r (point d) in between the plates of the capacitor?





Conceptual and Strategic Analysis

Charge Q_1 creates electric field between the plates of C

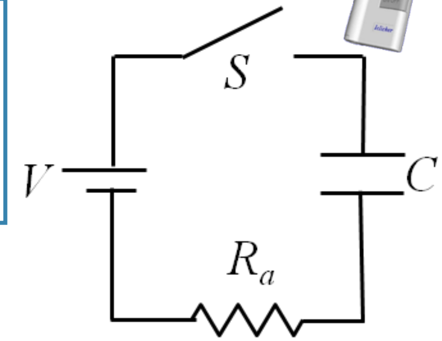
Charge Q_1 changing in time gives rise to a changing electric flux between the plates

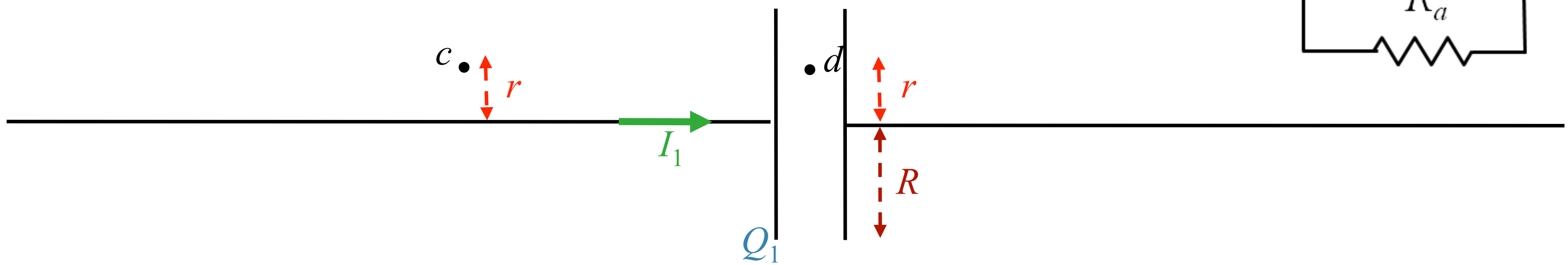
Changing electric flux gives rise to a displacement current I_D in between the plates

Apply (modified) Ampere's law using I_D to determine B

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.



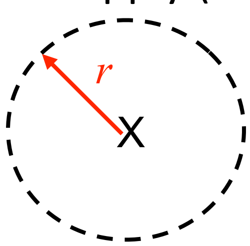


Compare the magnitudes of the B fields at points c and d.

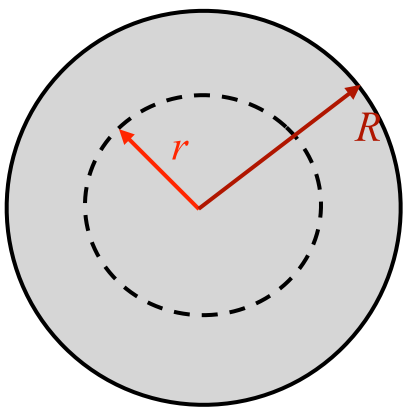
A) $B_c < B_d$ B) $B_c = B_d$ C) $B_c > B_d$

What is the difference?
Apply (modified) Ampere's Law

point c:
I(enclosed) = I_1

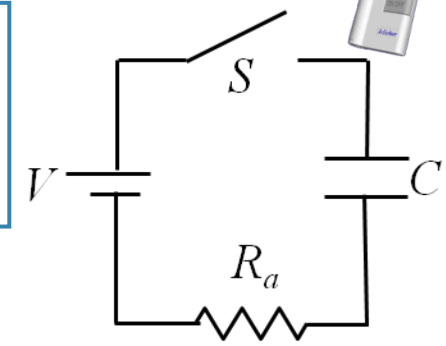


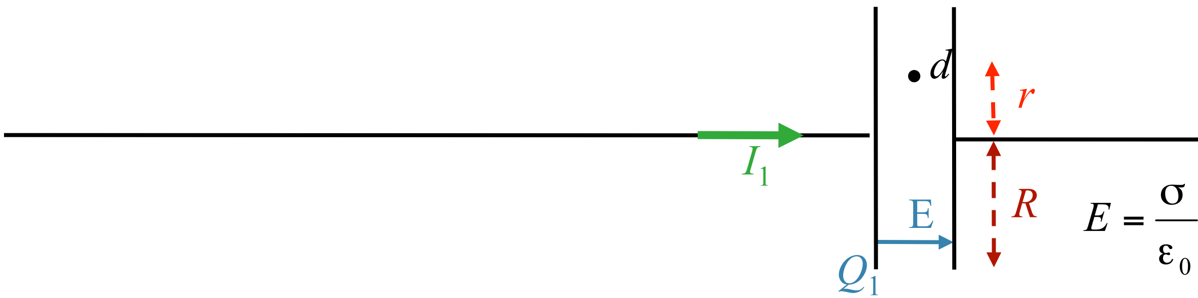
point d:
I_D(enclosed) < I_1



Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.





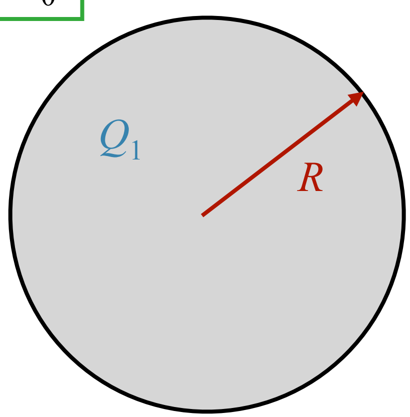
What is the magnitude of the electric field between the plates?

A) $E = \dfrac{Q_1}{\pi R^2 \varepsilon_0}$

B) $E = \dfrac{Q_1 \pi R^2}{\varepsilon_0}$

C) $E = \dfrac{Q_1}{\varepsilon_0}$

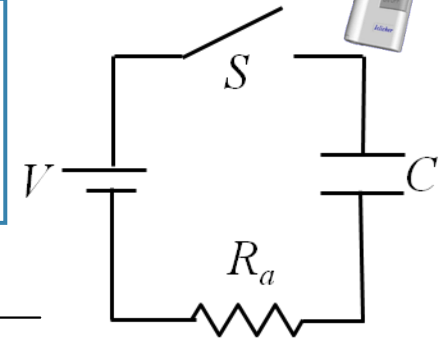
D) $E = \dfrac{Q_1}{r}$

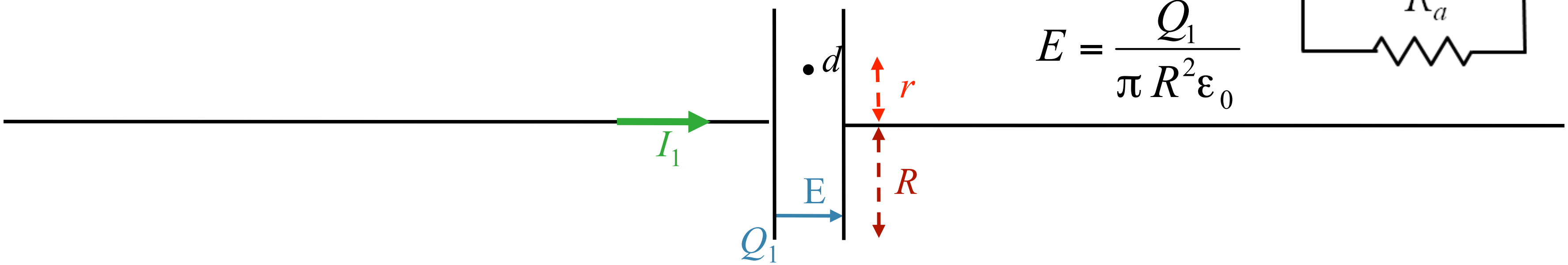


$$E = \frac{\sigma}{\varepsilon_0} \rightarrow \sigma = \frac{Q_1}{A} = \frac{Q_1}{\pi R^2} \rightarrow E = \frac{Q_1}{\varepsilon_0 \pi R^2}$$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.





$$E = \frac{Q_1}{\pi R^2 \varepsilon_0}$$

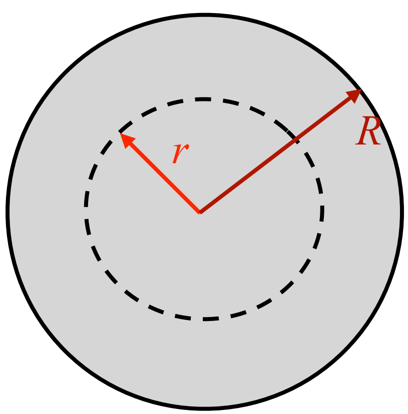
What is the electric flux through a circle of radius r in between the plates?

A) $\Phi_E = \frac{Q_1}{\varepsilon_0} \pi r^2$

B) $\Phi_E = \frac{Q_1}{\varepsilon_0} \pi R^2$

C) $\Phi_E = \frac{Q_1 r^2}{\varepsilon_0 R^2}$

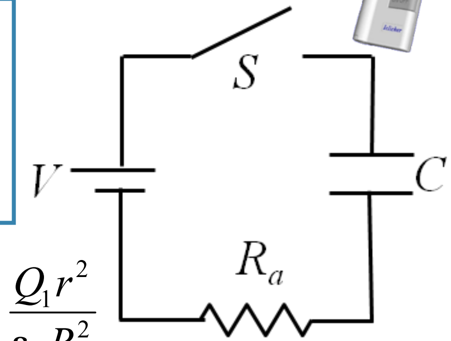
D) $\Phi_E = \frac{Q_1 \pi r^2}{\varepsilon_0 R^2}$

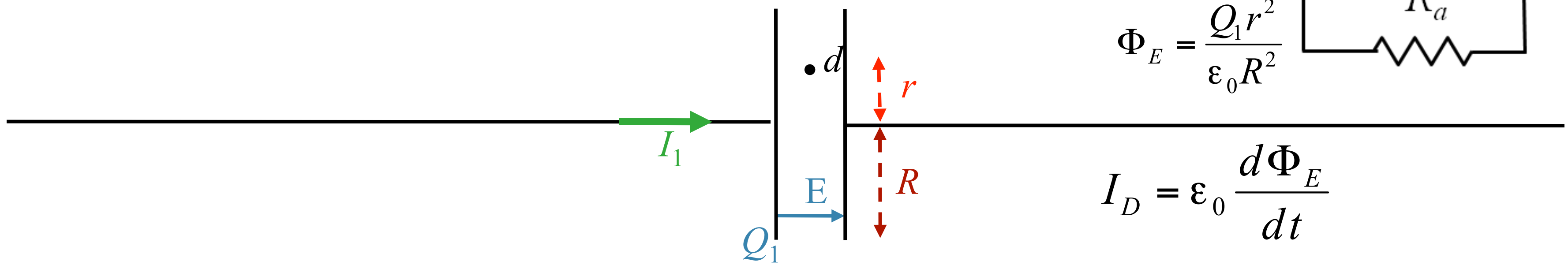


$$\Phi_E = \vec{E} \cdot \vec{A} \rightarrow \Phi_E = \frac{Q_1}{\varepsilon_0 \pi R^2} \pi r^2 \rightarrow \Phi_E = \frac{Q_1}{\varepsilon_0} \frac{r^2}{R^2}$$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.





$$\Phi_E = \frac{Q_1 r^2}{\varepsilon_0 R^2}$$

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

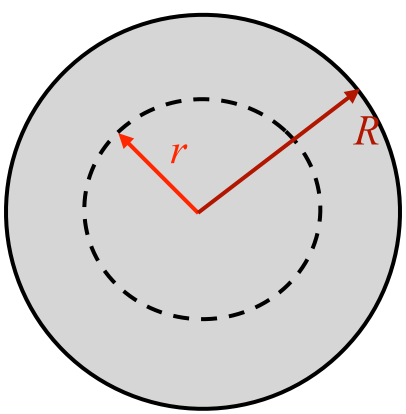
What is the displacement current enclosed by circle of radius r ?

A) $I_D = I_1 \frac{R^2}{r^2}$

B) $I_D = I_1 \frac{r}{R}$

C) $I_D = I_1 \frac{r^2}{R^2}$

D) $I_D = I_1 \frac{R}{r}$

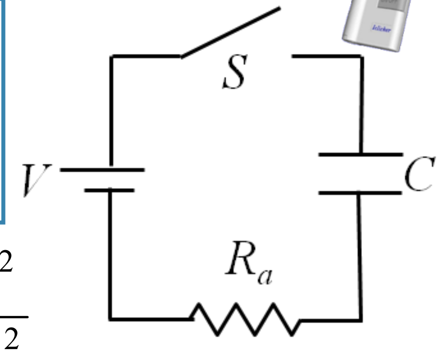


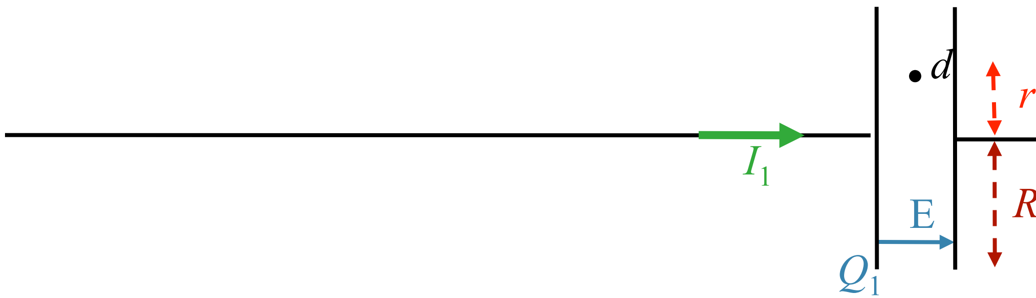
$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt} = \frac{dQ_1}{dt}\frac{r^2}{R^2} = I_1 \frac{r^2}{R^2}$$

$$\rightarrow \quad I_D = I_1 \frac{r^2}{R^2}$$

Calculation

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.





$$I_D = I_1 \frac{r^2}{R^2}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (I + I_D)$$

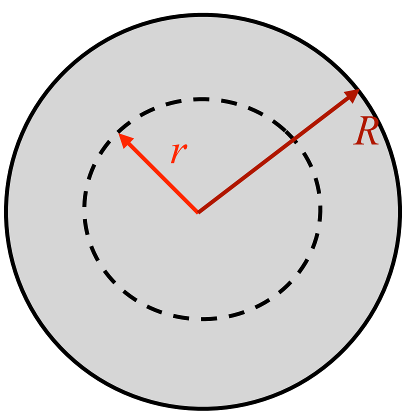
What is the magnitude of the B field at radius r ?

A) $B = \dfrac{\mu_0 I_1}{2\pi R}$

B) $B = \dfrac{\mu_0 I_1}{2\pi r}$

C) $B = \dfrac{\mu_0 I_1}{2\pi} \dfrac{R}{r^2}$

D) $B = \dfrac{\mu_0 I_1}{2\pi} \dfrac{r}{R^2}$



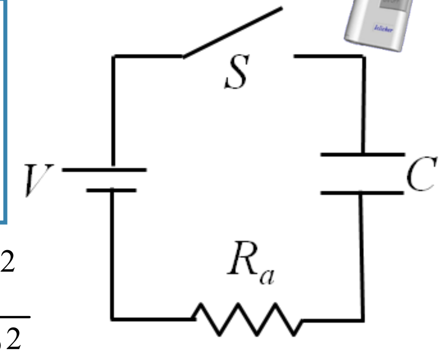
Ampere's Law: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 (I + I_D)$

$$\rightarrow \quad B(2\pi r) = \mu_0 \left(0 + I_1 \frac{r^2}{R^2}\right)$$

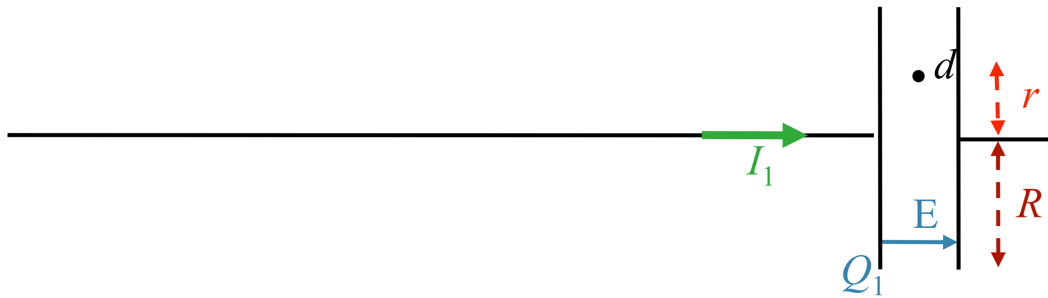
$$\rightarrow \quad B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$

Calculate

Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.



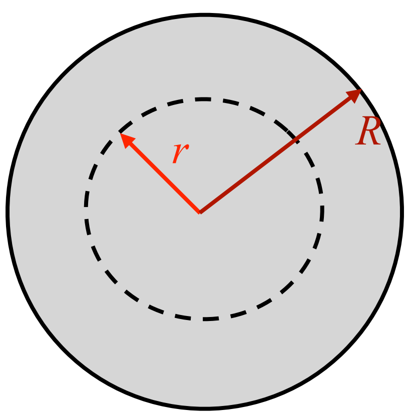
$$I_D = I_1 \frac{r^2}{R^2}$$



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0(I + I_D)$$

What is the magnitude of the B field at radius r ?

$$B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$



Let:

$I_1 = 1$ A
$R = 1$ m

What is B at $r = 0.5$ m ?
(answer on next page)

answer

$$B = \frac{\mu_0 I_1}{2\pi} \frac{r}{R^2}$$

Let:

I_1 = 1 A
R = 1 m

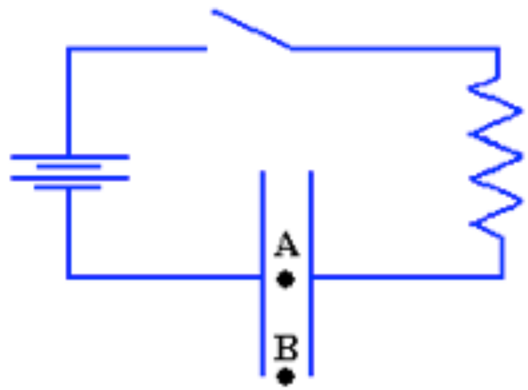
What is B at r = 0.5 m ?

$B = (2 \times 10^{-7})(1)(0.5)/1^2$

$B = 1 \times 10^{-7}$ T

CheckPoint 2

2) At time t = 0 the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; A is at the center and B is toward the outer edge.



After the switch is closed, there will be a magnetic field at point A which is proportional to the current in the circuit.

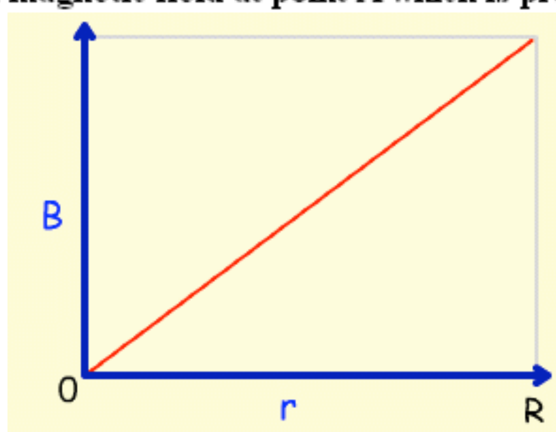
○ True ○ False

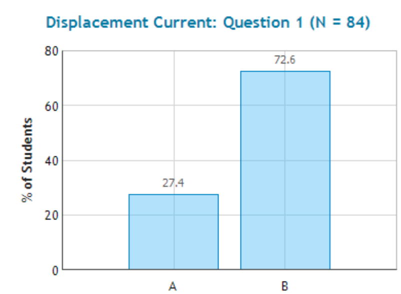
$$B = \frac{\mu_0 I_1}{2\pi}\frac{r}{R^2}$$

B is proportional to *I*

but

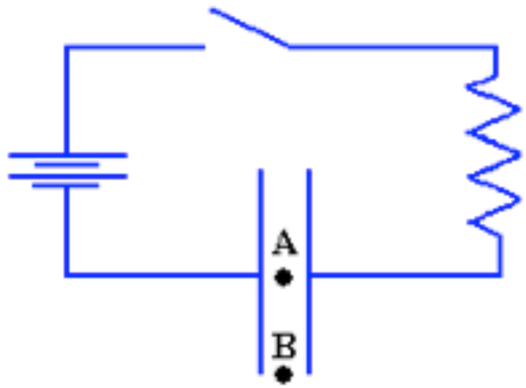
At *A*, *B* = 0 !!





CheckPoint 4

At time t = 0 the switch in the circuit shown below is closed. Points A and B lie inside the capacitor; A is at the center and B is toward the outer edge.



Compare the magnitudes of the magnetic fields at points A and B just after the switch is closed:

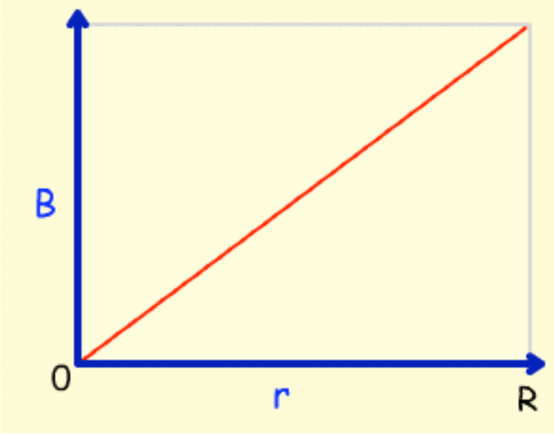
A ○ $B_A < B_B$

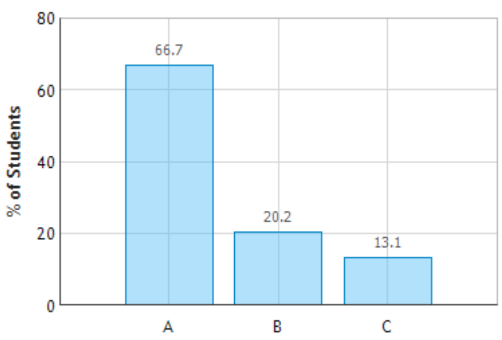
B ○ $B_A = B_B$

C ○ $B_A > B_B$

From the calculation we just did:

$$B = \frac{\mu_0 I_1}{2\pi}\frac{r}{R^2}$$

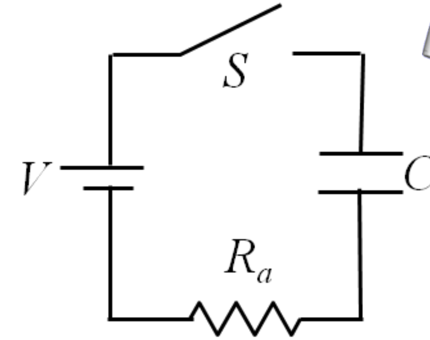




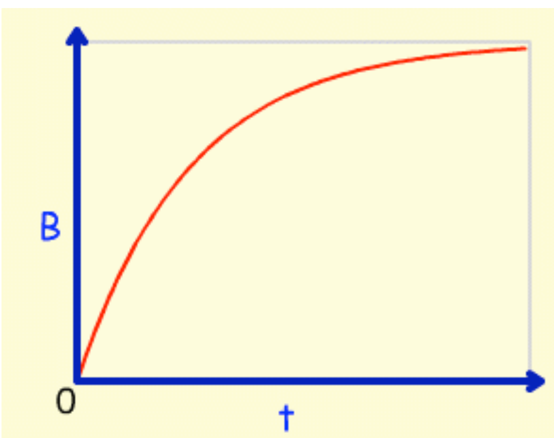
Follow-Up

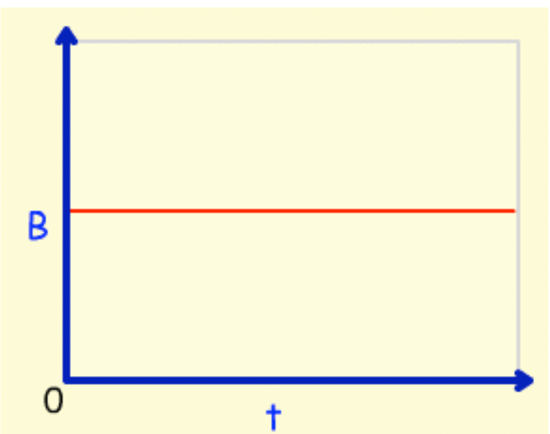
Switch S has been open a long time when at $t = 0$, it is closed. Capacitor C has circular plates of radius R. At time $t = t_1$, a current I_1 flows in the circuit and the capacitor carries charge Q_1.

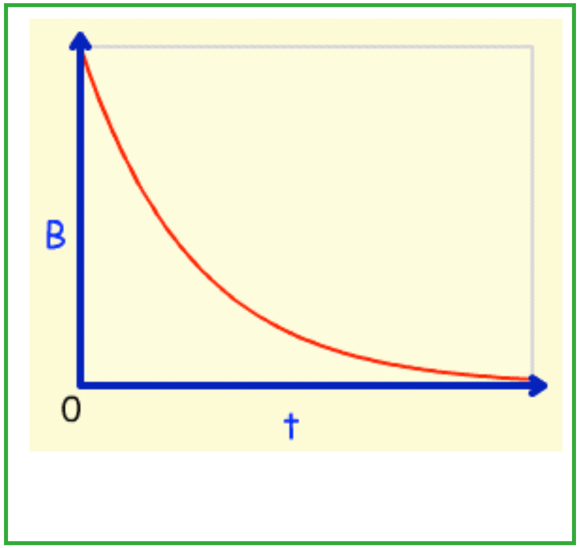
What is the time dependence of the magnetic field B at a radius r between the plates of the capacitor?



$$B_1 = \frac{\mu_0 I_1}{2\pi}\frac{r}{R^2}$$







B at fixed r is proportional to the current I

Close switch: $V_C = 0 \Rightarrow I = V/R_a$ (maximum)

I exponentially decays with time constant $\tau = R_a C$

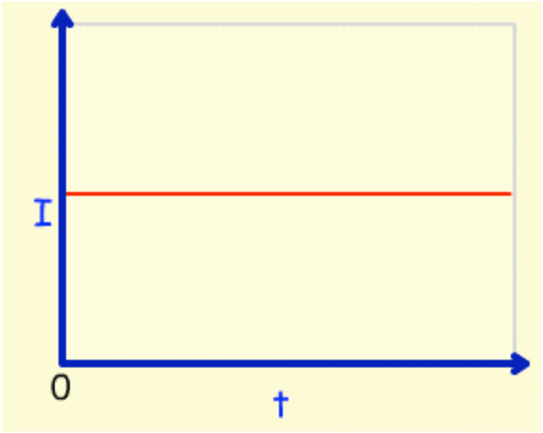
Follow-Up 2

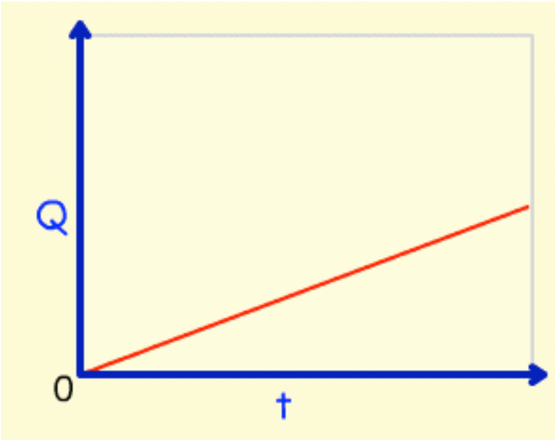
Suppose you were able to charge a capacitor with constant current (*I* does not change in time).

Does a B field exist in between the plates of the capacitor?

A) YES

B) NO





Constant current $\Rightarrow Q$ increases linearly with time

Therefore E increases linearly with time, $E = Q/(A\varepsilon_0)$

dE/dt is not zero $\Rightarrow$ Displacement current is not zero

$\Rightarrow B$ is not zero !

We learned about waves in Physics 120/140

1-D Wave Equation

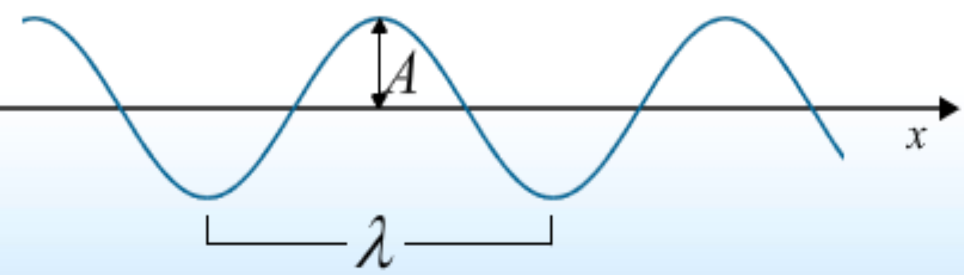
$$\frac{d^2h}{dx^2} = \frac{1}{v^2}\frac{d^2h}{dt^2}$$

Solution

$$h(x,t) = h_1(x - vt) + h_2(x + vt)$$

Common Example: Harmonic Plane Wave

$$h(x,t) = A\cos(kx - \omega t)$$



Variable Definitions

Amplitude: A

Wave Number: $k = \frac{2\pi}{\lambda}$

Wavelength: λ

Angular Frequency: $\omega = \frac{2\pi}{T}$

Period: T

Frequency: $f = \frac{1}{T}$

Velocity: $v = \lambda f = \frac{\omega}{k}$

Faraday's Law

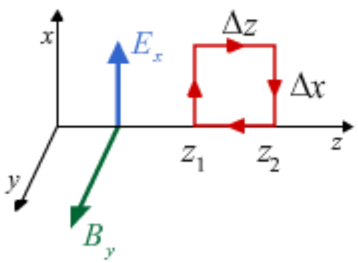
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{A}$$

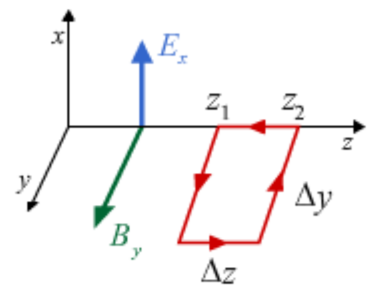
Plane Wave Solution

$$\vec{E} \to \vec{E}(z,t)$$
$$\vec{B} \to \vec{B}(z,t)$$

Modified Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \varepsilon_o \frac{d}{dt}\int \vec{E} \cdot d\vec{A}$$





$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial}{\partial z}\frac{\partial B_y}{\partial t}$$

$$\frac{\partial B_y}{\partial z} = -\mu_o \varepsilon_o \frac{\partial E_x}{\partial t}$$

$$\frac{\partial}{\partial t}\frac{\partial B_y}{\partial z} = -\mu_o \varepsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

Wave Equation

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

Speed of Electromagnetic Wave

$$v = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = c = 3.00 \times 10^8 \text{ m/s}$$

Speed of Light !

Special Relativity (1905)

Speed of Light is Constant

Albert Einstein

“How can light move at the same velocity in any inertial frame of reference? That's really trippy. ”

see PHYS 285

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 E_x}{\partial t^2}$$

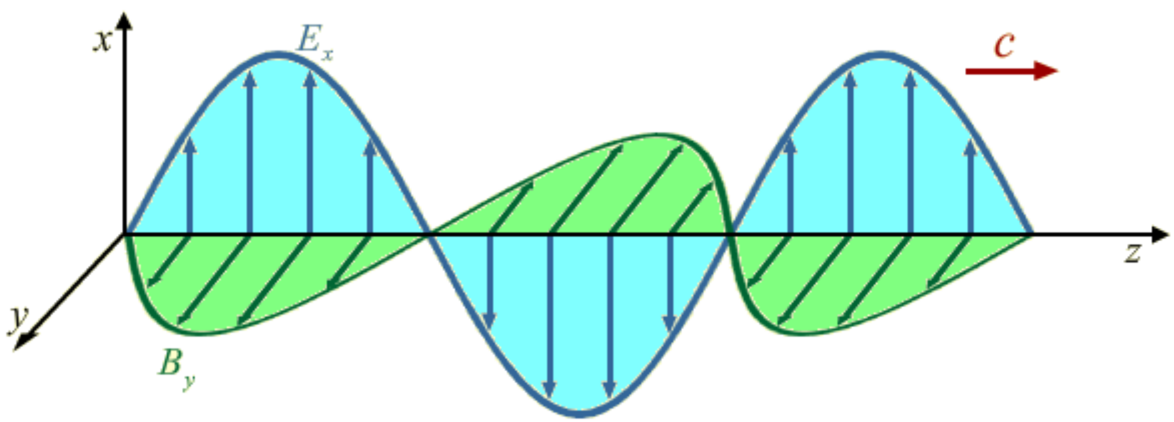
$$\frac{\partial^2 B_y}{\partial z^2} = \mu_o \varepsilon_o \frac{\partial^2 B_y}{\partial t^2}$$

Example: A Harmonic Solution

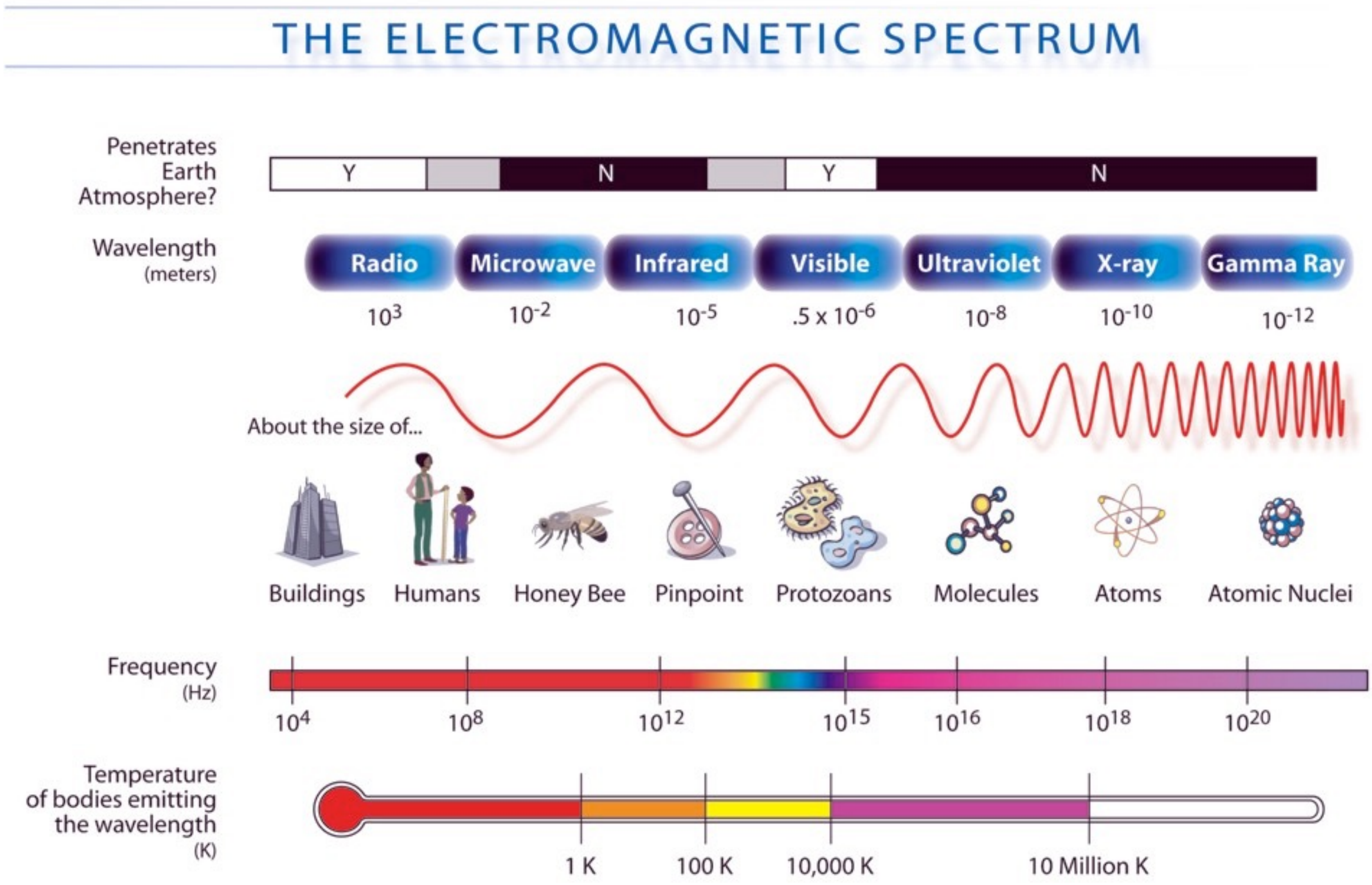
$$E_x = E_o \cos(kz - \omega t) \quad \xrightarrow{\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}} \quad B_y = \frac{k}{\omega} E_o \cos(kz - \omega t)$$

Two Important Features

1. B_y is in phase with E_x
2. $B_o = \frac{E_o}{c}$

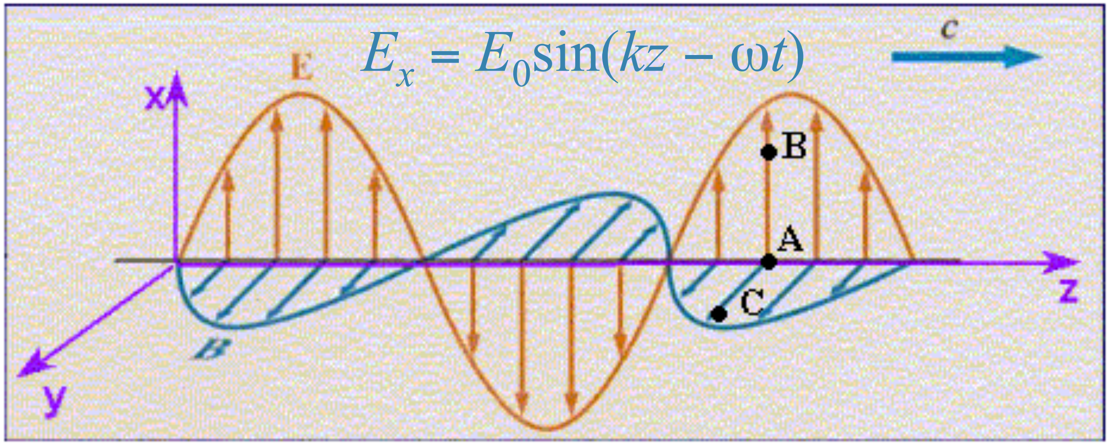


Wave simulations


THE ELECTROMAGNETIC SPECTRUM
Penetrates Earth Atmosphere?
Y
N
Y
N
Wavelength (meters)
Radio
Microwave
Infrared
Visible
Ultraviolet
X-ray
Gamma Ray
10^3
10^-2
10^-5
.5 x 10^-6
10^-8
10^-10
10^-12
About the size of...
Buildings
Humans
Honey Bee
Pinpoint
Protozoans
Molecules
Atoms
Atomic Nuclei
Frequency (Hz)
10^4
10^8
10^12
10^15
10^16
10^18
10^20
Temperature of bodies emitting the wavelength (K)
1 K
100 K
10,000 K
10 Million K

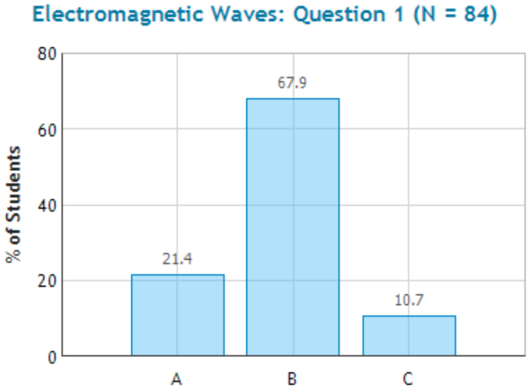
CheckPoint 6

6) An electromagnetic plane-wave is traveling in the +z direction. The illustration below shows this wave an some instant in time. Points A, B, and C have the same z coordinate.



Compare the magnitudes of the electric field at points A and B.

- $E_a < E_b$
- $E_a = E_b$
- $E_a > E_b$



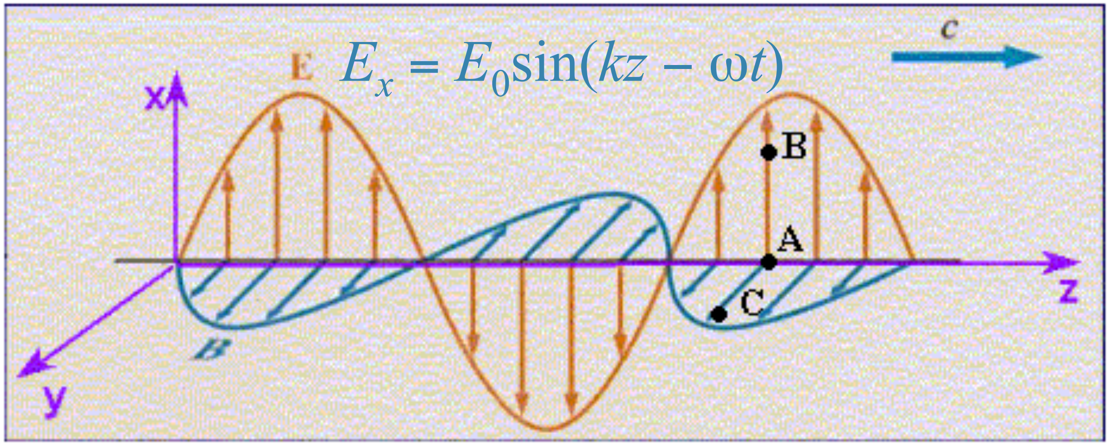
$E = E_0 \sin(kz - \omega t)$:

E depends only on z coordinate for constant t.

z coordinate is same for A, B, C.

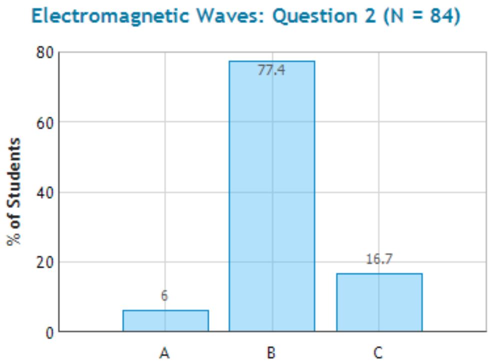
CheckPoint 7

An electromagnetic plane-wave is traveling in the +z direction. The illustration below shows this wave an some instant in time. Points A, B, and C have the same z coordinate.



Compare the magnitudes of the electric field at points A and C.

- $E_a < E_c$
- $E_a = E_c$
- $E_a > E_c$

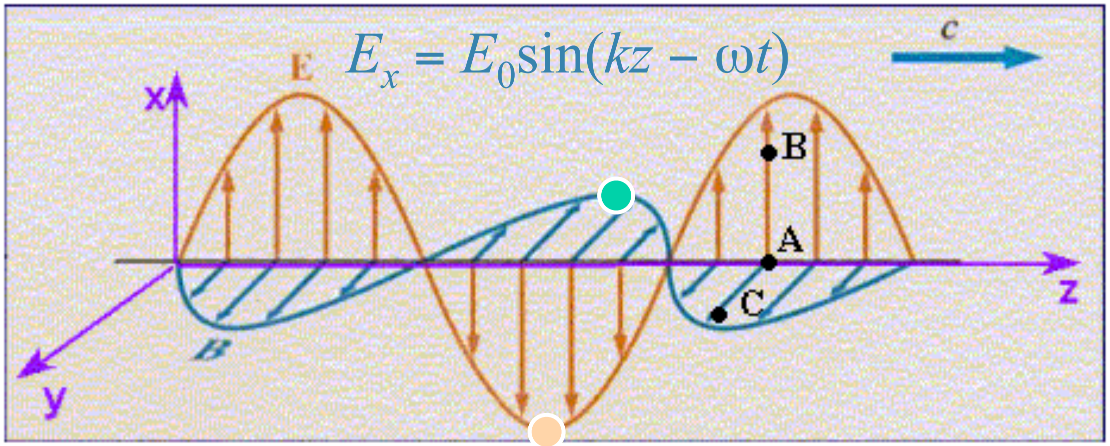


$E = E_0 \sin(kz - \omega t)$:

E depends only on z coordinate for constant t.

z coordinate is same for A, B, C.

Clicker Question



Consider a point (x,y,z) at time t when E_x is negative and has its maximum magnitude.

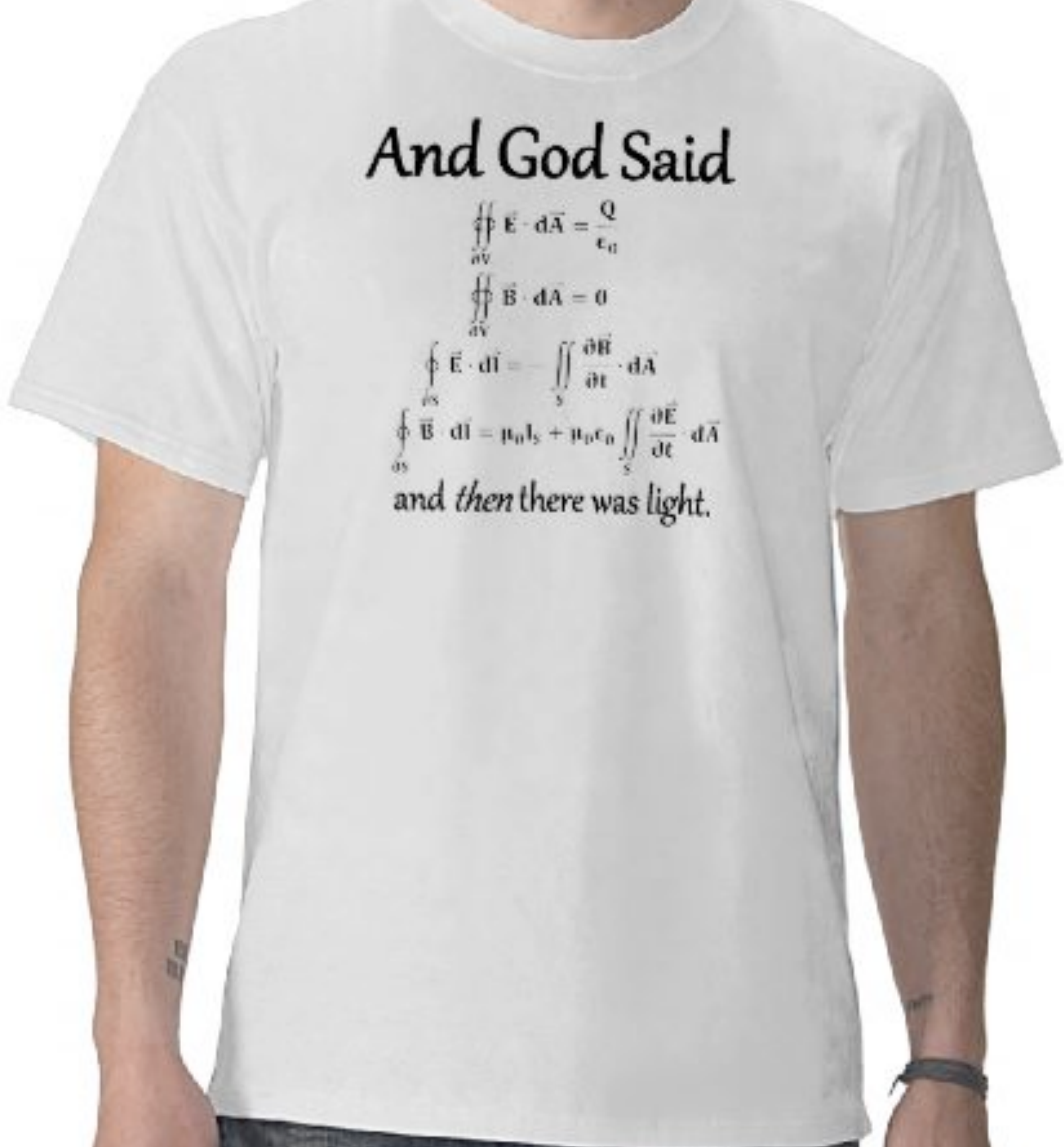
At (x,y,z) at time t, what is B_y?

A) B_y is positive and has its maximum magnitude

B) B_y is negative and has its maximum magnitude

C) B_y is zero

D) We do not have enough information



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