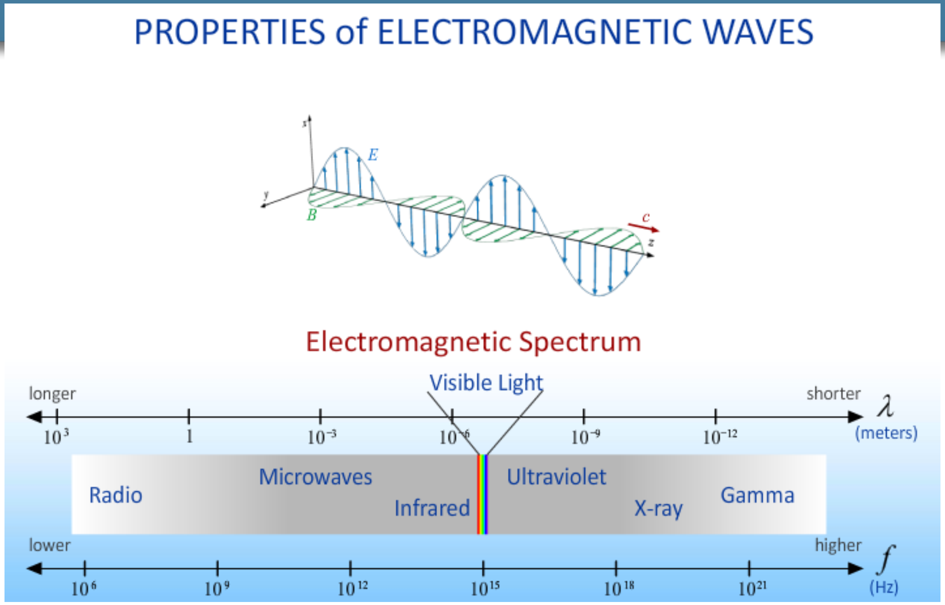
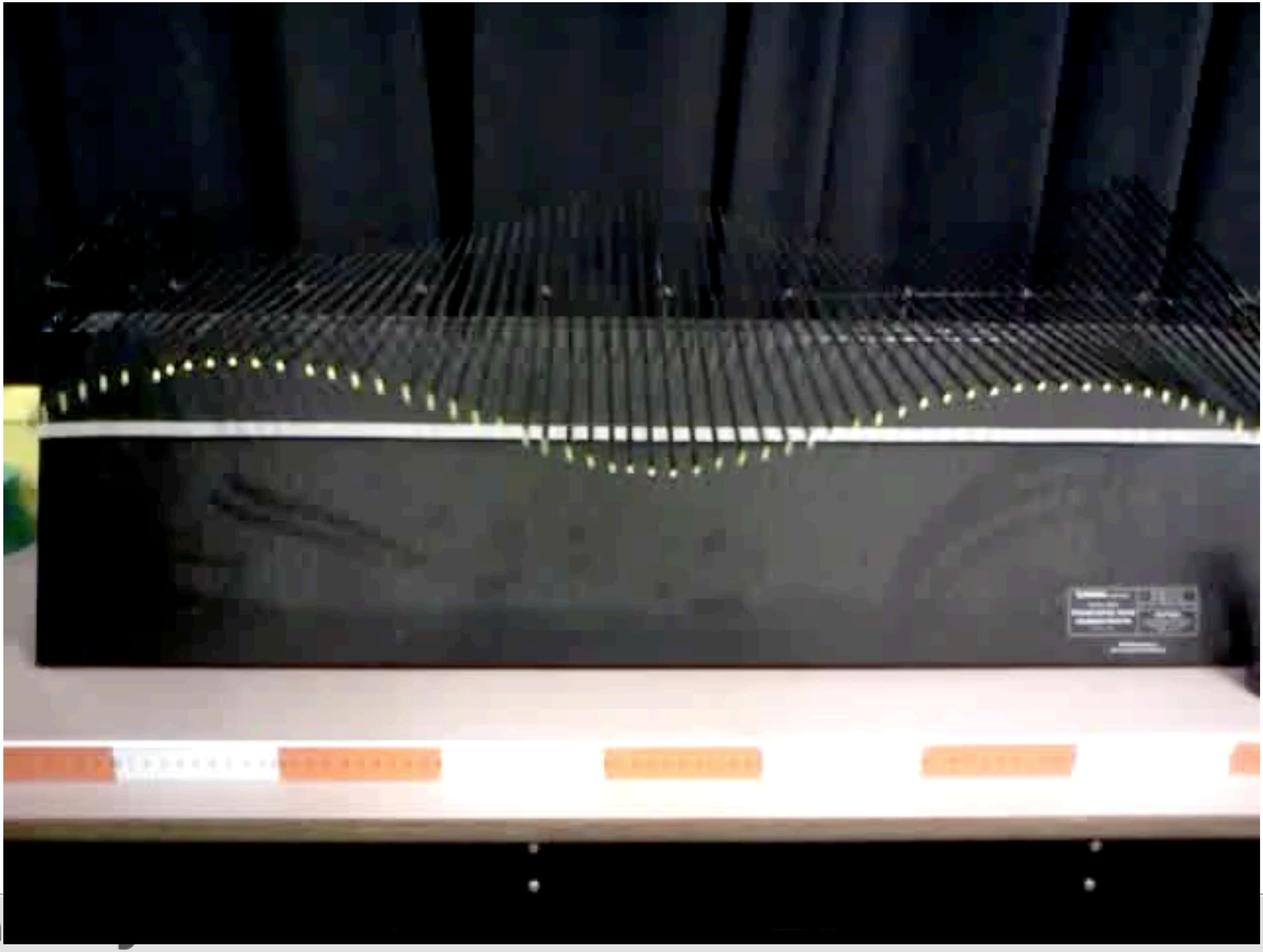
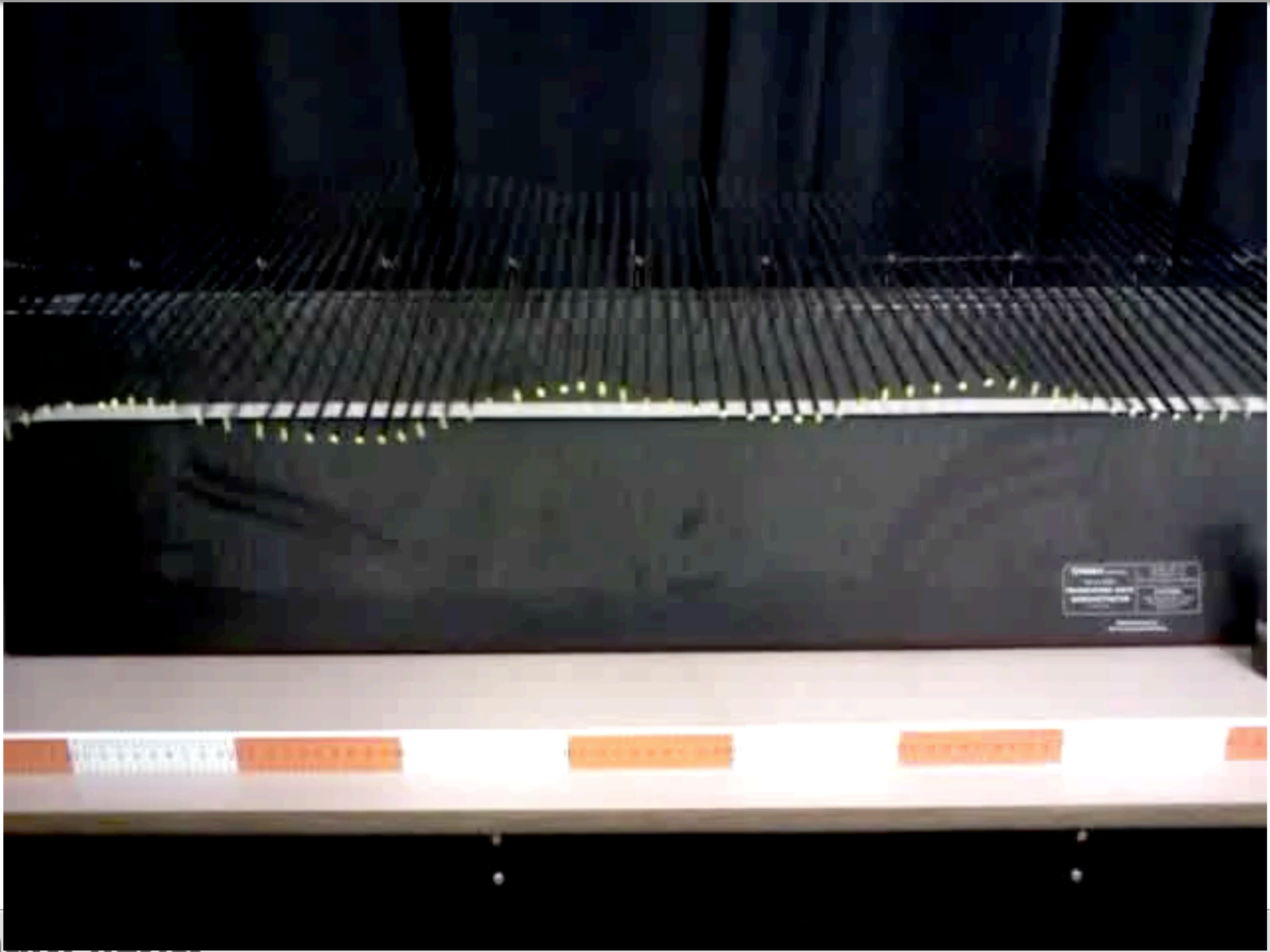
Electricity & Magnetism
Lecture 23



1 cycle per second

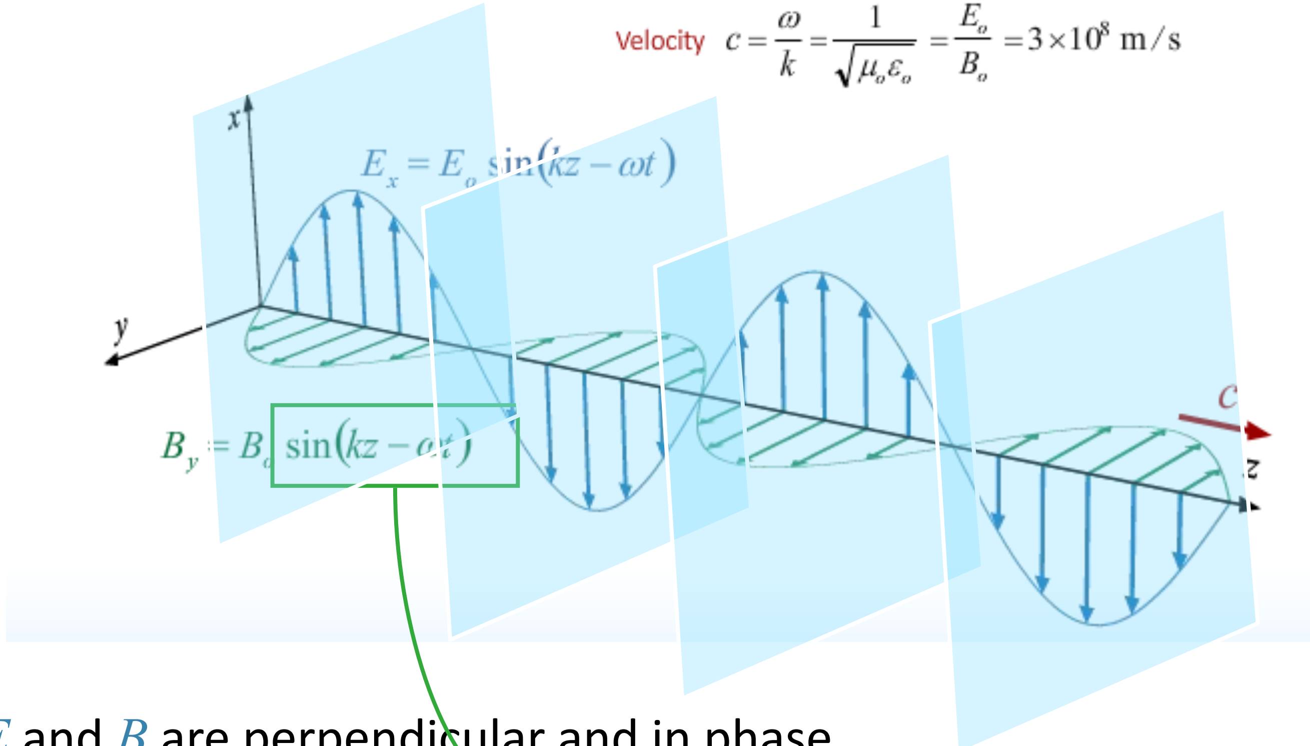


2 cycles per second



Plane Waves from Last Time

Velocity $c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = \frac{E_o}{B_o} = 3\times10^8 \text{ m/s}$



E and B are perpendicular and in phase

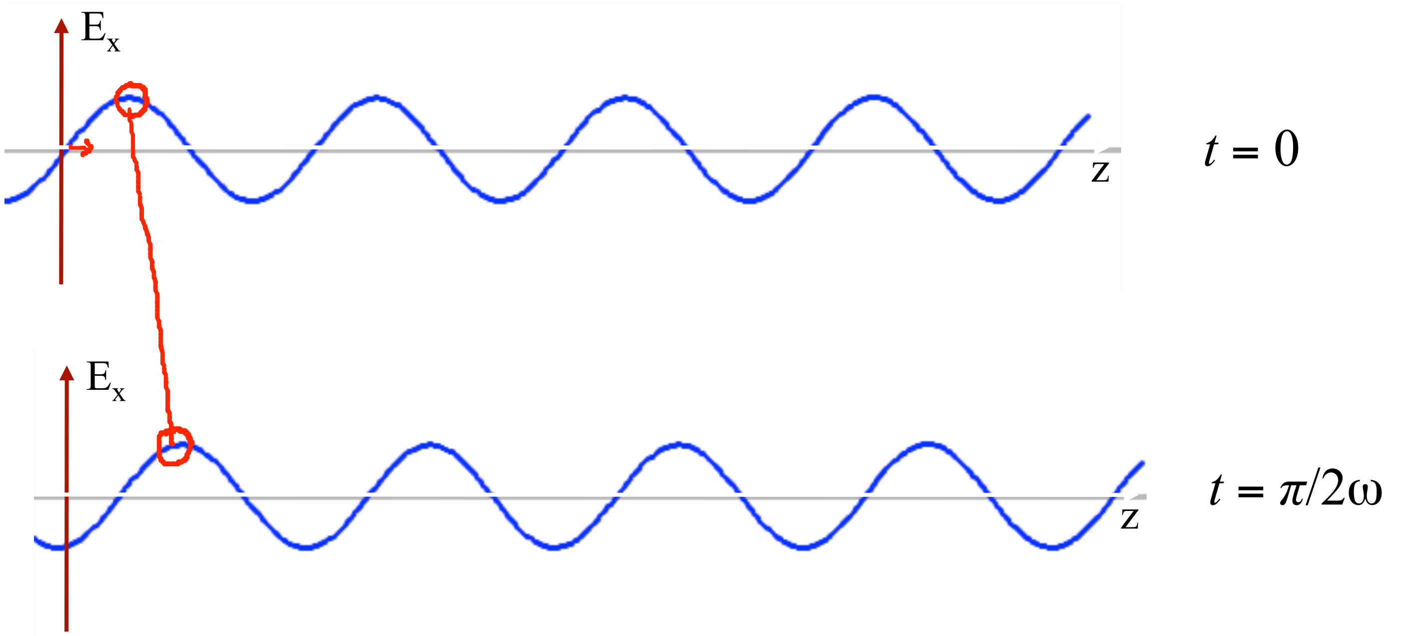
Oscillate in time and space

Direction of propagation given by $E \times B$

$B_0 = E_0/c$

Argument of sin/cos gives direction of propagation

$$E_x = E_o \sin(kz - \omega t)$$



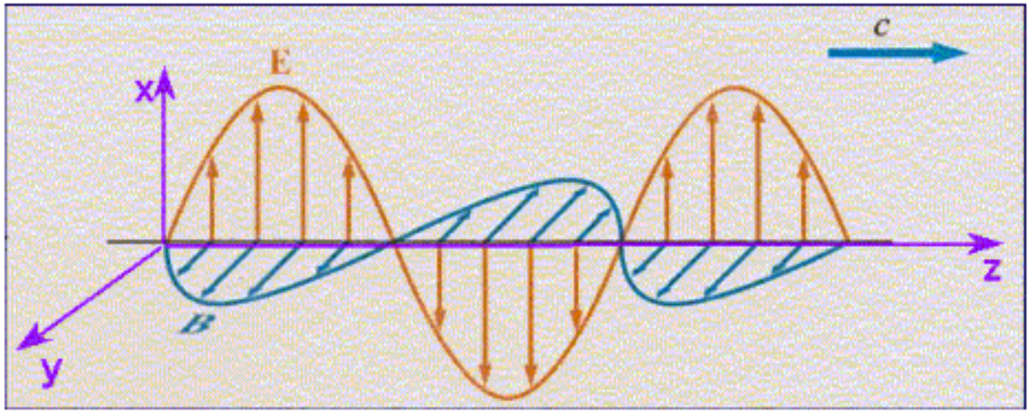
$$\sin(kz - \tfrac{\pi}{2}) = -\cos(kz)$$

What has happened to the wave form in this time interval?

It has MOVED TO THE RIGHT by $\lambda/4$

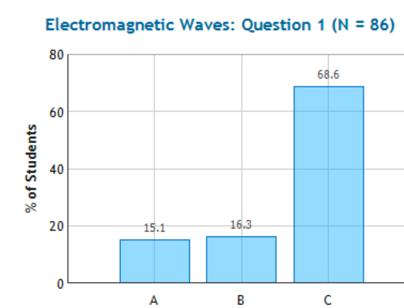
$$speed = c = \frac{\lambda / 4}{\pi / 2\omega} = \lambda \frac{\omega}{2\pi} = \lambda f$$

CheckPoint 2



Which equation correctly describes this electromagnetic wave?

- ○ $\mathbf{E}_x = \mathbf{E}_o \sin(kz + \omega t)$ No – moving in the minus *z* direction
- ○ $\mathbf{E}_y = \mathbf{E}_o \sin(kz - \omega t)$ No – has E_y rather than E_x
- ○ $\mathbf{B}_y = \mathbf{B}_o \sin(kz - \omega t)$



From the prelecture

The equation for the x-component of the electric field of a plane electromagnetic wave is given by: $E_x = E_o \sin(kz - \omega t)$

Which of the following equations describes the associated magnetic field?

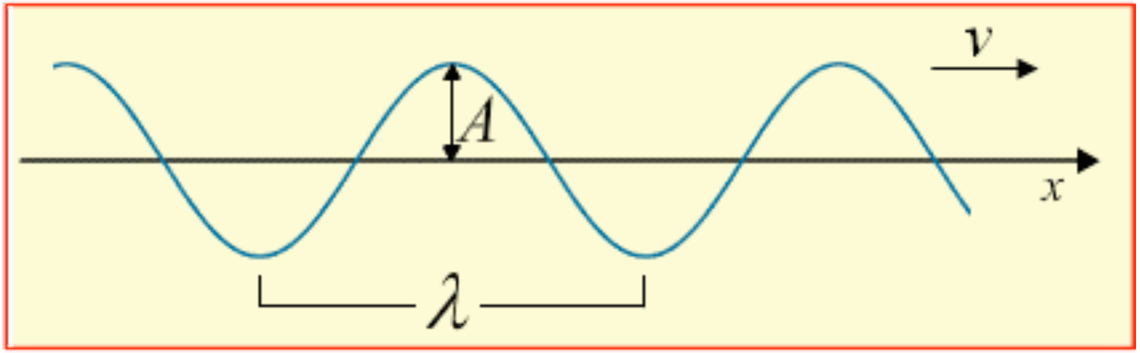
A) $B_y = E_o c \sin(kz - \omega t)$

B) $B_y = (E_o/c) \sin(kz - \omega t)$

C) $B_y = E_o c \cos(kz - \omega t)$

D) $B_y = (E_o/c) \cos(kz - \omega t)$

Prelecture question



Which of the following statements does not correctly describe a harmonic plane wave traveling in some medium.

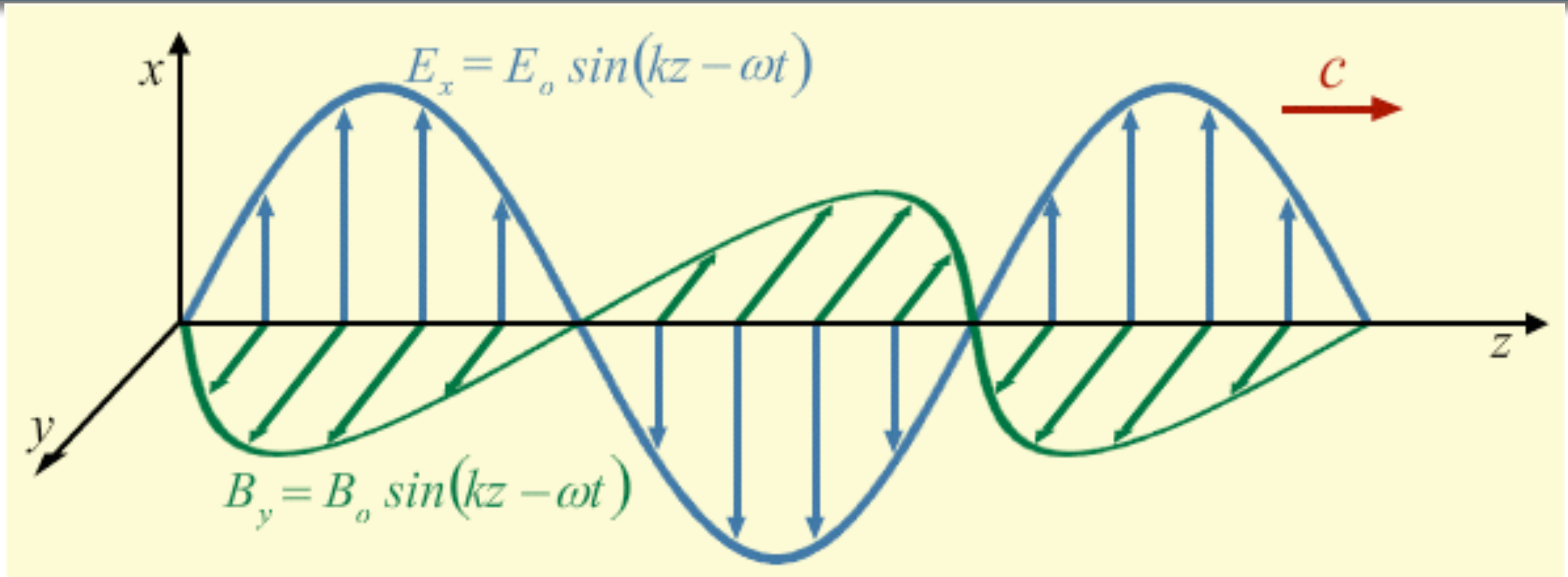
A) The time taken by any point of the wave to make one complete oscillation does not depend on the amplitude.

B) Doubling the wavelength of the wave will halve its frequency.

C) Doubling the amplitude has no effect on the wavelength.

D) Doubling the frequency of the wave will double its speed.

Prelecture



An electromagnetic wave is traveling through free space and the magnitudes of its electric and magnetic fields are E_o and B_o respectively. It then passes through a filter that cuts the magnitude of the electric field by a quarter ($E = E_o/4$). What happens to the magnitude of the magnetic field?

A. $B = B_o/4$

B. $B = B_o/2$

C. $B = B_o$

Prelecture Question

The color of the stars we observe in galaxies can be used to deduce the velocity of the galaxy relative to Earth.

Suppose the average color of the stars in a newly discovered galaxy is **bluer** than the average color of stars in our own galaxy. What would be a sensible conclusion about the motion of the new galaxy relative to our own?

A. That it is moving toward us.

B. That it is moving away from us.

CheckPoint 6

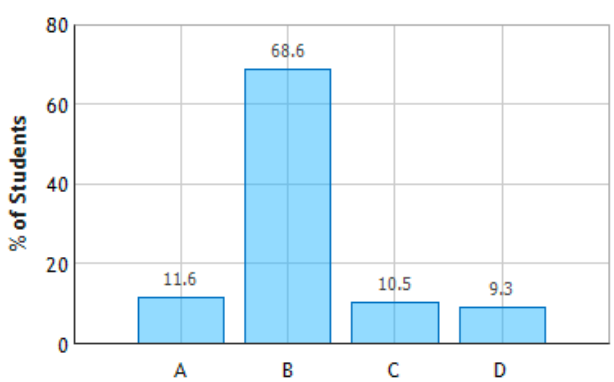
Your iclicker operates at a frequency of approximately 900 MHz (900x10^6 Hz). What is the approximately wavelength of the EM wave produced by your iclicker?

- 0.03 meters
- 0.3 meters
- 3.0 meters
- 30. meters

$C = 3.0 \times 10^8$ m/s

Wavelength is equal to the speed of light divided by the frequency.

$$\lambda = \frac{c}{f} = \frac{300{,}000{,}000}{900{,}000{,}000} = \frac{1}{3}$$

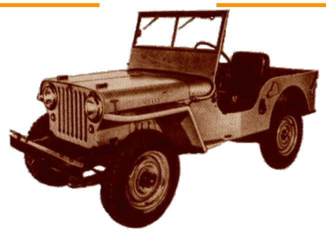


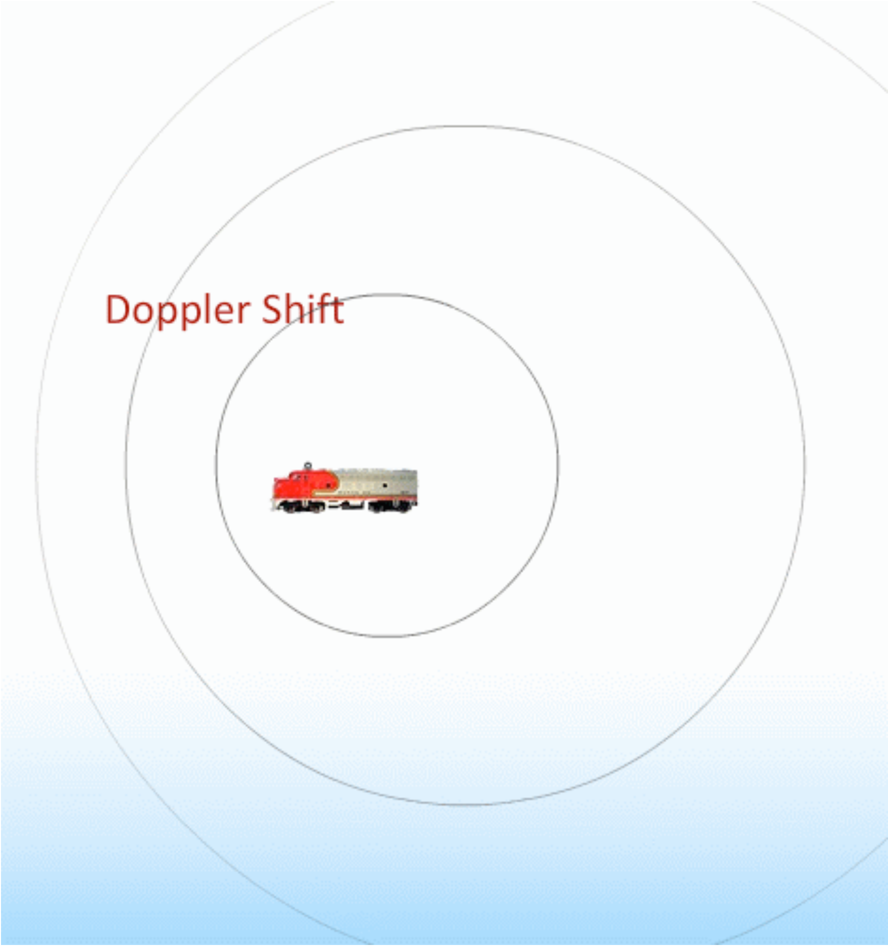
Check:

Look at size of antenna on base unit

Doppler Shift

Dr Chai Demo





BigBang

The Big Idea

As source approaches:

Wavelength decreases

Frequency Increases

Doppler Shift for E-M Waves

What's Different from Sound or Water Waves ?

Sound /Water Waves :

You can calculate (no relativity needed)

BUT

Result is somewhat complicated: is source or observer moving wrt medium?

Electromagnetic Waves :

You need relativity (time dilation) to calculate

BUT

Result is simple: only depends on relative motion of source & observer

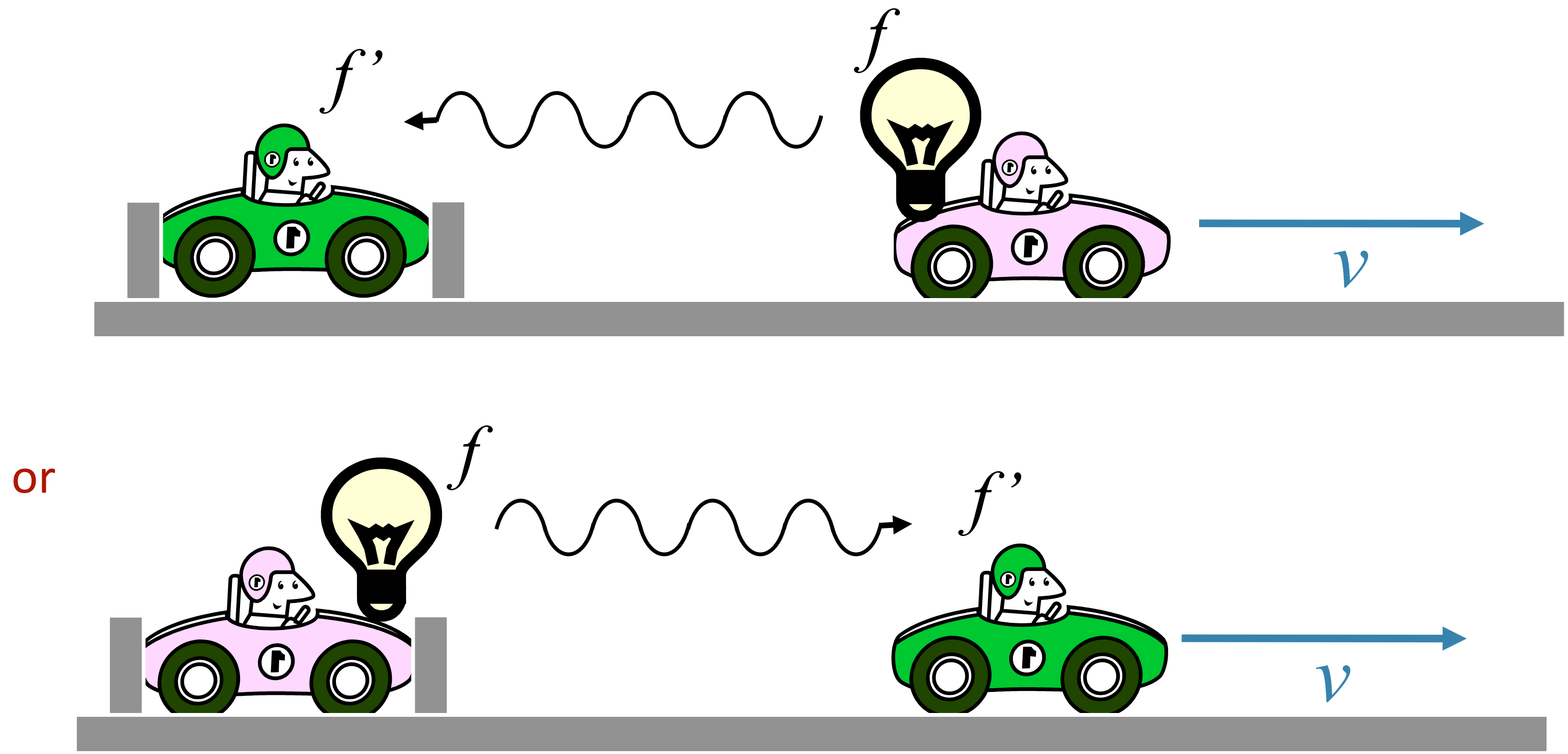
$$f' = f\sqrt{\frac{1+\beta}{1-\beta}}$$

$\beta = v/c$

$\beta > 0$ if source & observer are approaching

$\beta < 0$ if source & observer are separating

Doppler Shift for E-M Waves



The Doppler Shift is the SAME for both cases!

f'/f only depends on the relative velocity

$$f' = f\sqrt{\frac{1+\beta}{1-\beta}}$$

Doppler Shift for E-M Waves

A Note on Approximations

$$f' = f\sqrt{\frac{1+\beta}{1-\beta}}$$

$\beta << 1$ →

$$f' \approx f(1+\beta)$$

why?

Taylor Series: Expand $F(\beta) = \left(\frac{1+\beta}{1-\beta}\right)^{1/2}$ around $\beta = 0$

$$F(\beta) = F(0) + \frac{F'(0)}{1!}\beta + \frac{F''(0)}{2!}\beta^2 + \ldots$$

Evaluate:

$F(0) = 1$

$F'(0) = 1$

→

$$F(\beta) \approx 1 + \beta$$

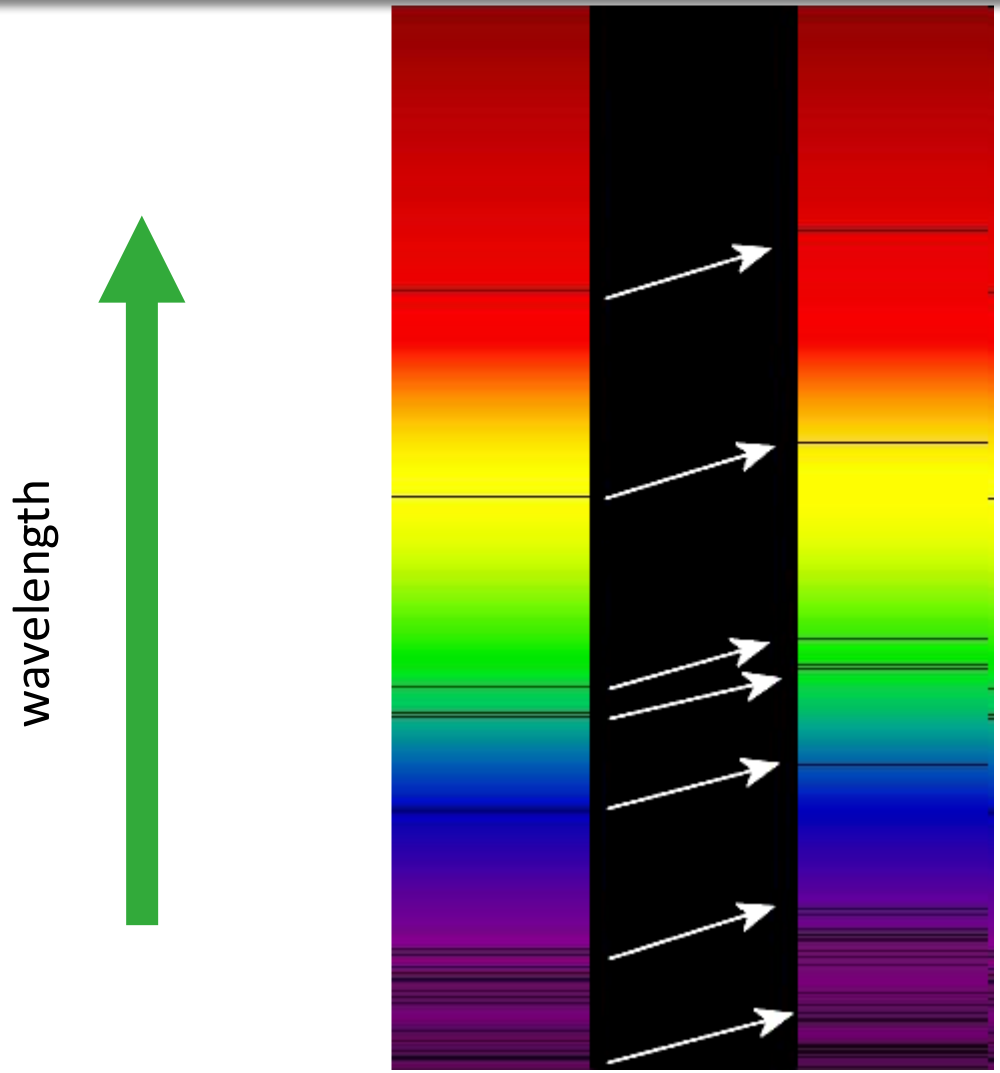
NOTE:

$F(\beta) = (1+\beta)^{1/2}$

↓

$F(\beta) \approx 1 + \frac{1}{2}\beta$

Red Shift of Stellar Spectra



Our Sun

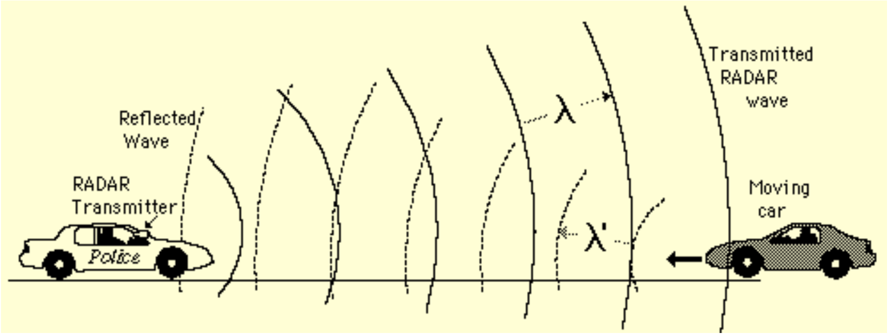
Star in a distant galaxy

Wavelengths shifted higher

Frequencies shifted lower

Star separating from us
(Expanding Universe)

Example



Police radars get twice the effect since the EM waves make a round trip:

$$f' \approx f(1 + 2\beta)$$

If f = 24,000,000,000 Hz (k-band radar gun)

c = 300,000,000 m/s

v	β	f'	$f'-f$
30 m/s (108 km/h)	1.000×10^{-7}	24,000,004,800	4800 Hz
31 m/s (112 km/h)	1.033×10^{-7}	24,000,004,959	4959 Hz

Waves Carry Energy

Total Energy Density

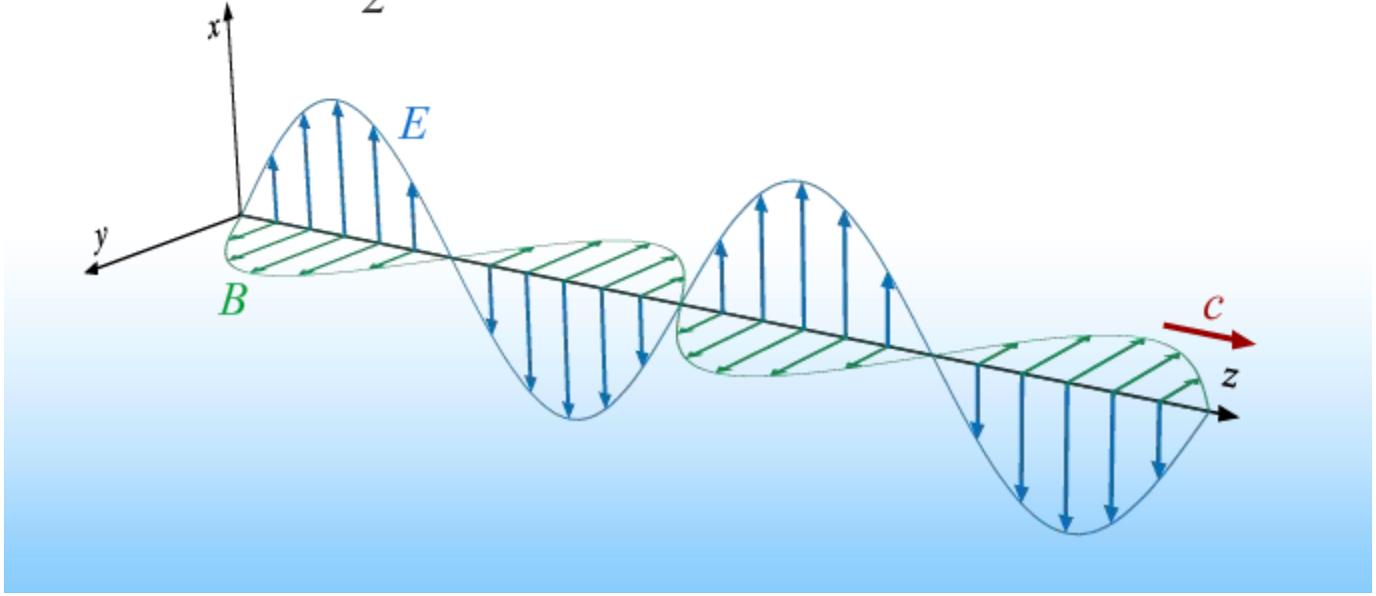
$$u = \varepsilon_o E^2$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o^{\,2}$$

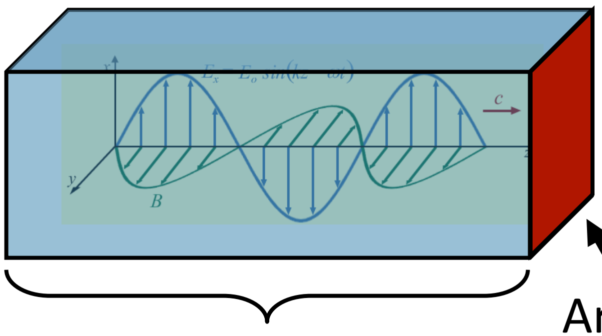
Intensity

$$I = \frac{1}{2} c\,\varepsilon_o E_o^{\,2} = c\,\langle u \rangle$$



Intensity

Intensity = Average energy delivered per unit time, per unit area



$$I \equiv \frac{1}{A}\left\langle \frac{dU}{dt} \right\rangle$$

$$\langle dU \rangle = \langle u \rangle \times volume = \langle u \rangle A c dt$$

Total Energy Density

$$u = \varepsilon_o E^2$$

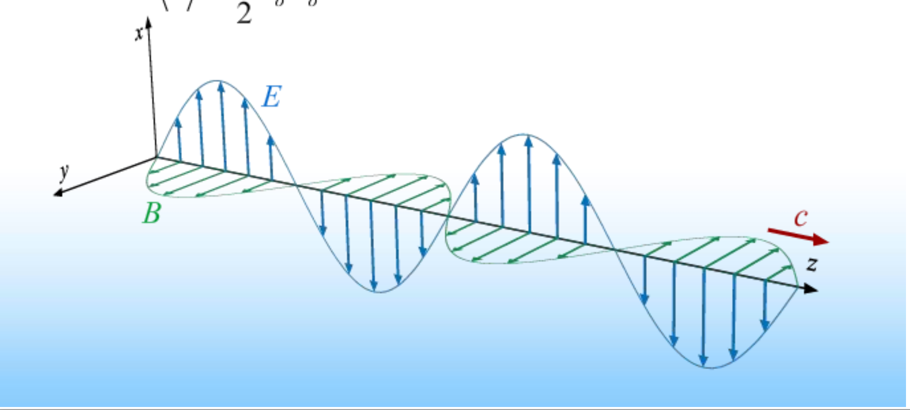
Average Energy Density

$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o^{\,2}$$

Intensity

$$I = \frac{1}{2} c \varepsilon_o E_o^{\,2} = c\,\langle u \rangle$$

$$I = c\langle u \rangle$$



Sunlight on Earth:

$I \sim 1000$ J/s/m^2

~ 1 kW/m^2

Waves Carry Energy

Total Energy Density

$$u = \varepsilon_o E^2$$

Intensity

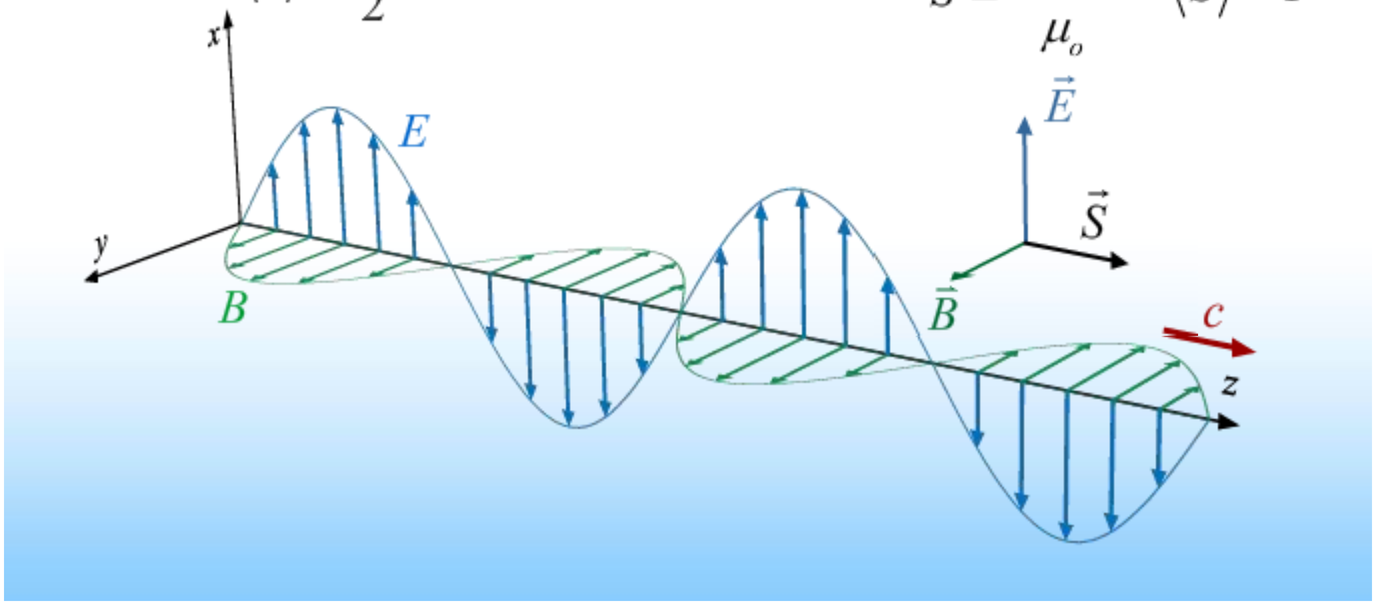
$$I = \frac{1}{2} c \varepsilon_o E_o^{\,2} = c \langle u \rangle$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2} \varepsilon_o E_o^{\,2}$$

Poynting Vector

$$\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_o} \qquad \langle S \rangle = I$$



Comment on Poynting Vector

Just another way to keep track of all this:

Its magnitude is equal to I

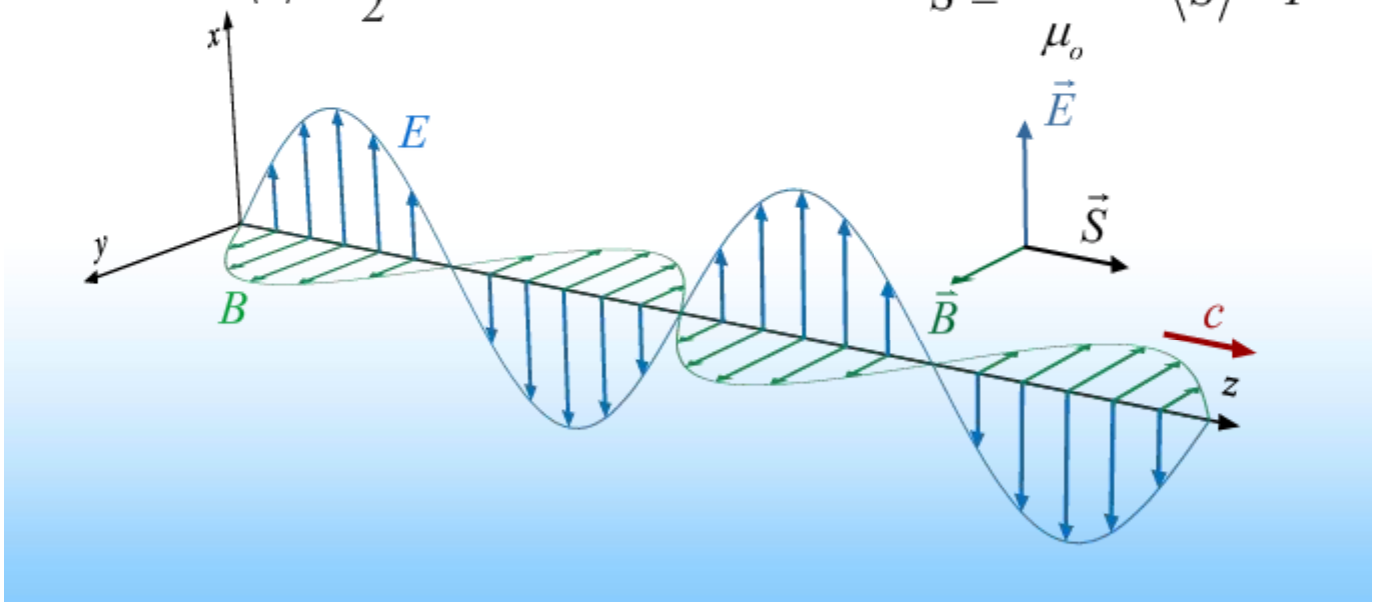
Its direction is the direction of propagation of the wave

Average Energy Density

$$\langle u \rangle = \frac{1}{2}\varepsilon_o E_o^{\ 2}$$

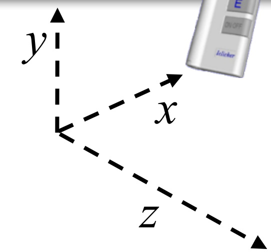
Poynting Vector

$$\vec{S} \equiv \frac{\vec{E} \times \vec{B}}{\mu_o} \qquad \langle S \rangle = I$$

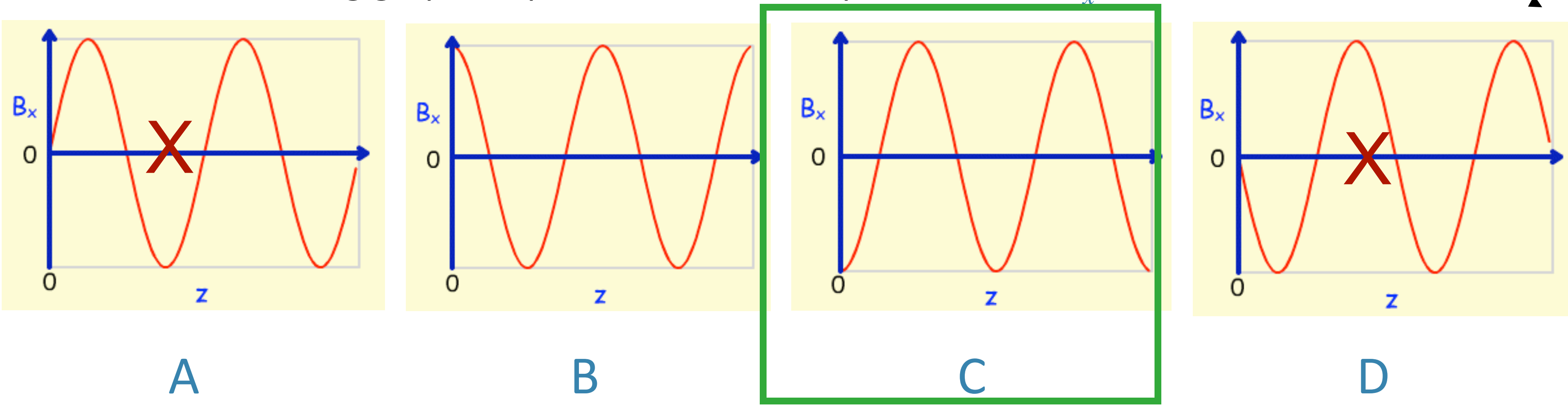


Exercise

An electromagnetic wave is described by: $\vec{E} = \hat{j}E_0 \cos(kz - \omega t)$
where $\hat{j}$ is the unit vector in the $+y$ direction.



Which of the following graphs represents the z – dependence of B_x at $t = 0$?

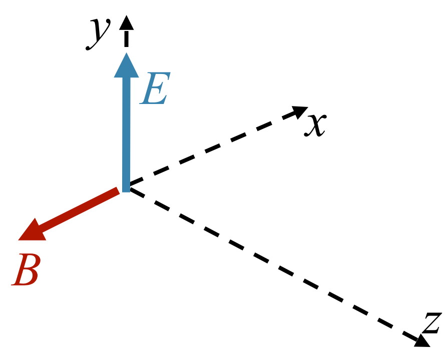


A B C D

E and B are "in phase" (or 180^o out of phase)

$\vec{E} = \hat{j}E_0 \cos(kz - \omega t)$ → Wave moves in $+z$ direction

$\vec{E} \times \vec{B}$ Points in direction of propagation



$\vec{B} = -\hat{i}B_0 \cos(kz - \omega t)$

Light has Momentum!

If it has energy and its moving, then it also has momentum:

Analogy from mechanics:

$$E = \frac{p^2}{2m}$$

$$\frac{dE}{dt} = \frac{\cancel{2}p}{\cancel{2}m}\frac{dp}{dt} = \frac{\cancel{m}v}{\cancel{m}}\frac{dp}{dt} = vF$$

For $E - M$ waves: $\frac{dE}{dt} \rightarrow \frac{dU}{dt} = IA$

$v \rightarrow c$

$$IA = cF$$

$$\frac{I}{c} = \frac{F}{A}$$

pressure

$$P = \frac{I}{c}$$

Radiation pressure

CheckPoint 4

4) An electromagnetic wave has electric field amplitude E, wavelength λ, and frequency ω. Which should we increase if we want the energy carried by the wave to increase (you can mark more than one answer).

A) ☐ E

B) ☐ λ

C) ☐ ω

Intensity

$$I = \frac{1}{2} c \varepsilon_o E_o^{\ 2}$$

But then again, what are we keeping constant here?

WHAT ABOUT PHOTONS?

Photons

We believe the energy in an e-m wave is carried by photons

Question: What are Photons?

Answer: Photons are Photons.

Photons possess both wave and particle properties

Particle:

Energy and Momentum localized

Wave:

They have definite frequency & wavelength $(f\lambda = c)$

Connections seen in equations:

$$E = hf$$

$$p = h/\lambda$$

Planck's constant

$$h = 6.63\times10^{-34}\ J\cdot s$$

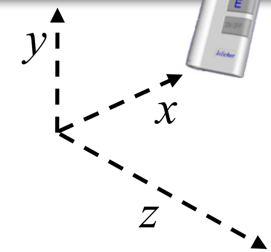
Question: How can something be both a particle and a wave?

Answer: It can't (when we observe it)

What we see depends on how we choose to measure it!

The mystery of quantum mechanics: More on this in PHYS 285

Exercise



An electromagnetic wave is described by: $\vec{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} E_0 \cos(kz + \omega t)$

What is the form of B for this wave?

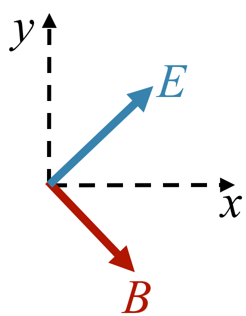
A) $\vec{B} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

B) $\vec{B} = \frac{\hat{i} - \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

C) $\vec{B} = \frac{-\hat{i} + \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

D) $\vec{B} = \frac{-\hat{i} - \hat{j}}{\sqrt{2}} (E_0 / c) \cos(kz + \omega t)$

$\vec{E} = \frac{\hat{i} + \hat{j}}{\sqrt{2}} E_0 \cos(kz + \omega t)$ → Wave moves in $-z$ direction



$+z$ points out of screen

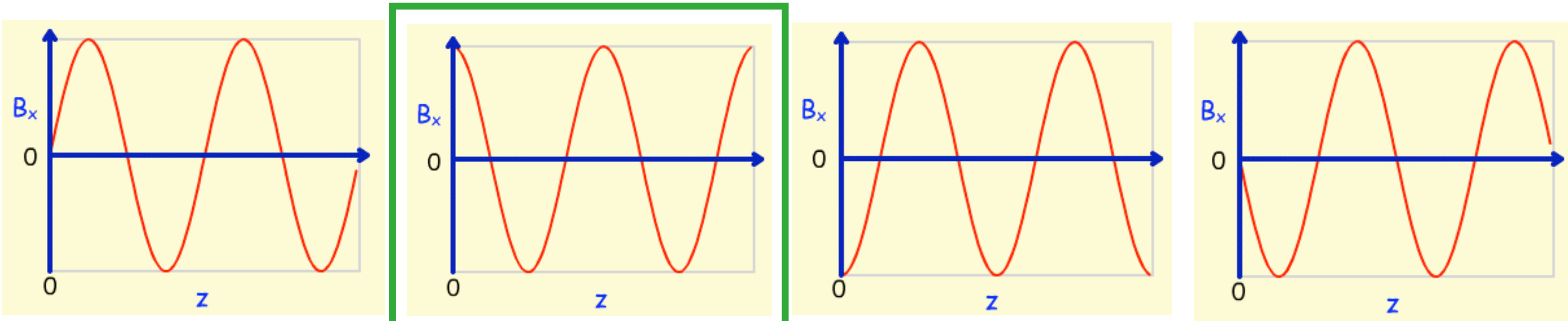
$-z$ points into screen

$\vec{E} \times \vec{B}$ Points in direction of propagation

Exercise

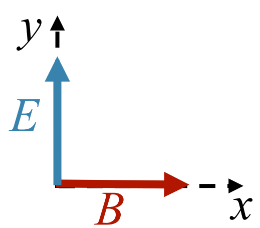
An electromagnetic wave is described by: $\vec{E} = \hat{j}E_0 \sin(kz + \omega t)$

Which of the following plots represents $B_x(z)$ at time $t = \pi/2\omega$?



A B C D

Wave moves in negative z direction → $\vec{B} = \hat{i}(E_0/c)\sin(kz + \omega t)$



+ z points out of screen

– z points into screen

at $\omega t = \pi/2$:

$$B_x = (E_0/c)\sin(kz + \pi/2)$$

$$B_x = (E_0/c)\{\sin kz \cos(\pi/2) + \cos kz \sin(\pi/2)\}$$

$$B_x = (E_0/c)\cos(kz)$$

$\vec{E} \times \vec{B}$ Points in direction of propagation