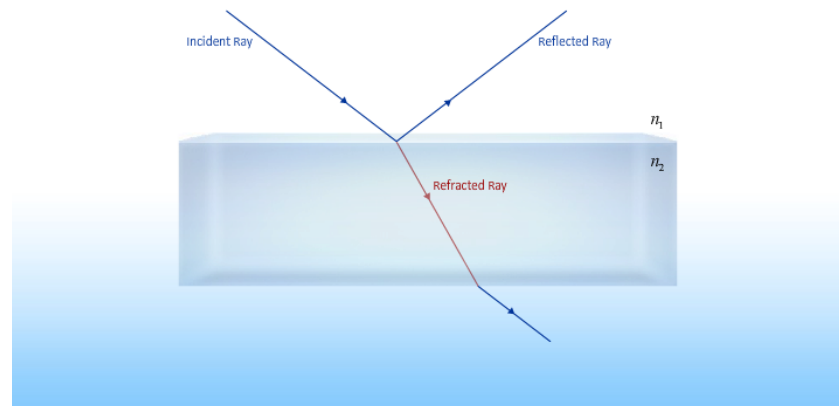


# Optics

## Lecture 25

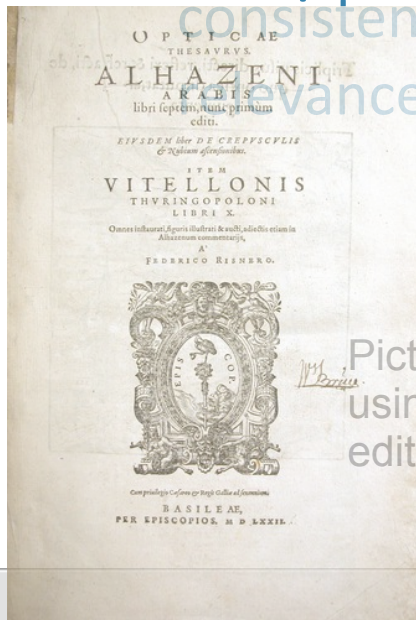
### REFLECTION and REFRACTION



# Geometrical Optics

## Father of Geometrical Optics

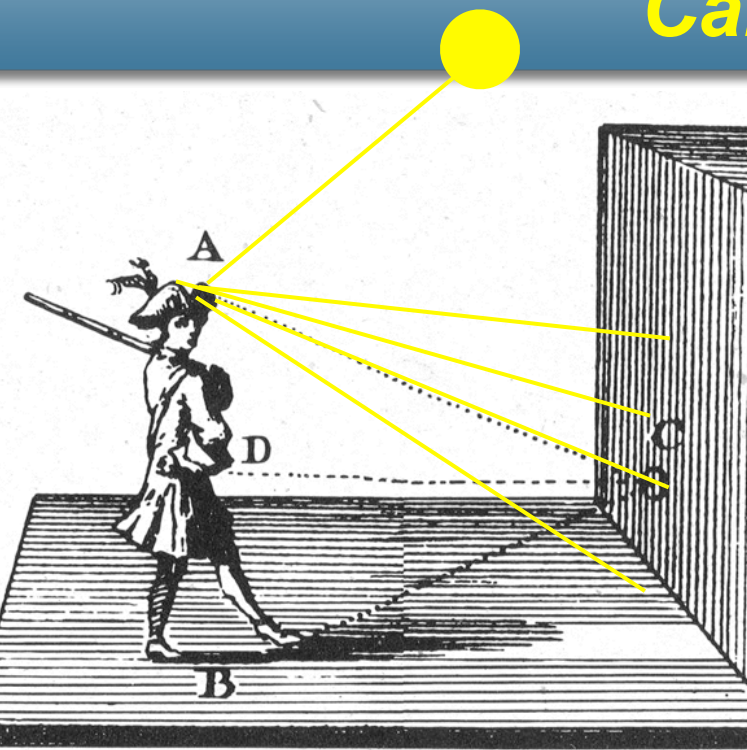
- “Al Hazen”: Abū ‘Alī al-Ḥasan ibn al-Ḥasan ibn al-Haytham (Arabic : ابن الهيثم )
- Kitab al-Manazir كتاب المناظر (Book of Optics), written from 1011 to 1021
- Translated to latin in 1200’s.
- Electromagnetic and Quantum Theories of optics are consistent with geometrical optics in its domain of relevance.



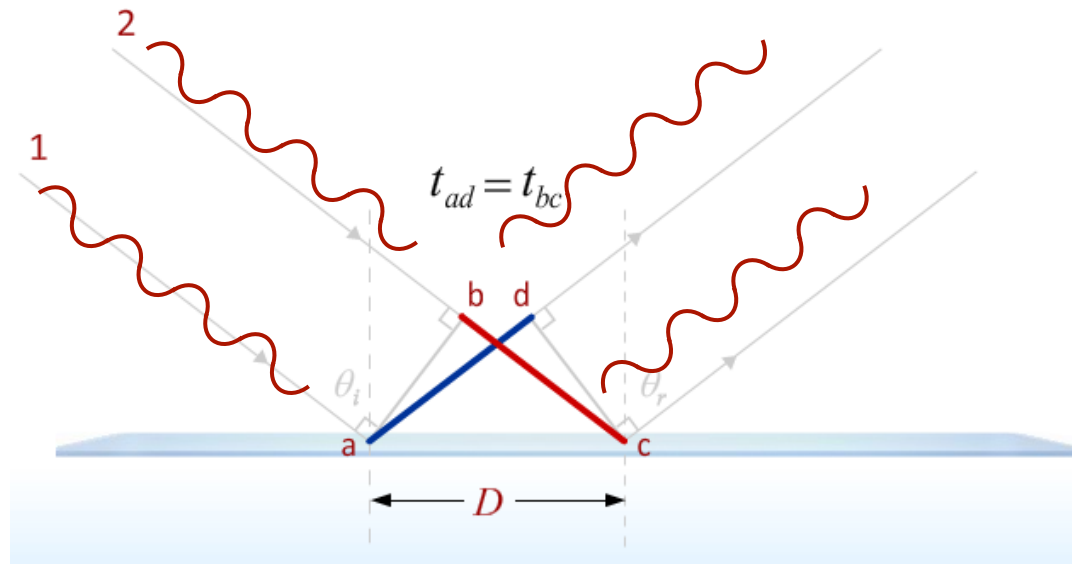
Picture of how Archimedes burnt Roman ships using parabolic mirrors. From the 1572 Latin edition.



# Camera Obscura

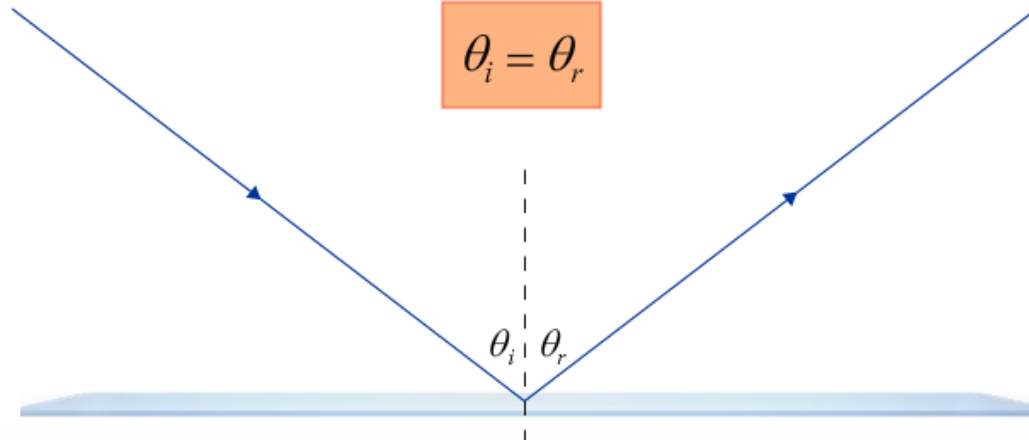


# Reflection



Law of Reflection

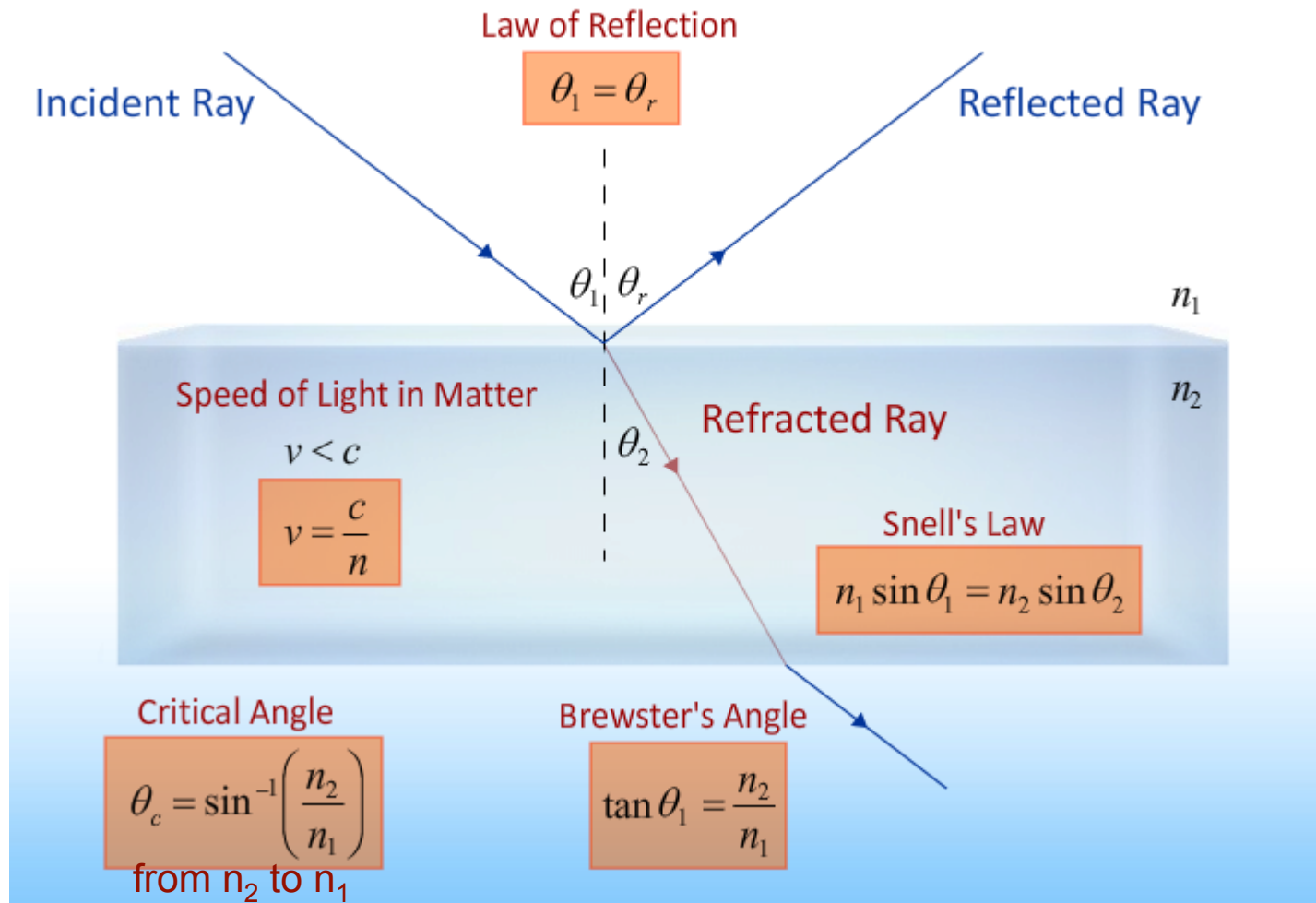
$$\theta_i = \theta_r$$



# *Mirrors in Movies*

- A) Duck Soup (1933) Marx Brothers
  - B) The Lady from Shanghai (1947) Orson Welles
  - C) Enter the Dragon (1973) Bruce Lee
- and many more

# Let's Start with a Summary:



The speed of light in a medium is slower than in empty space:

Speed of Light

$$v = \frac{1}{\sqrt{\mu\epsilon}} < \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

Index of Refraction

$$n \equiv \frac{c}{v} = \frac{\sqrt{\mu\epsilon}}{\sqrt{\mu_0\epsilon_0}} \approx \sqrt{\frac{\epsilon}{\epsilon_0}} \approx \sqrt{K}$$

Examples for Visible Light

$$n_{air} = 1.0$$

$$n_{glass} = 1.5$$

$$n_{diamond} = 2.4$$

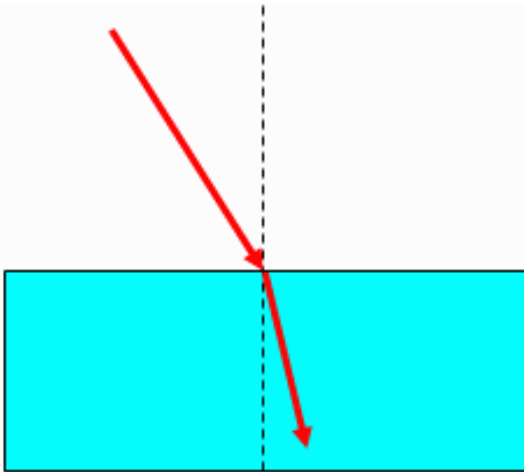
$$v_{medium} = c / n_{medium}$$



# CheckPoint 2



2) A ray of light passes from air into water with an angle of incidence of 30 degrees.

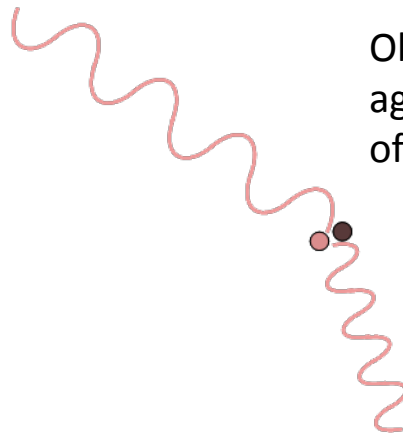


Which of the following quantities does not change as the light enters the water. Mark all correct answers.

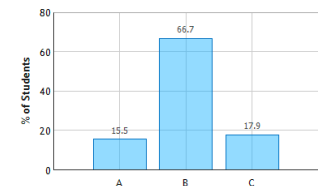
- ☐ A wavelength
- ☒ B frequency
- ☐ C speed of propagation

What about the wave must be the same on either side?

Observers in both media must agree on the frequency of vibration of the molecules

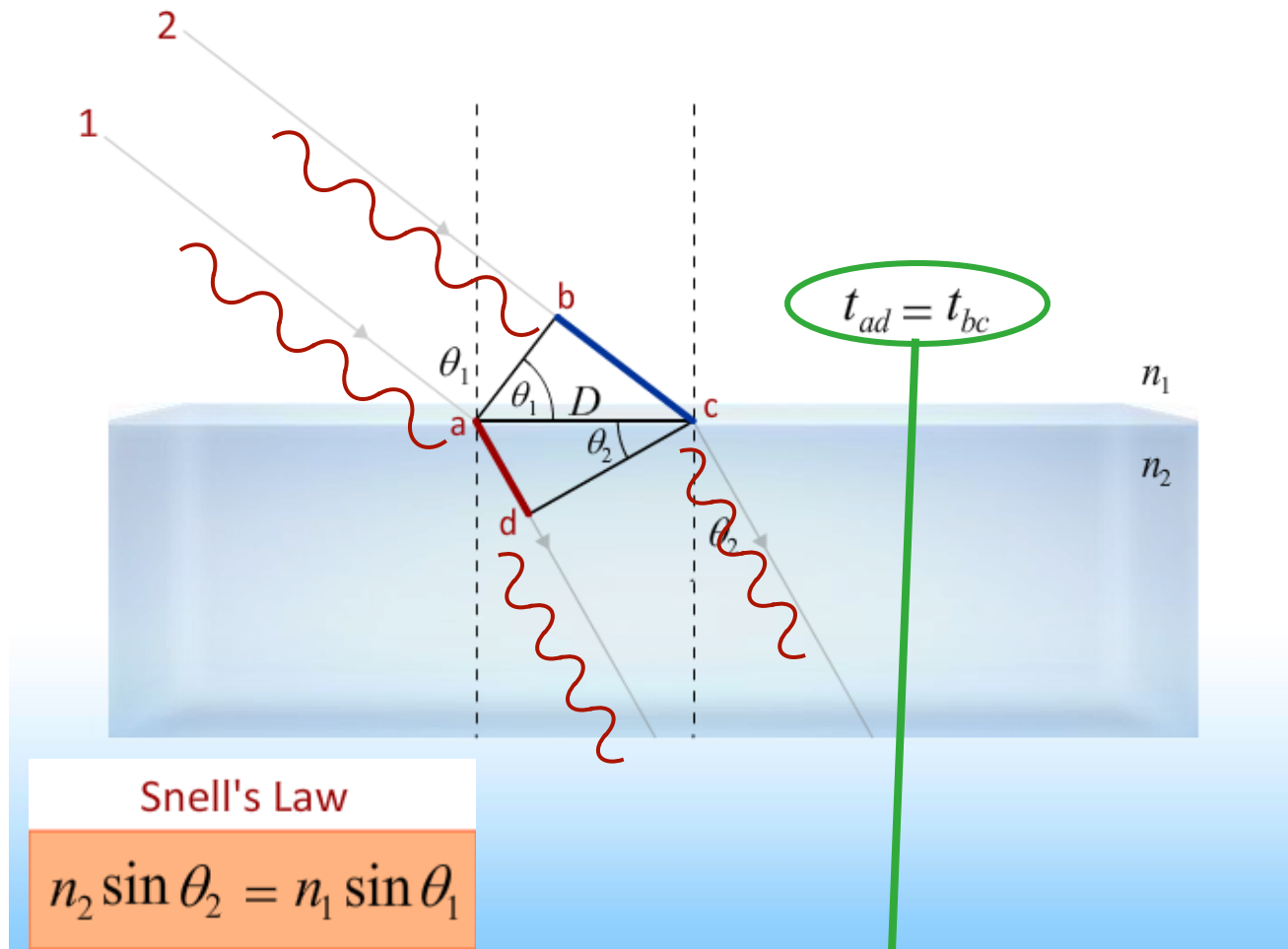


Light Passing from Air into Water: Question 1 (N = 84)





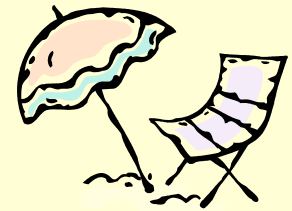
# Refraction: Snell's Law



$$\frac{D \sin \theta_2}{c/n_2} = \frac{D \sin \theta_1}{c/n_1} \rightarrow n_2 \sin \theta_2 = n_1 \sin \theta_1$$

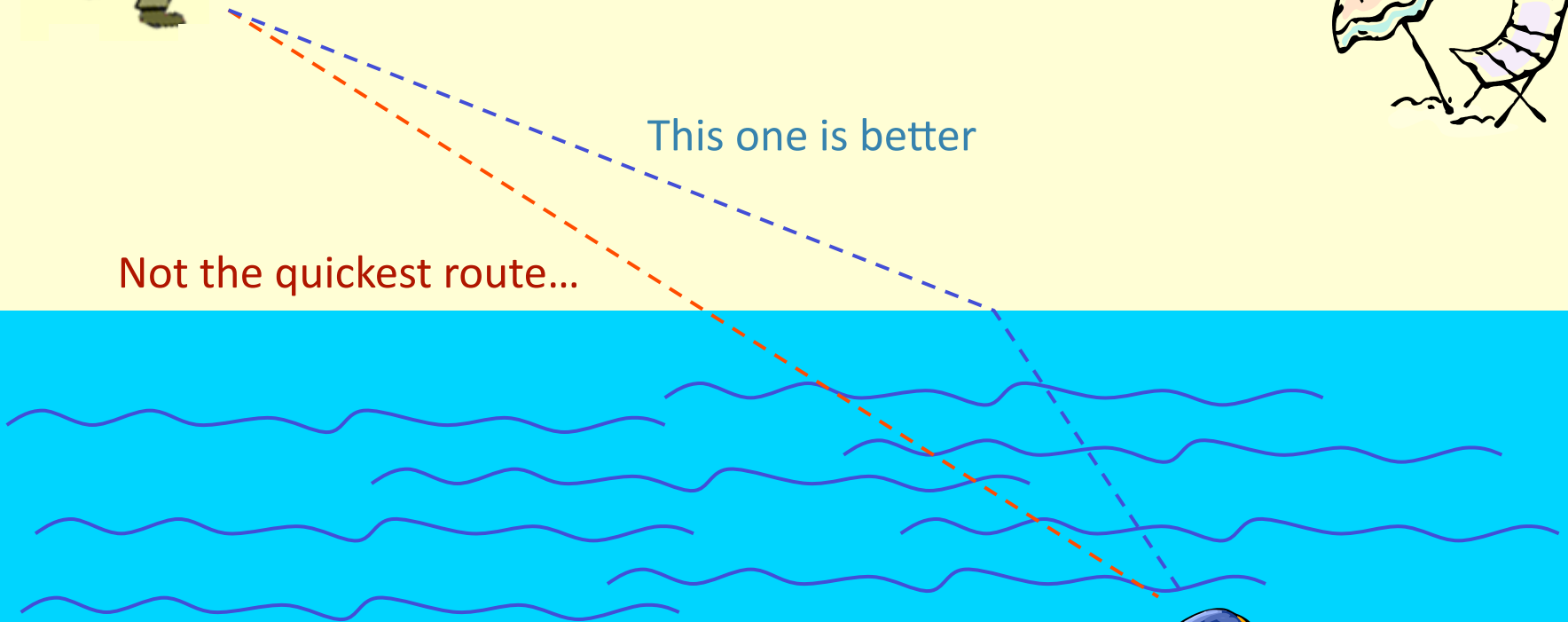
# Think of a Day at the Beach

What's the fastest path to the ball knowing you can run faster than you can swim?

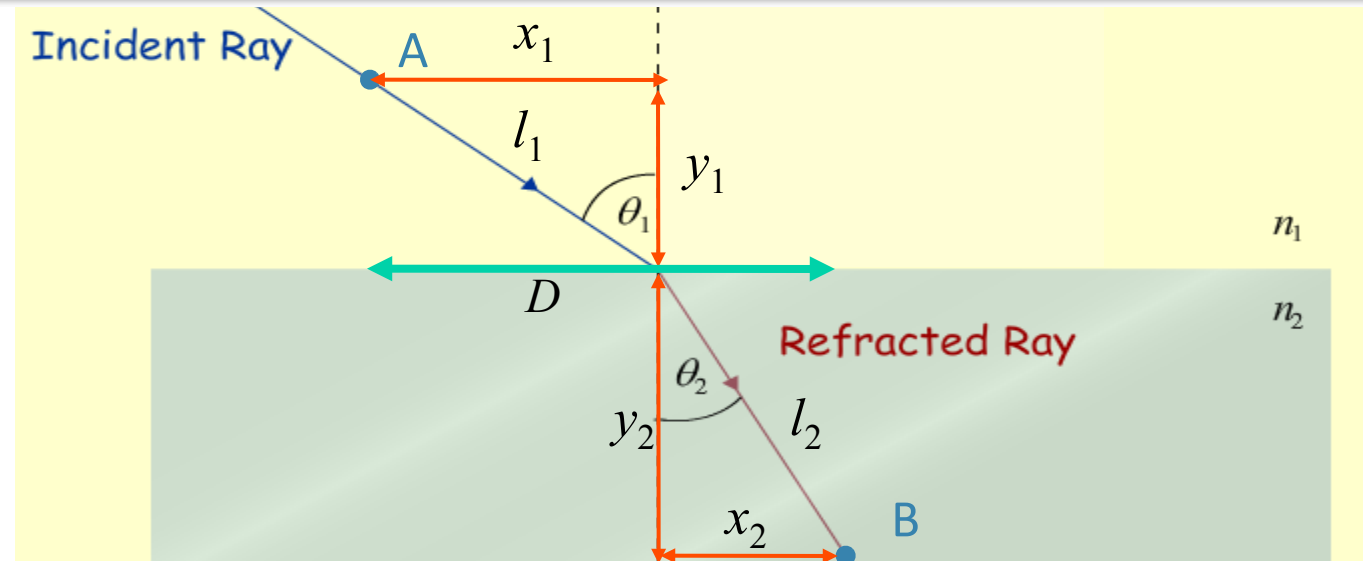


This one is better

Not the quickest route...



Same Principle  
works for Light!



Time from A to B :

$$t = \frac{l_1}{v_1} + \frac{l_2}{v_2} = \frac{\sqrt{x_1^2 + y_1^2}}{v_1} + \frac{\sqrt{x_2^2 + y_2^2}}{v_2}$$

To find minimum time,  
differentiate  $t$  wrt  $x_1$  and set  $= 0$

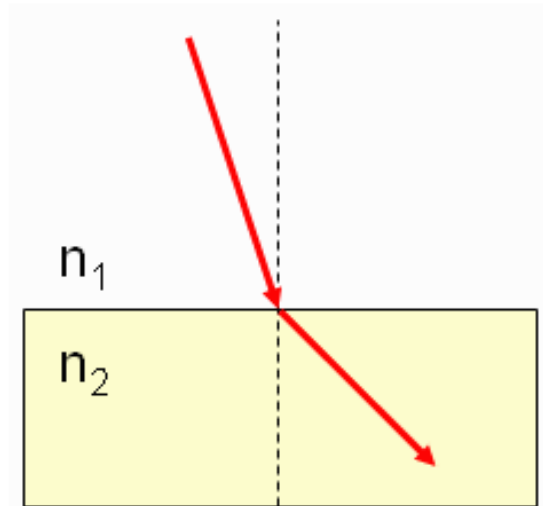
$$\frac{dt}{dx_1} = \frac{x_1}{v_1 \sqrt{x_1^2 + y_1^2}} + \frac{x_2}{v_2 \sqrt{x_2^2 + y_2^2}} \frac{dx_2}{dx_1}$$

How is  $x_2$  related to  $x_1$ ?  $x_2 = D - x_1 \rightarrow \frac{dx_2}{dx_1} = -1$

Setting  $\frac{dt}{dx_1} = 0 \rightarrow \frac{x_1}{v_1 l_1} - \frac{x_2}{v_2 l_2} = 0 \rightarrow \frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2} \xrightarrow{v = c/n} n_1 \sin \theta_1 = n_2 \sin \theta_2$

# CheckPoint 6

☐ The path of light is bent as passes from medium 1 to medium 2.



Compare the indexes of refraction in the two mediums.

☒  $n_1 > n_2$

☐  $n_1 = n_2$

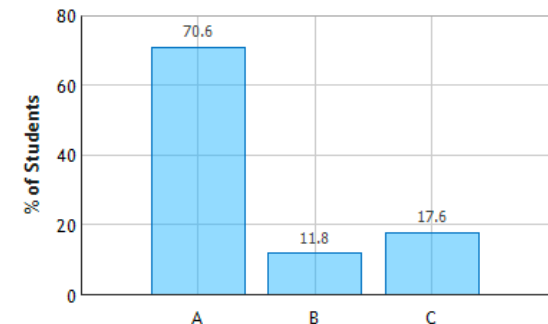
☐  $n_1 < n_2$

**Snell's Law:**

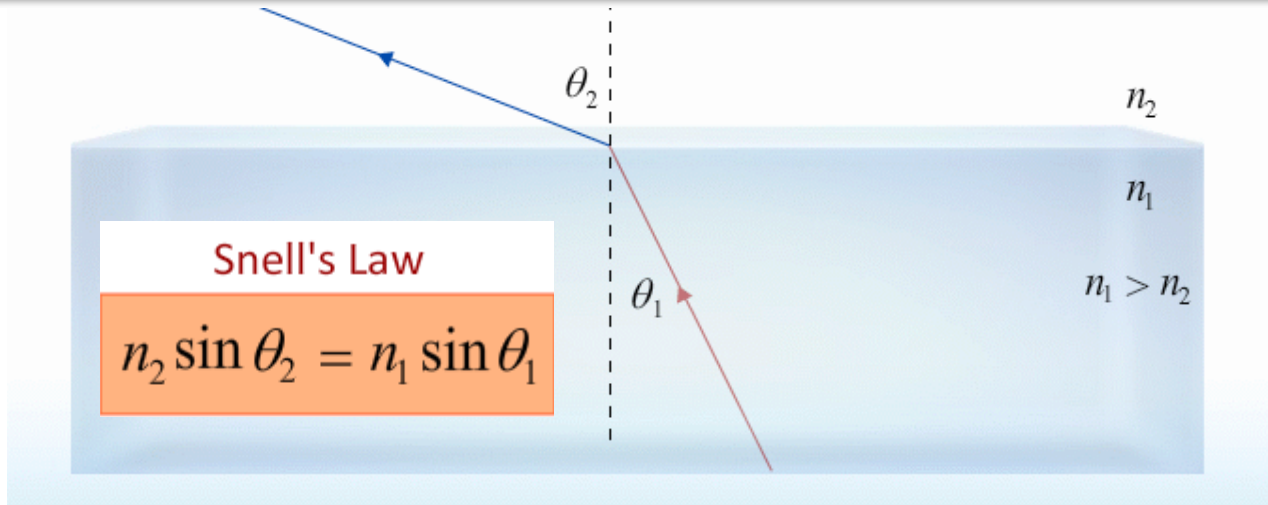
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$n$  decreases  $\Rightarrow \theta$  increases

Reflection and Refraction at a Surface:  
Question 1 (N = 85)

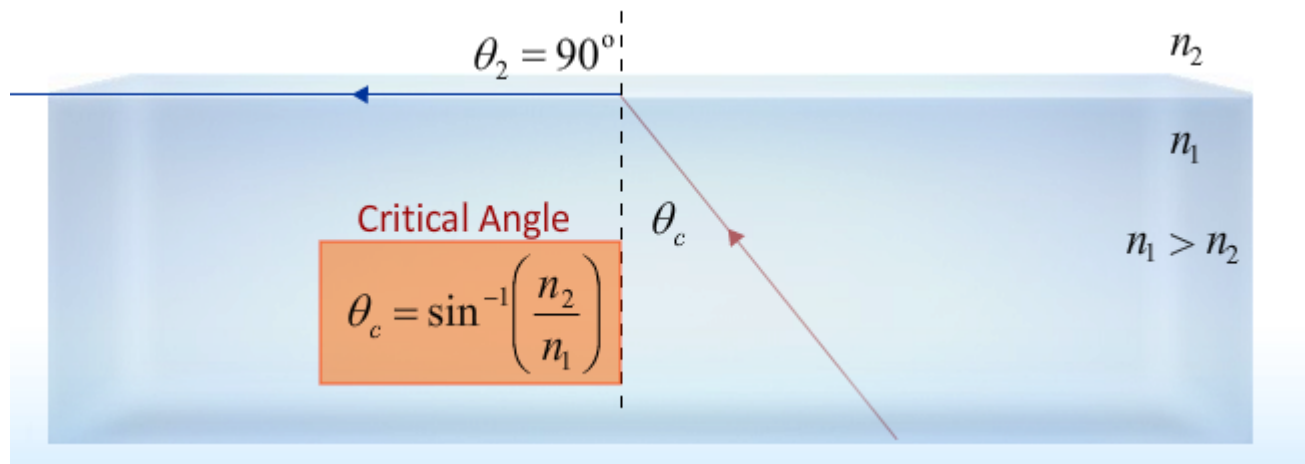


# Total Internal Reflection



NOTE:  $n_1 > n_2$  implies  $\theta_2 > \theta_1$

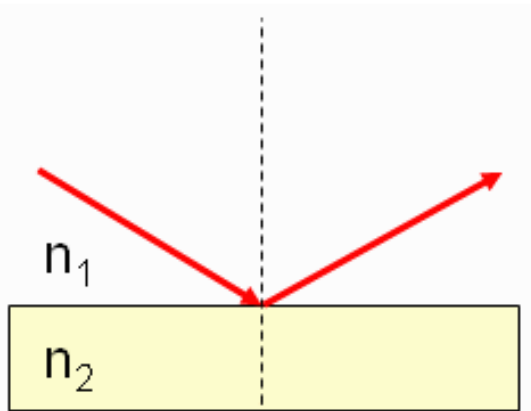
BUT:  $\theta_2$  has max value =  $90^\circ$ !



$\theta_1 > \theta_c \rightarrow$  Total Internal Reflection

# CheckPoint 8

A light ray travels in a medium with  $n_1$  and completely reflects from the surface of a medium with  $n_2$ .



The critical angle depends on:

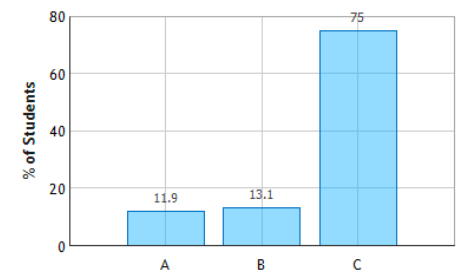
- ☐  $n_1$  only
- ☐  $n_2$  only
- ☒ both  $n_1$  and  $n_2$

Critical Angle

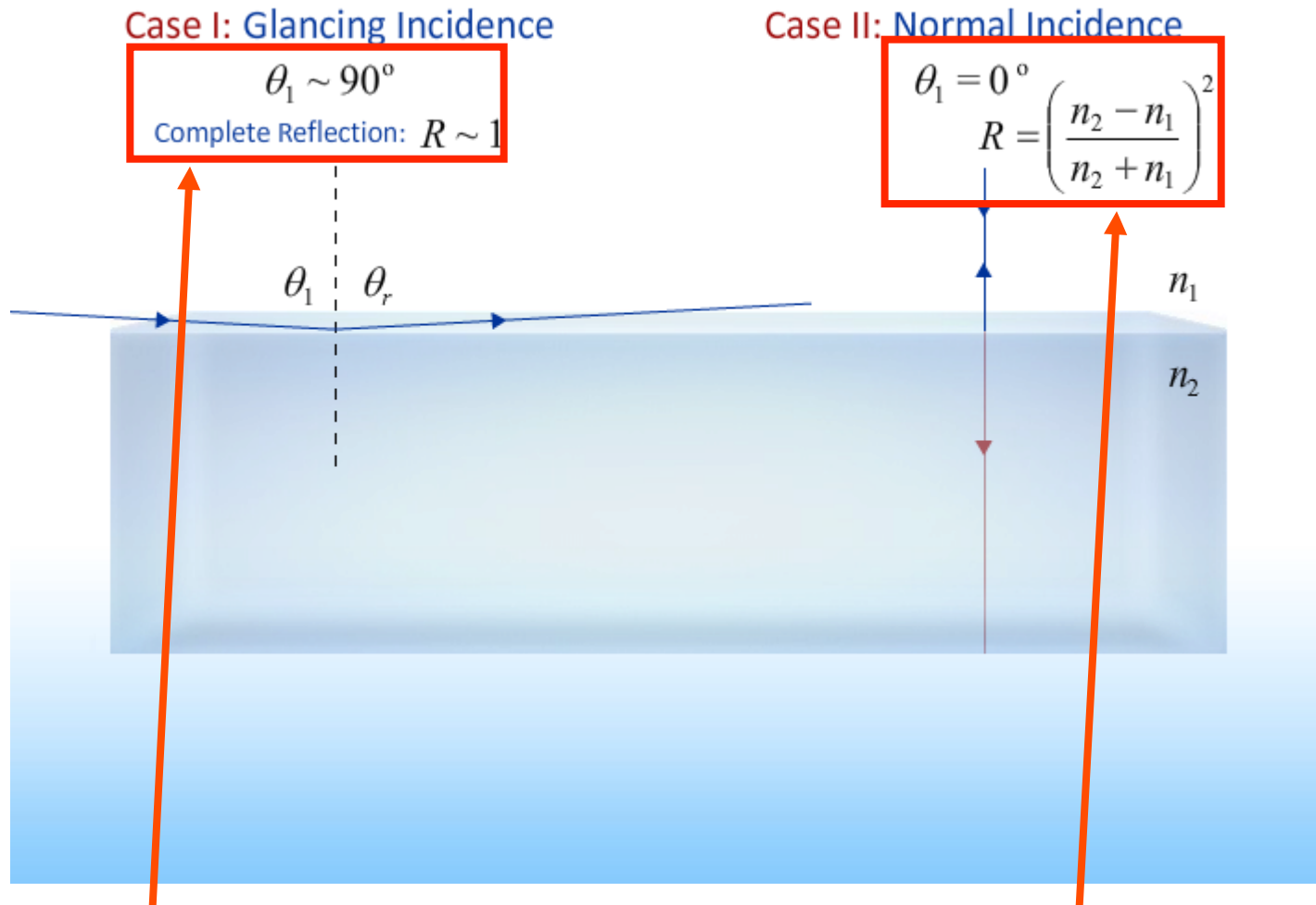
$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$\theta_c$  clearly depends on both  $n_2$  and  $n_1$

Reflection and Refraction at a Surface:  
Question 3 (N = 84)



# Intensity



Anything looks like a mirror  
if light is just glancing off it.

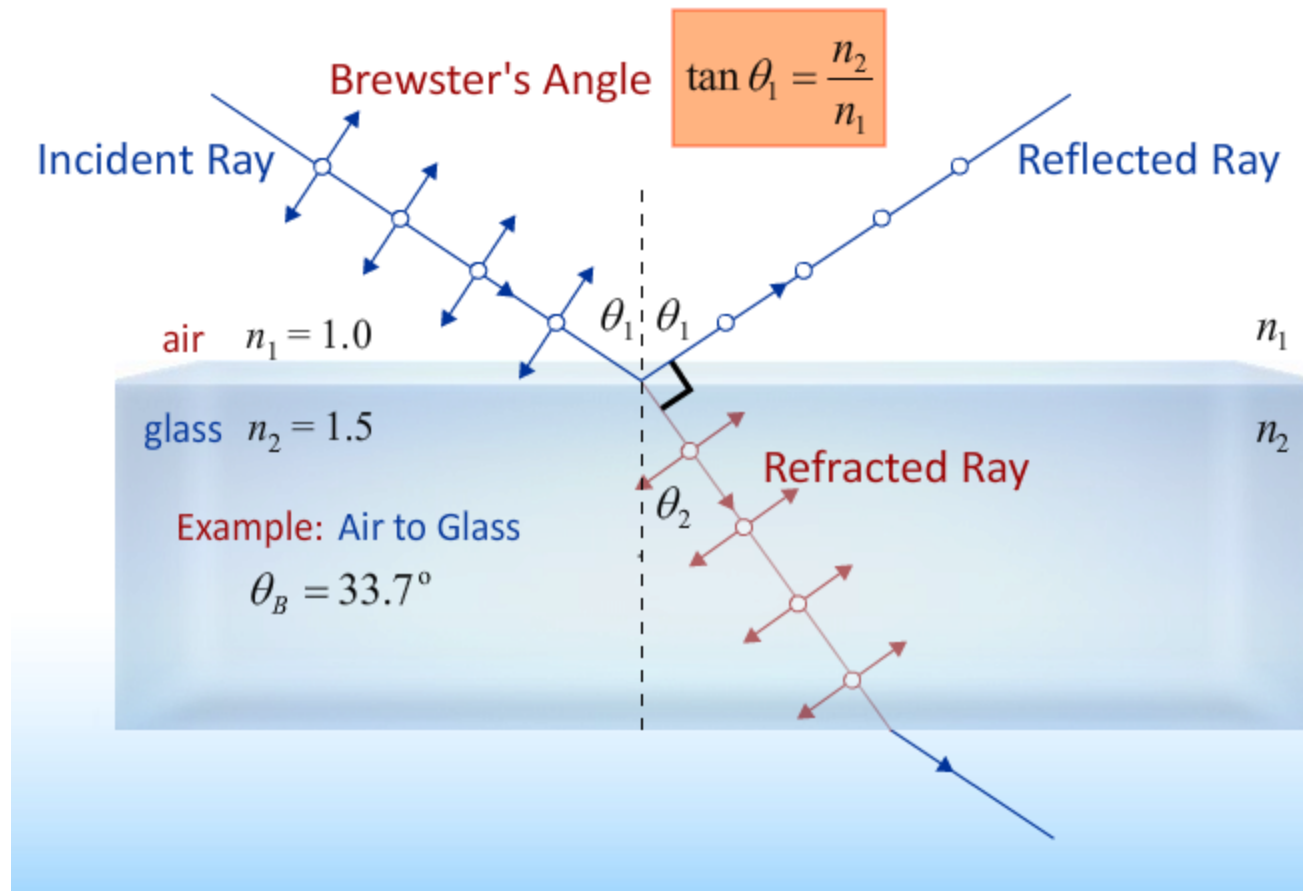
Demo

If two materials have the same  $n$   
then its hard to tell them apart.



“Please re-explain Brewster's angle in layman's terms. I don't really understand what they are trying to say in the pre-lecture.”

# Polarization



Only works for non-metallic surfaces.

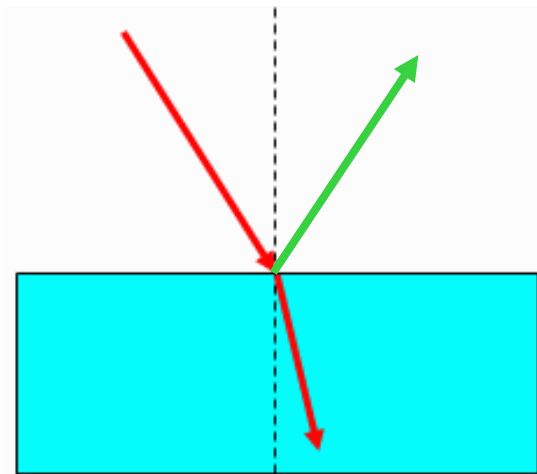
$$\theta_1 + \theta_2 = 90^\circ \quad \longrightarrow \quad \sin \theta_2 = \sin(90^\circ - \theta_1) = \cos \theta_1$$

$$\text{Snell's Law: } n_1 \sin \theta_1 = n_2 \sin \theta_2 = n_2 \cos \theta_1 \quad \longrightarrow \quad \tan \theta_1 = \frac{n_2}{n_1}$$

# CheckPoint 4

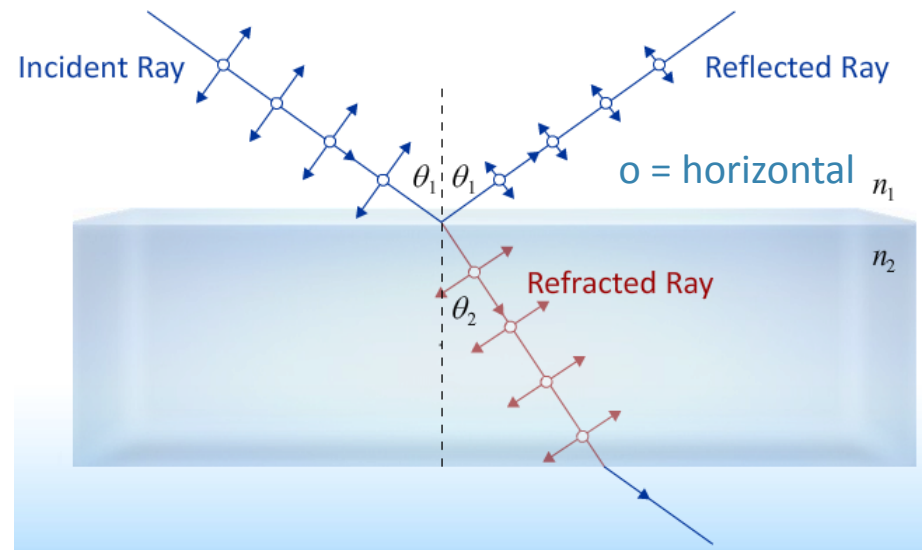


A ray of light passes from air into water with an angle of incidence of 30 degrees.



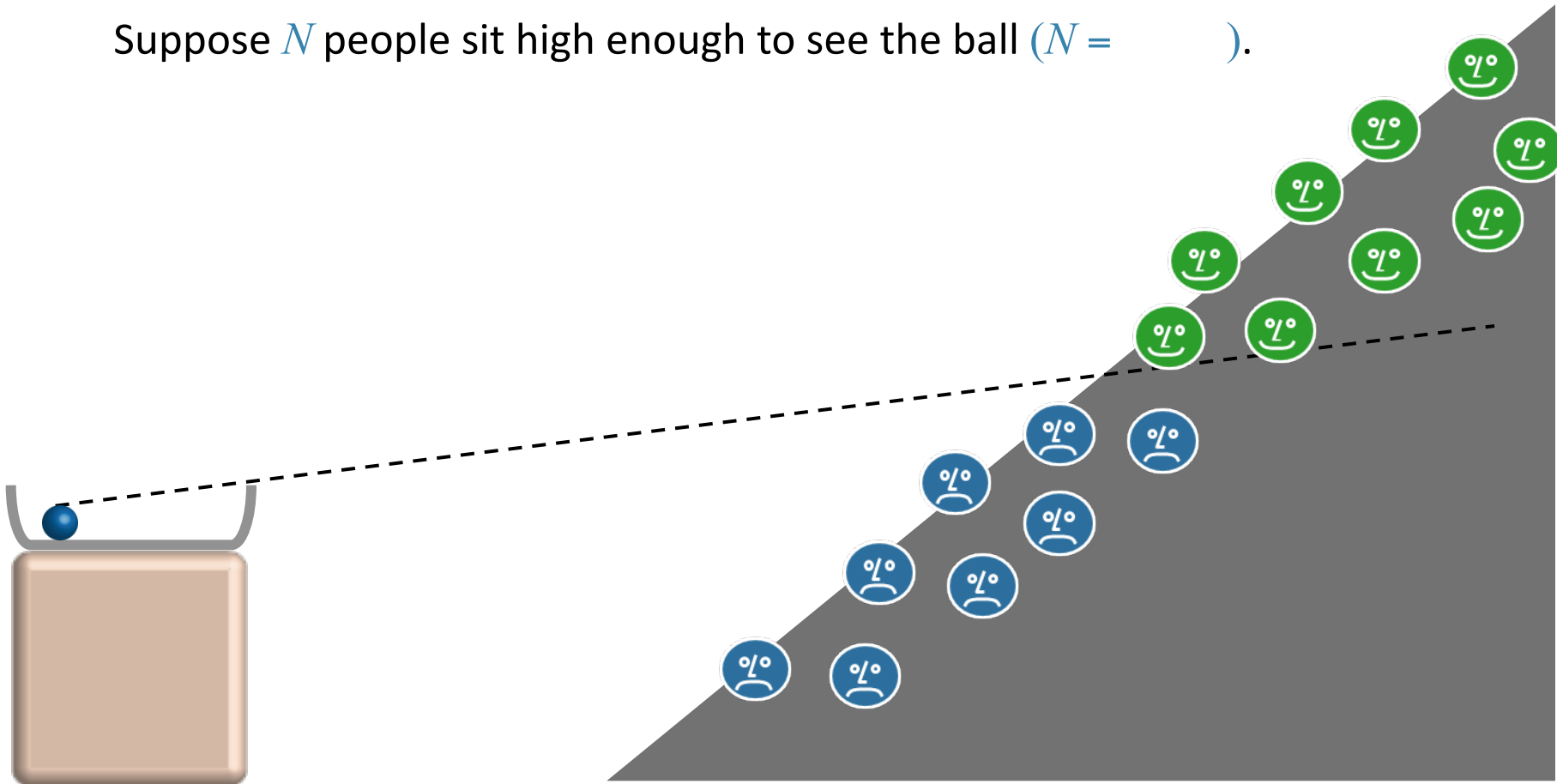
Some of the light also reflects off the surface of the water. If the incident light is initially unpolarized, the reflected light will be

- ☐ unpolarized
- ☒ somewhat horizontally polarized
- ☐ somewhat vertically polarized



A ball sits in the bottom of an otherwise empty tub at the front of the room.

Suppose  $N$  people sit high enough to see the ball ( $N =$  ).

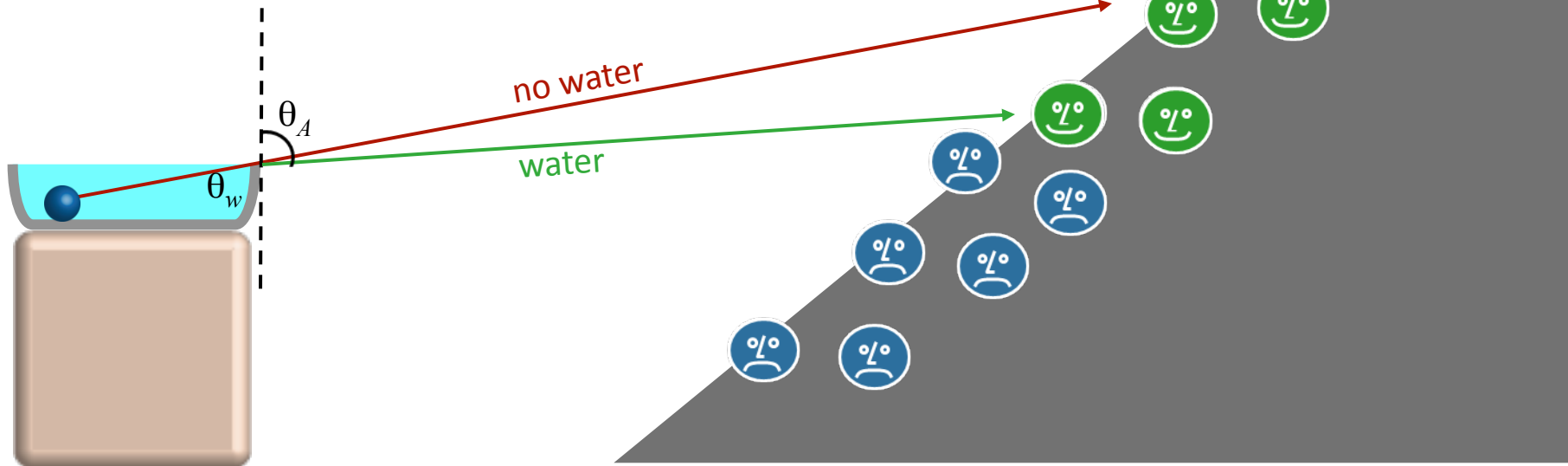


A ball sits in the bottom of an otherwise empty tub at the front of the room.  
Suppose  $N$  people sit high enough to see the ball ( $N =$  ).

Suppose I fill the tub with water but the ball doesn't move.

Will more or less people see the ball?

- A) More people will see the ball
- B) Same # will see the ball
- C) Less people will see the ball

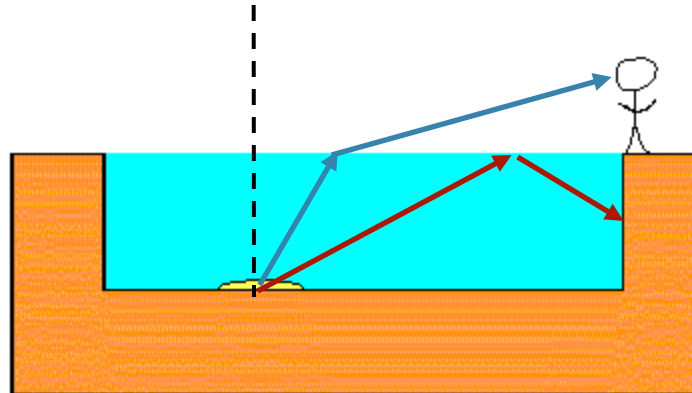


Snell's Law: ray bent away from normal going from water to air

# Checkpoint 10



A light is shining at the bottom of a swimming pool (shown in yellow in the figure ). A person is standing at the edge of the pool.



Can the person standing on the edge of the pool be prevented from seeing the light by total internal reflection at the water-air surface?

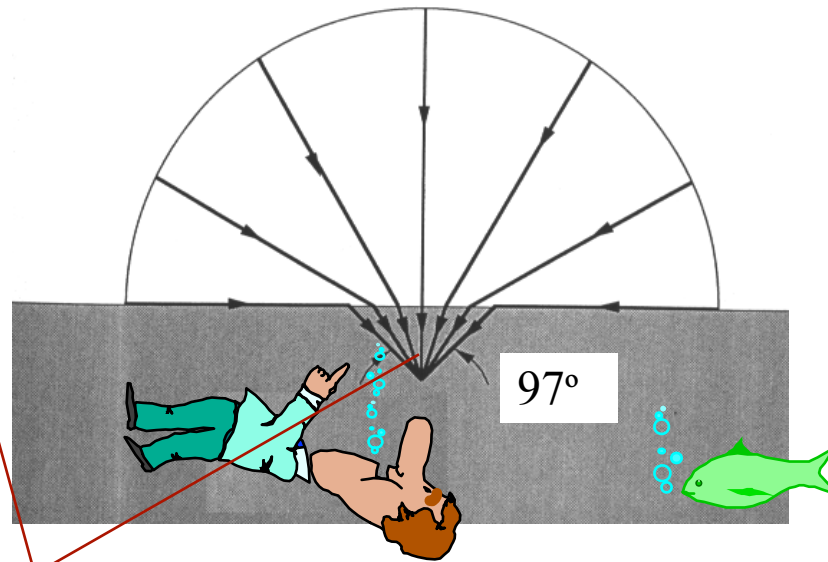
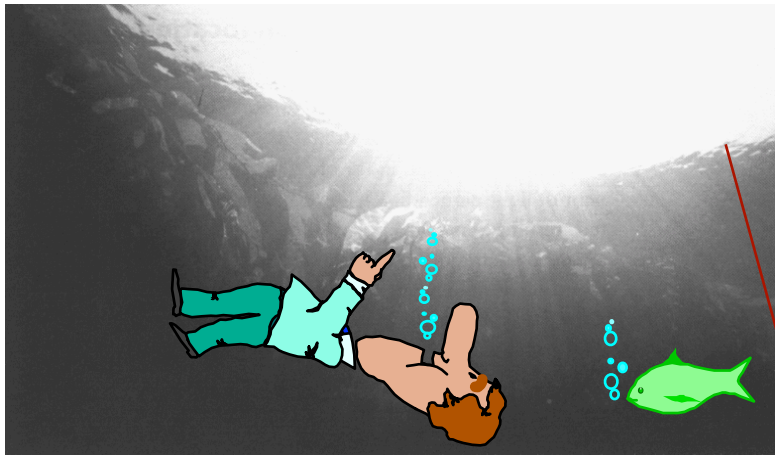
☐ yes ☒ no

The light would go out in all directions, so only some of it would be internally reflected. The person would see the light that escaped after being refracted.

Draw some rays

# Example: Refraction at Water/Air Interface

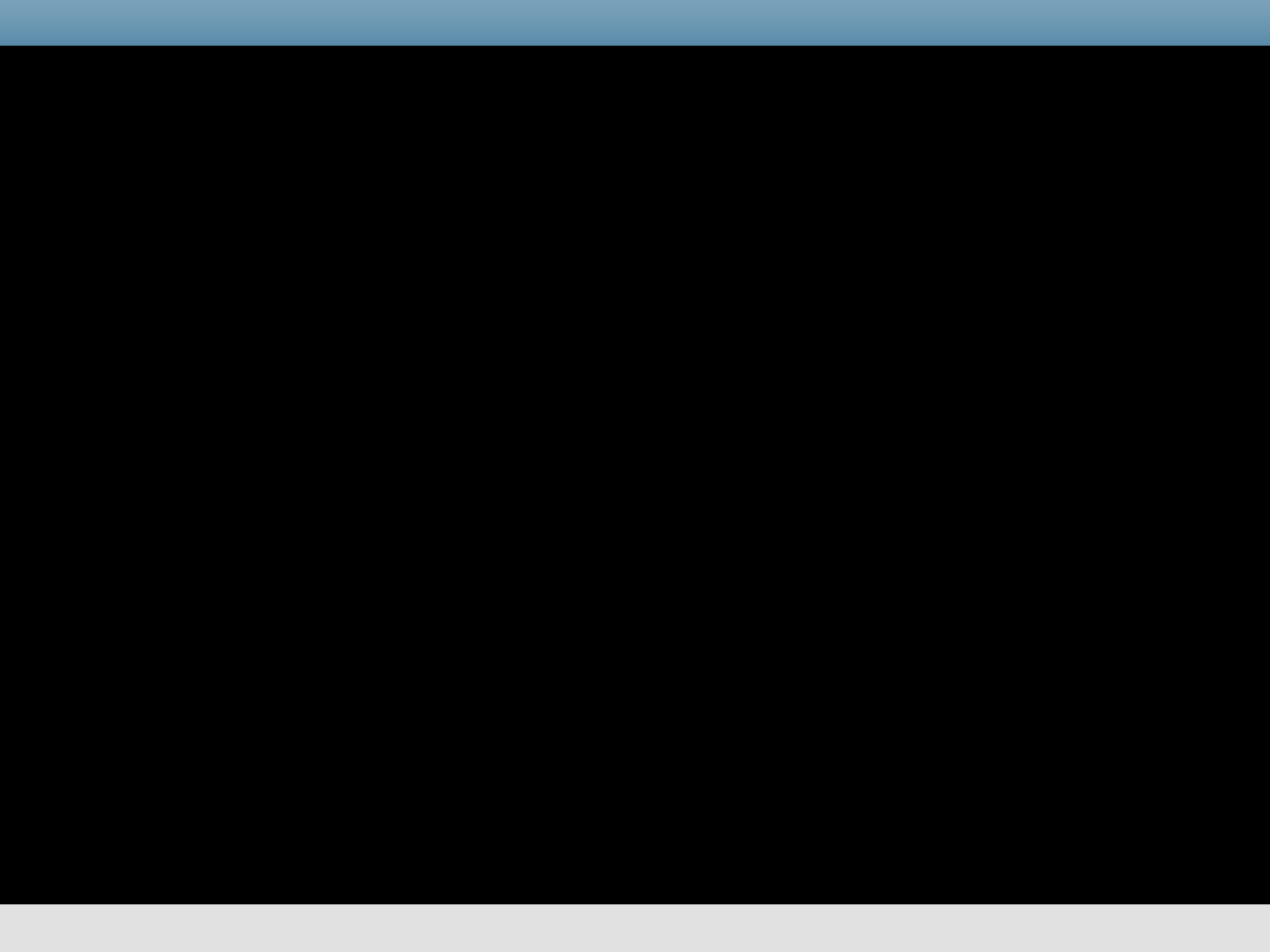
## Diver's illusion



Diver sees all of horizon  
refracted into a  $97^\circ$  cone

$$\theta_1 = 90^\circ \quad \rightarrow \quad \sin \theta_2 = \frac{n_1}{n_2} \sin 90^\circ = \frac{n_1}{n_2} = \frac{1}{1.33} \quad \rightarrow \quad \theta_2 = 48.5^\circ$$

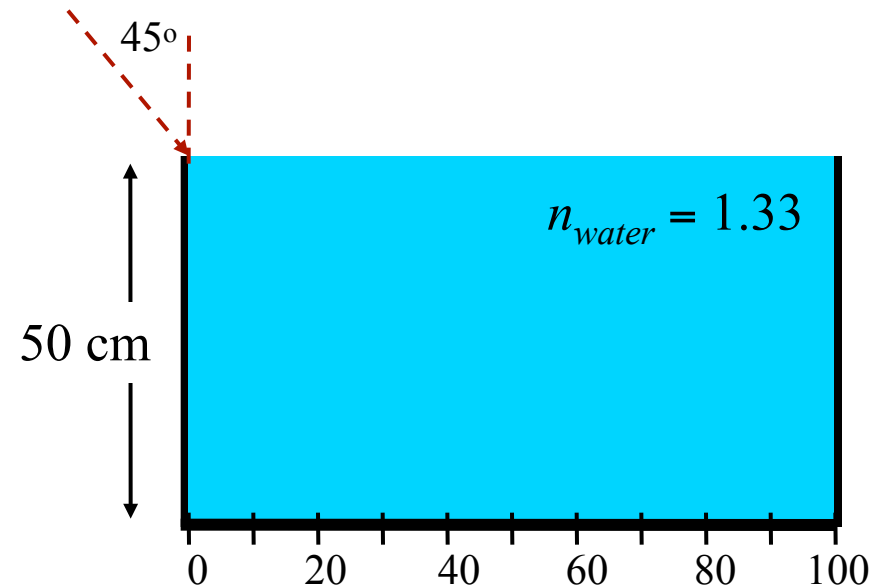




# Exercise

A meter stick lies at the bottom of a rectangular water tank of height 50cm. You look into the tank at an angle of  $45^\circ$  relative to vertical along a line that skims the top edge of the tank.

What is the smallest number on the ruler that you can see?



## Conceptual Analysis:

- Light is refracted at the surface of the water

## Strategy:

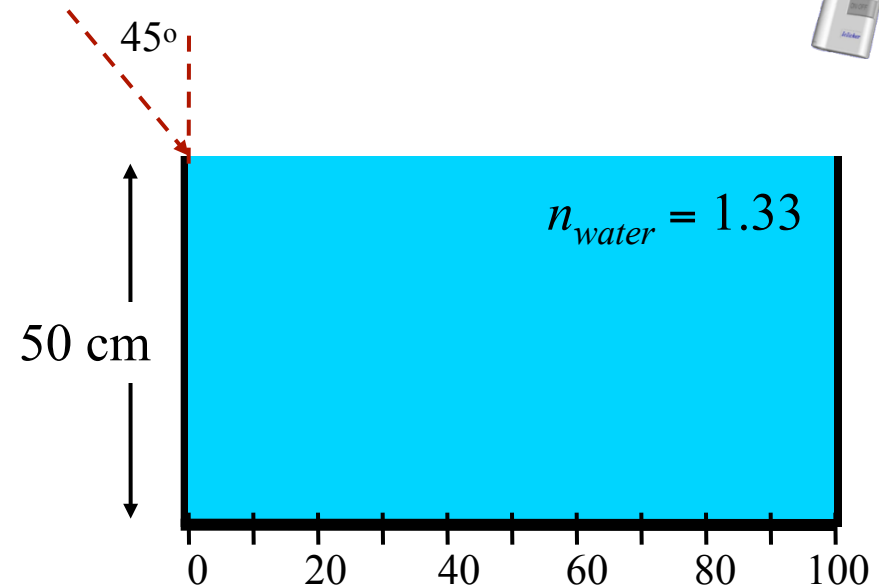
- Determine the angle of refraction in the water and extrapolate this to the bottom of the tank.

# Exercise



A meter stick lies at the bottom of a rectangular water tank of height 50cm. You look into the tank at an angle of  $45^\circ$  relative to vertical along a line that skims the top edge of the tank.

What is the smallest number on the ruler that you can see?



If you shine a laser into the tank at an angle of  $45^\circ$ , what is the refracted angle  $\theta_R$  in the water ?

A)  $\theta_R = 28.3^\circ$

B)  $\theta_R = 32.1^\circ$

C)  $\theta_R = 38.7^\circ$

**Snell's Law:**  $n_{air} \sin(45) = n_{water} \sin(\theta_R)$

→  $\sin(\theta_R) = n_{air} \sin(45^\circ) / n_{water} = 0.532$

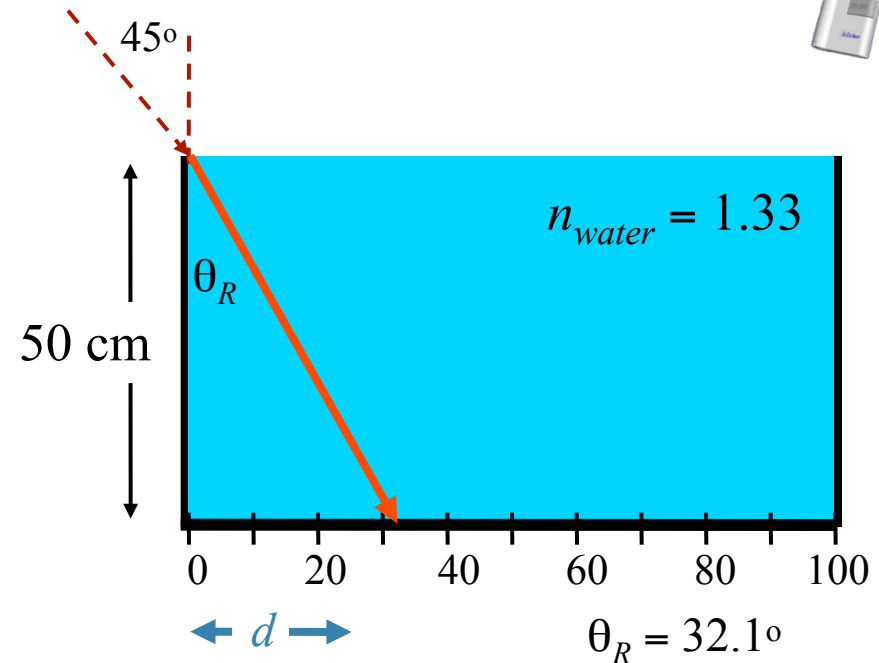
→  $\theta_R = \sin^{-1}(0.532) = 32.1^\circ$

# Exercise



A meter stick lies at the bottom of a rectangular water tank of height 50cm. You look into the tank at an angle of  $45^\circ$  relative to vertical along a line that skims the top edge of the tank.

What is the smallest number on the ruler that you can see?



What number on the ruler does the laser beam hit?

A) 31.4 cm

B) 37.6 cm

C) 44.1 cm

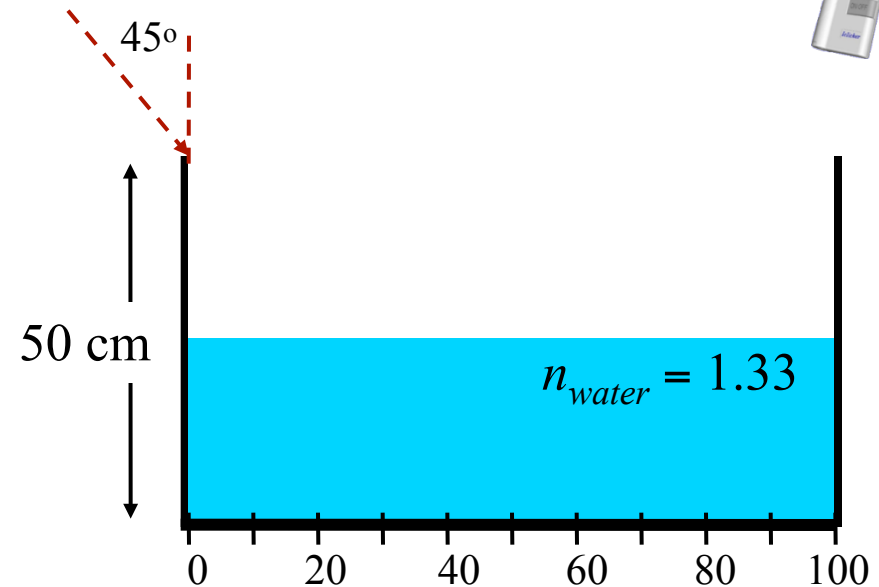
$$\tan(\theta_R) = d/50$$

$$\rightarrow d = \tan(32.1^\circ) \times 50\text{cm} = 31.4\text{cm}$$

# Follow-Up

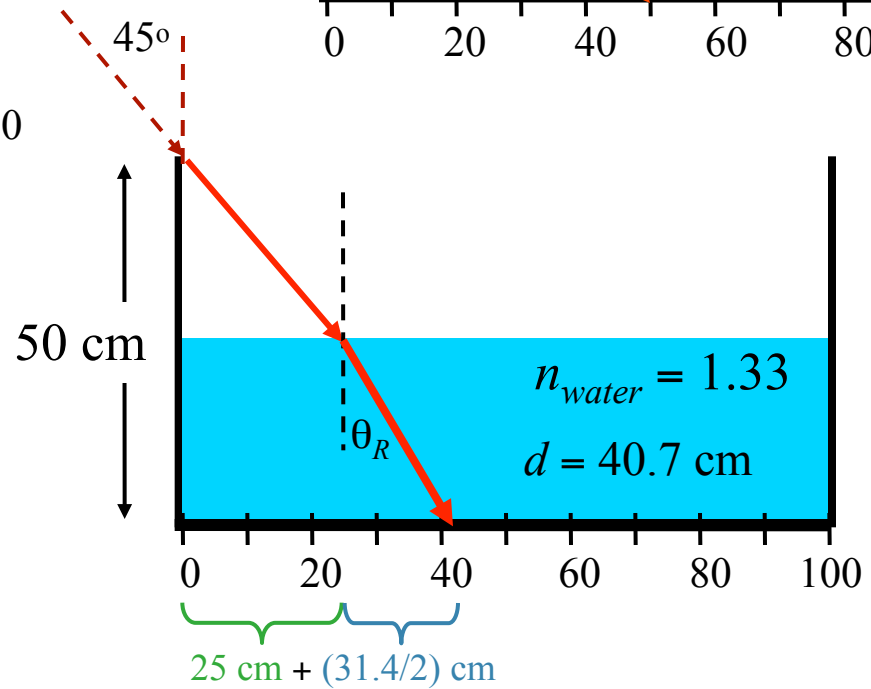
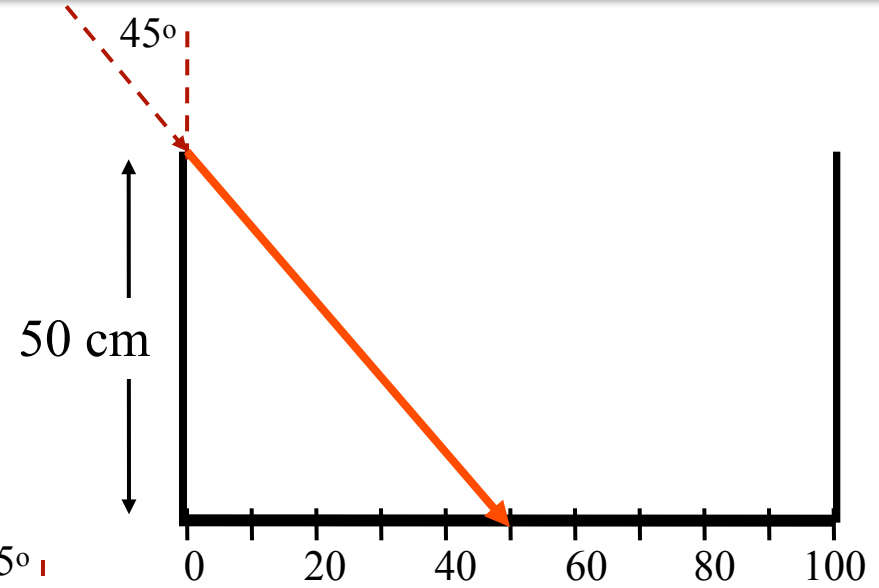
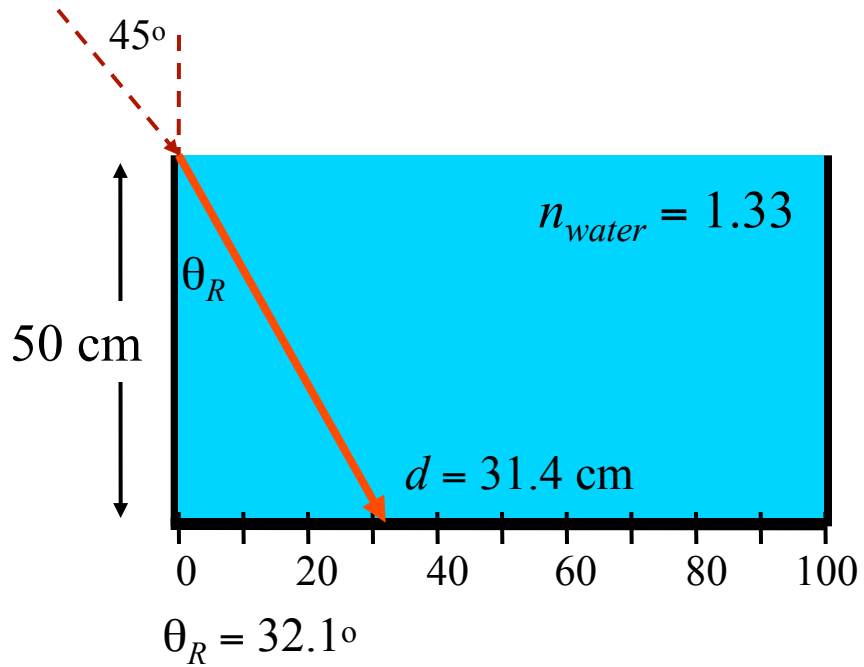


A meter stick lies at the bottom of a rectangular water tank of height 50cm. You look into the tank at an angle of  $45^\circ$  relative to vertical along a line that skims the top edge of the tank.



If the tank were half full of water, what number would the laser hit?  
(When full, it hit at 31.4 cm)

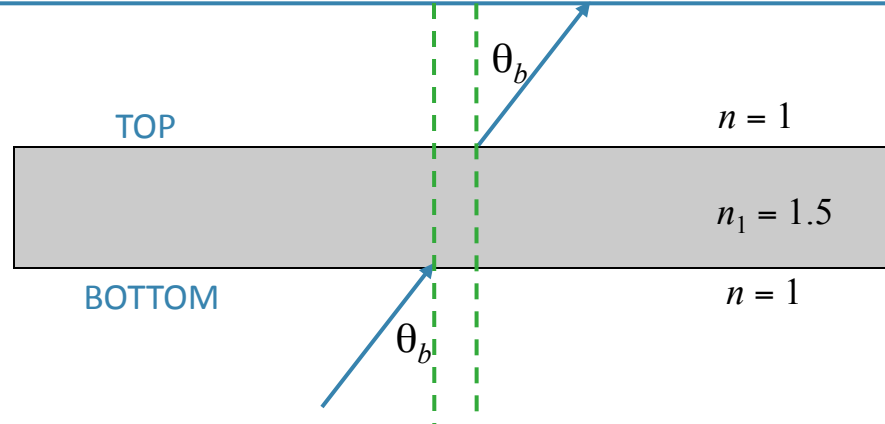
- A) 25 cm      B) 31.4 cm      C) 32.0 cm      D) 40.7 cm      E) 44.2 cm



# More Practice



A monochromatic ray enters a slab with  $n_1 = 1.5$  at an angle  $\theta_b$  as shown.



- A) Total internal reflection at the top occurs for all angles  $\theta_b$ , such that  $\sin\theta_b < 2/3$
- B) Total internal reflection at the top occurs for all angles  $\theta_b$ , such that  $\sin\theta_b > 2/3$
- C) There is no angle  $\theta_b$  ( $0 < \theta_b < 90^\circ$ ) such that total internal reflection occurs at top.

Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



$n \sin\theta$  is "conserved"



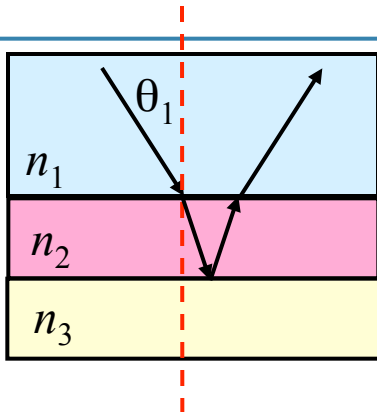
Ray exits to air with same angle as it entered!



# Follow-Up



A ray of light moves through a medium with index of refraction  $n_1$  and is incident upon a second material ( $n_2$ ) at angle  $\theta_1$  as shown. This ray is then totally reflected at the interface with a third material ( $n_3$ ). Which statement must be true?

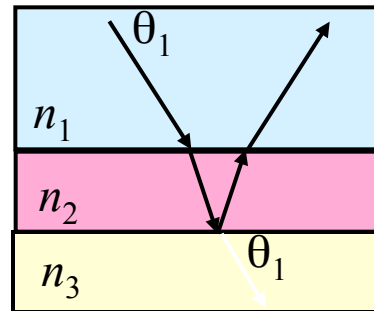


A)  $n_3 < n_1$

B)  $n_1 < n_3 \leq n_2$

C)  $n_3 \geq n_2$

If  $n_1 = n_3$



Want larger angle of refraction in  $n_3$



$n_3 < n_1$