- Calculate the x and y components of the e field due to the semicircle.

  Linear charge density on the semicircle assuming that $Q = 1\mu C$

$$\lambda = \frac{3Q}{a\pi/2} = \frac{6Q}{a\pi}$$

Find the x component of the E field at the origin:

$$E_{1x} = \frac{1}{4\pi\epsilon_0}\int_0^{\pi/2}\frac{\lambda}{a^2}a\cos\theta d\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{a} = \frac{1}{4\pi\epsilon_0}\frac{1.9098Q}{a^2}$$

$$E_{1y} = \frac{1}{4\pi\epsilon_0}\int_0^{\pi/2}\frac{\lambda}{a^2}a\sin\theta d\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{a} = \frac{1}{4\pi\epsilon_0}\frac{1.9098Q}{a^2}$$

Magnitude of E_1

$$E_1 = \sqrt{E_x^2 + E_y^2} = \frac{\sqrt{2}}{4\pi\epsilon_0}\frac{\lambda}{a} = \frac{\sqrt{2}}{4\pi\epsilon_0}\frac{1}{a}\frac{6Q}{a\pi}$$

For the 3 Q's find the x and y components o the E field. Note that all the Q's are the same distance from the origin.

$$E_{2x} = -\frac{1}{4\pi\epsilon_0}\left(\frac{Q}{a^2} + \frac{Q}{a^2}\cos 45^\circ\right) = -\frac{1}{4\pi\epsilon_0}\left(\frac{1.707Q}{a^2}\right)$$

Similarly

$$E_{2y} = -\frac{1}{4\pi\epsilon_0}\left(\frac{Q}{a^2} + \frac{Q}{a^2}\sin 45^\circ\right) = -\frac{1}{4\pi\epsilon_0}\left(\frac{1.707Q}{a^2}\right)$$

The net electric field at the origin is

$$E_x = E_y = E_{1x} + E_{2x} = \frac{1}{4\pi\epsilon_0}\frac{0.203Q}{a^2}$$

- First convert the surface charge density to linear charge density

$$\lambda = 2\pi r_1\sigma = 0.314\ \mu\text{C/m}$$

(a) The field $r < r_1$ $E_0 = 0$ because it is conductor at equilibrium.

(b) for $r_1 > r > r_2$

$$E_1 = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

Surface charge density at r_2 :

$$\sigma_2 = \frac{\lambda}{2\pi r_2} = 16.67\ \mu\text{C/m}^2$$

Surface charge density at r_3 must balance charge at r_2

$$\sigma_2 = \frac{\lambda}{2\pi r_3} = 12.5\ \mu\text{C/m}^2$$

E field inside shield is zero. Outside: $r > r_4$

$$E_4 = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

(h) Potential difference: integrate from r_1 to r_2. Note that the potential doesn't change inside the conducting shield so no need to integrate from r_2 to r_3.

$$\Delta V = \frac{1}{2\pi\epsilon_0}\int_{r_1}^{r_2}\frac{\lambda}{r}dr = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{r_2}{r_1}$$

(i) Capacitance per unit length

$$C/L = \frac{\lambda}{\Delta V} = \frac{2\pi\epsilon_0}{\ln(3/2)} = 137\ \text{pF/m}$$

For a 30 cm length

$$C = 137 \times 0.30 = 41\ \text{pF}$$

- (a) After a long time the capacitor charges to the potential difference across R_2 at which time there is no current thru R_3 (therefore no voltage across it) and into the capacitor, .

  (b) The voltage of the battery is divided among R_1, R_1, R_2 and R_4 so the voltage across R_2 is

$$V_2 = V\frac{R_2}{R_1 + R_2 + R_3} = 30\frac{20}{10 + 20 + 10} = 15V$$

  (c) immediately after the switch is opened current through R_1 and R_4 is blocked and is zero. The capacitor discharges through R_3 and R_2.

$$I_2 = I_3 = 15V/1020\Omega = 0.0147A$$

  (d) The decay time constant of the capacitor $\tau = (R_2 + R_3)C = (1020\Omega)(10\mu F) = 10200\mu s$

$$15e^{-t/\tau} = 5$$
$$t = -\tau \ln 5/15 = 11.2ms$$

- Node equation for the three currents

  (%o1) $$I2 = I3 + I1$$

  KVL for the left hand loop:

  (%o2) $$-V1 + I2\,R2 + I1\,R1 = 0$$

  KVL for the outside loop:

  (%o3) $$V3 + V1 + I3\,R3 - I1\,R1 = 0$$

  Solving these three equations (by hook or crook–student should explain method)

  (%o5) $$[[I1 = \frac{R2\,V3 + (R3 + R2)\ V1}{R2\ (R3 + R1) + R1\,R3}, I2 = \frac{R3\,V1 - R1\,V3}{R2\ (R3 + R1) + R1\,R3}, I3 = -\frac{R2\,V3 + R1\,V3 + R2\,V1}{R2\ (R3 + R1) + R1\,R3}]]$$

  Plugging in the numbers....

  (%o11) $$[[I1 = 3.5, I2 = -0.5, I3 = -4.0]]$$

  in amperes.