# *Electricity & Magnetism*
# *Lecture 2: Electric Fields*

## Today's Concepts:

A) The Electric Field

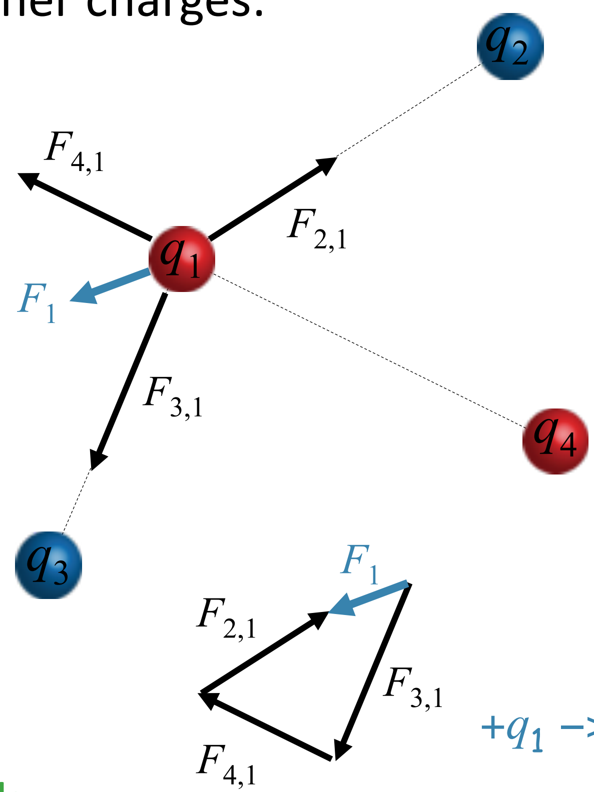
B) Continuous Charge Distributions

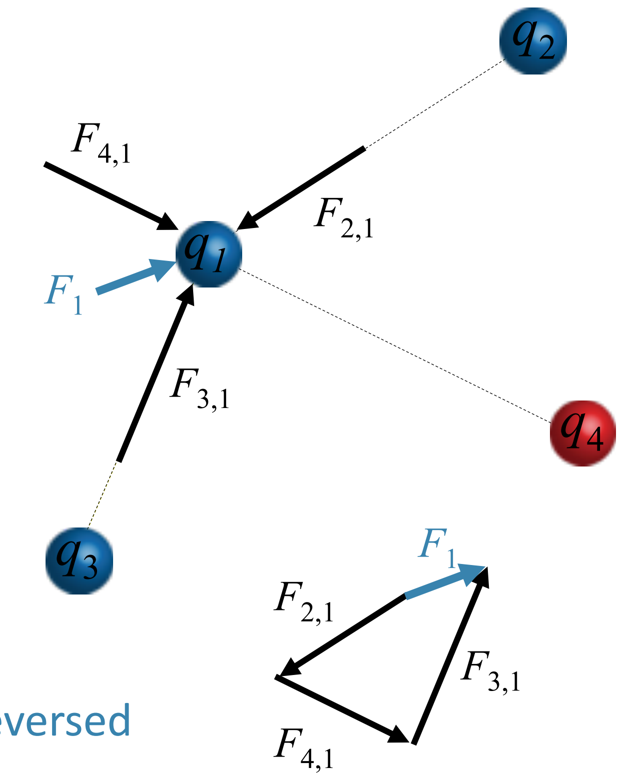
# *Your Comments and Questions*

- Will  Coulomb's law ever be used on objects that aren't point particles? If so, are we to assume the centre is where the charge is located?
  - yes
- id say this lecture was a decent recovery from the last one. by far easier to understand in my opinion. lets hope the homework isnt bad tho
- I dont get anything. Please help
  - Conceptual Physics — Hewitt

# Coulomb's Law (from last time)

If there are more than two charges present, the total force on any given charge is just the vector sum of the forces due to each of the other charges:



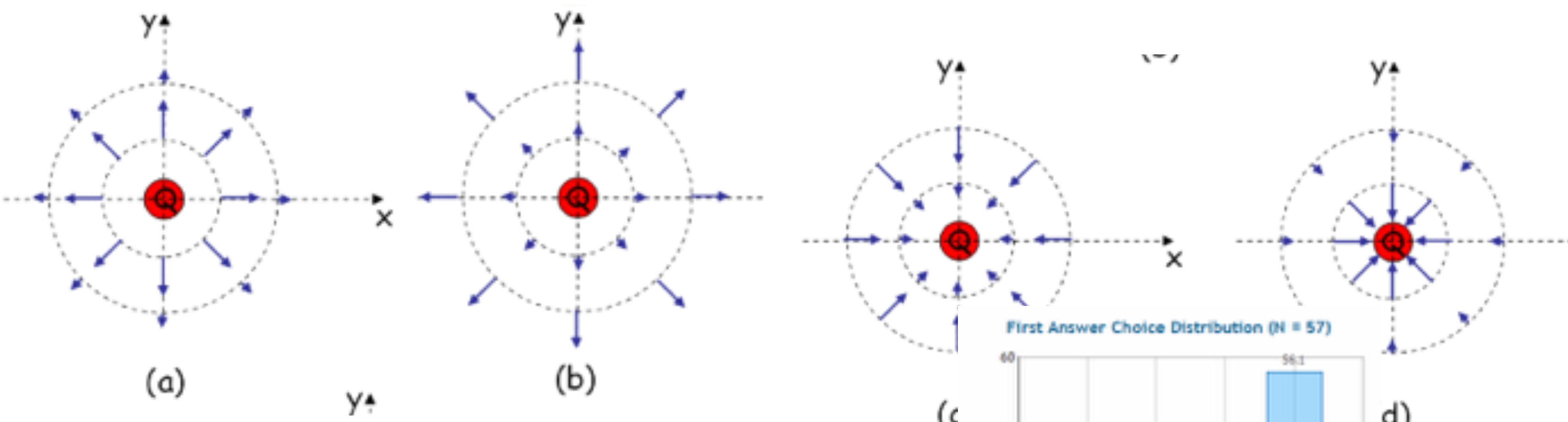



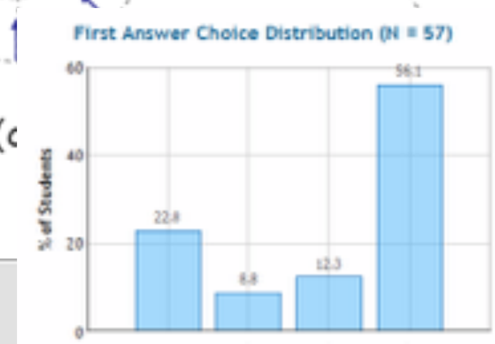

$+q_1$ –> $-q_1$ Þ direction reversed

MATH:

$$\vec{F}_1 = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} + \frac{kq_1q_3}{r_{13}^2}\hat{r}_{13} + \frac{kq_1q_4}{r_{14}^2}\hat{r}_{14} \longrightarrow \vec{E} = \frac{\vec{F}_1}{q_1} = \frac{kq_2}{r_{12}^2}\hat{r}_{12} + \frac{kq_3}{r_{13}^2}\hat{r}_{13} + \frac{kq_4}{r_{14}^2}\hat{r}_{14}$$

In the diagrams below, the magnitude and direction of the electric field is represented by the length and direction of the blue arrows. Which of the diagrams best represents the electric field from a **negative** charge?

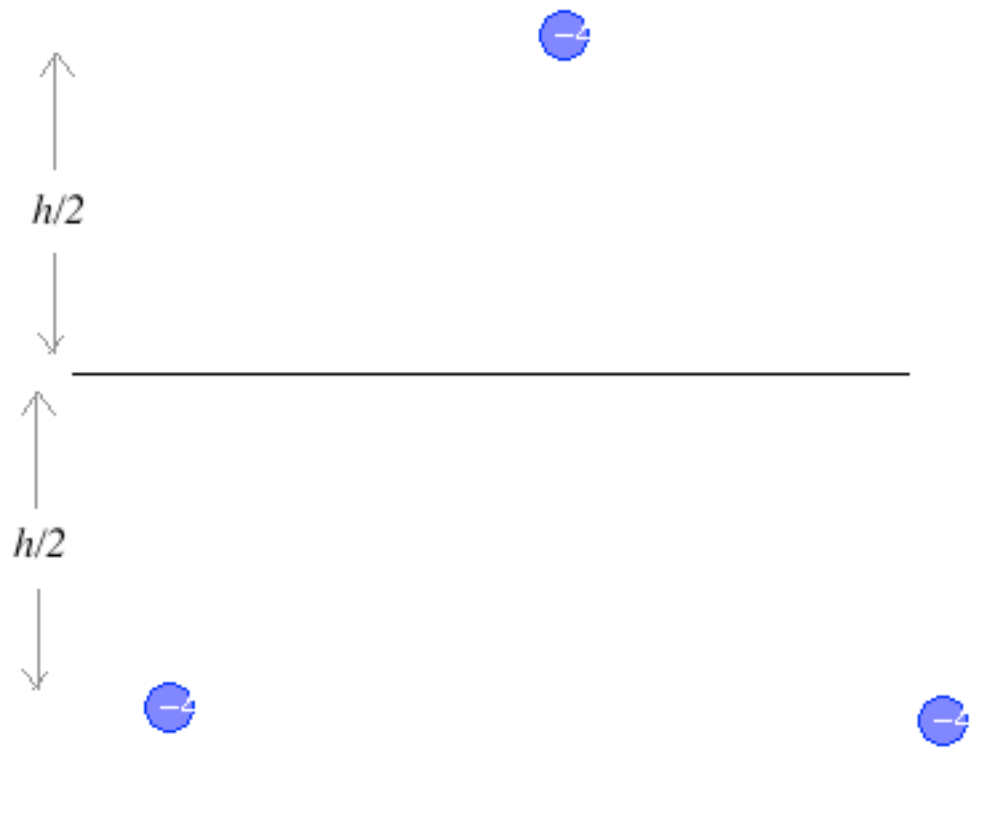


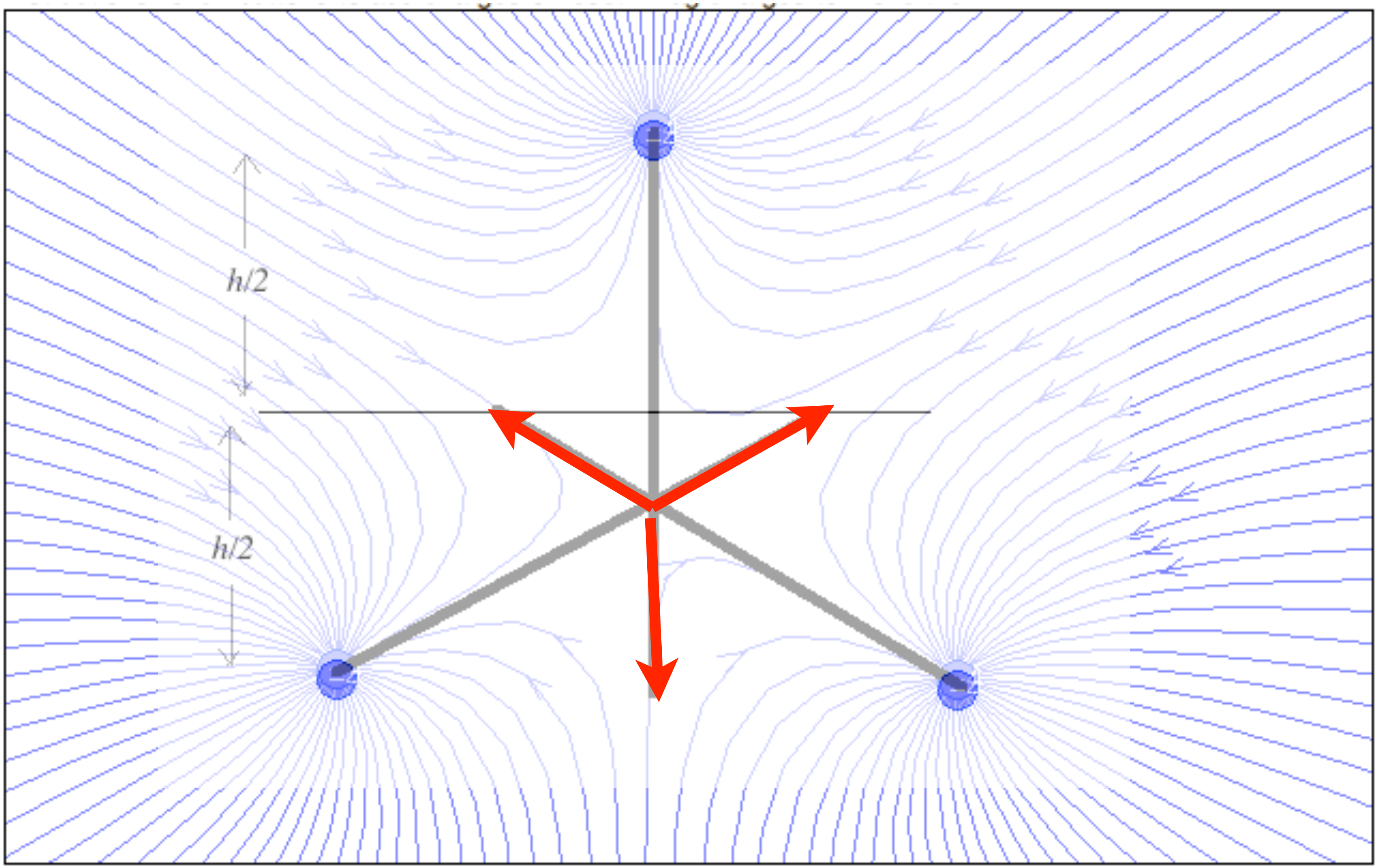
3 equal – charges are at the corners of an equilateral triangle

Where is the E field 0? The black line is 1/2 way between the base and the top charge.

A. Above the black line
B. Below the black line
C. on the black line
D. nowhere except infinity
E. somewhere else




Electric Field PhET


h/2
h/2

# Electric Field

"What exactly does the electric field that we calculate mean/represent? "

"What is the essence of an electric field? "

The electric field $E$ at a point in space is simply the force per unit charge at that point.

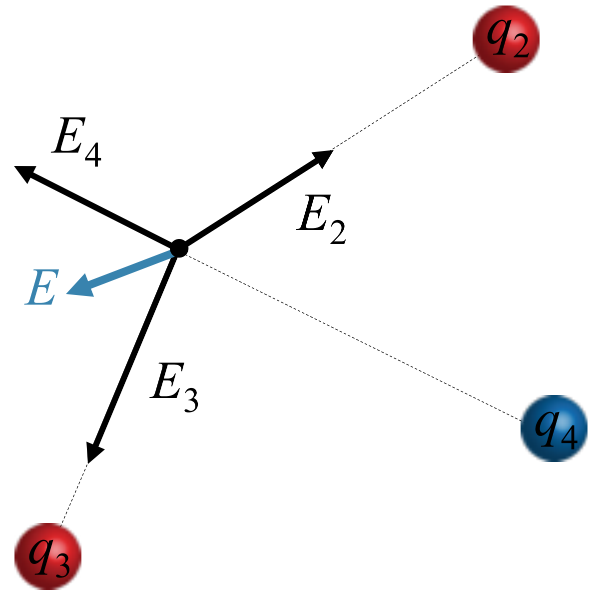
$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric field due to a point charged particle

$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$

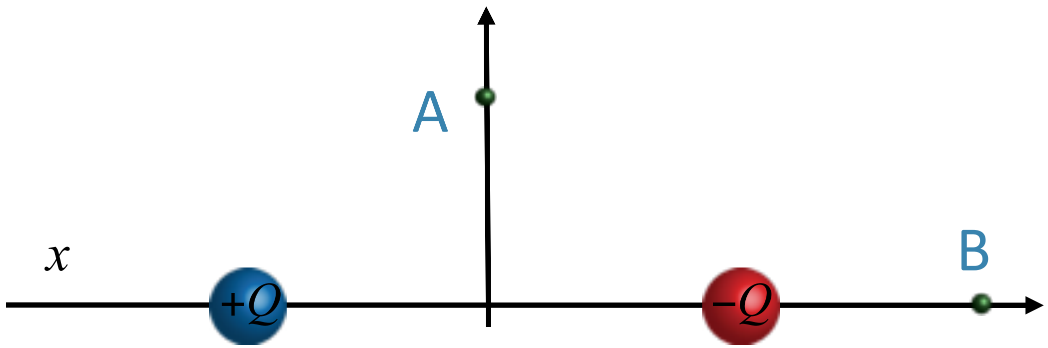
$$\vec{E} = \sum_i k\frac{Q_i}{r_i^2}\hat{r}_i$$

Superposition




Field points toward negative and Away from positive charges.

# CheckPoint: Electric Fields1




Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

What is the direction of the electric field at point A?

A) Up

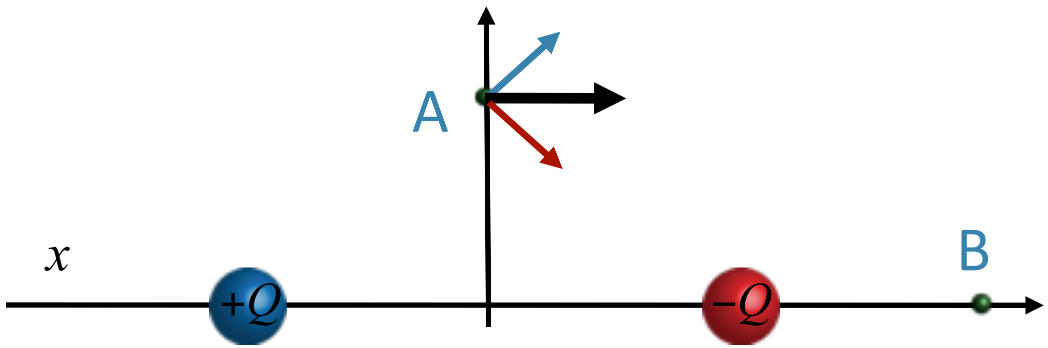
B) Down

C) Left

D) Right

E) Zero

# CheckPoint Results: Electric Fields1

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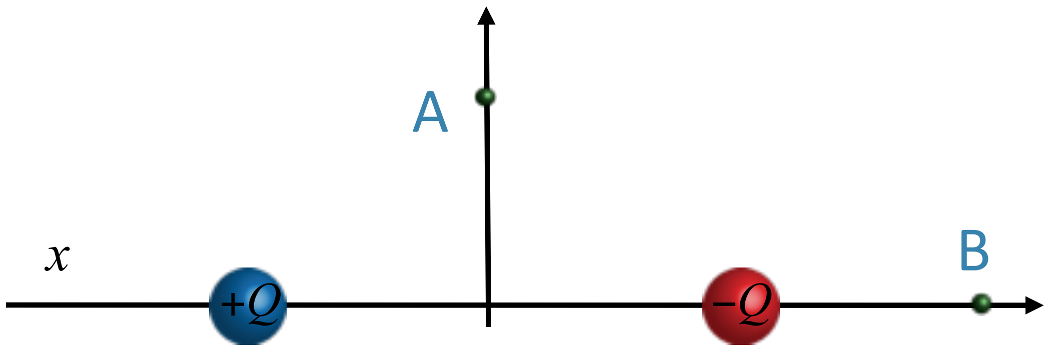
What is the direction of the electric field at point A?

A) Up
B) Down
C) Left
D) Right
E) Zero

# CheckPoint: Electric Fields2

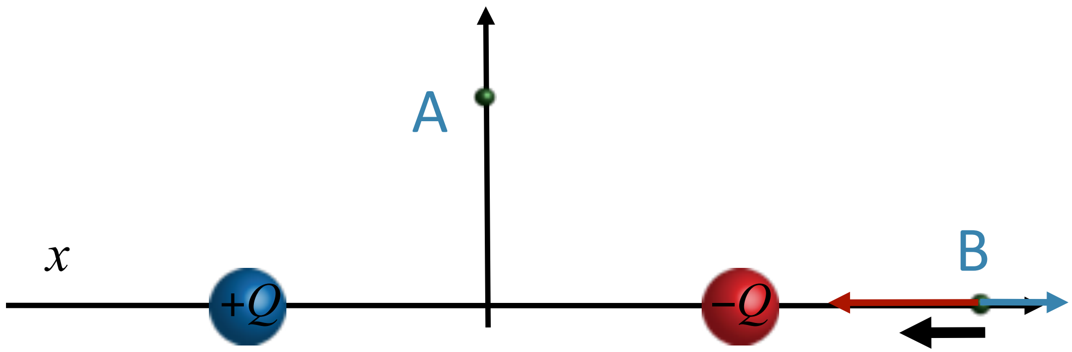



Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

What is the direction of the electric field at point B?

A) Up
B) Down
C) Left
D) Right
E) Zero

# CheckPoint Results: Electric Fields2




Two equal, but opposite charges are placed on the x axis. The positive charge is placed to the left of the origin and the negative charge is placed to the right, as shown in the figure above.

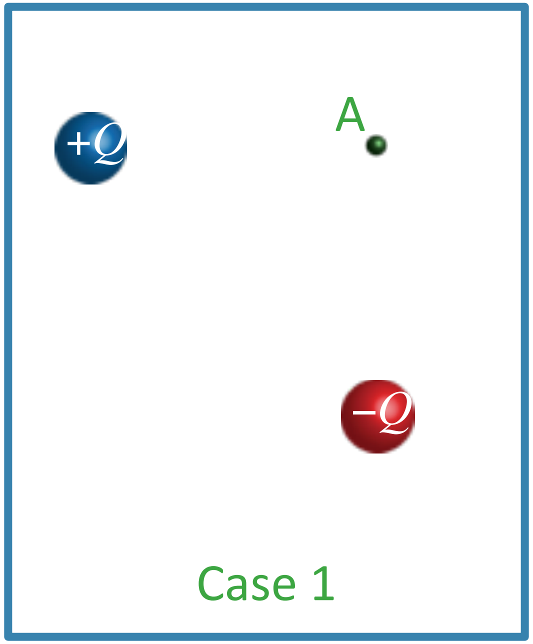
What is the direction of the electric field at point B?

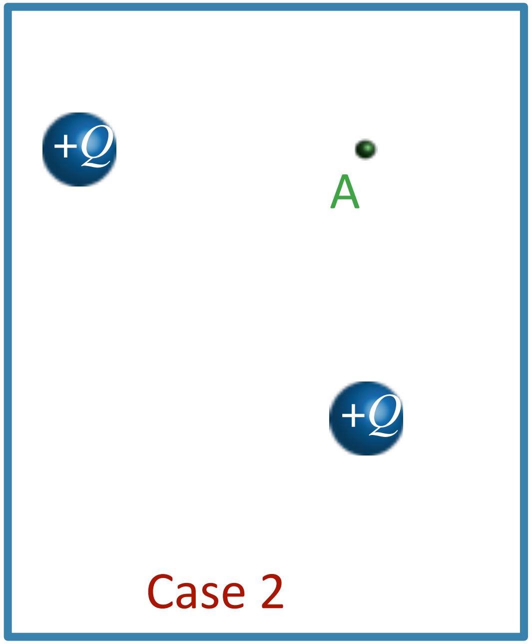
A) Up
B) Down
C) Left
D) Right
E) Zero

Polarization Demo

# CheckPoint: Magnitude of Field (2 Charges)

In which of the two cases shown below is the magnitude of the electric field at the point labeled A the largest?

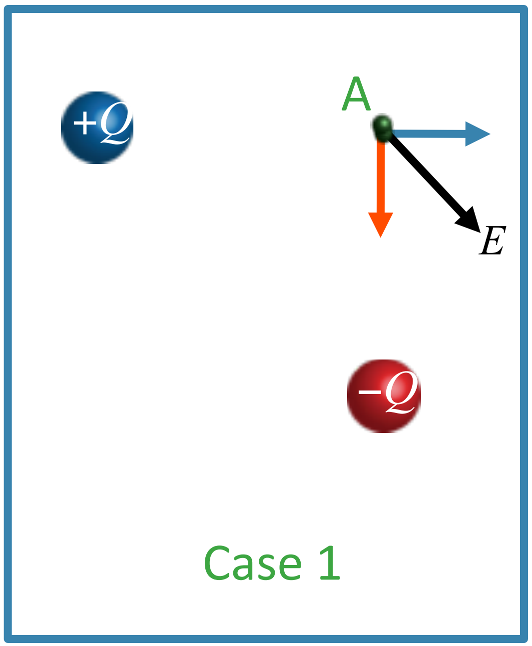


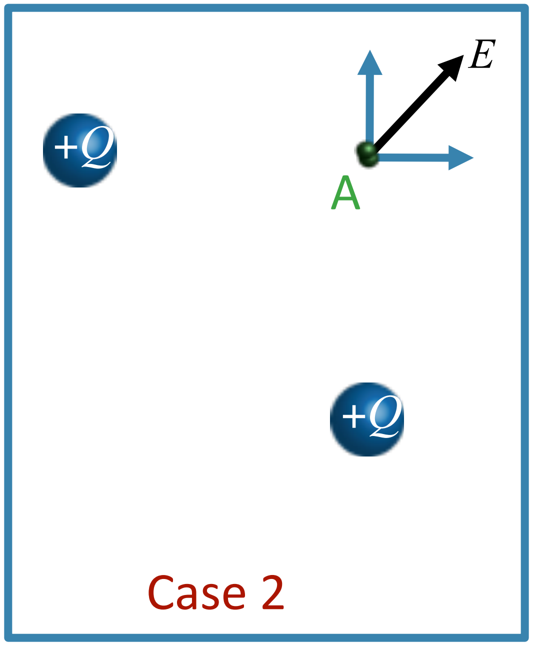




A) Case 1
B) Case 2
C) Equal

# CheckPoint Results: Magnitude of Field (2 Chrg)

In which of the two cases shown below is the magnitude of the electric field at the point labeled A the largest?



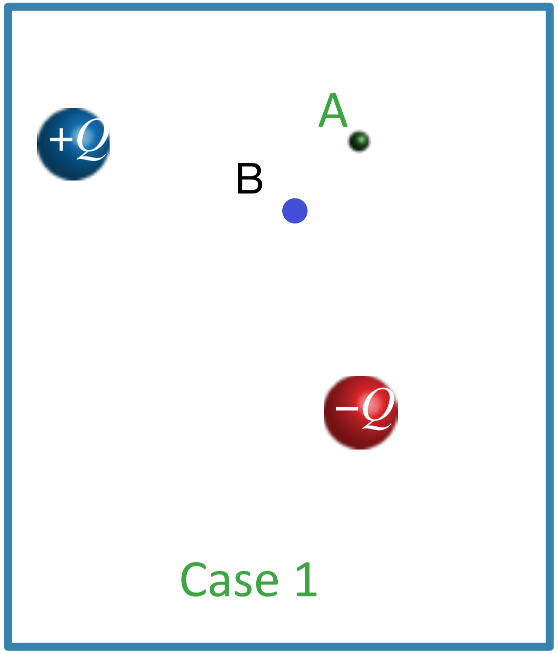



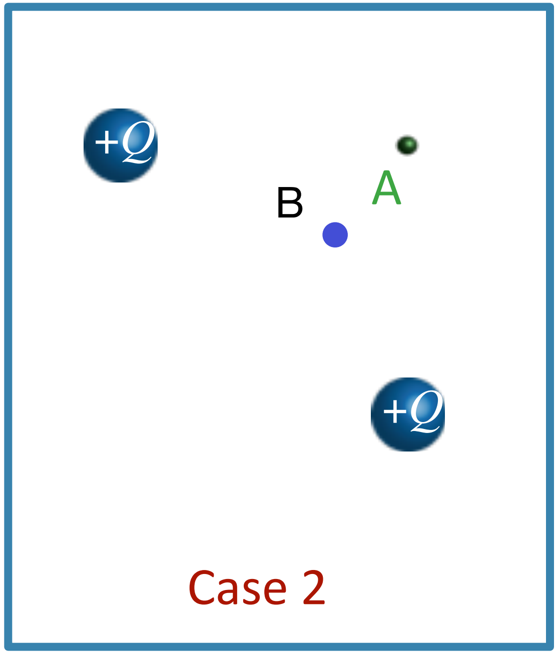

A) Case 1
B) Case 2
C) Equal

"The upper left +Q only affects the x direction in both and the lower right (+/-)Q only affects the y direction so in both, nothing cancels out, so they'll have the same magnitude. "

# CheckPoint: Magnitude of Field (2 Charges)

In which of the two cases shown below is the magnitude of the electric field at the point labeled B the largest?



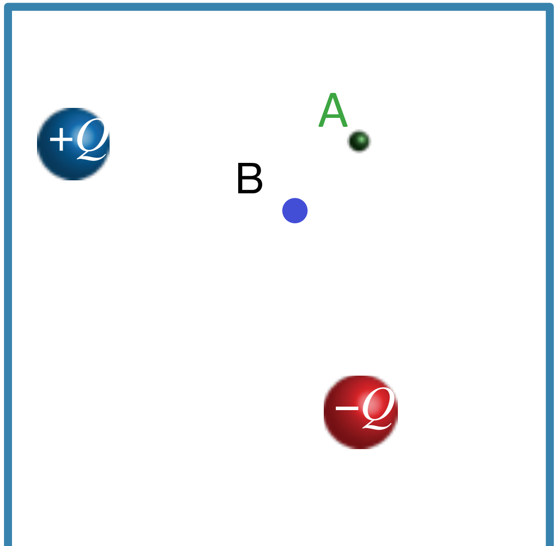

A) Case 1

B) Case 2

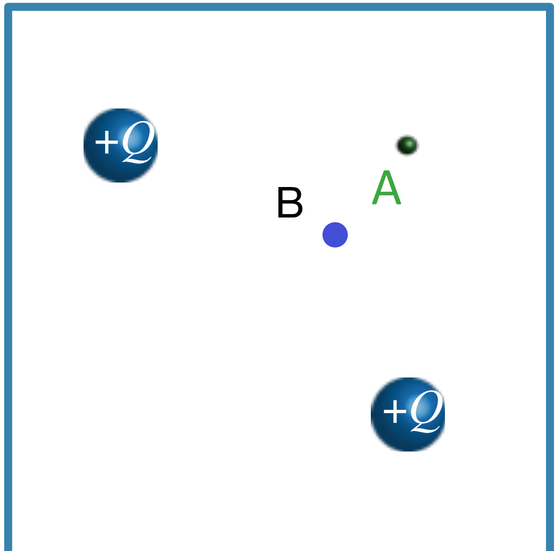
C) Equal

# CheckPoint: Magnitude of Field (2 Charges)

In which of the two cases shown below is the magnitude of the electric field at the point labeled B the largest?




Case 1

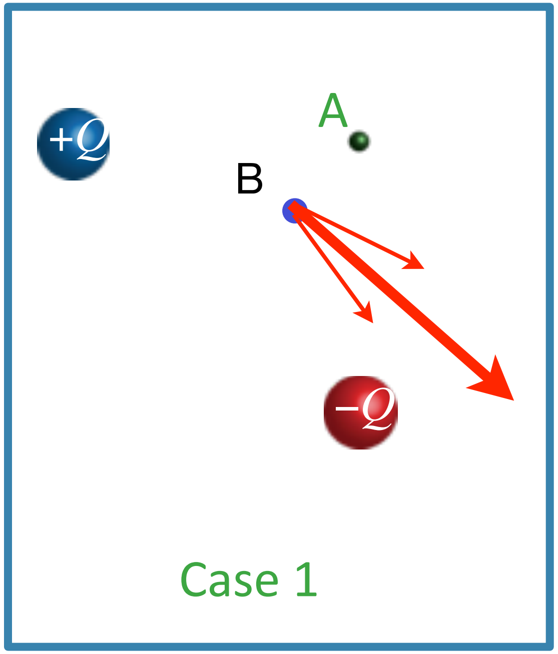


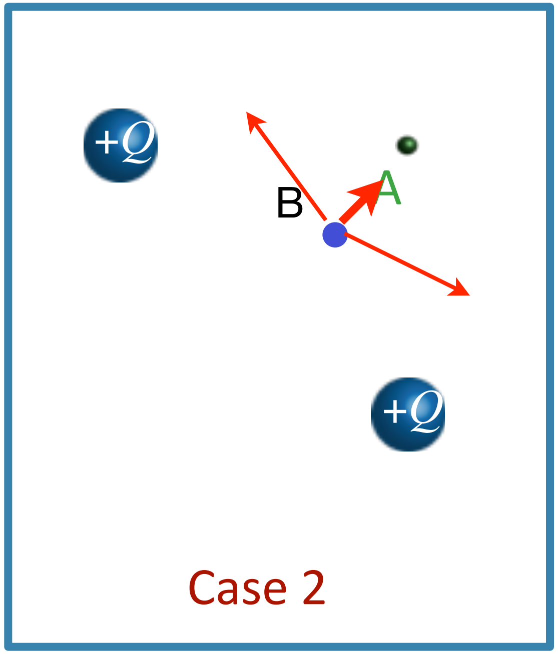

Case 2

A) Case 1
B) Case 2
C) Equal

# CheckPoint: Magnitude of Field (2 Charges)

In which of the two cases shown below is the magnitude of the electric field at the point labeled B the largest?




A) Case 1

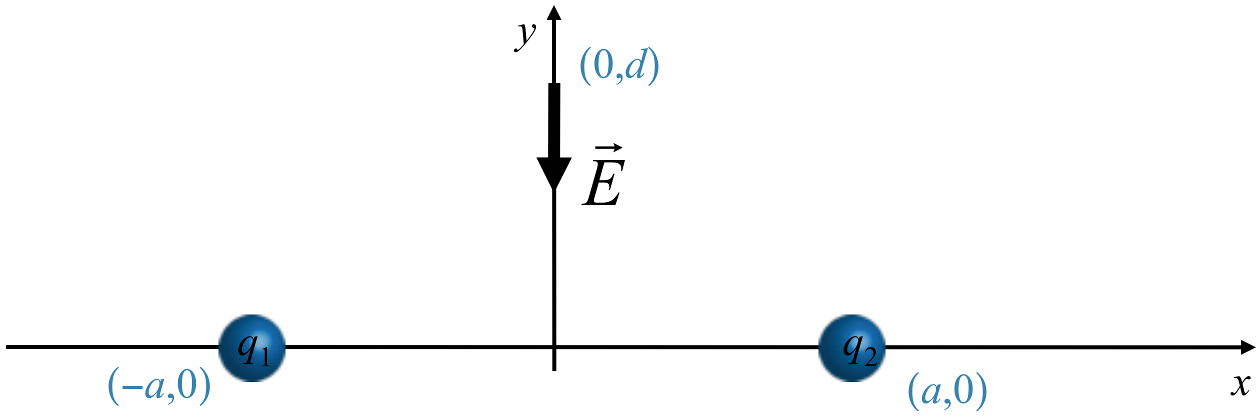
B) Case 2

C) Equal

# Clicker Question: Two Charges

Two charges $q_1$ and $q_2$ are fixed at points $(-a,0)$ and $(a,0)$ as shown.

Together they produce an electric field at point $(0,d)$ which is directed along the negative *y*-axis.



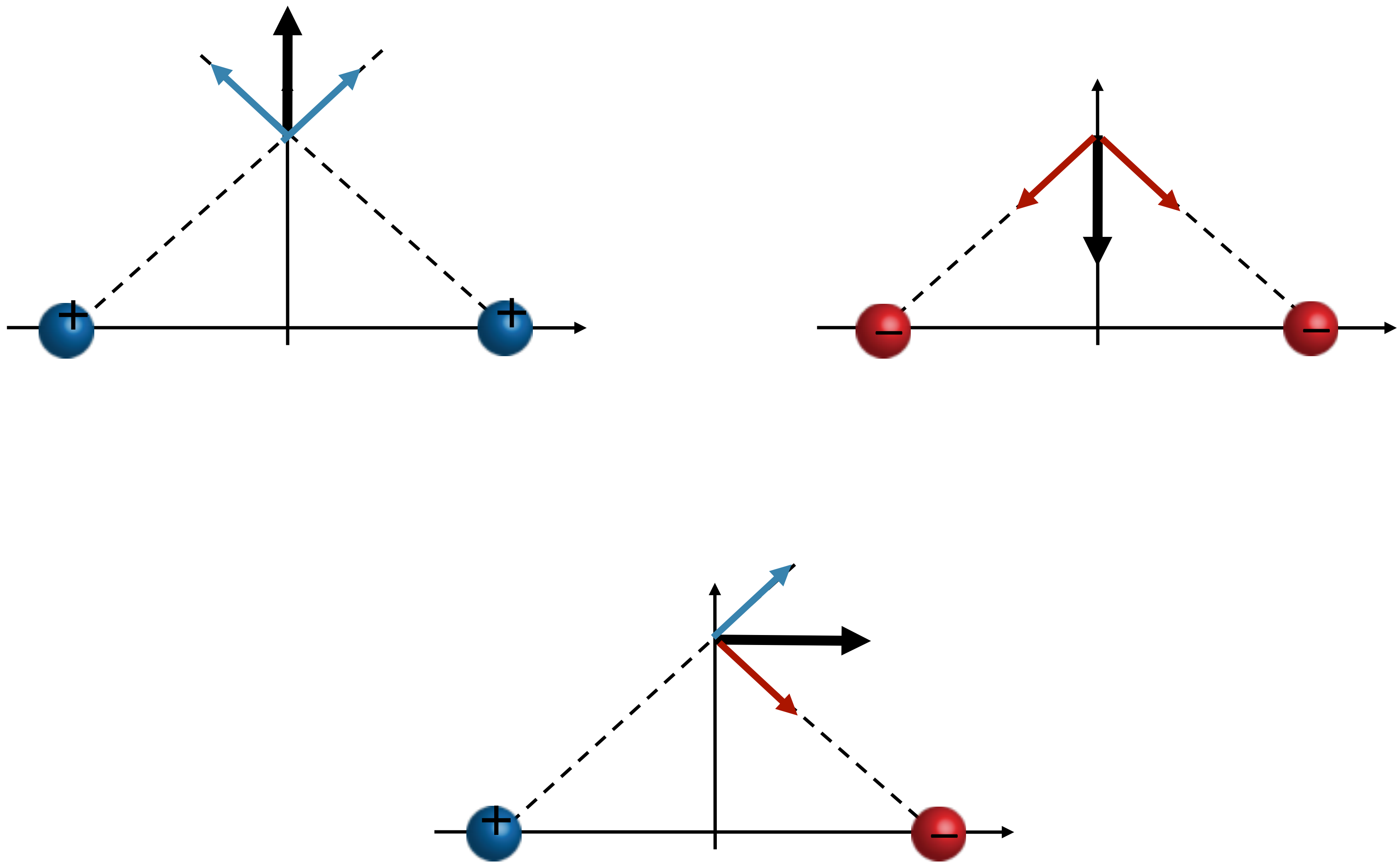

Which of the following statements is true:

A) Both charges are negative

B) Both charges are positive

C) The charges are opposite

D) There is not enough information to tell how the charges are related



# CheckPoint Results: Motion of Test Charge


A positive test charge q is released from rest at distance r away from a charge of +Q and a distance 2r away from a charge of +2Q. How will the test charge move immediately after being released?

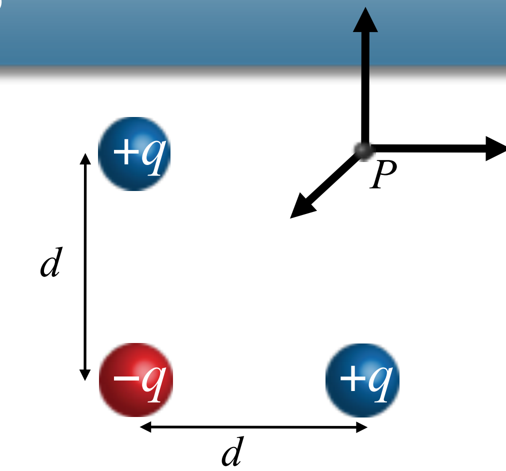
A) To the left
B) To the right
C) Stay still
D) Other

"The force is proportional to the charge divided by the square of the distance. Therefore, the force of the 2Q charge is 1/2 as much as the force of the Q charge. "

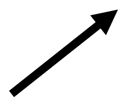
"Even though the charge on the right is larger, it is twice as far away, which makes the force it exherts on the test charge half that as the charge on the left, causing the charge to move to the right."

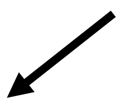
The ratio between the R and Q on both sides is 1:1 meaning they will result in the same magnitude of electric field acting in opposite directions, causing q to remain still.

# Electric Field Example




What is the direction of the electric field at point $P$, the unoccupied corner of the square?

A) 

B) 

C) $E = 0$

D) Need to know $d$

E) Need to know $d$ & $q$

Calculate $E$ at point $P$.

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i$$

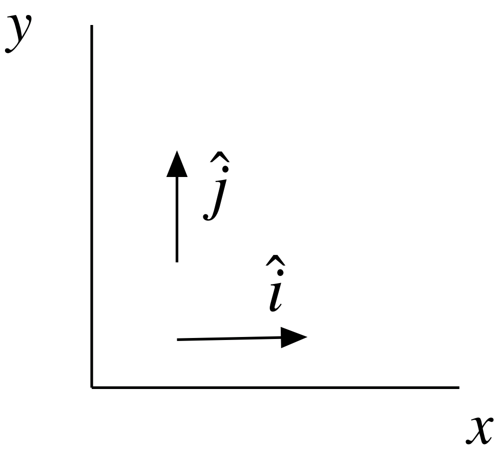
$$E_x = k\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2} \cos\frac{\pi}{4}\right)$$

$$E_y = k\left(\frac{q}{d^2} - \frac{q}{\left(\sqrt{2}d\right)^2} \sin\frac{\pi}{4}\right)$$

# Comment

What does the r with a hat mean? Is it like the unit vectors i-hat, j-hat and k-hat, or does it mean something else? The video didn't really explain it that well.

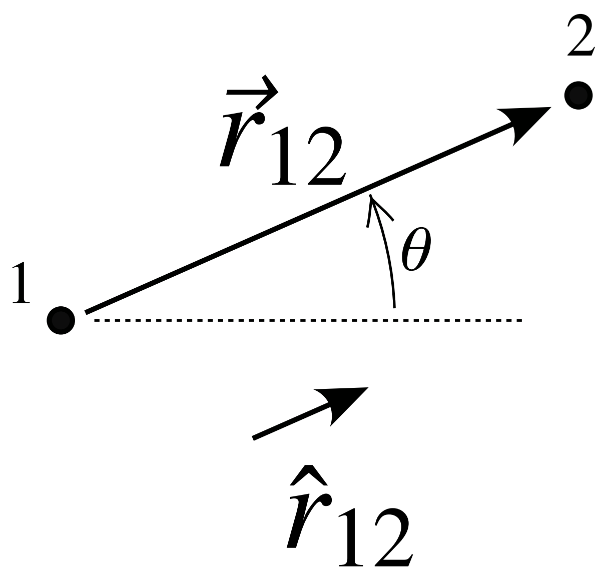
# More About $\hat{r}$



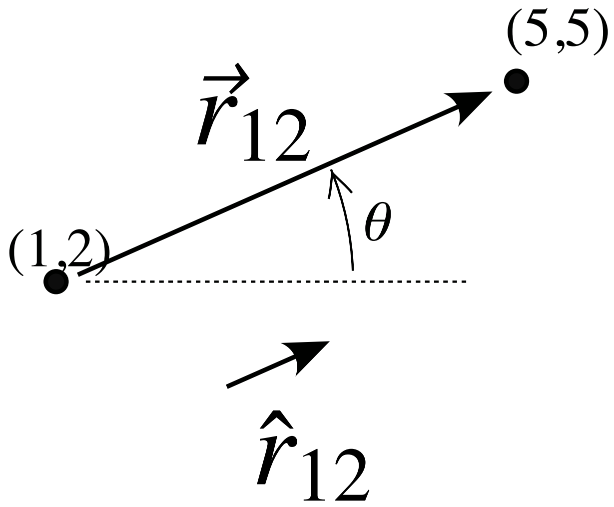

$$\vec{r}_{12} = r_{12}\cos\theta\,\hat{i} + r_{12}\sin\theta\,\hat{j}$$

$$\hat{r}_{12} = \frac{r_{12}\cos\theta\,\hat{i} + r_{12}\sin\theta\,\hat{j}}{r_{12}}$$

$$\hat{r}_{12} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$



# For example




$$\vec{r}_{12} = 4\hat{i} + 3\ \hat{j}$$

$$r_{12} = \sqrt{4^2 + 3^2} = 5$$

$$\cos\theta = \tfrac{4}{5}$$

$$\sin\theta = \tfrac{3}{5}$$

$$\hat{r}_{12} = \frac{r_{12}\cos\theta\ \hat{i} + r_{12}\sin\theta\ \hat{j}}{r_{12}}$$

$$\hat{r}_{12} = \tfrac{4}{5}\ \hat{i} + \tfrac{3}{5}\ \hat{j}$$

# Continuous Charge Distributions

"I don't understand the whole dq thing and lambda."

Summation becomes an integral (be careful with vector nature)

$$\vec{E} = \sum_i k \frac{Q_i}{r_i^2} \hat{r}_i \quad \longrightarrow \quad \vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

WHAT DOES THIS MEAN ?

Integrate over all charges $(dq)$

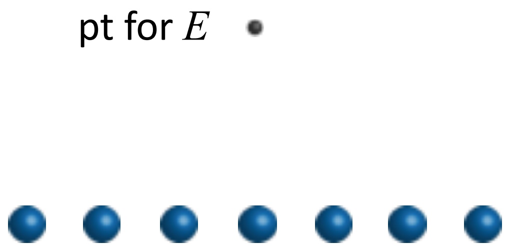
$r$ is vector from $dq$ to the point at which $E$ is defined

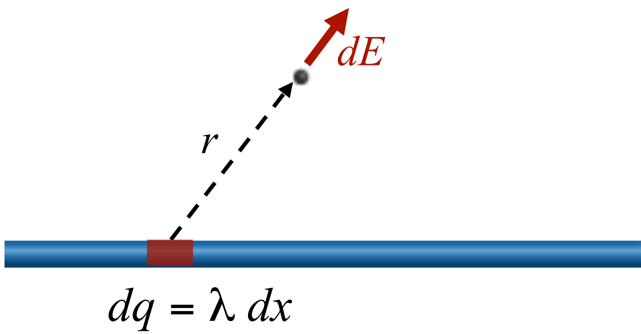
## Linear Example:

$$\lambda = Q/L$$

pt for $E$

charges





# Clicker Question: Charge Density

"I would like to know more about the charge density."

Linear ($\lambda = Q/L$) Coulombs/meter

Surface ($\sigma = Q/A$) Coulombs/meter$^2$

Volume ($\rho = Q/V$) Coulombs/meter$^3$

### Some Geometry

$$A_{sphere} = 4\pi R^2 \qquad A_{cylinder} = 2\pi RL$$

$$V_{sphere} = \tfrac{4}{3}\pi R^3 \qquad V_{cylinder} = \pi R^2 L$$

## What has more net charge?.

A) A sphere w/ radius 2 meters and volume charge density $\rho$ = 2 C/m$^3$

B) A sphere w/ radius 2 meters and surface charge density $\sigma$ = 2 C/m$^2$

C) Both A) and B) have the same net charge.

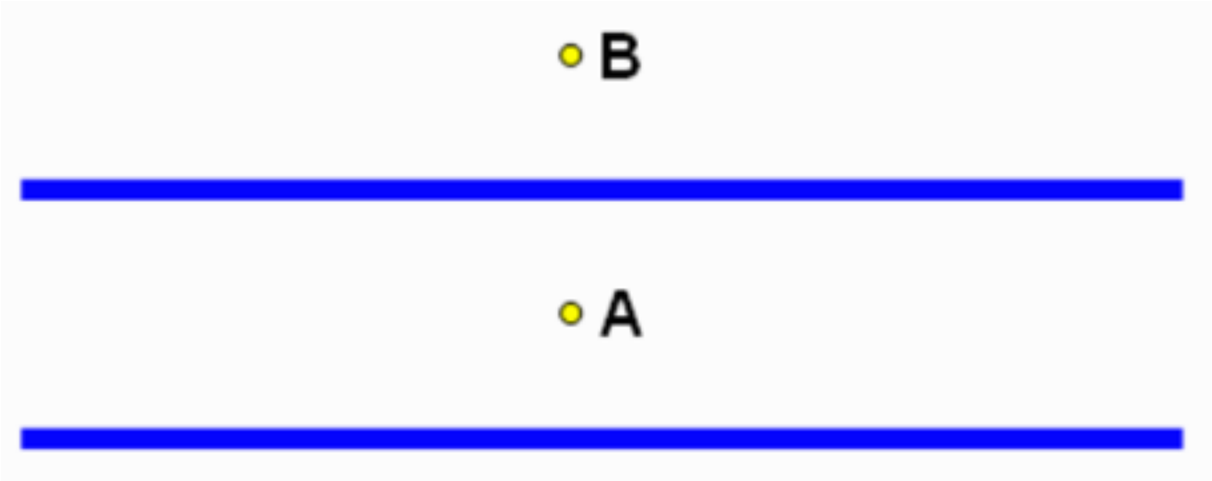
$$Q_A = \rho V = \rho \tfrac{4}{3}\pi R^3$$

$$Q_B = \sigma A = \sigma\, 4\pi R^2$$

$$\frac{Q_A}{Q_B} = \frac{\rho \tfrac{4}{3}\pi R^3}{\sigma\, 4\pi R^2} = \frac{1}{3}\frac{\rho}{\sigma} R$$

# CheckPoint: Two Lines of Charge

10) Two infinite lines of charge are shown below.




Both lines have identical charge densities $+\lambda$ C/m. Point A is equidistant from both lines and Point B is located above the top line as shown.

How does $E_A$, the magnitude of the electric field at point A, compare to $E_B$, the magnitude of the electric field at point B?

A) $E_A < E_B$

B) $E_A = E_B$

C) $E_A > E_B$

# CheckPoint: Two Lines of Charge

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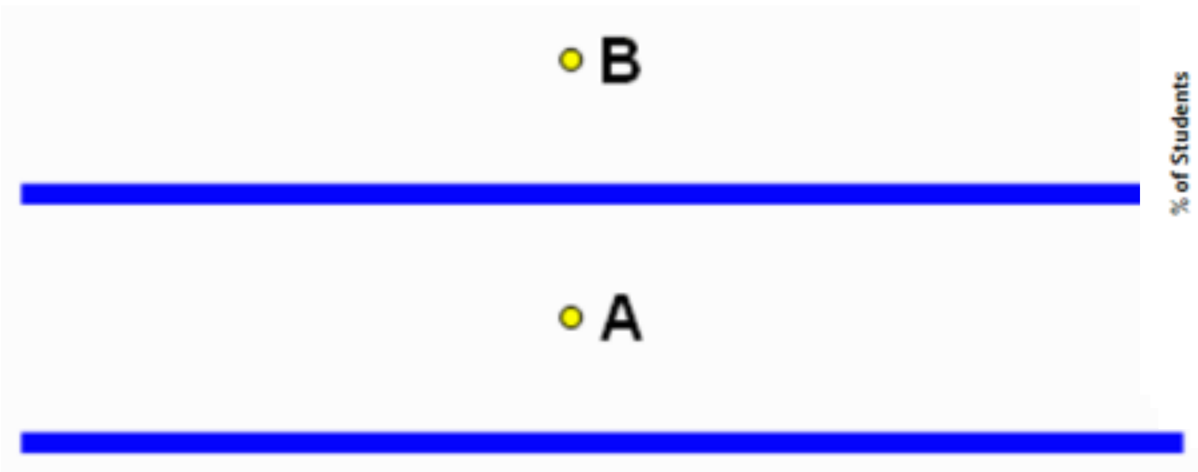
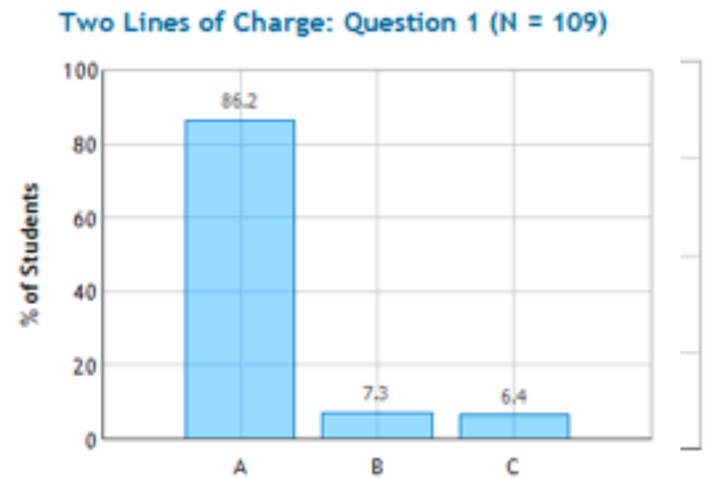



Both lines have identical charge densities +$\lambda$ C/m. Point A is equidistant from both lines and Point B is located a above the top line as shown.

How does $E_A$, the magnitude of the electric field at point A, compare to $E_B$, the magnitude of the electric field at point B?

A) $E_A < E_B$

B) $E_A = E_B$

C) $E_A > E_B$

Since point A is located inside both lines, the electric field at point A will be canceled out => E_A = 0, while at point B, the electric will be added up => E_A < E_B

# *Many comments like this…*

Please go over the infinite line.

# Infinite Line of Charge…

$$E_y = \int_{x=-\infty}^{x=\infty} k\frac{\lambda\,dx}{s^2}\cos\theta$$

substitute

$$s = \frac{r}{\cos\theta}$$

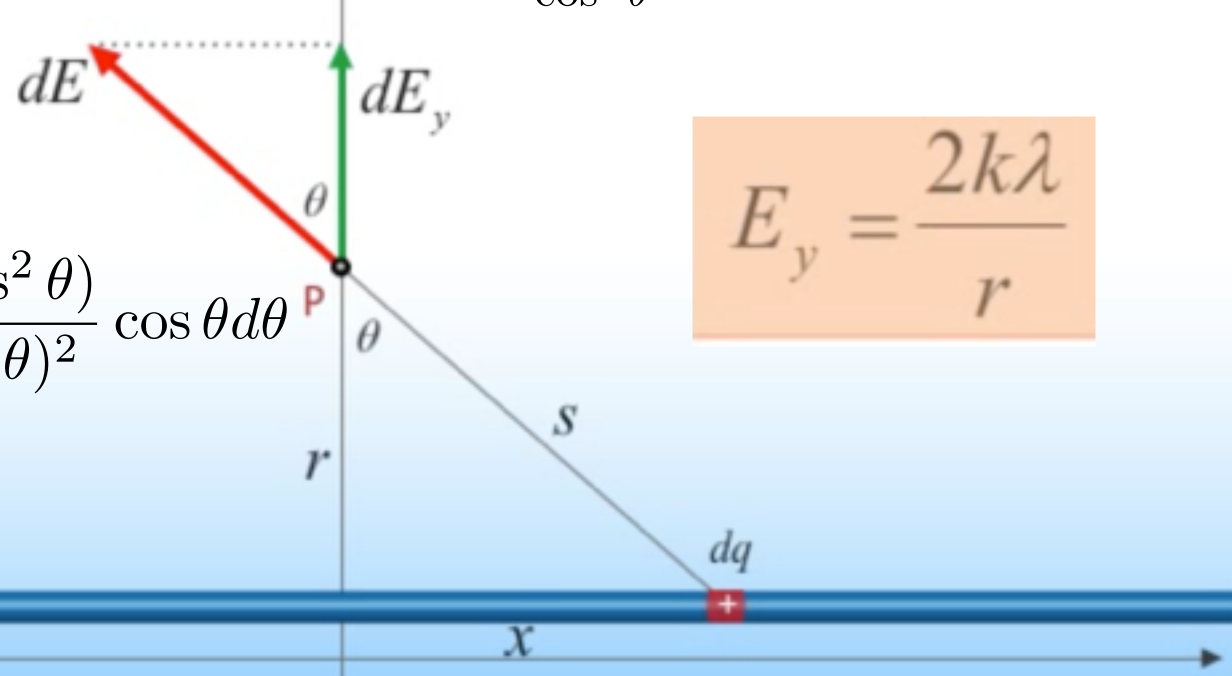
$$x = r\tan\theta$$

$$E_y = \frac{k\lambda}{r}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta\,d\theta$$

$$\frac{dx}{d\theta} = r\frac{1}{\cos^2\theta}$$

$$dx = r\frac{1}{\cos^2\theta}d\theta$$

$$E_y = \int_{\theta=-\pi/2}^{\theta=\pi/2} k\frac{\lambda(r/\cos^2\theta)}{(r/\cos\theta)^2}\cos\theta d\theta$$




$$E_y = \frac{2k\lambda}{r}$$

# Prelecture Question

What is the electric field at point a?

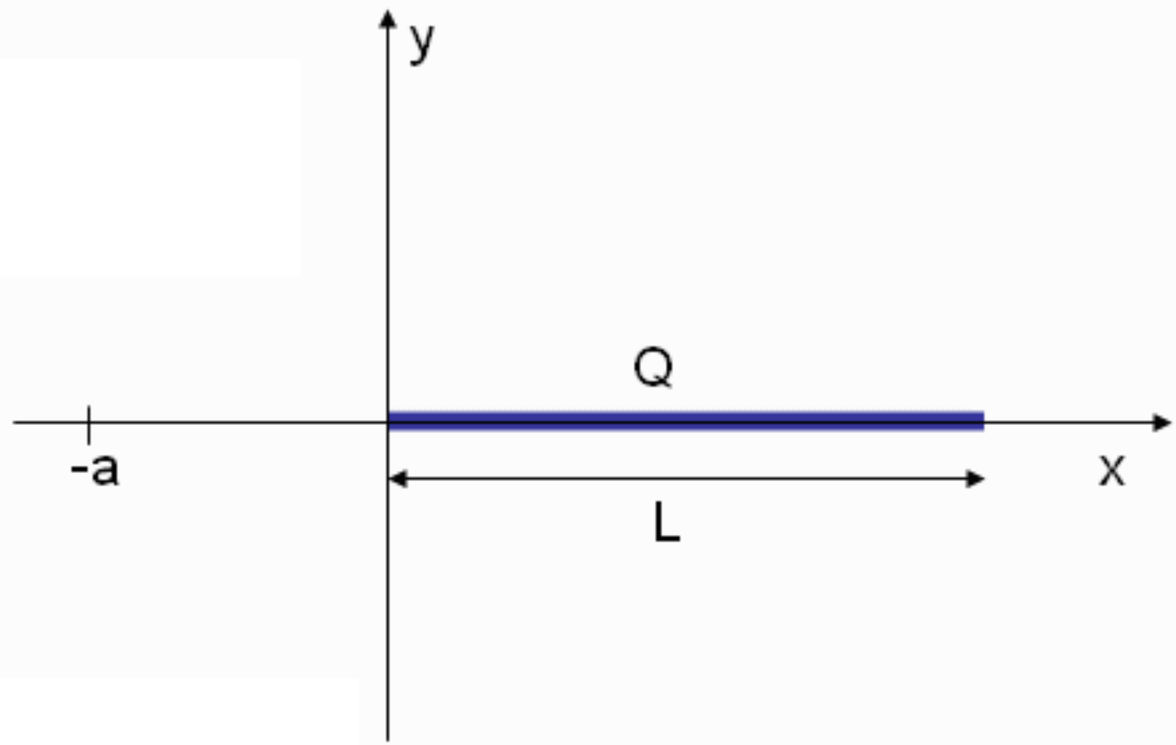
A) $E_x = -\int_0^L k \frac{Q}{L(a+x)^2} dx$

B) $E_x = -\int_0^L k \frac{Q}{Lx^2} dx$

C) $E_x = -\int_0^L k \frac{Q}{x^2} dx$

D) $E_x = -\int_0^L k \frac{Q}{L(x-a)^2} dx$

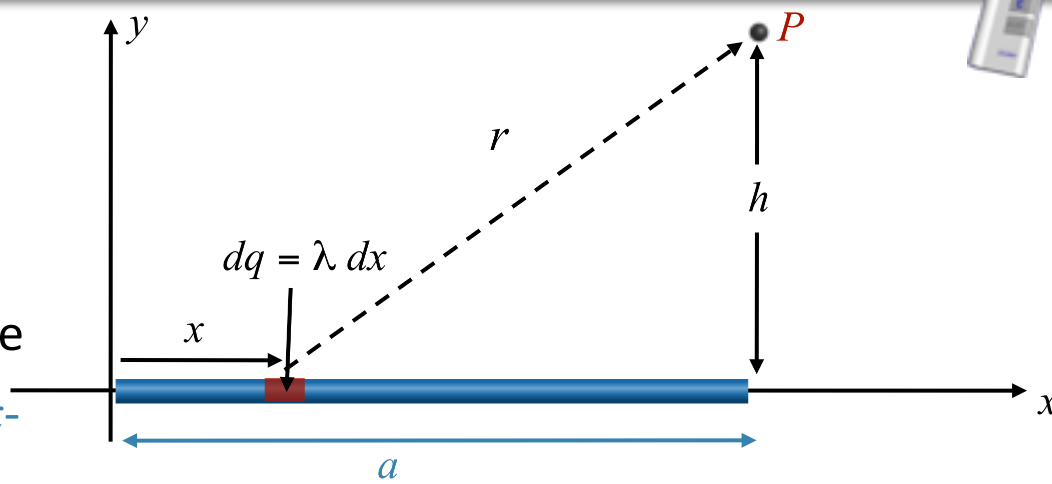
E) $E_x = -\int_a^{L+a} k \frac{Q}{x^2} dx$




Please go over question 3 from the prelecture if you could, and give some incite on the

# Clicker Question: Calculation

"How is the integration of $dE$ over $L$ worked out, step by step?"




Charge is uniformly distributed along the $x$-axis from the origin to $x = a$. The charge density is $\lambda$ C/m. What is the $x$-component of the electric field at point $P$: $(x,y) = (a,h)$?

We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

What is $\dfrac{dq}{r^2}$ ?

A) $\dfrac{dx}{x^2}$

B) $\dfrac{dx}{a^2 + h^2}$

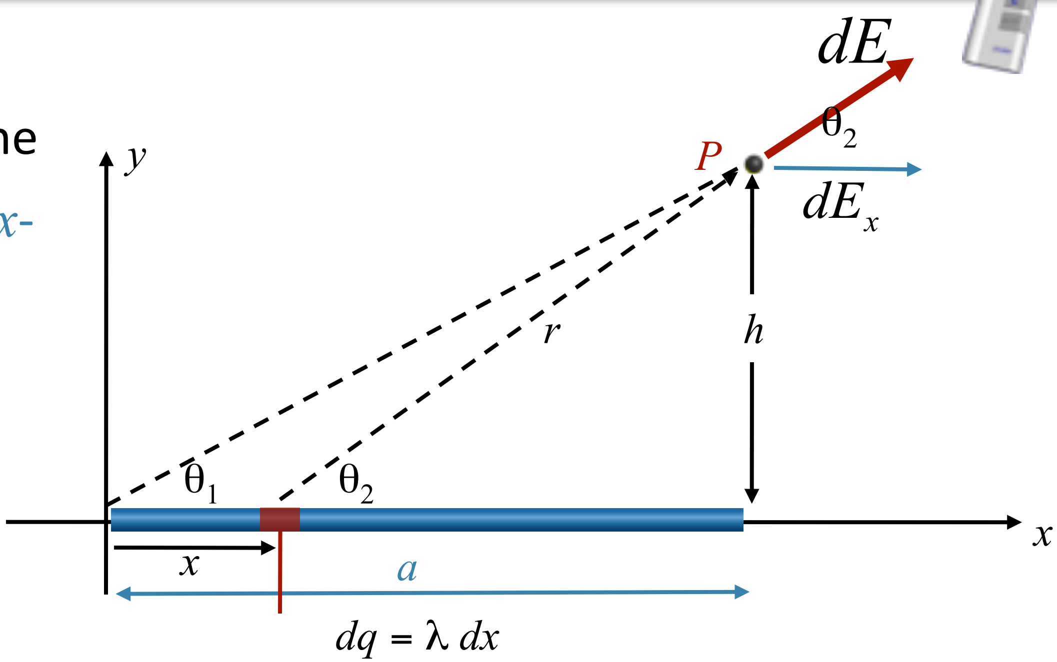
C) $\dfrac{\lambda dx}{a^2 + h^2}$

D) $\dfrac{\lambda dx}{(a - x)^2 + h^2}$

E) $\dfrac{\lambda dx}{x^2}$

# Clicker Question: Calculation

Charge is uniformly distributed along the $x$-axis from the origin to $x = a$. The charge density is $\lambda$ C/m. What is the $x$-component of the electric field at point $P$: $(x,y) = (a,h)$?




We know:

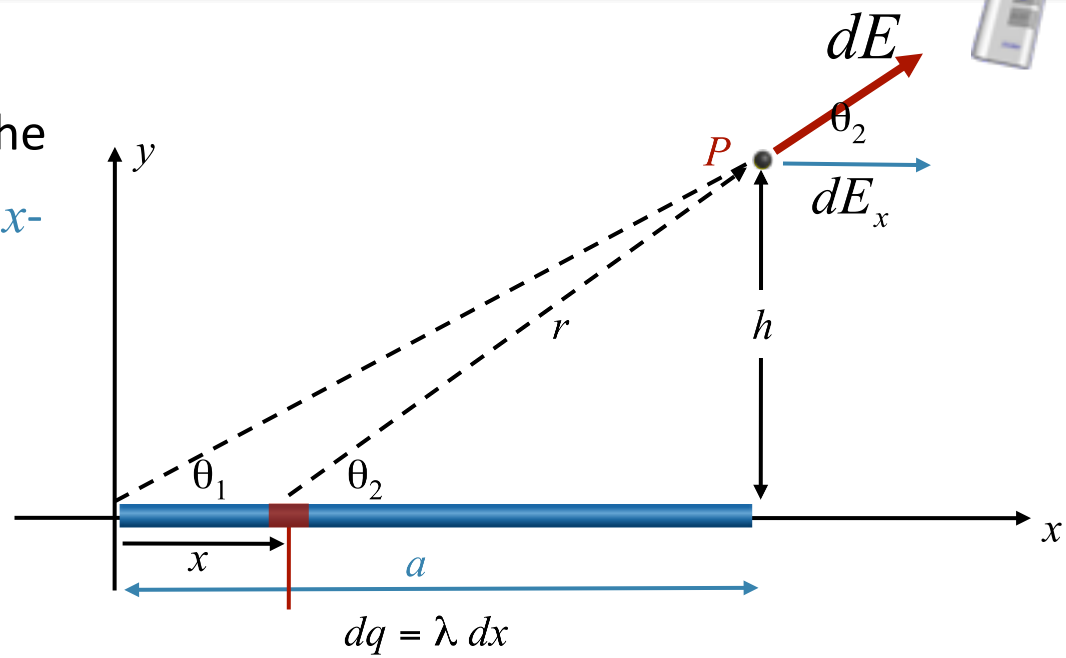
$$\vec{E} = \int k \frac{dq}{r^2}\hat{r} \qquad \frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2} \qquad E_x = \int dE_x$$

What is $dE_x$?

A) $dE\cos\theta_1$ B) $dE\cos\theta_2$ C) $dE\sin\theta_1$ D) $dE\sin\theta_2$

# Clicker Question: Calculation

Charge is uniformly distributed along the $x$-axis from the origin to $x = a$. The charge density is $\lambda$ C/m. What is the $x$-component of the electric field at point $P$: $(x,y) = (a,h)$?




We know:

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE_x = \int dE \cos\theta_2$$

What is $E_x$ ?

A) $k\lambda \cos\theta_2 \int_{-\infty}^{\infty} \frac{dx}{(a-x)^2 + h^2}$

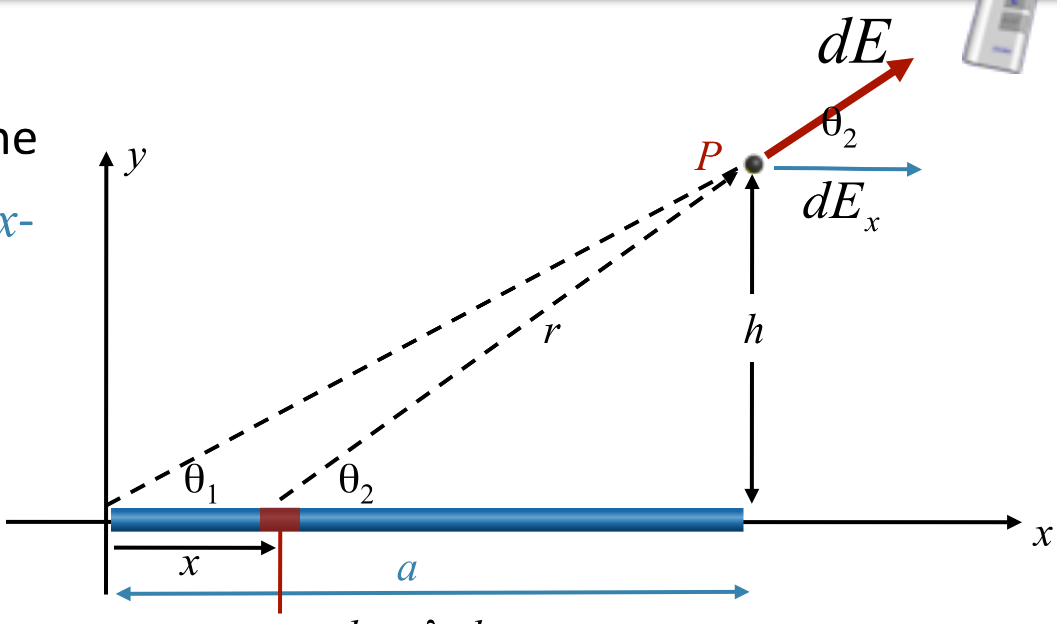
B) $k\lambda \cos\theta_2 \int_{0}^{a} \frac{dx}{(a-x)^2 + h^2}$

C) none of the above

$\cos\theta_2$ DEPENDS ON $x$!

# Clicker Question: Calculation

Charge is uniformly distributed along the $x$-axis from the origin to $x = a$. The charge density is $\lambda$ C/m. What is the $x$-component of the electric field at point $P$: $(x,y) = (a,h)$?




$$\vec{E} = \int k \frac{dq}{r^2}\hat{r}$$

We know:

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2}$$

$$E_x = \int dE_x = \int dE \cos\theta_2$$

What is $\cos\theta_2$ ?

A) $\dfrac{x}{\sqrt{a^2 + h^2}}$

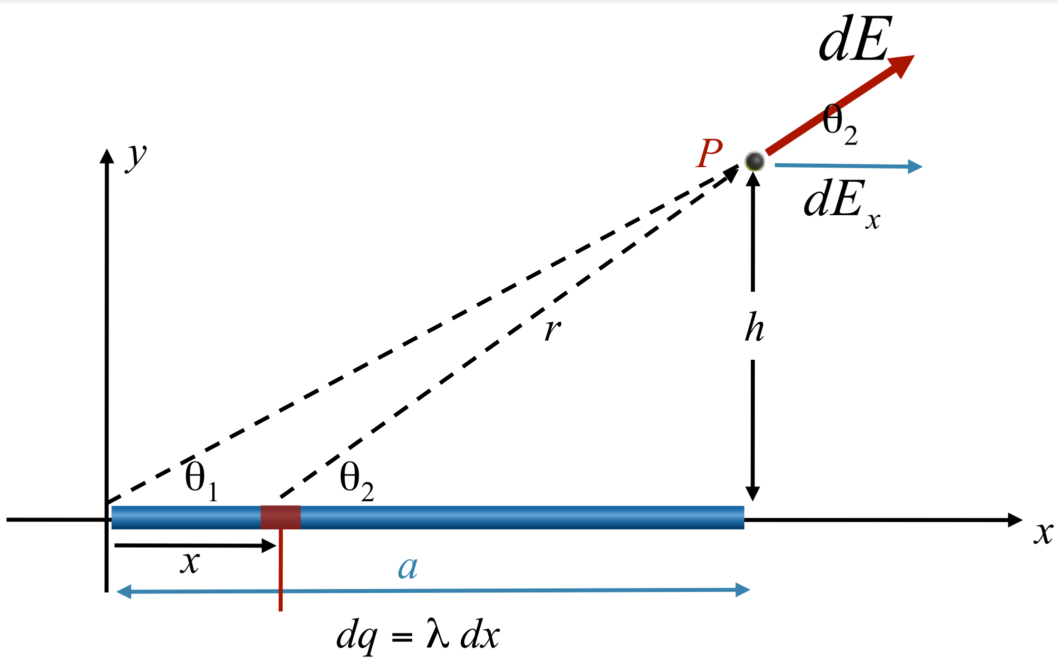
B) $\dfrac{a-x}{\sqrt{(a-x)^2 + h^2}}$

C) $\dfrac{a}{\sqrt{a^2 + h^2}}$

D) $\dfrac{a}{\sqrt{(a-x)^2 + h^2}}$

# Calculation

Charge is uniformly distributed along the $x$-axis from the origin to $x = a$. The charge density is $\lambda$ C/m. What is the $x$-component of the electric field at point $P$: $(x,y) = (a,h)$?




We know: $$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

$$\frac{dq}{r^2} = \frac{\lambda dx}{(a-x)^2 + h^2} \qquad E_x = \int dE_x = \int dE \cos\theta_2 \qquad \cos\theta_2 = \frac{a-x}{\sqrt{(a-x)^2 + h^2}}$$

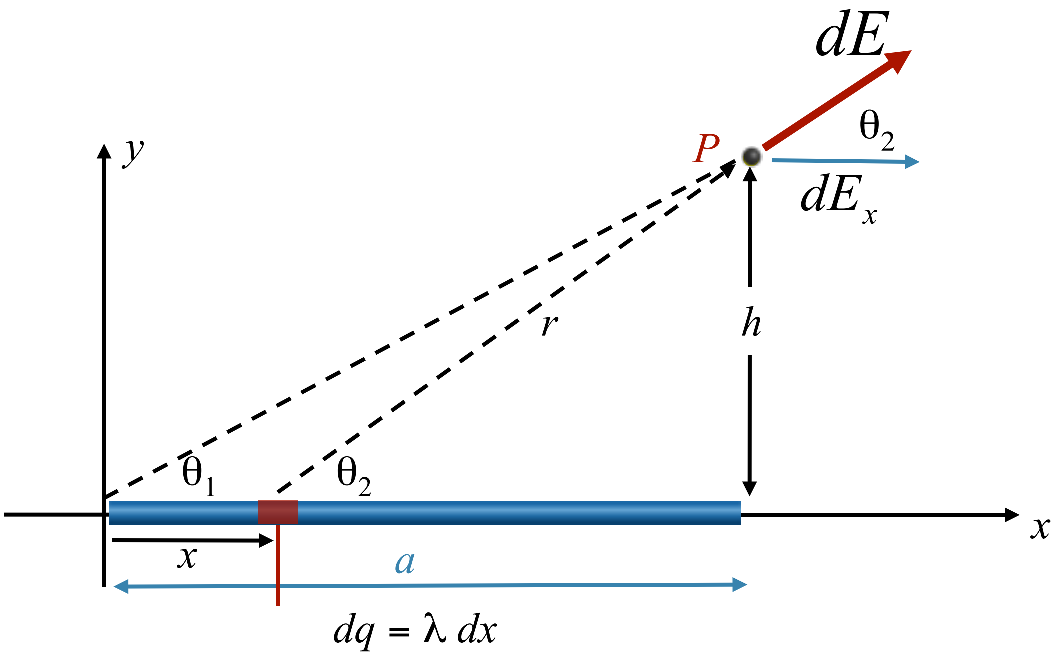
What is $E_x(P)$ ?

$$E_x(P) = k\lambda \int_0^a dx \frac{a-x}{\sqrt{(a-x)^2 + h^2}} \rightarrow E_x(P) = k\lambda \left(1 - \frac{h}{\sqrt{a^2 + h^2}}\right)$$

# Observation

Charge is uniformly distributed along the $x$-axis from the origin to $x = a$. The charge density is $\lambda$ C/m. What is the $x$-component of the electric field at point $P$: $(x,y) = (a,h)$?

Note that our result can be rewritten more simply in terms of $\theta_1$.




$$E_x(P) = \frac{k\lambda}{h}\left(1 - \frac{h}{\sqrt{h^2 + a^2}}\right) \quad \longrightarrow \quad E_x(P) = \frac{k\lambda}{h}(1 - \sin\theta_1)$$

Exercise for student:

Change variables: write $x$ in terms of $\theta$ $\longrightarrow$ $E_x(P) = \frac{k\lambda}{h}\int_{\theta_1}^{\pi/2} d\theta \cos\theta$

Result: obtain simple integral in $\theta$