

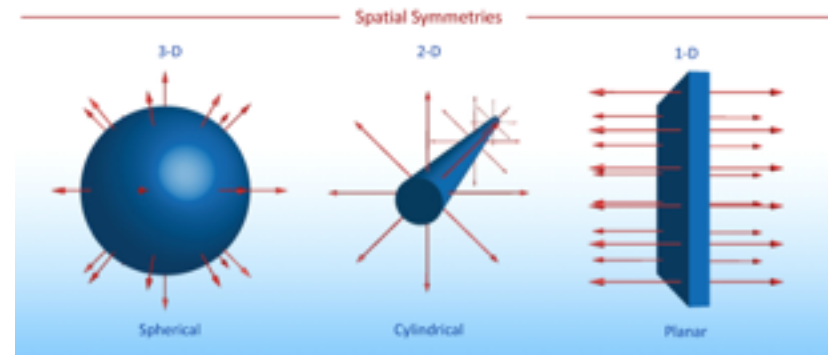
Electricity & Magnetism

Lecture 4: Gauss' Law

Today's Concepts:

A) Conductors

B) Using Gauss' Law







Stuff you asked about:

- if the conductor allows charges to move freely then why $E_{\text{inside}} = 0$?
- “Will we have to do any integrals?”
- I'd like to see a breakdown of why exactly it "makes sense" that the magnitude of an electric field is independent of its distance away from an infinitely long plane
- Could you explain why the E field in the conductive sphere example is 0? what does equilibrium mean in this case? Also how does E field not depend on distance from the plane in the case of infinite planes of charge? would a charge halfway across the galaxy experience the same electric force from an infinite plane of charge as a charge right next to the plane?
- I think this lecture was great. I think we should discuss conducting shells.

More videos

“Are there like any reliable video sources on youtube for this course?”

- ★ Electric Flux (MIT)
- ★ The Mechanical Universe: Electric Fields

Conductors = Charges Free to Move

Claim: $E = 0$ inside any conductor at equilibrium

Charges in conductor move to make E field zero inside. (Induced charge distribution).

If $E \neq 0$, then charge feels force and moves!

Claim: Excess charge on conductor only on surface at equilibrium

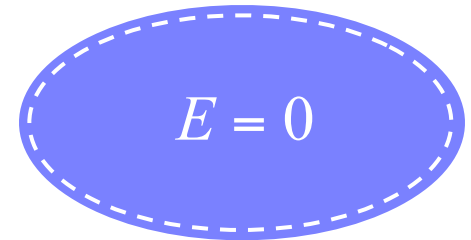
Why?

➤ Apply Gauss' Law

➤ Take Gaussian surface to be just inside conductor surface

➤ $E = 0$ everywhere inside conductor $\rightarrow \oint_{\text{surface}} \vec{E} \cdot d\vec{A} = 0$

➤ Gauss' Law: $\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \rightarrow Q_{\text{enc}} = 0$



Simulation

Gauss' Law + Conductors + Induced Charges

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

ALWAYS TRUE!

If choose a **Gaussian surface** that is entirely in metal, then $E = 0$ so Q_{enclosed} must also be zero!

$$E = \frac{Q_{\text{enc}}}{A\epsilon_o}$$

How Does This Work?

Charges in conductor move to surfaces to make $Q_{\text{enclosed}} = 0$.

We say charge is induced on the surfaces of conductors

Clicker Question: Charge in Cavity of Conductor



A particle with charge $+Q$ is placed in the center of an uncharged conducting hollow sphere. How much charge will be induced on the inner and outer surfaces of the sphere?

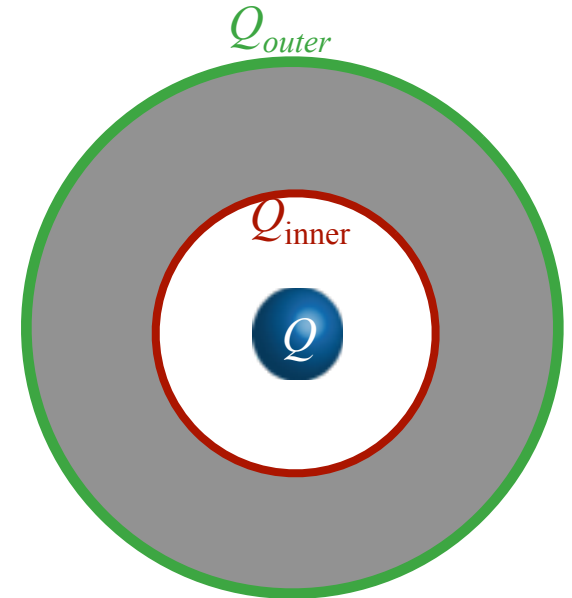
A) inner = $-Q$, outer = $+Q$

B) inner = $-Q/2$, outer = $+Q/2$

C) inner = 0, outer = 0

D) inner = $+Q/2$, outer = $-Q/2$

E) inner = $+Q$, outer = $-Q$



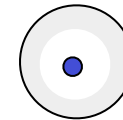
Since $E = 0$ in conductor

➤ Gauss' Law: $\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \rightarrow Q_{\text{enc}} = 0$

Clicker Question: Infinite Cylinders

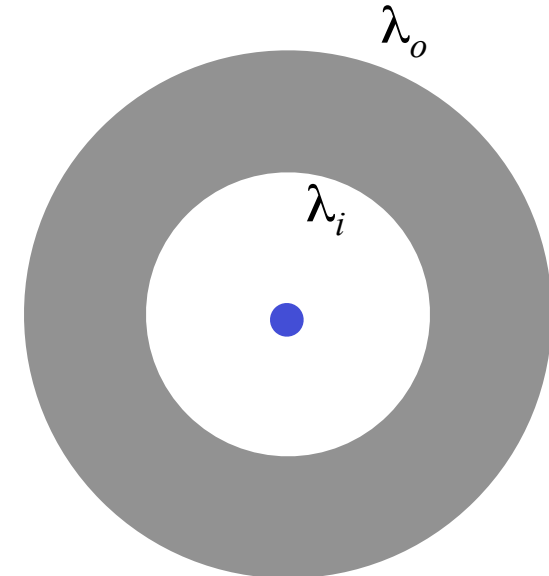


A long thin wire has a uniform positive charge density of 2.5 C/m . Concentric with the wire is a long thick conducting cylinder, with inner radius 3 cm , and outer radius 5 cm . The conducting cylinder has a net linear charge density of -4 C/m .



What is the linear charge density of the induced charge on the inner surface of the conducting cylinder (λ_i) and on the outer surface (λ_o)?

λ_i :	$+2.5 \text{ C/m}$	-4 C/m	-2.5 C/m	-2.5 C/m	0
λ_o :	-6.5 C/m	0	$+2.5 \text{ C/m}$	-1.5 C/m	-4 C/m
	A	B	C	D	E



Gauss' Law

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

ALWAYS TRUE!

In cases with symmetry can pull E outside and get $E = \frac{Q_{\text{enc}}}{A\epsilon_0}$

In General, integral to calculate flux is difficult.... and not useful!

To use Gauss' Law to calculate E , need to choose surface carefully!

1) Want E to be constant and equal to value at location of interest

OR

2) Want E dot $A = 0$ so doesn't add to integral

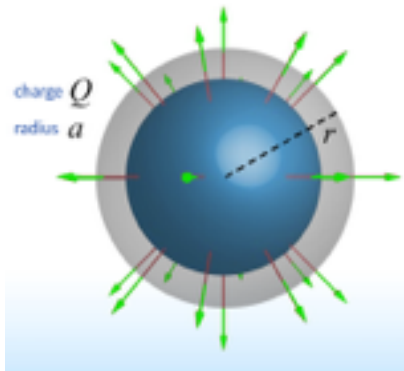
Gauss' Law Symmetries

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

ALWAYS TRUE!

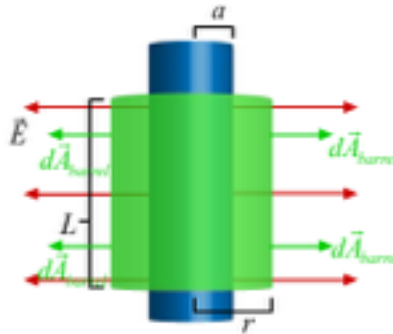
In cases with symmetry can pull E outside and get $E = \frac{Q_{\text{enc}}}{A\epsilon_0}$

Spherical



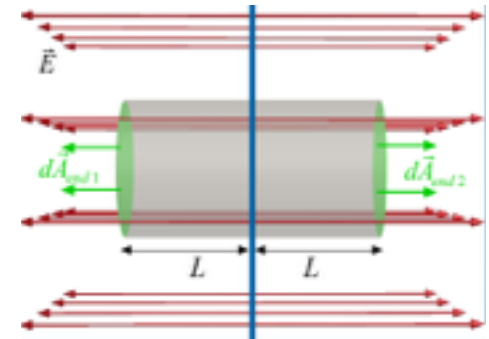
$$A = 4\pi r^2$$
$$E = \frac{Q_{\text{enc}}}{4\pi r^2 \epsilon_0}$$

Cylindrical



$$A = 2\pi rL$$
$$E = \frac{\lambda}{2\pi r\epsilon_0}$$

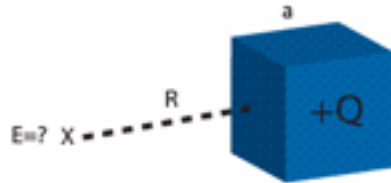
Planar



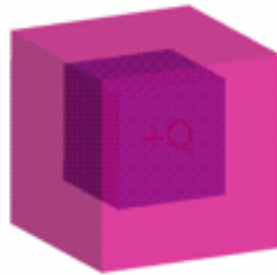
$$A = 2\pi r^2$$
$$E = \frac{\sigma}{2\epsilon_0}$$

CheckPoint: Gaussian Surface Choice

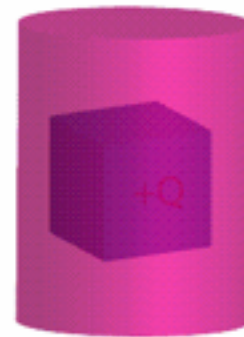
You are told to use Gauss' Law to calculate the electric field at a distance R away from a charged cube of dimension a . Which of the following Gaussian surfaces is best suited for this purpose?



(A)



(B)

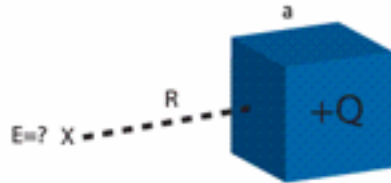


(C)

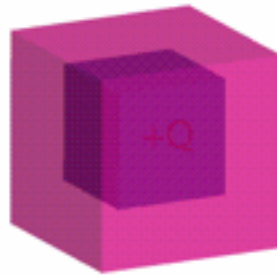
- A. a sphere of radius $R + \frac{1}{2}a$
- B. a cube of dimension $R + \frac{1}{2}a$
- C. a cylinder with cross sectional radius of $R + \frac{1}{2}a$ and arbitrary length
- D. This field cannot be calculated using Gauss' law
- E. None of the above

CheckPoint Results: Gaussian Surface Choice

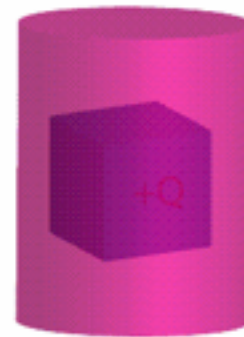
You are told to use Gauss' Law to calculate the electric field at a distance R away from a charged cube of dimension a . Which of the following Gaussian surfaces is best suited for this purpose?



(A)



(B)



(C)

latest

THE CUBE HAS NO GLOBAL SYMMETRY!

THE FIELD AT THE FACE OF THE CUBE

IS NOT

PERPENDICULAR OR PARALLEL

- A. a sphere of radius $R + \frac{1}{2}a$
- B. a cube of dimension $R + \frac{1}{2}a$
- C. a cylinder with cross sectional radius of $R + \frac{1}{2}a$ and arbitrary length
- D. This field cannot be calculated using Gauss' law**
- E. None of the above

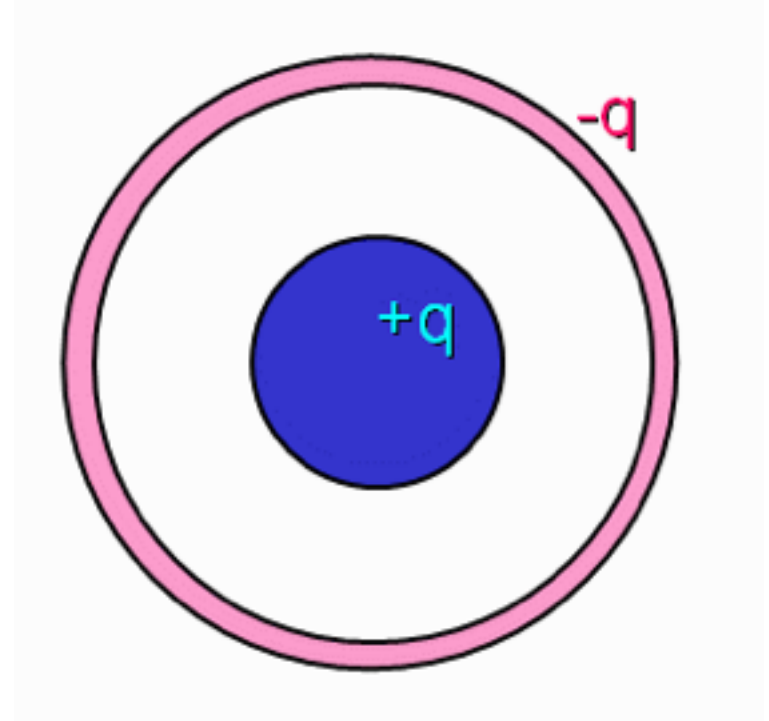
3D	POINT	®	SPHERICAL
2D	LINE	®	CYLINDRICAL
1D	PLANE	®	PLANAR

please explain this checkpoint question again:...

CheckPoint: Charged Conducting Sphere & Shell 1

A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same. Which of the following statements best describes the electric field in the region **between** the spheres?

- A. The field points radially outward
- B. The field points radially inward
- C. The field is zero



CheckPoint: Charged Conducting Sphere & Shell 2

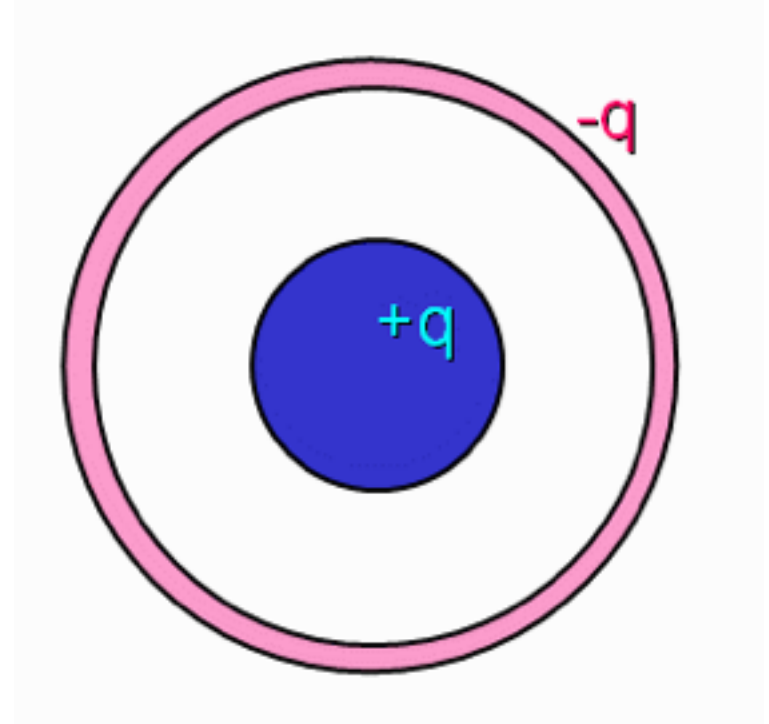
A positively charged solid conducting sphere is contained within a negatively charged conducting spherical shell as shown. The magnitude of the total charge on each sphere is the same. Which of the following statements best describes the electric field in the region **outside** the red sphere?

- A. The field points radially outward
- B. The field points radially inward
- C. The field is zero

“Since they have the same charge, the field from the red sphere is larger than the field from the blue sphere. So the red field points inwards, the blue sphere points outwards so the resultant is outward”

“ closest influence is inwards”

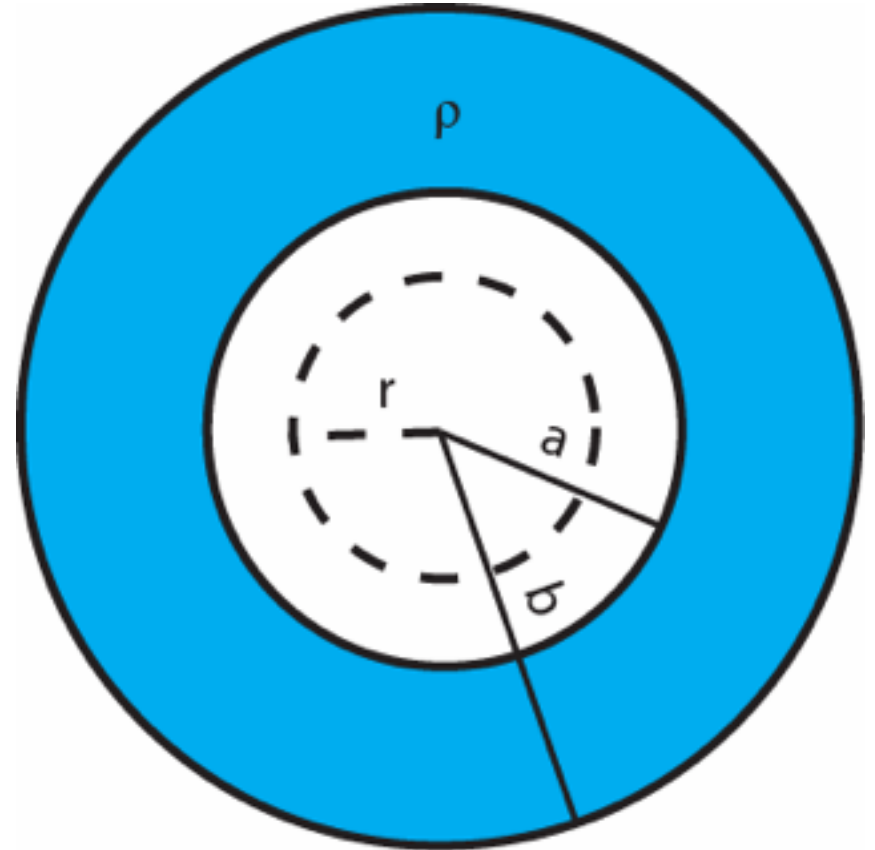
“if $+q = -q$, the field is zero, because the enclosed charge inside the Gaussian surface is $+q + (-q) = 0$, since $E = \text{total } q / A$, and q is 0, E is also 0”



CheckPoint: Charged Spherical Shell

A charged spherical insulating shell has inner radius a and outer radius b . The charge density on the shell is ρ . What is the magnitude of the E-field at a distance r away from the center of the shell where $r < a$?

- A. ρ/ϵ_0
- B. zero
- C. $\rho(b^3 - a^3)/(3\epsilon_0 r^2)$
- D. none of the above



CheckPoint Results: Charged Spherical Shell

A charged spherical insulating shell has inner radius a and outer radius b . The charge density on the shell is ρ . What is the magnitude of the E-field at a distance r away from the center of the shell where $r < a$?

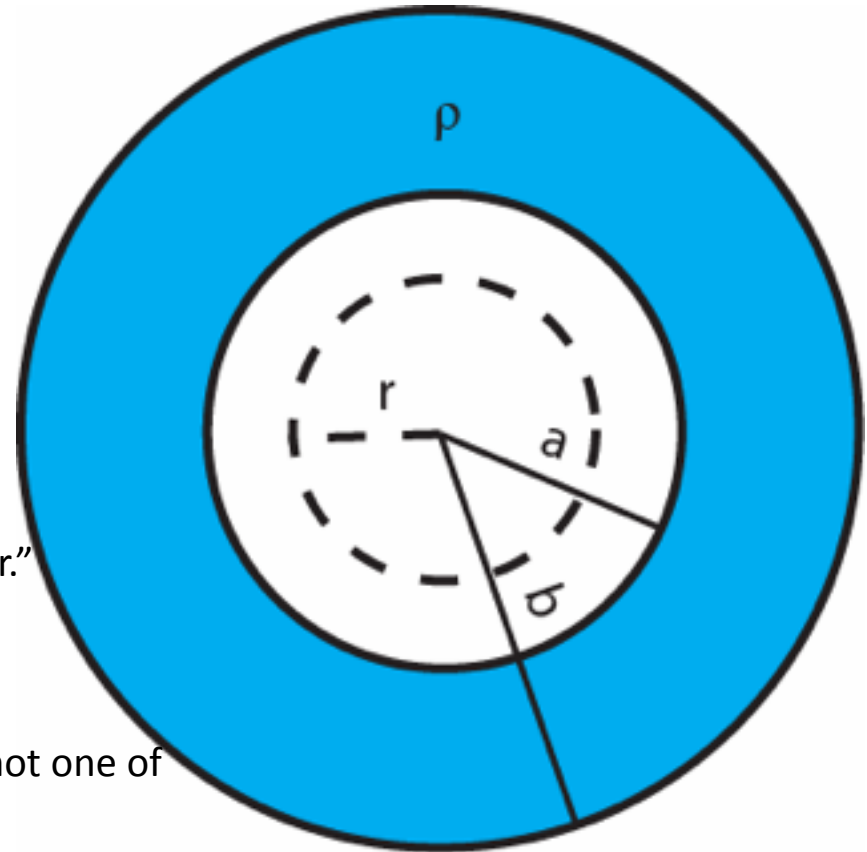
- A. ρ/ϵ_0
- ☒ B. zero
- C. $\rho(b^3-a^3)/(3\epsilon_0 r^2)$
- D. none of the above

Latest

"I'm not actually sure but it seems like the right answer."

Trick!!!! There is no enclosed charge in $r < a$ HA!

The e-field at r is given by $\rho * r / (3 * \epsilon_0)$, which is not one of the options.



Applications?

On the Effectiveness of Aluminium Foil Helmets:

An Empirical Study

Ali Rahimi¹, Ben Recht², Jason Taylor², Noah Vawter²
17 Feb 2005

1: Electrical Engineering and Computer Science department, MIT.

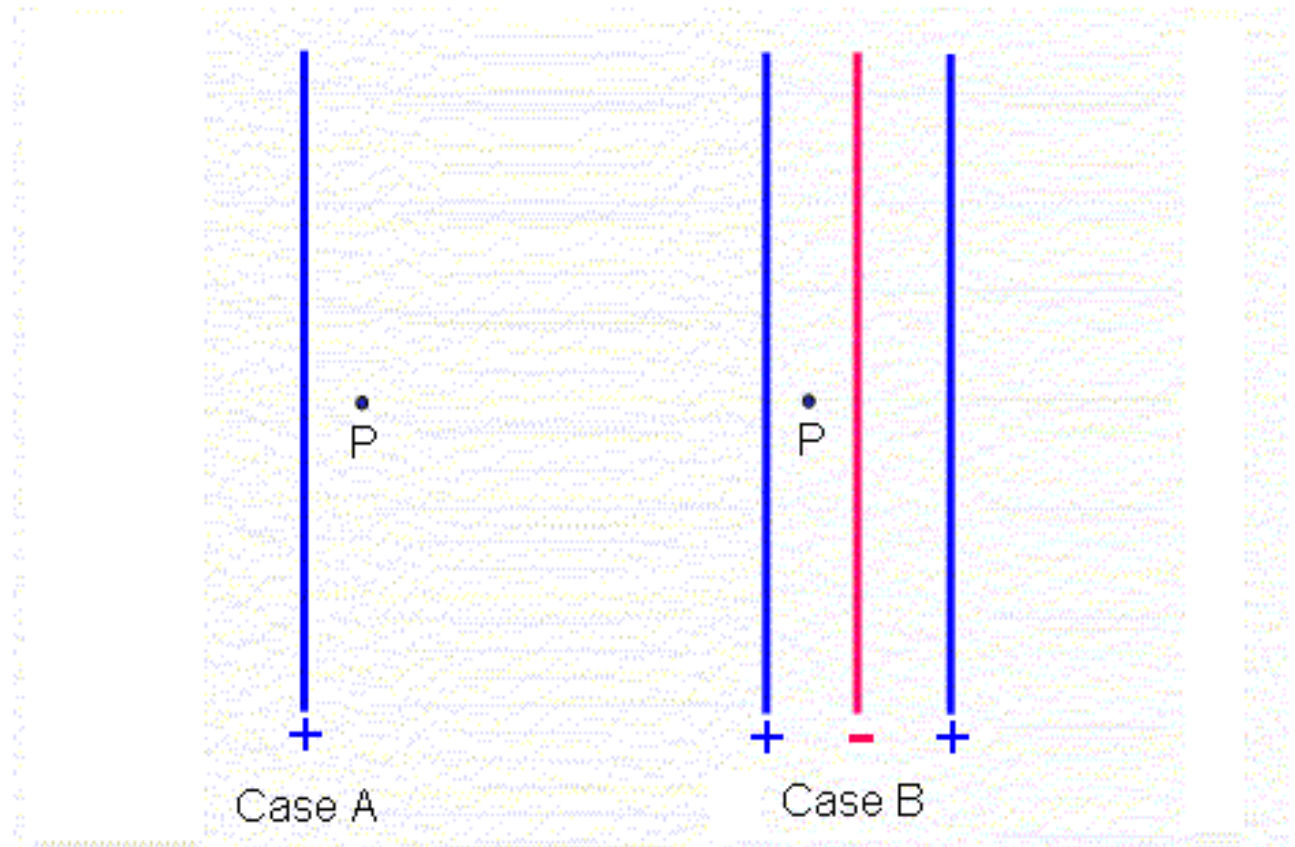
2: Media Laboratory, MIT.



Checkpoint: Infinite Sheets of Charge

In both cases shown below, the colored lines represent positive (blue) and negative (red) charged planes. The magnitudes of the charge per unit area on each plane is the same. In which case is the magnitude of the electric field at point P bigger?

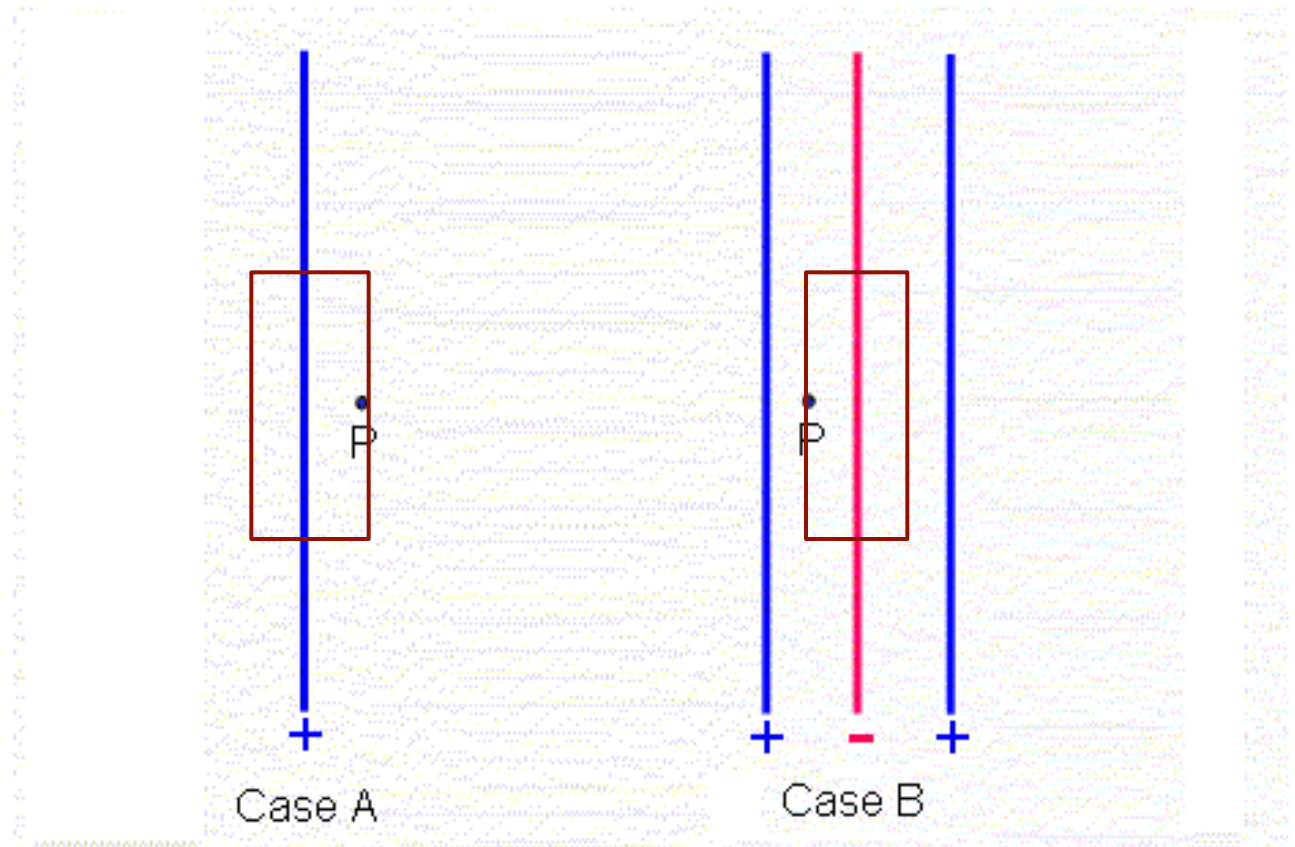
- A. Case A
- B. Case B
- C. They are the same

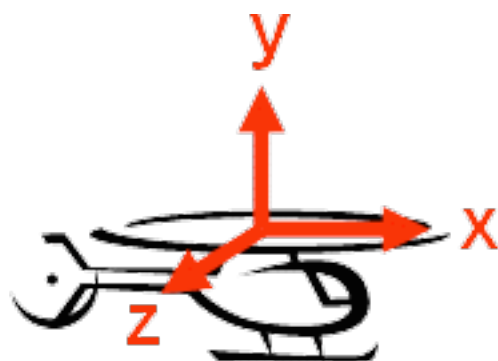


CheckPoint Results: Infinite Sheets of Charge

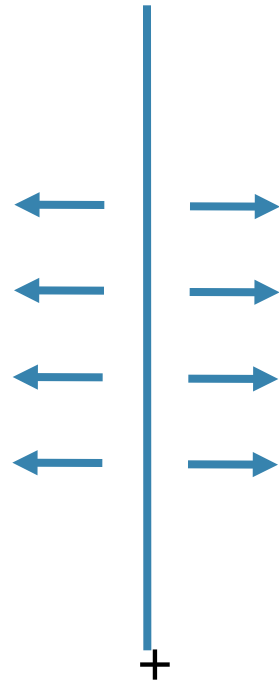
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- A. Case A
- B. Case B
- C. They are the same

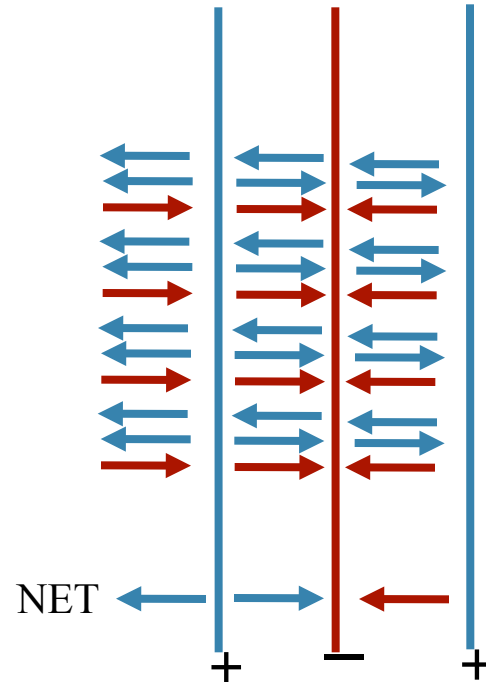




Superposition:

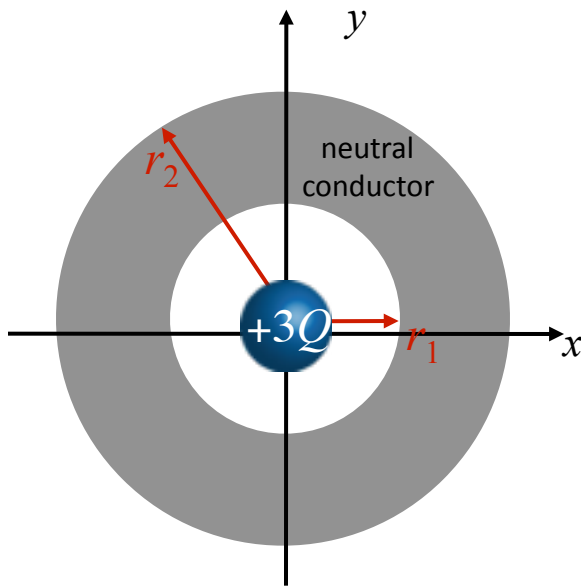


Case A



Case B

Calculation



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2 .

a) What is E everywhere?

First question: Do we have enough symmetry to use **Gauss' Law** to determine E ?

Yes, **Spherical Symmetry** (what does this mean???)

A) Magnitude of E is *fcn* of r

B) Magnitude of E is *fcn* of $(r-r_1)$

C) Magnitude of E is *fcn* of $(r-r_2)$

D) None of the above

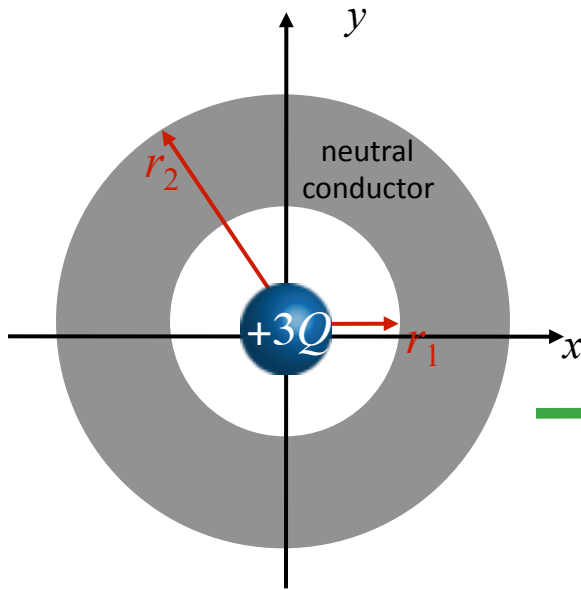
A) Direction of E is along $\hat{x}$

B) Direction of E is along $\hat{y}$

C) Direction of E is along $\hat{r}$

D) None of the above

Calculation



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2 .

A) What is E everywhere?

We know:

magnitude of E is fcn of r
direction of E is along $\hat{r}$

We can use Gauss' Law to determine E

Use Gaussian surface = sphere centered on origin

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} = E4\pi r^2$$

$r < r_1$

$$Q_{\text{enc}} = +3Q$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

$$\text{A) } E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

$r_1 < r < r_2$

$$\text{B) } E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r_1^2}$$

$$\text{C) } E = 0$$

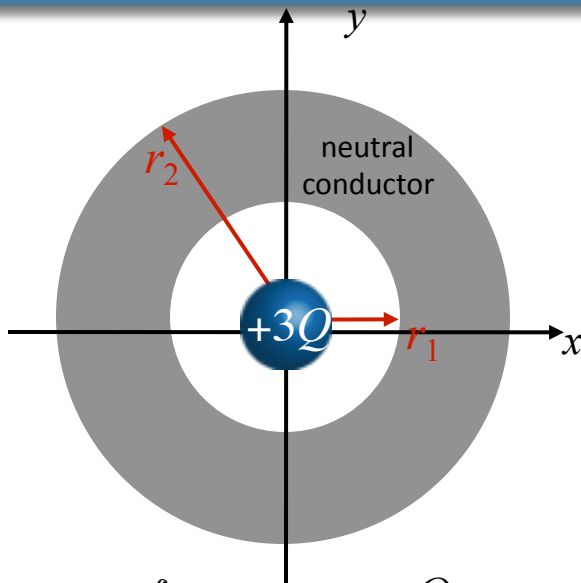
$$\text{A) } E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

$r > r_2$

$$\text{B) } E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{(r - r_2)^2}$$

$$\text{C) } E = 0$$

“Based on the graph obtained of E against r , please repeat the explanation of how the graph changes when $r < a$, $r = a$, $r > a$ (in/outside shell)”



Point charge $+3Q$ at center of neutral conducting shell of inner radius r_1 and outer radius r_2 .

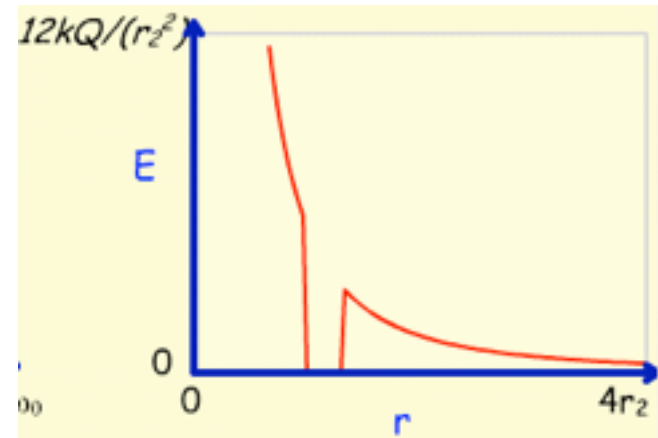
A) What is E everywhere?

We know:

$$r < r_1 \quad E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

$$r > r_2$$

$$r_1 < r < r_2 \quad E = 0$$



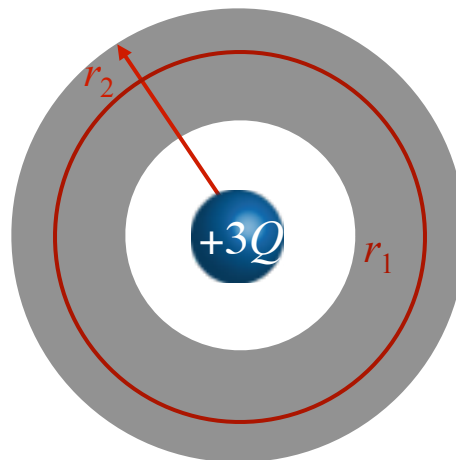
$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

B) What is charge distribution at r_1 ?

A) $\sigma < 0$

B) $\sigma = 0$

C) $\sigma > 0$



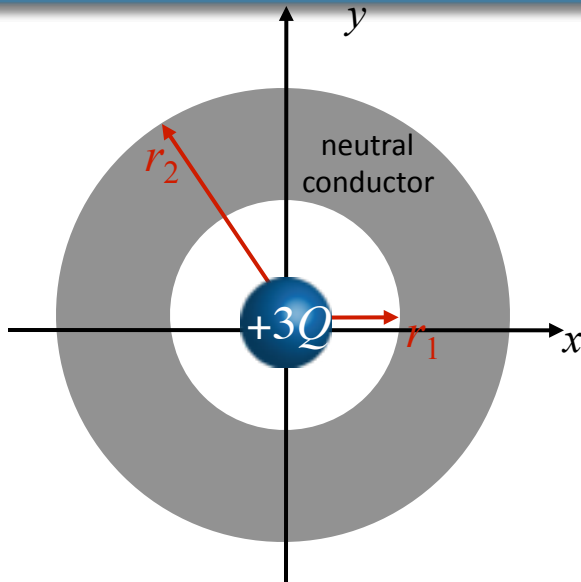
Gauss' Law:

$$E = 0 \rightarrow Q_{\text{enc}} = 0 \rightarrow \sigma_1 = \frac{-3Q}{4\pi r_1^2}$$

Similarly:

$$\sigma_2 = \frac{+3Q}{4\pi r_2^2}$$

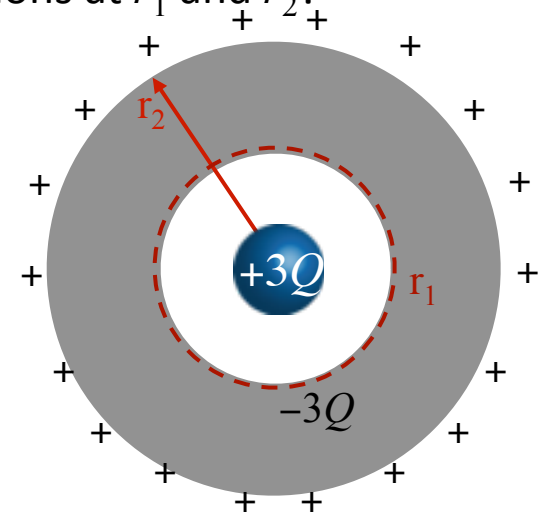
Calculation



Suppose the conductor has a charge of $-Q$

- A) What is E everywhere?
 B) What are charge distributions at r_1 and r_2 ?

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$



$$r < r_1$$

$$\text{A) } E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

$$\text{B) } E = \frac{1}{4\pi\epsilon_0} \frac{2Q}{r^2}$$

$$\text{C) } E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$r > r_2$$

$$\text{A) } E = \frac{1}{4\pi\epsilon_0} \frac{3Q}{r^2}$$

$$\text{B) } E = \frac{1}{4\pi\epsilon_0} \frac{2Q}{r^2}$$

$$\text{C) } E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$r_1 < r < r_2$$

$$E = 0$$