Electricity & Magnetism
Lecture 5: Electric Potential Energy

Today...

- Electric Potential Energy

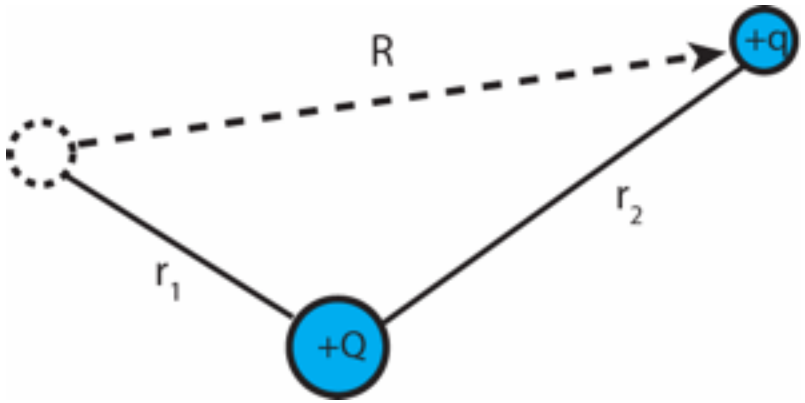
Stuff you asked about:

- "I noticed the formula sheet does not have the potential energy equation used in the prelecture. Will this be changed on the sheet we will use for exams or are we supposed to be able to find this equation through the given force equation?"
  - You should be able to figure out Potential Energy from the Electric Potential and the principle of superposition.
- "How much harder does this course get down the line? This course already has me shook."
  - It doesn't necessarily get easy but most people make it through. Gauss' Law is usually puzzling at first.

CheckPoint Result: EPE of Point Charge

A charge of +Q is fixed in space. A second charge of +q was first placed at a distance r_1 away from +Q. Then it was moved along a straight line to a new position at a distance R away from its starting position. The final location of +q is at a distance r_2 from +Q. What is the change in the potential energy of charge +q during this process?

A. kQq/R
B. $kQqR/r_1^2$
C. $kQqR/r_2^2$
D. $kQq((1/r_2)-(1/r_1))$
E. $kQq((1/r_1)-(1/r_2))$



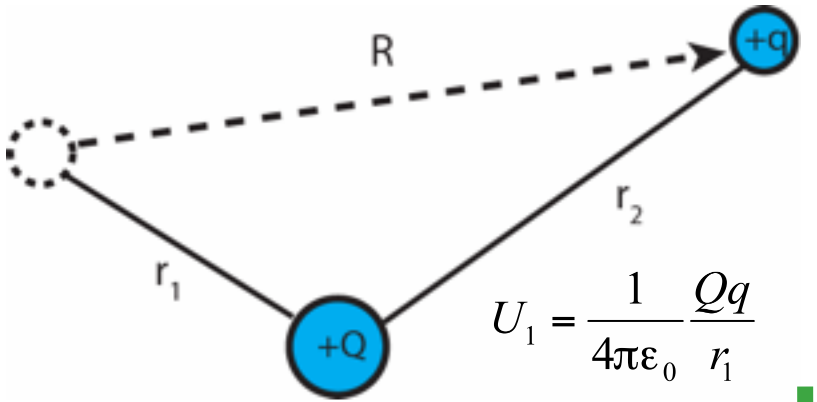
"kQq/r is the potential at a point so the difference in these two potentials will be found by doing kQq((1/r2)-(1/r1))"

"Since the particle is moved away from the fixed charge, the potential energy must increase. The part 1/r1-1/r2 would yield a positive answer because 1/r1>1/r2"

CheckPoint Result: EPE of Point Charge

A charge of +Q is fixed in space. A second charge of +q was first placed at a distance r_1 away from +Q. Then it was moved along a straight line to a new position at a distance R away from its starting position. The final location of +q is at a distance r_2 from +Q. What is the change in the potential energy of charge +q during this process?

A. kQq/R
B. $kQqR/r_1^2$
C. $kQqR/r_2^2$
D. $kQq((1/r_2)-(1/r_1))$
E. $kQq((1/r_1)-(1/r_2))$



$$U_1 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_1} \qquad U_2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r_2}$$

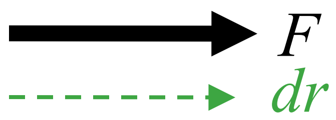
$$\Delta U \equiv U_2 - U_1 = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Note: $+q$ moves AWAY from $+Q$.
Its Potential energy MUST DECREASE
$\Delta U < 0$

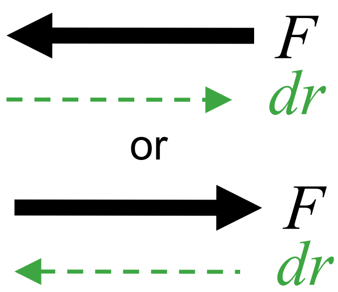
Recall from Mechanics:

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$$

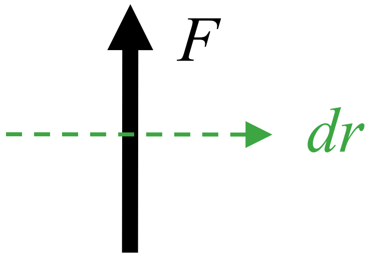
$$W_{TOT} = \Delta K$$



$W > 0$ Object speeds up ($\Delta K > 0$)



$W < 0$ Object slows down ($\Delta K < 0$)



$W = 0$ Constant speed ($\Delta K = 0$)

Dot Product (review)

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$$

And given that $\hat{i} \cdot \hat{i} = 1$ and $\hat{i} \cdot \hat{j} = 0$ etc then...

$$\vec{A} \cdot \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$

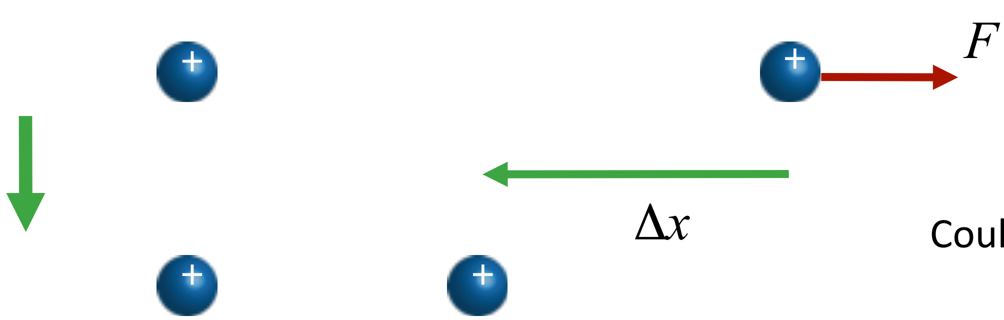
If you know $\vec{A}$ and $\vec{B}$ in rectangular coordinates, then you can find the angle between them.

Potential Energy

$$\Delta U \equiv -W_{conservative}$$

If gravity does negative work, potential energy increases!

Same idea for Coulomb force… if Coulomb force does negative work, potential energy increases.



Coulomb force does negative work
Potential energy increases

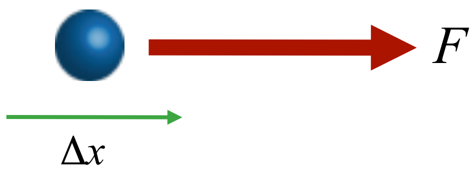
CheckPoint: Motion of Point Charge Electric Field

A charge is released from rest in a region of electric field. The charge will start to move

A) In a direction that makes its potential energy increase.

B) In a direction that makes its potential energy decrease.

C) Along a path of constant potential energy.



It will move in the same direction as F

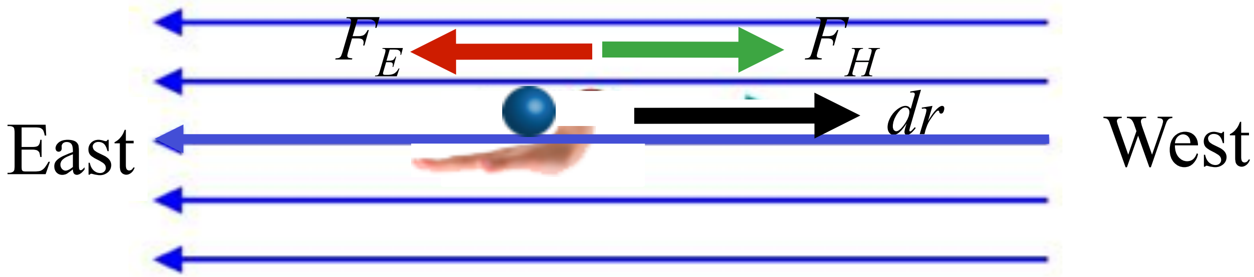
Work done by force is positive

ΔU = –Work is negative

Nature wants things to move in such a way that PE decreases

Clicker Question

You hold a positively charged ball and walk due west in a region that contains an electric field directed due east.



W_H is the work done by the hand on the ball

W_E is the work done by the electric field on the ball

Which of the following statements is true:

A) $W_H > 0$ and $W_E > 0$

B) $W_H > 0$ and $W_E < 0$

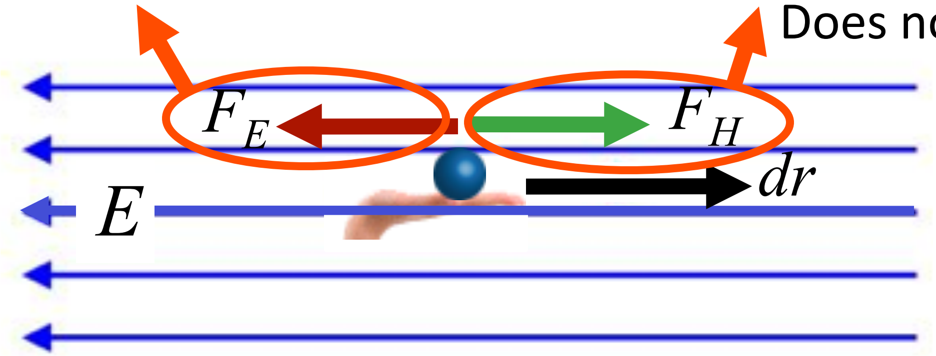
C) $W_H < 0$ and $W_E < 0$

D) $W_H < 0$ and $W_E > 0$

Conservative force: $\Delta U = -W_E$

Not a conservative force.

Does not have any ΔU



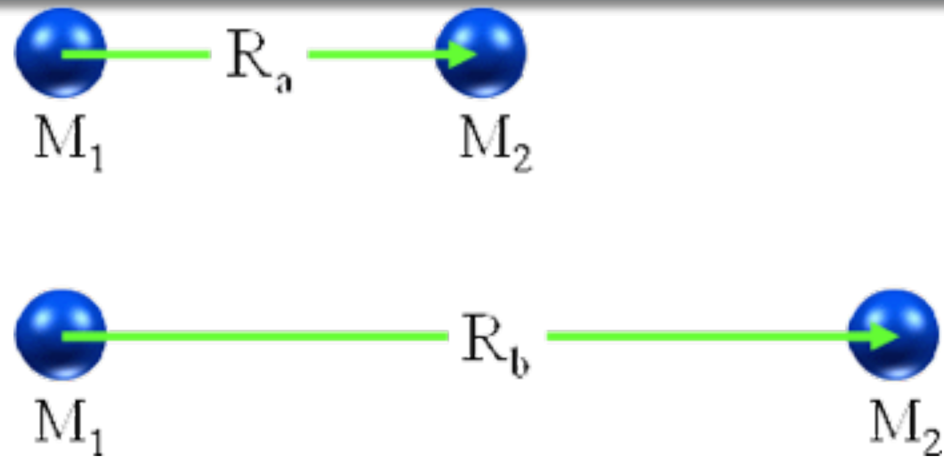
B) $W_H > 0$ and $W_E < 0$

Is ΔU positive or negative?

A) Positive

B) Negative

Checkpoint



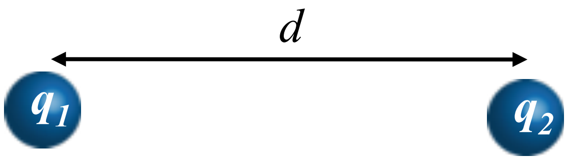
Masses M_1 and M_2 are initially separated by a distance R_a. Mass M_2 is now moved further away from mass M_1 such that their final separation is R_b. Which of the following statements best describes the work W_{ab} **done by the force of gravity** on M_2 as it moves from R_a to R_b?

A) $W_{ab} > 0$ B) $W_{ab} = 0$ C) $W_{ab} < 0$

Example: Two Point Charges

Calculate the change in potential energy for two point charges originally very far apart moved to a separation of "*d*"

$$\Delta U = -\int_{\infty}^{d} k \frac{q_1 q_2}{r_{12}^2} dr$$



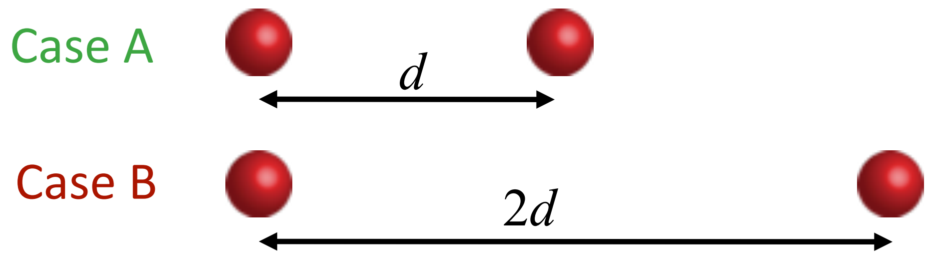
$$\Delta U = k \frac{q_1 q_2}{d} \equiv \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{d}$$

ϵ is another epsilon

Charged particles with the same sign have an increase in potential energy when brought closer together.

For point charges often choose r = infinity as "zero" potential energy.

Clicker Question



In case A two negative charges which are equal in magnitude are separated by a distance d. In case B the same charges are separated by a distance $2d$. Which configuration has the highest potential energy?

A) Case A

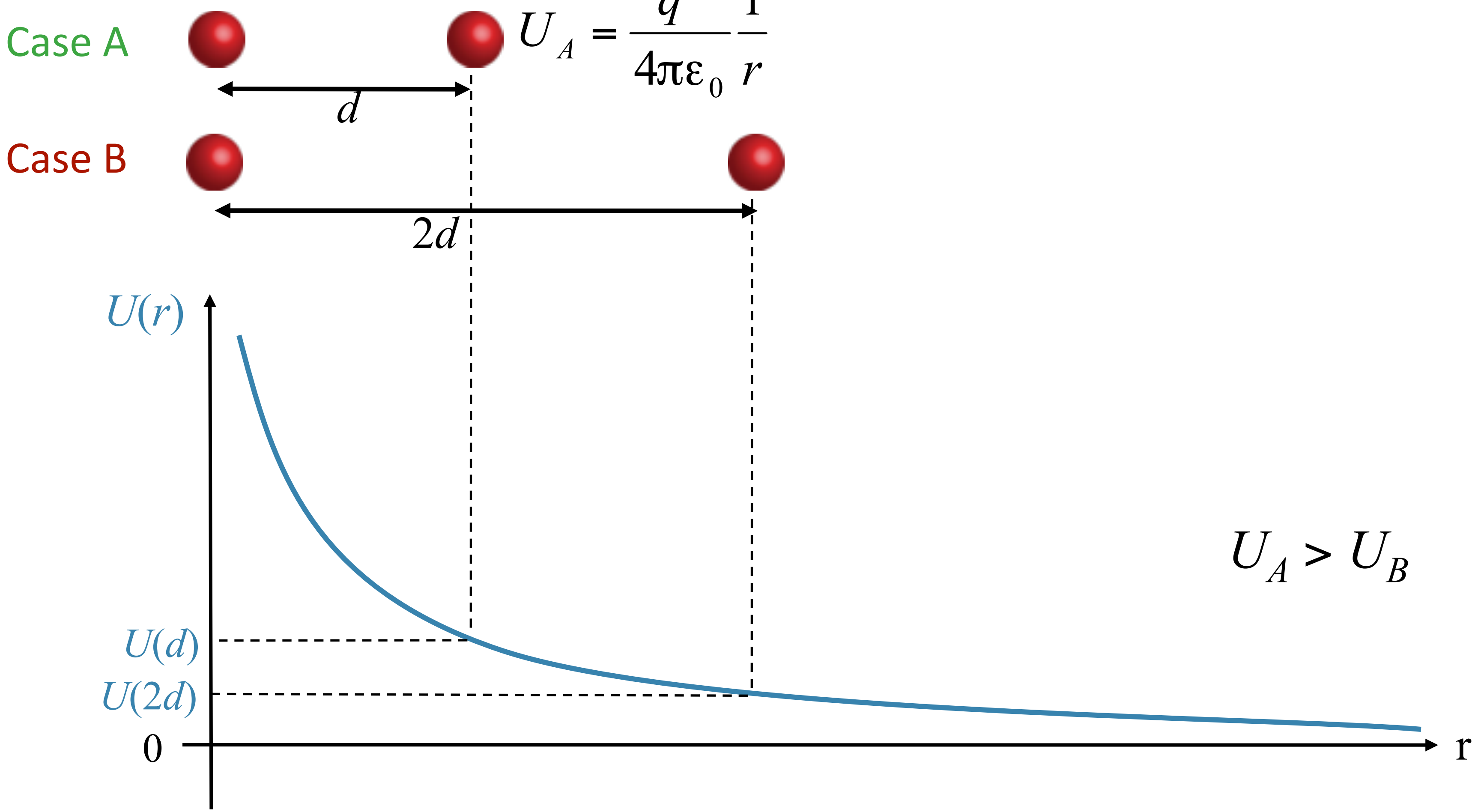
B) Case B

Clicker Question Discussion

As usual, choose $U = 0$ to be at infinity:

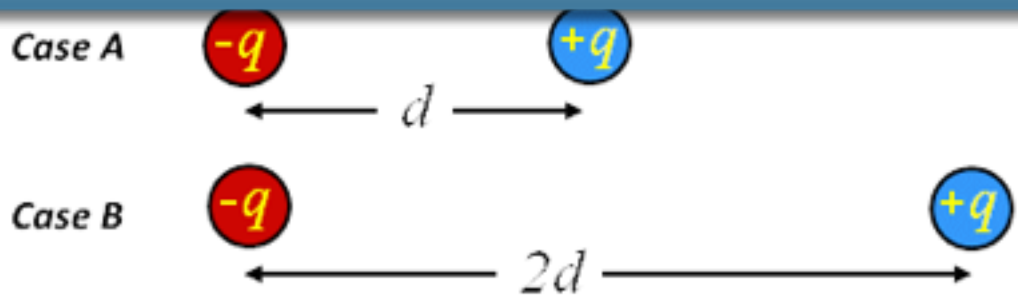
$$U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}$$

$$U_A = \frac{q^2}{4\pi\varepsilon_0}\frac{1}{r}$$



$$U_A > U_B$$

Checkpoint

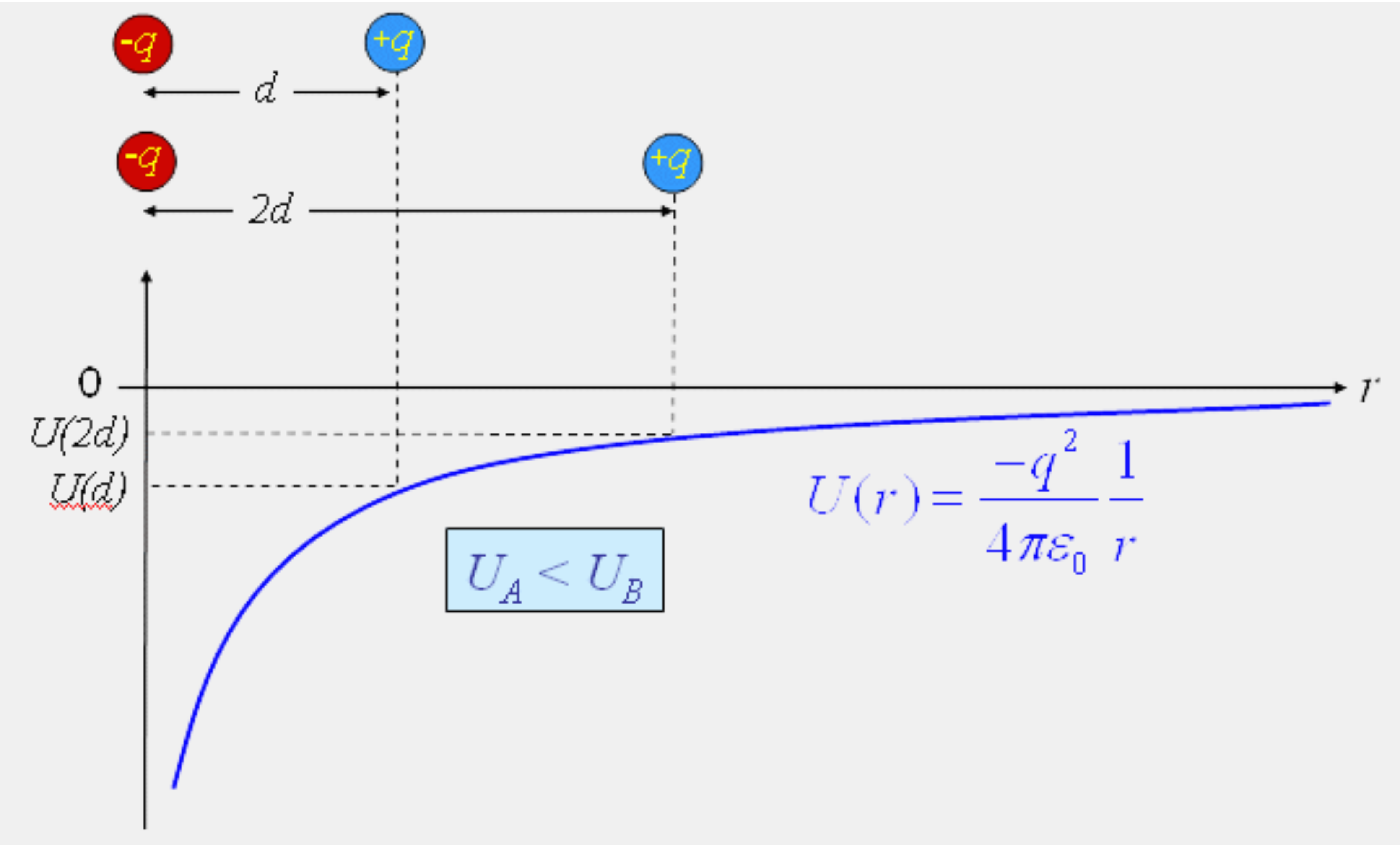


In Case A two charges which are equal in magnitude but opposite in charge are separated by a distance d. In Case B the same charges are separated by a distance $2d$. Which configuration has the highest potential energy U?

A) Case A has the highest potential energy

B) Case B has the highest potential energy

C) Both cases have the same potential energy



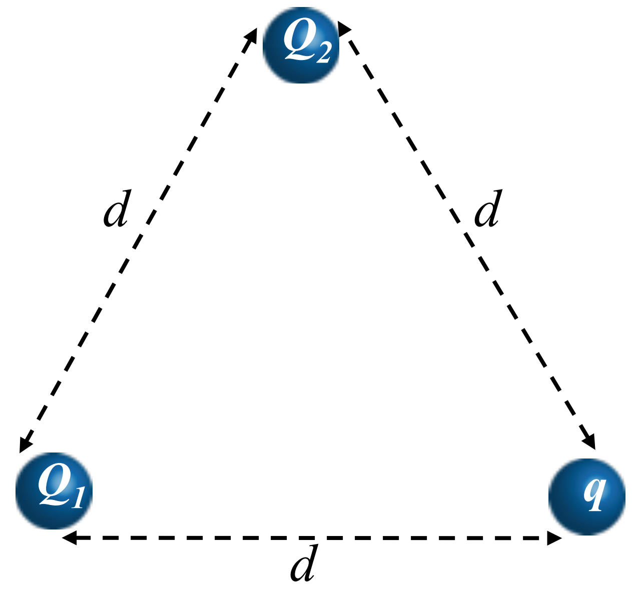
-q
+q
d
-q
+q
2d
0
r
U(2d)
U(d)
$U_A < U_B$
$U(r) = \frac{-q^2}{4\pi\varepsilon_0}\frac{1}{r}$

Potential Energy of Many Charges

Two charges are separated by a distance d. What is the change in potential energy when a third charge q is brought from far away to a distance d from the original two charges?

$$\Delta U = \frac{qQ_1}{4\pi\varepsilon_0}\frac{1}{d} + \frac{qQ_2}{4\pi\varepsilon_0}\frac{1}{d}$$

(superposition)



Potential Energy of Many Charges

What is the total energy required to bring in three identical charges, from infinitely far away to the points on an equilateral triangle shown.

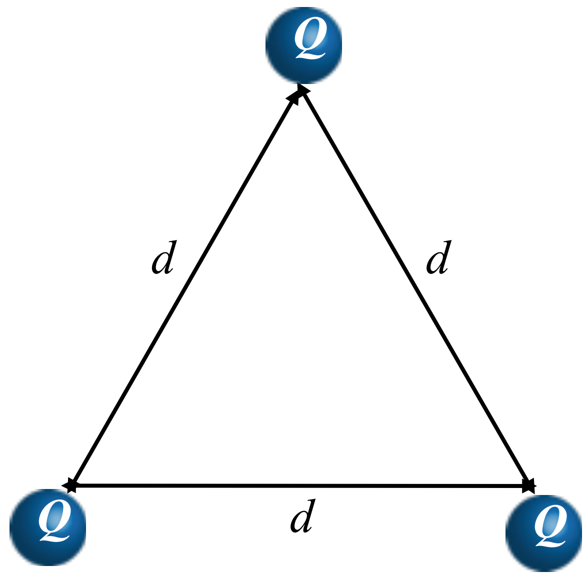
A) 0

B) $\Delta U = \frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

C) $\Delta U = 2\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

D) $\Delta U = 3\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

E) $\Delta U = 6\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$



$$W = \sum W_i = -\frac{3}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

$$\Delta U = +\frac{3}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

Work to bring in first charge: $W_1 = 0$

Work to bring in second charge : $W_2 = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$

Work to bring in third charge : $W_3 = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} - \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} = -\frac{2}{4\pi\varepsilon_0}\frac{Q^2}{d}$

Potential Energy of Many Charges

Suppose one of the charges is negative. Now what is the total energy required to bring the three charges in infinitely far away?

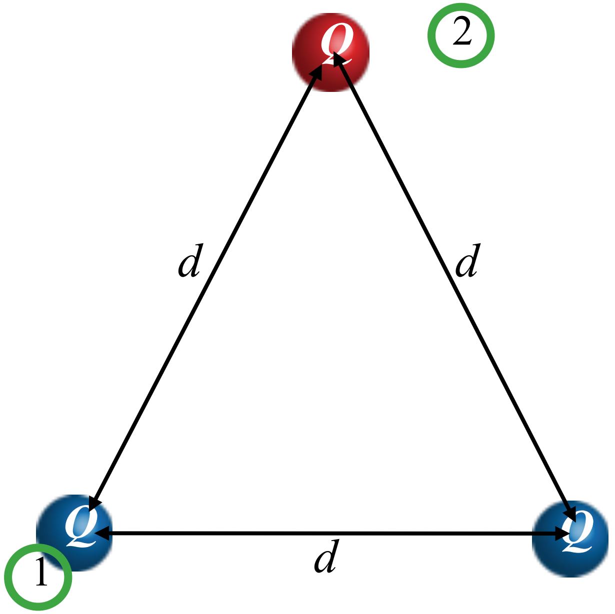
A) 0

B) $\Delta U = +1\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

C) $\Delta U = -1\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

D) $\Delta U = +2\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$

E) $\Delta U = -2\frac{Q^2}{4\pi\varepsilon_0}\frac{1}{d}$



$$W = \sum W_i = +\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

$$\Delta U = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$$

Work to bring in first charge: $W_1 = 0$

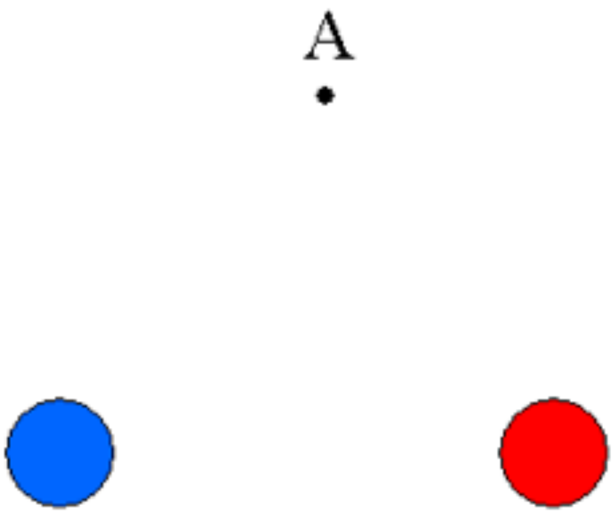
Work to bring in second charge : $W_2 = +\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d}$

Work to bring in third charge : $W_3 = +\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} - \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{d} = 0$

CheckPoint: EPE of a System of Point Charges 1

Two charges which are equal in magnitude, but opposite in sign, are placed at equal distances from point A as shown. If a third charge is added to the system and placed at point A, how does the electric potential energy of the charge collection change?

A. Potential energy increases
B. Potential energy decreases
C. Potential energy does not change
D. The answer depends on the sign of the third charge



If the new charge has a potential energy caused by charge 1 equal to charge 2, when we sum them up, it will add up to 0, resulting in no change in total potential energy.

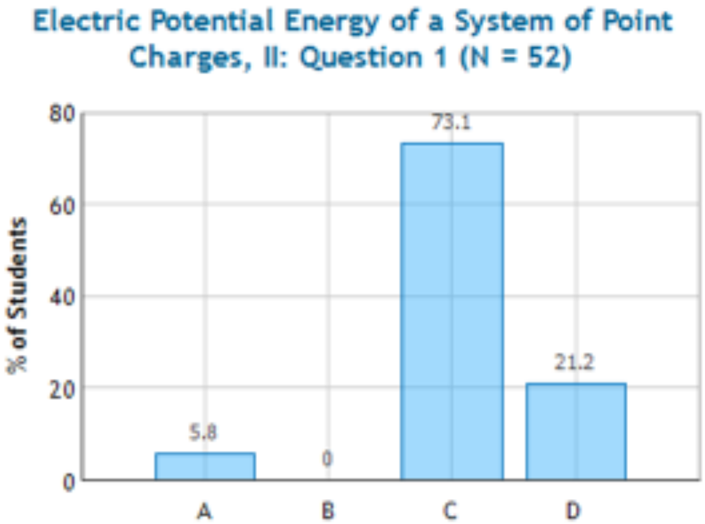
third charge will add kq_1q_3/d and kq_2q_3/d to the total U, unless q_3 is zero, it will definitely change the total potential energy of the system.

Question

“Could you go over the third checkpoint, in some detail and show how the calculation would look like for this kind of problem”

CheckPoint: EPE of a System of Point Charges 2

Two point charges are separated by some distance as shown. The charge of the first is positive. The charge of the second is negative and its magnitude is **twice as large** as the first. Is it possible find a place to bring a third charge in from infinity without changing the total potential energy of the system?



A. YES, as long as the third charge is positive
B. YES, as long as the third charge is negative
C. YES, no matter what the sign of the third charge
D. NO

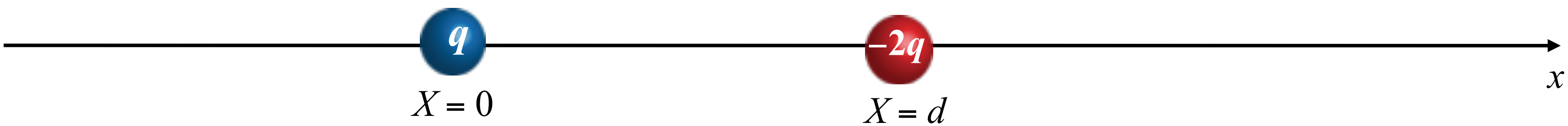
HOW? LET'S DO THE CALCULATION.

It is possible to bring a third charge in from infinity without changing the total potential energy of the system if this third charge is placed a distance twice as far way from the negative charge than from the positive charge (and q1 and q2 must be equal in magnitude?).

"The potential energies the third charge will contribute to the system have opposite signs but NOT equal magnitudes. So the net potential energy will not equal 0."

Example

A positive charge q is placed at $x = 0$ and a negative charge $-2q$ is placed at $x = d$. At how many different places along the x axis could another positive charge be placed without changing the total potential energy of the system?



A) 0

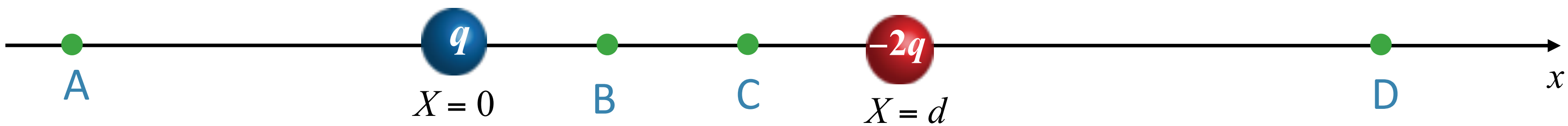
B) 1

C) 2

D) 3

Example

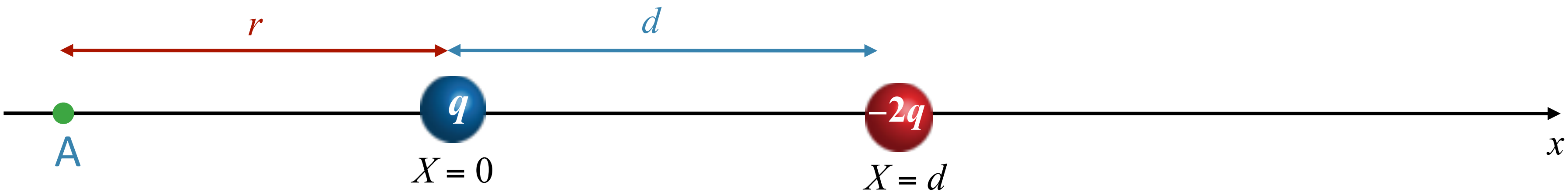
At which two places can a positive charge be placed without changing the total potential energy of the system?



A) A & B
B) A & C
C) B & C
D) B & D
E) A & D

Let's calculate the positions of A and B

Lets work out where A is



$$\Delta U = +\frac{1}{4\pi\varepsilon_0}\frac{Qq}{r} - \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{r+d}$$

Set $\Delta U = 0$

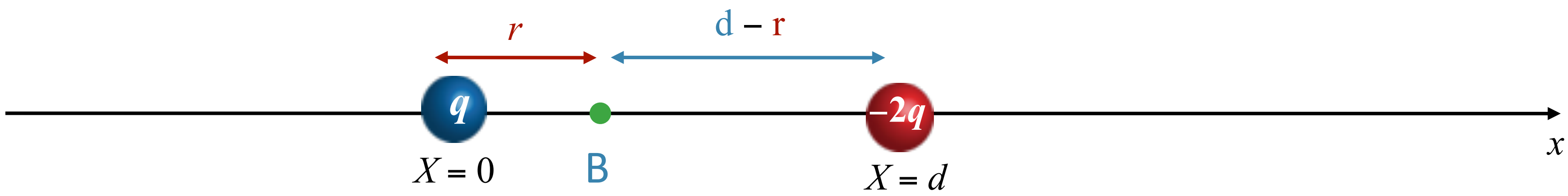
$$\frac{1}{r} = \frac{2}{r+d}$$

$$\boxed{r = d}$$

Makes Sense!

Q is twice as far from $-2q$ as it is from $+q$

Lets work out where B is



Setting $\Delta U = 0$ → $$\frac{1}{r} = \frac{2}{d-r}$$

$$2r = d - r$$

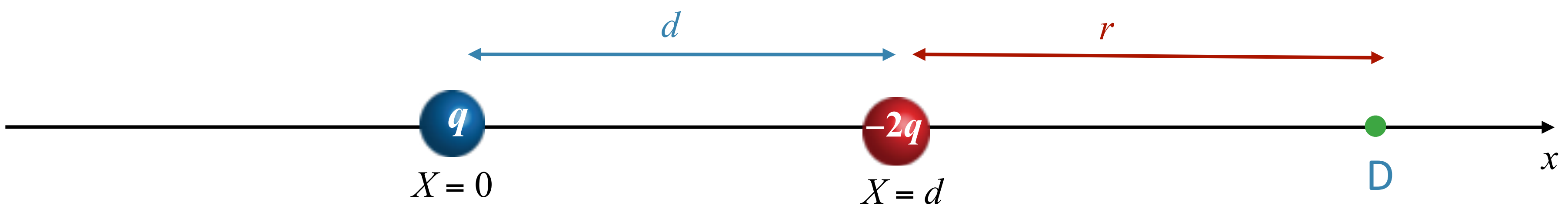
$$r = \frac{d}{3}$$

Makes Sense!

Q is twice as far from $-2q$ as it is from $+q$

What about D?

Can you prove that is *not* possible to put another charge at position D without changing *U*?



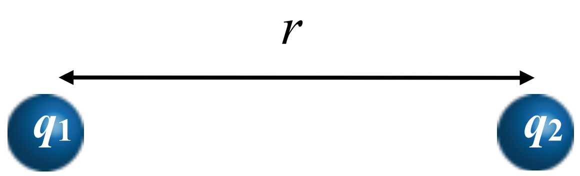
$$\frac{1}{r+d} = \frac{2}{r}$$

(*r* must be positive)

Summary

For a pair of charges:

Just evaluate $U = k\dfrac{q_1 q_2}{r}$



(We usually choose $U = 0$ to be where the charges are far apart)

For a collection of charges:

Sum up $U = k\dfrac{q_1 q_2}{r}$ for all pairs