Electricity & Magnetism
Lecture 6: Electric Potential

Today's Concept:

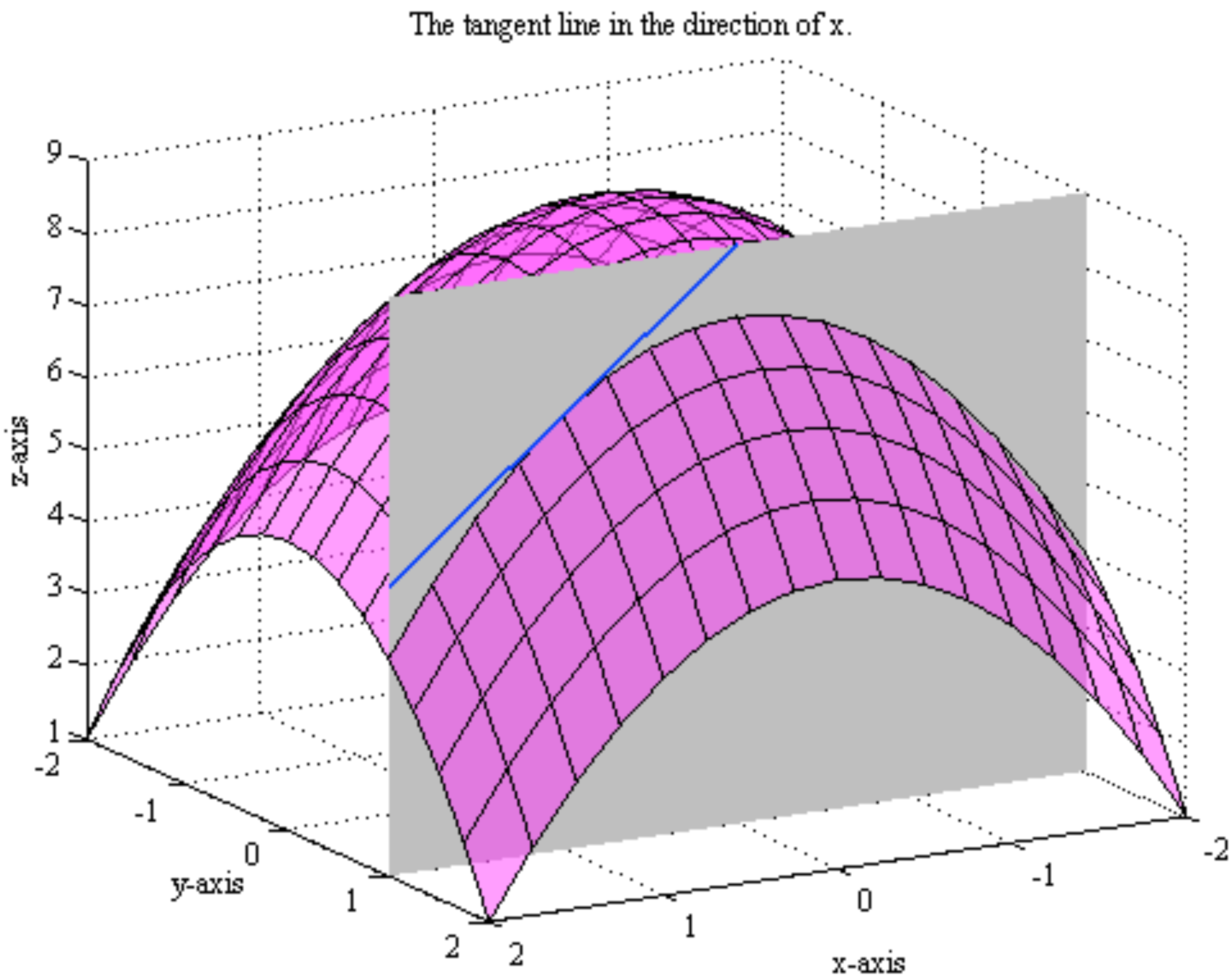
Electric Potential

(Defined in terms of Path Integral of Electric Field)

Stuff you asked about:

- "Explain more why E is negative of delta V"
- "Are we required to know partial derivatives? I think they came up in the pre-lecture and I haven't learnt them yet. ."
- "So we just need to superimpose the radial field lines which are found by taking the negative of the gradient of the electric potential in 3d cartesian/spherical/cylindrical coordinate system and are perpendicular to equipotentials, the locus point of all point with the same potential difference. Simple enough... We'll just do that! . . .. Ohhhhh wait... WHAT? ."
- "so electric potential is the antiderivative of electric field is that correct? in other words the area under electric field function gives the electric potential?
- What is the difference between an integral of a dot product and a integral of a simple product?

Partial Derivative



Applications

- How a van de Graaff machine works: https://youtu.be/EsZQS2GOMQE
- How an electrostatic photocopier works: https://youtu.be/Sy5PPaxaBCI

Big Idea

Last time we defined the electric potential energy of charge q in an electric field:

$$\Delta U_{a\to b} = -\int_a^b \vec{F}\cdot d\vec{l} = -\int_a^b q\vec{E}\cdot d\vec{l}$$

The only mention of the particle was through its charge q.

We can obtain a new quantity, the electric potential, which is a PROPERTY OF THE SPACE, as the potential energy per unit charge.

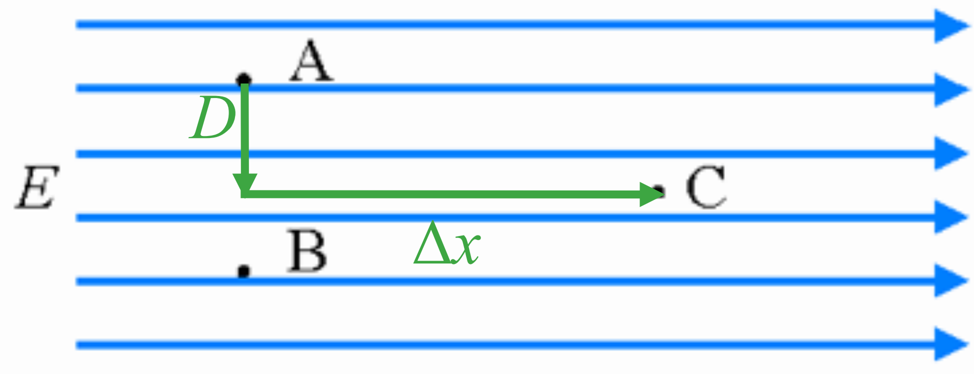
$$\Delta V_{a\to b} \equiv \frac{\Delta U_{a\to b}}{q}$$

Note the similarity to the definition of another quantity which is also a PROPERTY OF THE SPACE, the electric field.

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

Electric Potential from E field

Consider the three points A, B, and C located in a region of constant electric field as shown.



What is the sign of $\Delta V_{AC} = V_C - V_A$?

A) $\Delta V_{AC} < 0$ B) $\Delta V_{AC} = 0$ C) $\Delta V_{AC} > 0$

Remember the definition: $\Delta V_{a \to b} = -\int_a^b \vec{E} \cdot d\vec{l}$

Choose a path (any will do!)

$$\Delta V_{A \to C} = -\int_A^D \vec{E} \cdot d\vec{l} - \int_D^C \vec{E} \cdot d\vec{l} \quad \longrightarrow \quad \Delta V_{A \to C} = -0 - \int_D^C \vec{E} \cdot d\vec{l} = -E\Delta x < 0$$

CheckPoint: Zero Electric Field

Suppose the electric field is zero in a certain region of space. Which of the following statements best describes the electric potential in this region?

A. The electric potential is zero everywhere in this region.

B. The electric potential is zero at least one point in this region.

C. The electric potential is constant everywhere in this region.

D. There is not enough information given to distinguish which of the above answers is correct.

Remember the definition

$$\Delta V_{a\to b} = -\int_a^b \vec{E}\cdot d\vec{l}$$

$$\vec{E} = 0 \longrightarrow \Delta V_{A\to B} = 0 \longrightarrow V \text{ is constant!}$$

E from V

If we can get the potential by integrating the electric field:

$$\Delta V_{a \to b} = -\int_a^b \vec{E} \cdot d\vec{l}$$

We should be able to get the electric field by differentiating the potential?

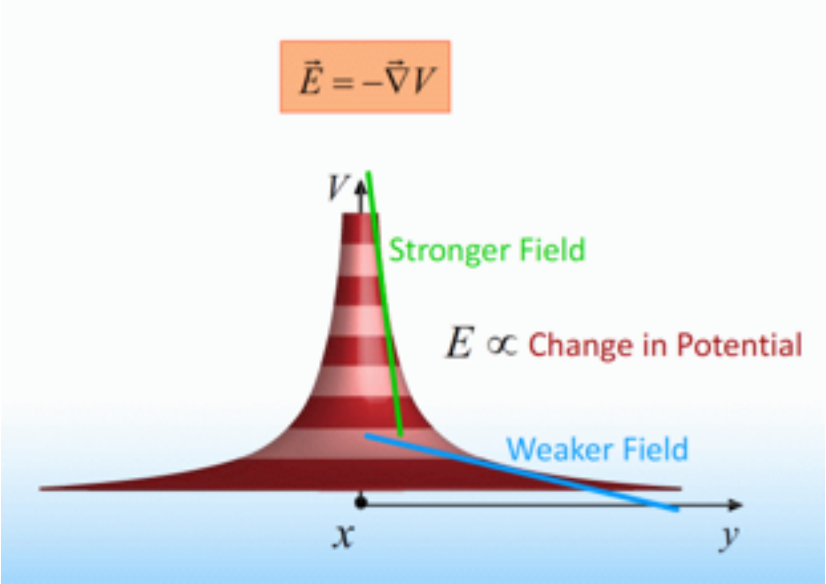
$$\vec{E} = -\vec{\nabla} V$$

In Cartesian coordinates:

$$E_x = -\frac{dV}{dx}$$

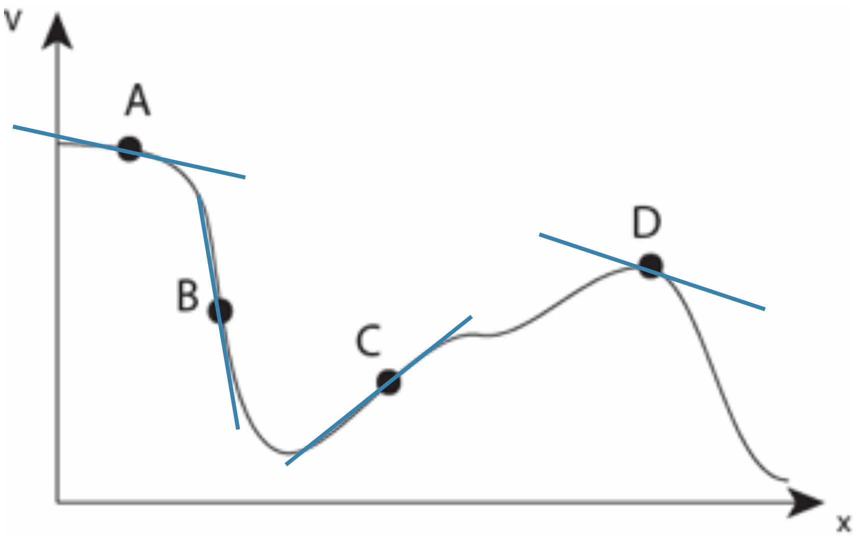
$$E_y = -\frac{dV}{dy}$$

$$E_z = -\frac{dV}{dz}$$



CheckPoint: Spatial Dependence of Potential 1

The electric potential in a certain region is plotted in the following graph



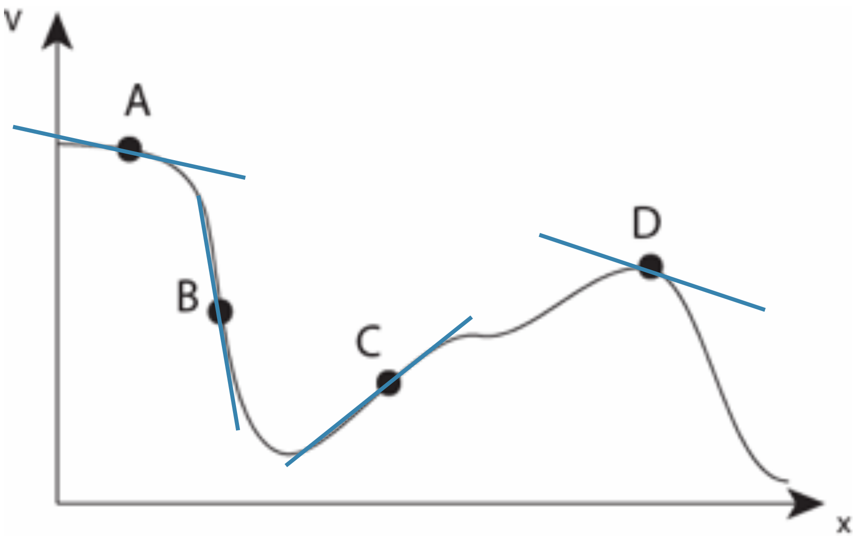
At which point is the magnitude of the E-FIELD greatest?

- ○ A
- ○ B
- ○ C
- ○ D

$$\vec{E} = -\vec{\nabla} V$$

CheckPoint: Spatial Dependence of Potential 1

The electric potential in a certain region is plotted in the following graph



How do we get E from V?

At which point is the magnitude of the E-FIELD greatest?

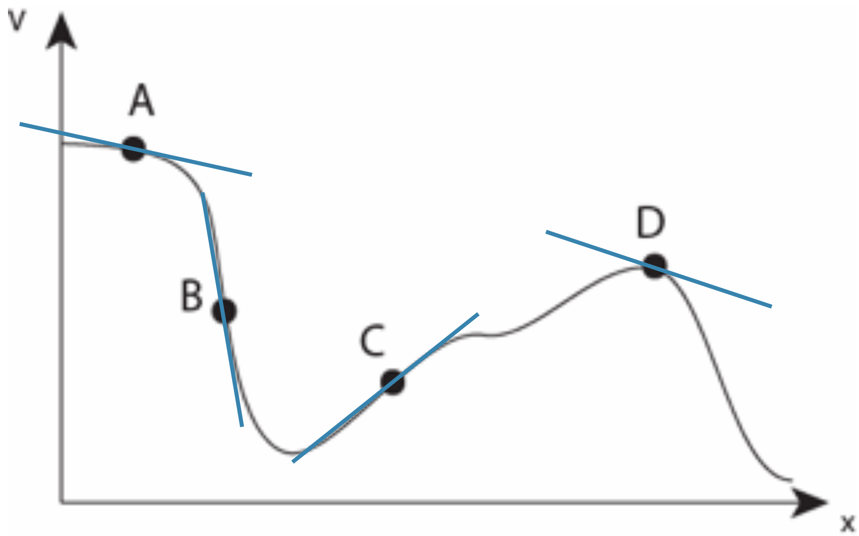
- ○ A
- ○ B
- ○ C
- ○ D

$$\vec{E} = -\vec{\nabla}V \rightarrow E_x = -\frac{\partial V}{\partial x} \rightarrow$$

Look at slopes!

CheckPoint: Spatial Dependence of Potential 2

The electric potential in a certain region is plotted in the following graph



At which point is the direction of the E-field along the negative x-axis?

- ○ A
- ○ B
- ○ C
- ○ D

"At B, the slope is decreasing (-) so the direction of the E field is negative "

"E is negative when the slope of V is positive (E=-dV/dx). Therefore E is directed along the x-axis at point C. "

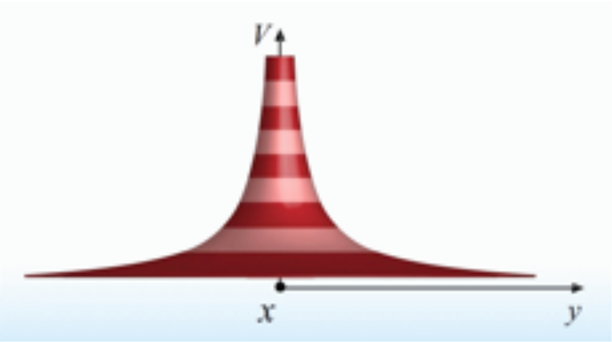
How do we get E from V?

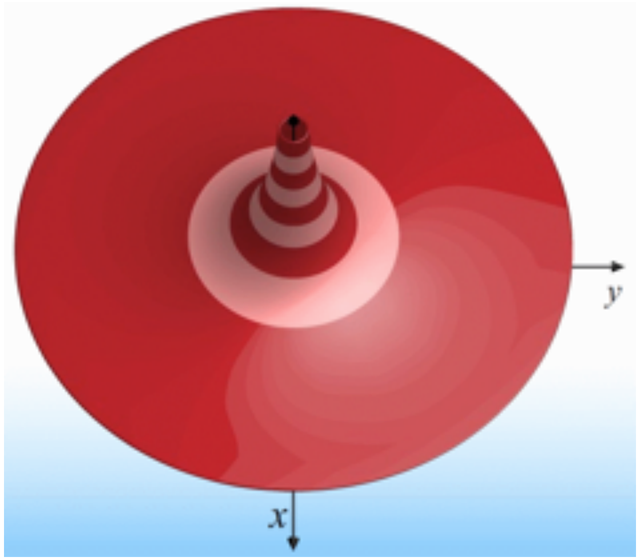
$$\vec{E} = -\vec{\nabla} V \longrightarrow E_x = -\frac{\partial V}{\partial x} \longrightarrow$$

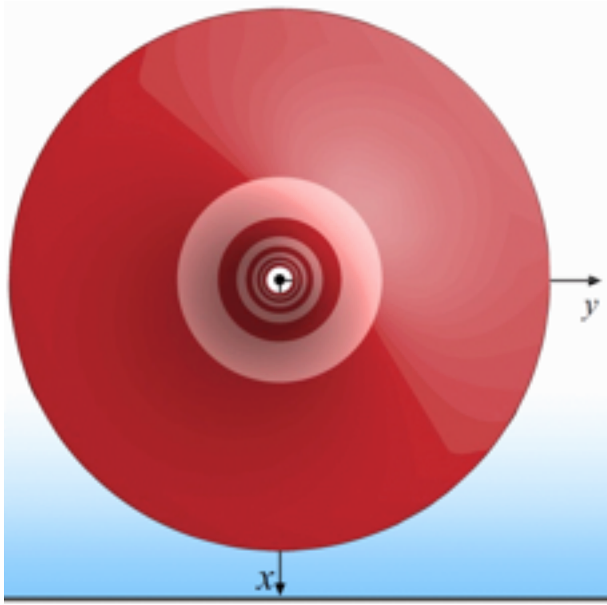
Look at slopes!

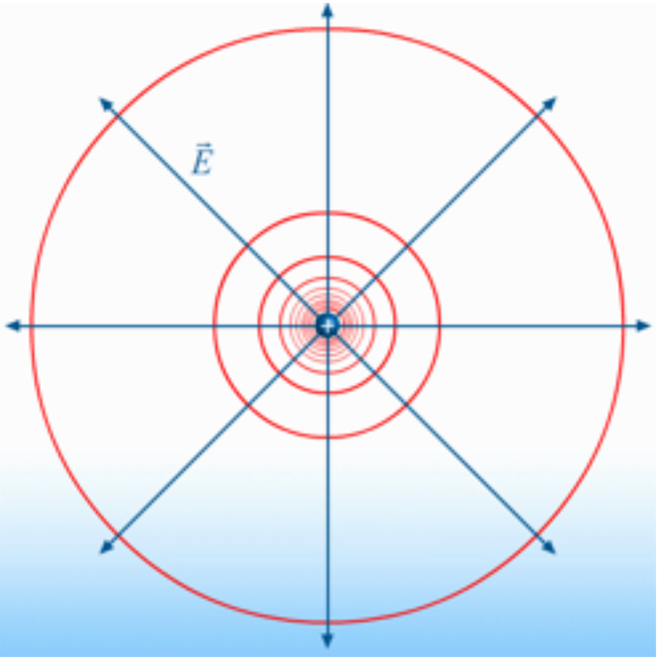
Equipotentials

Equipotentials are the locus of points having the same potential.





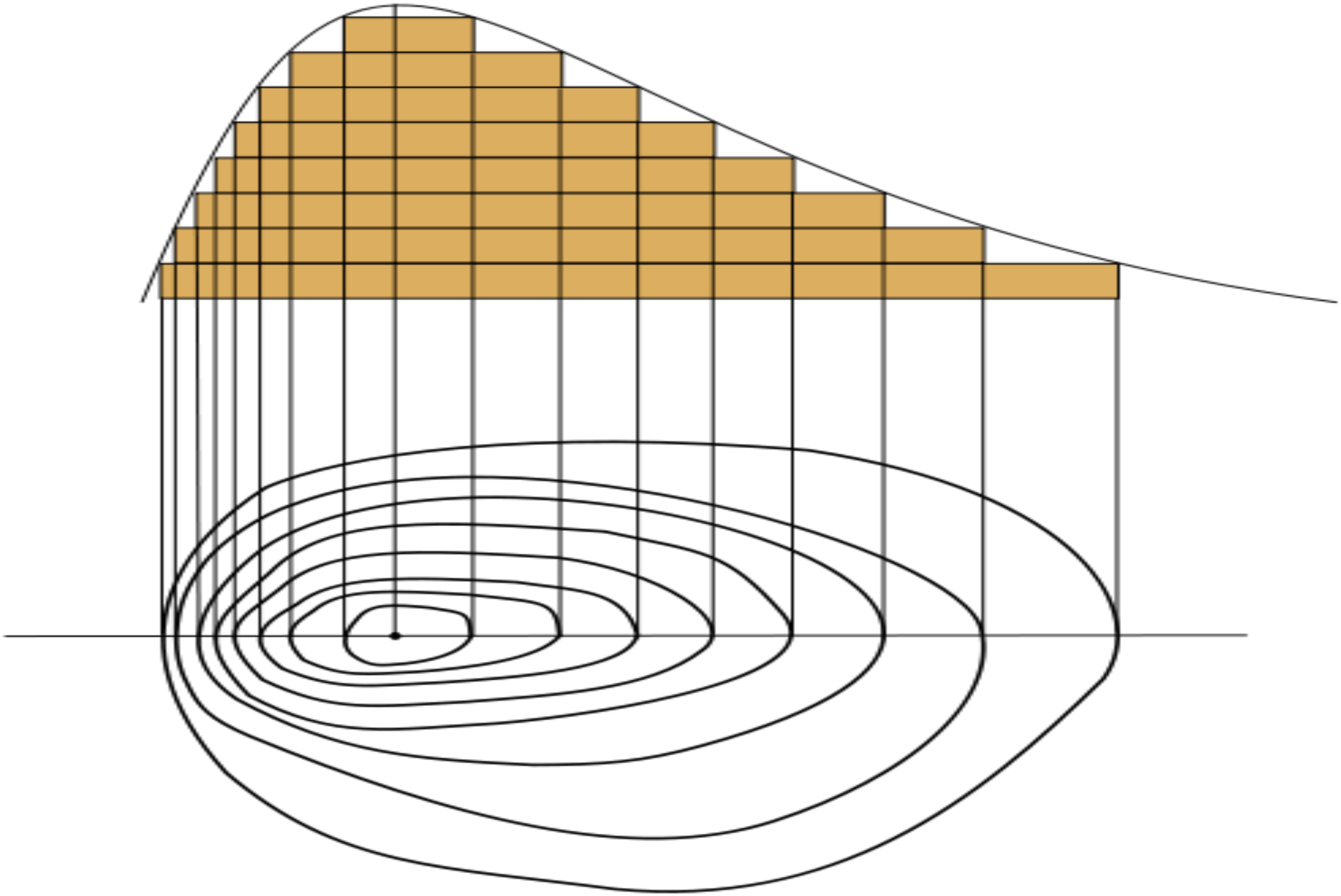




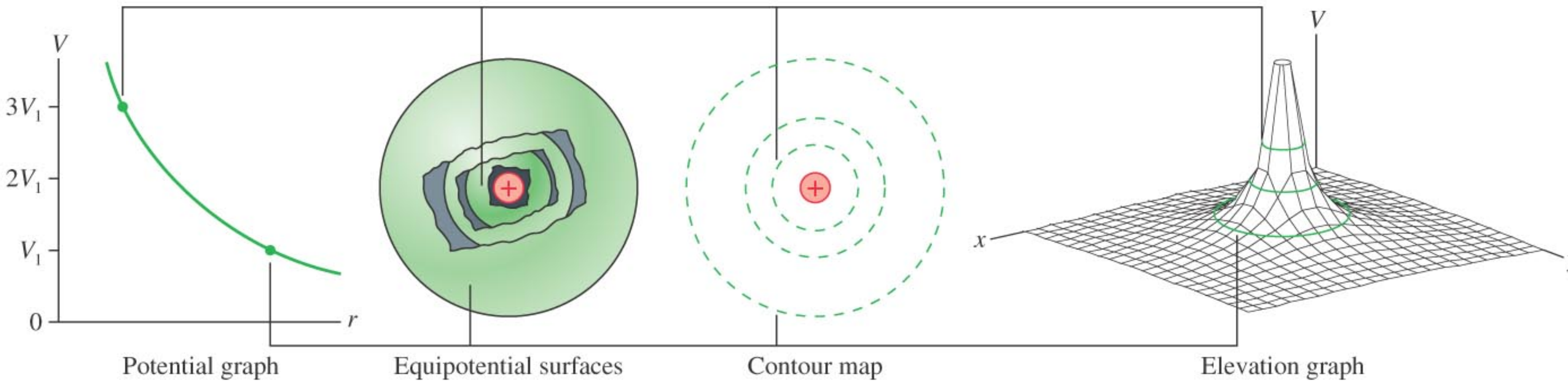
Equipotentials are
ALWAYS
perpendicular to the electric field lines.

The SPACING of the equipotentials indicates
The STRENGTH of the electric field.

Contour Lines on Topographic Maps

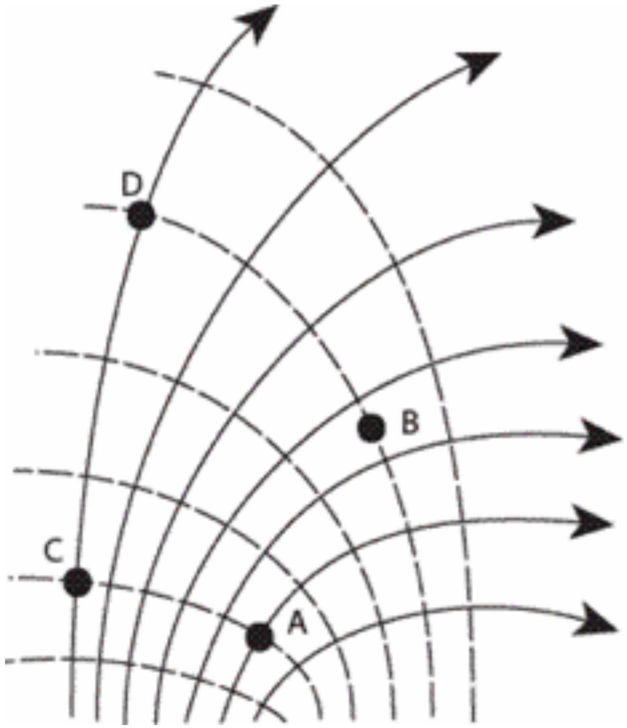


Visualizing the Potential of a Point Charge



CheckPoint: Electric Field Lines 1

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



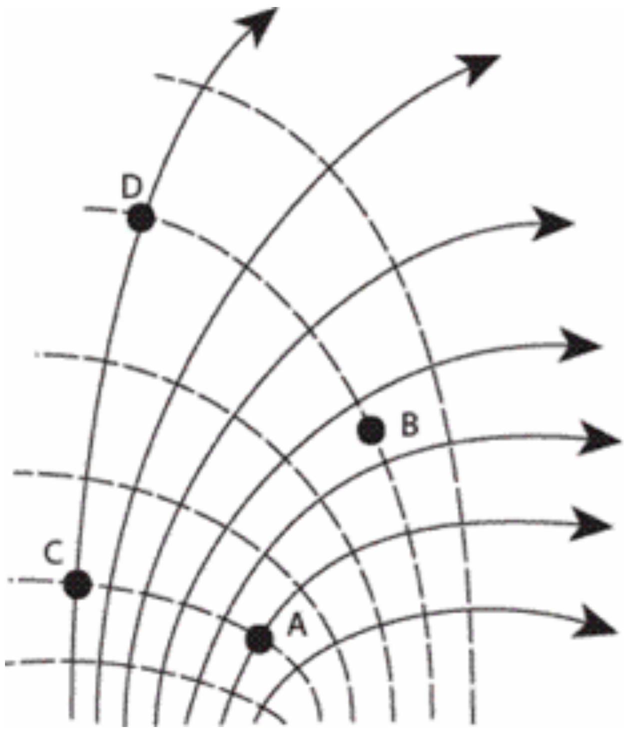
At which point in space is the E-field the weakest?

- ○ A
- ○ B
- ○ C
- ○ D

"The electric field lines are the least dense at D"
" From what I know, the answer should be D"
" D is where the electric field lines are the least dense "
" I'm pretty sure the electric field lines are the least dense at D"
" I'd guess D "

CheckPoint: Electric Field Lines 2

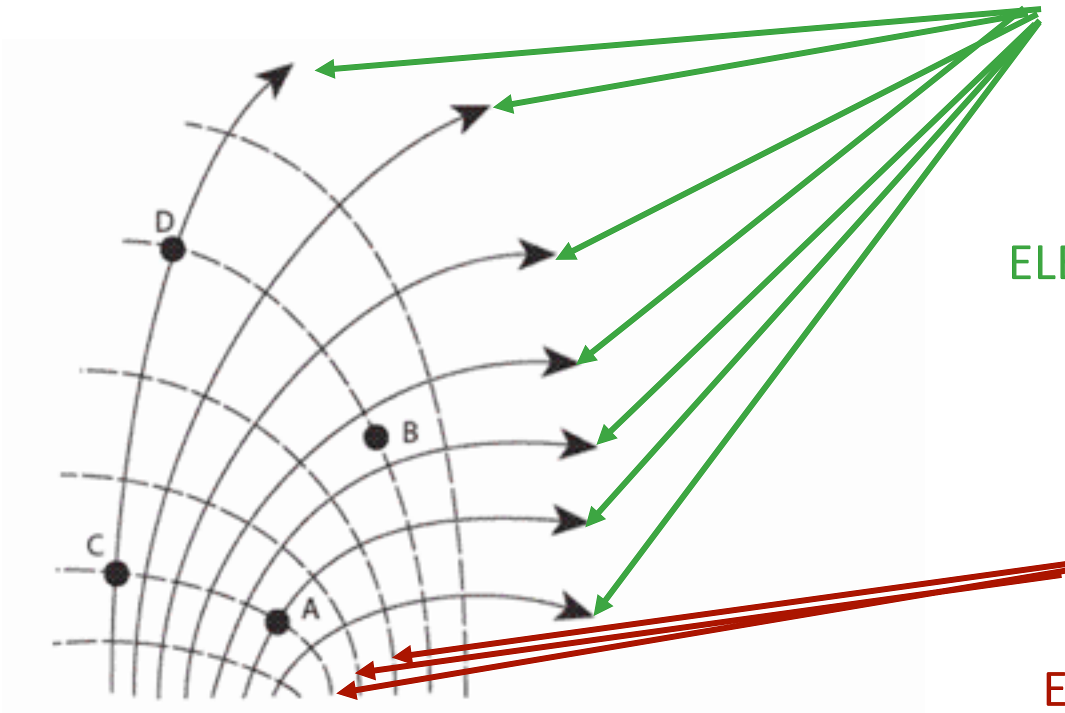
The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



Compare the work done moving a negative charge from A to B and from C to D. Which one requires more work?

A. More work is required to move a negative charge from A to B than from C to D
B. More work is required to move a negative charge from C to D than from A to B
C. The same amount of work is required to move a negative charge from A to B as to move it from C to D
D. Cannot determine without performing the calculation

Clicker Question: Electronic Field 2



What are these?

ELECTRIC FIELD LINES!

What are these?

EQUIPOTENTIALS!

What is the sign of W_{AC} = work done by E field to move negative charge from A to C ?

A) $W_{AC} < 0$

B) $W_{AC} = 0$

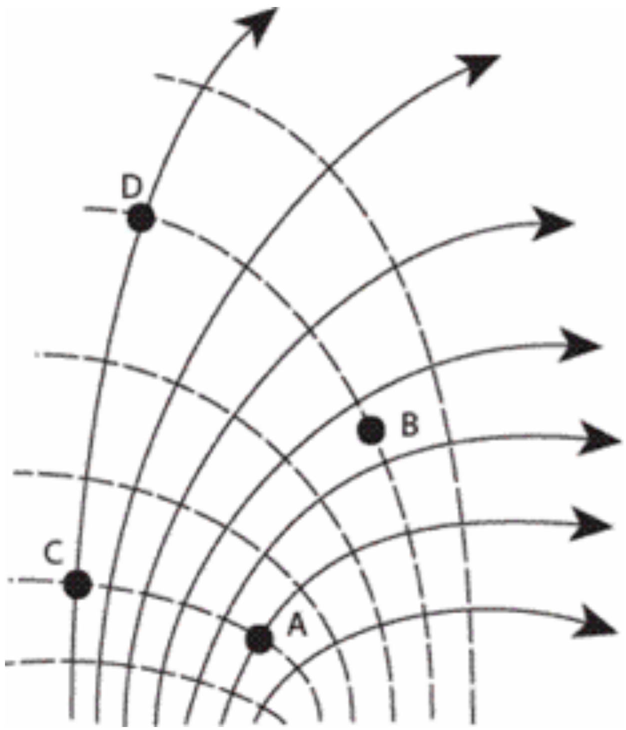
C) $W_{AC} > 0$

A and C are on the same equipotential → $W_{AC} = 0$

Equipotentials are perpendicular to the E field: No work is done along an equipotential

CheckPoint Results: Electric Field Lines 2

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.



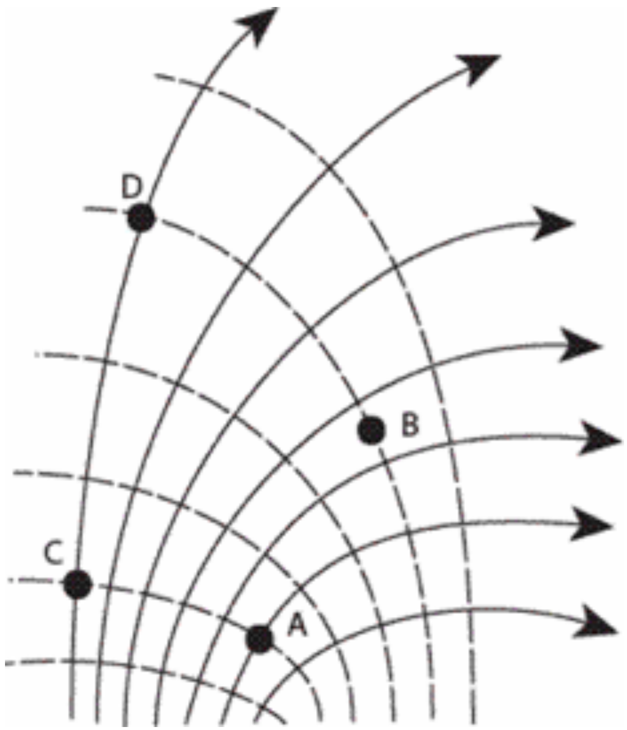
Compare the work done moving a negative charge from A to B and from C to D. Which one requires more work?

A. More work is required to move a negative charge from A to B than from C to D
B. More work is required to move a negative charge from C to D than from A to B
C. The same amount of work is required to move a negative charge from A to B as to move it from C to D
D. Cannot determine without performing the calculation

- A and C are on the same equipotential
- B and D are on the same equipotential
- Therefore the potential difference between A and B is the SAME as the potential between C and D

CheckPoint: Electric Field Lines 3

The field-line representation of the E-field in a certain region in space is shown below. The dashed lines represent equipotential lines.

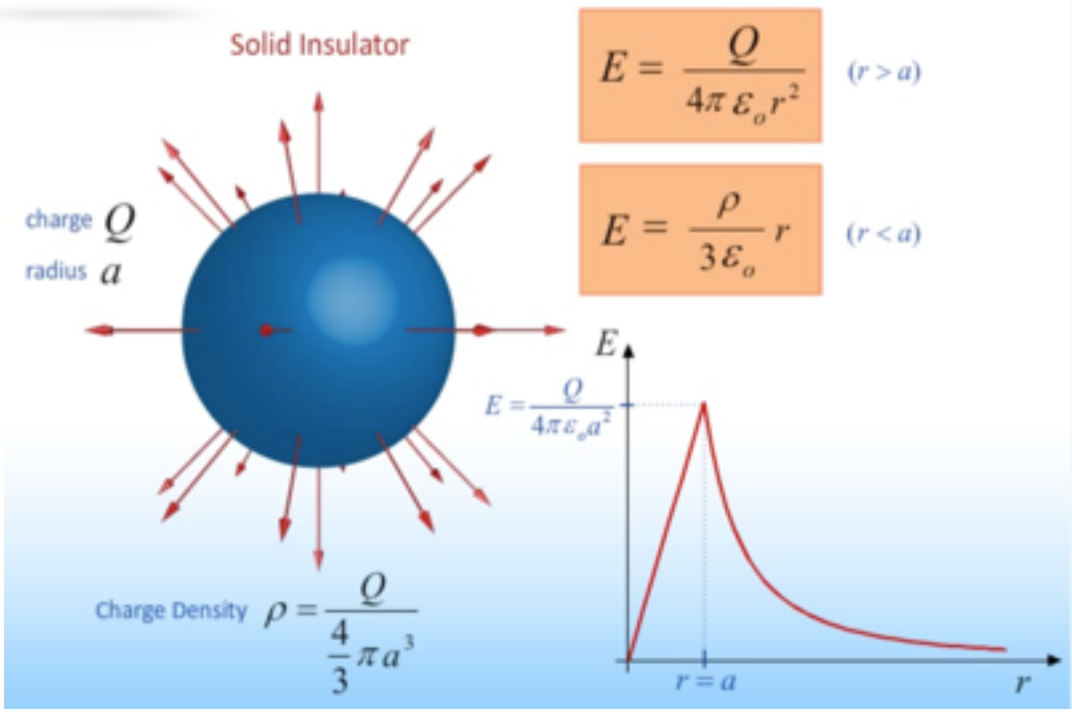


Compare the work done moving a negative charge from A to B and from **A to D**. Which one requires more work?

A. More work is required to move a negative charge from A to B than from A to D
B. More work is required to move a negative charge from A to D than from A to B
C. The same amount of work is required to move a negative charge from A to B as to move it from A to D
D. Cannot determine without performing the calculation

Insulating Charged Sphere

last week



$$E = \frac{\rho}{3\varepsilon_o} r$$

$$E = \frac{\frac{Q}{\frac{4}{3}\pi a^3}}{3\varepsilon_0} r$$

$$E = \frac{Q}{4\pi\varepsilon_0 a^3} r$$

$$E = k\frac{Q}{a^3} r$$

Insulating charged sphere

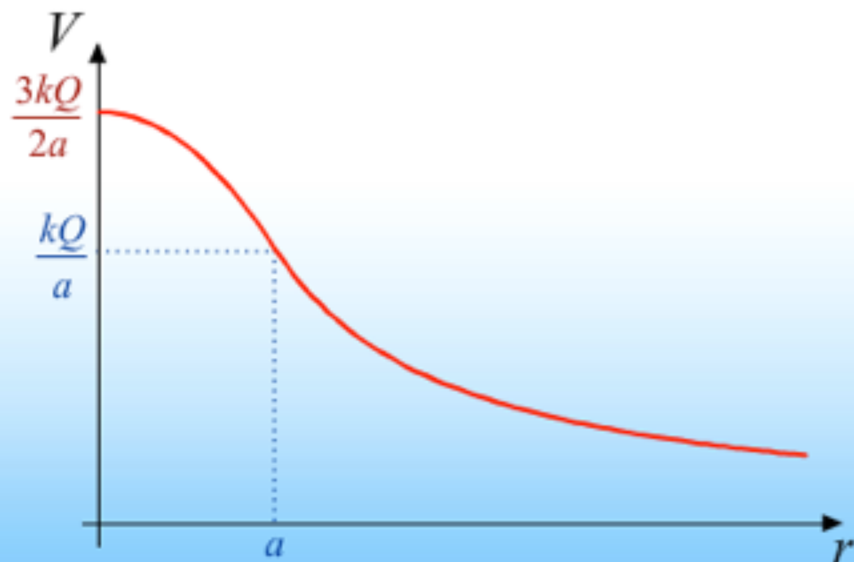
Solid Insulator

charge Q

radius a

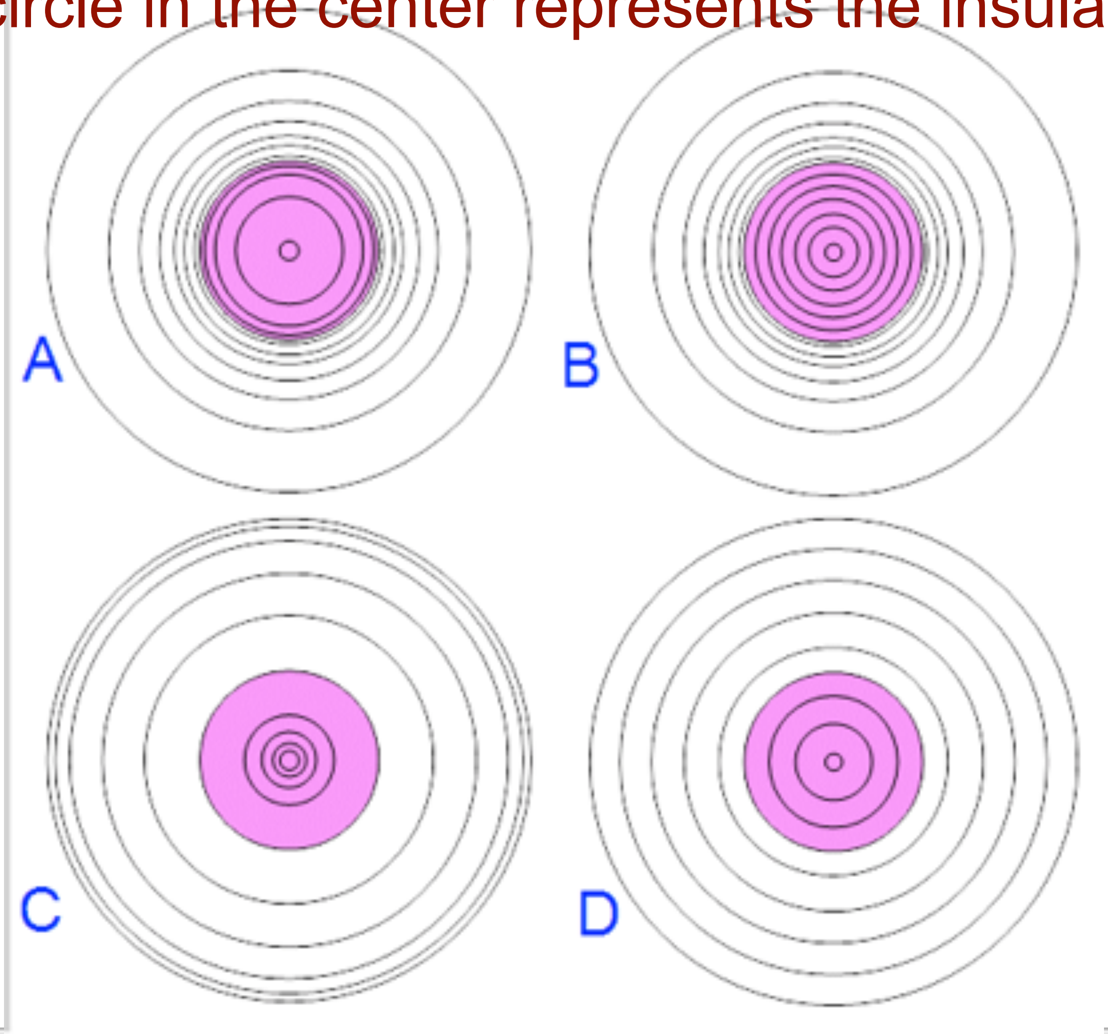
$$V(r) = k\frac{Q}{r}$$ For $r > a$

$$V(r) = k\frac{Q}{2a^3}(3a^2 - r^2)$$ For $r < a$



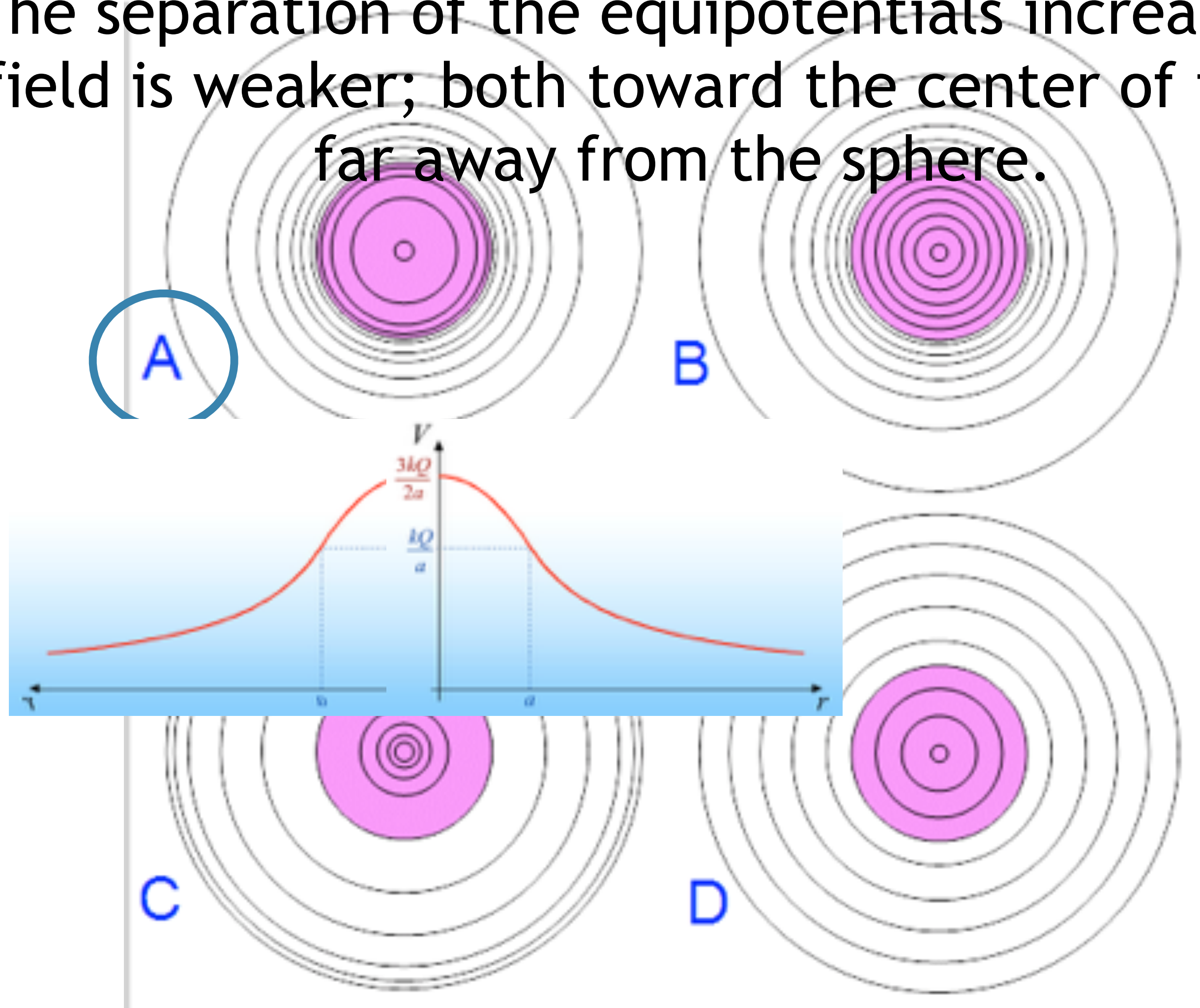
Prelecture Question

Which of the following equipotential diagrams best describes the spherical insulator in the previous example? (The colored circle in the center represents the insulator.)



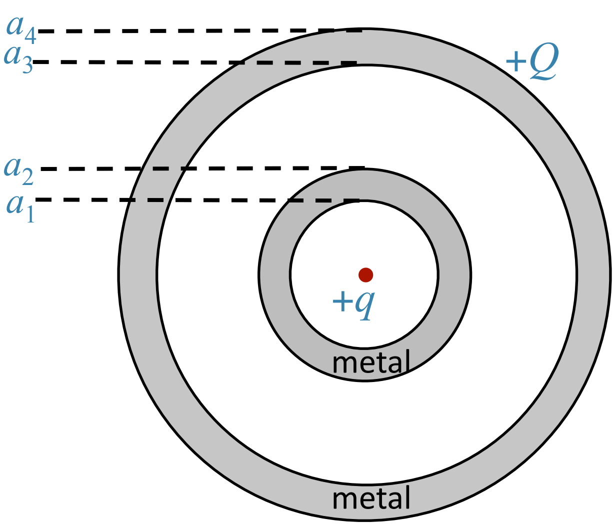
Prelecture Question

The separation of the equipotentials is smallest at the outer radius of the pink sphere since the electric field is strongest there. The separation of the equipotentials increases where the electric field is weaker; both toward the center of the sphere and far away from the sphere.



Calculation for Potential

cross-section



Point charge q at center of concentric conducting spherical shells of radii a_1, a_2, a_3, and a_4. The inner shell is uncharged, but the outer shell carries charge Q.

What is V as a function of r?

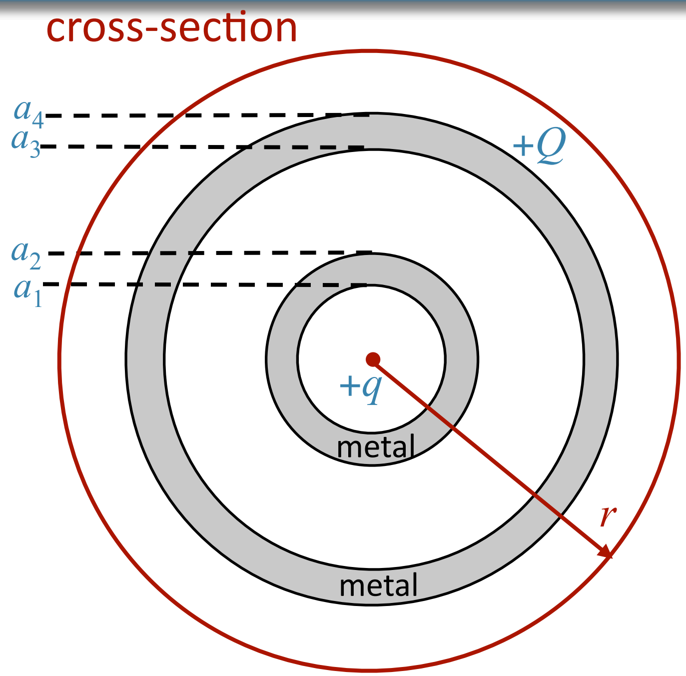
Conceptual Analysis:

- Charges q and Q will create an E field throughout space
- $$V(r) = -\int_{r_0}^{r} \vec{E} \cdot d\vec{\ell}$$

Strategic Analysis:

- Spherical symmetry: Use Gauss' Law to calculate E everywhere
- Integrate E to get V

Calculation: Quantitative Analysis



$r > a_4$: What is $E(r)$?

A) 0

B) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

C) $\frac{1}{2\pi\varepsilon_0}\frac{Q+q}{r}$

D) $\frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r^2}$

E) $\frac{1}{4\pi\varepsilon_0}\frac{Q-q}{r^2}$

Why?

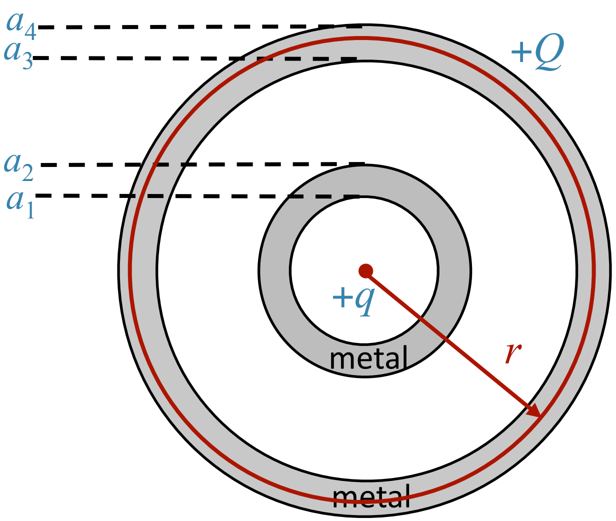
Gauss' law: $\int \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_0}$

$$E 4\pi r^2 = \frac{Q+q}{\varepsilon_0}$$

$$\rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r^2}$$

Calculation: Quantitative Analysis

cross-section



$a_3 < r < a_4$: What is $E(r)$?

A) 0

B) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

C) $\frac{1}{2\pi\varepsilon_0}\frac{q}{r}$

D) $\frac{1}{4\pi\varepsilon_0}\frac{-q}{r^2}$

E) $\frac{1}{4\pi\varepsilon_0}\frac{Q-q}{r^2}$

Applying Gauss' law, what is $Q_{enclosed}$ for red sphere shown?

A) q B) $-q$ C) 0

How is this possible?

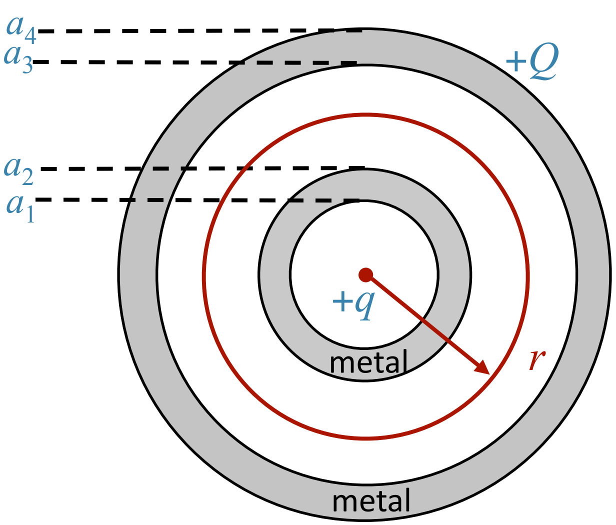
$-q$ must be induced at $r = a_3$ surface → charge at $r = a_4$ surface $= Q + q$

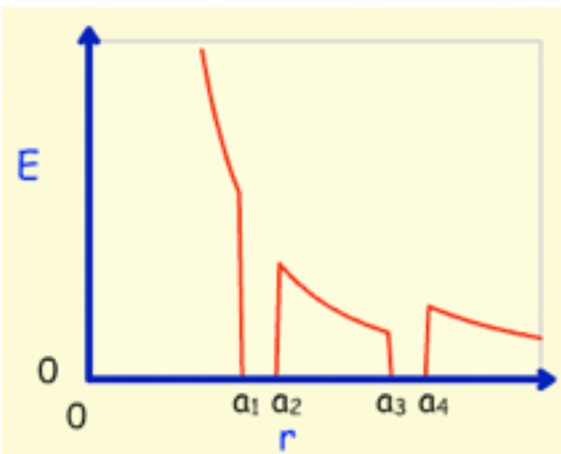
$$E4\pi r^2 = \frac{Q+q}{\varepsilon_0}$$

$$\sigma_4 = \frac{Q+q}{4\pi a_4^2}$$

Calculation: Quantitative Analysis

cross-section





Continue on in...

$a_2 < r < a_3$: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

$a_1 < r < a_2$: $E = 0$

$r < a_1$: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

$r > a_4$: $V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r}$

To find V:

1) Choose r_0 such that $V(r_0) = 0$ (usual: r_0 = infinity)
2) Integrate!

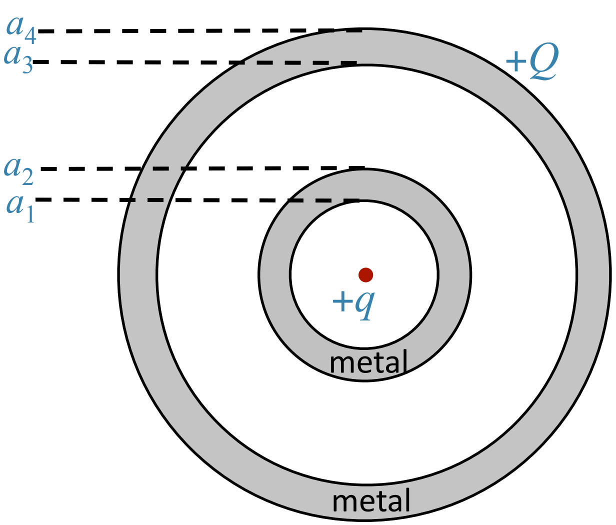
$a_3 < r < a_4$:

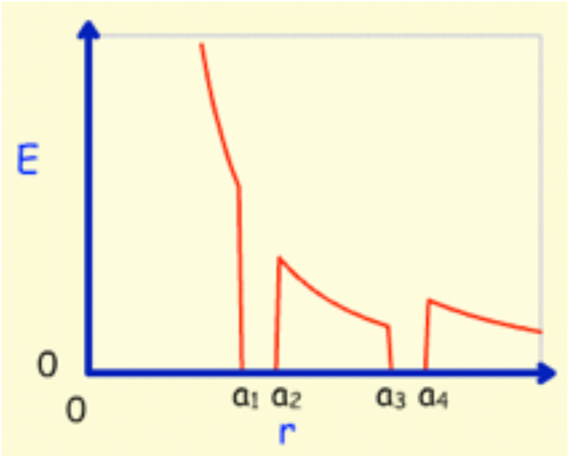
A) $V = 0$

B) $V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{a_4} = \Delta V(\infty \rightarrow a_4) + 0$

C) $V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{a_3}$

Calculation: Quantitative Analysis





$$r > a_4: \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{r}$$

$$a_3 < r < a_4: \quad V = \frac{1}{4\pi\varepsilon_0}\frac{Q+q}{a_4}$$

$$a_2 < r < a_3: \quad V(r) = \Delta V(\infty \to a_4) + 0 + \Delta V(a_3 \to r)$$

$$V(r) = \frac{Q+q}{4\pi\varepsilon_0 a_4} + 0 + \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{a_3}\right) \rightarrow V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_4} + \frac{q}{r} - \frac{q}{a_3}\right)$$

$$a_1 < r < a_2: \quad V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_4} + \frac{q}{a_2} - \frac{q}{a_3}\right)$$

$$0 < r < a_1: \quad V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q+q}{a_4} + \frac{q}{a_2} - \frac{q}{a_3} + \frac{q}{r} - \frac{q}{a_1}\right)$$