Electricity & Magnetism
Lecture 9: Conductors and Capacitance

Today's Concept:

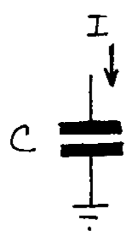
A) Conductors

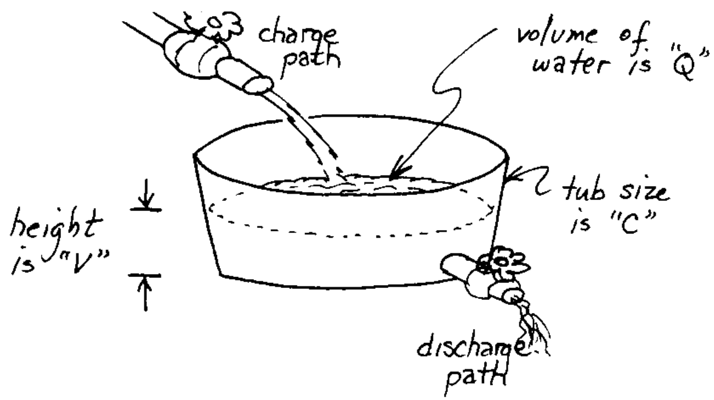
B) Capacitance

(

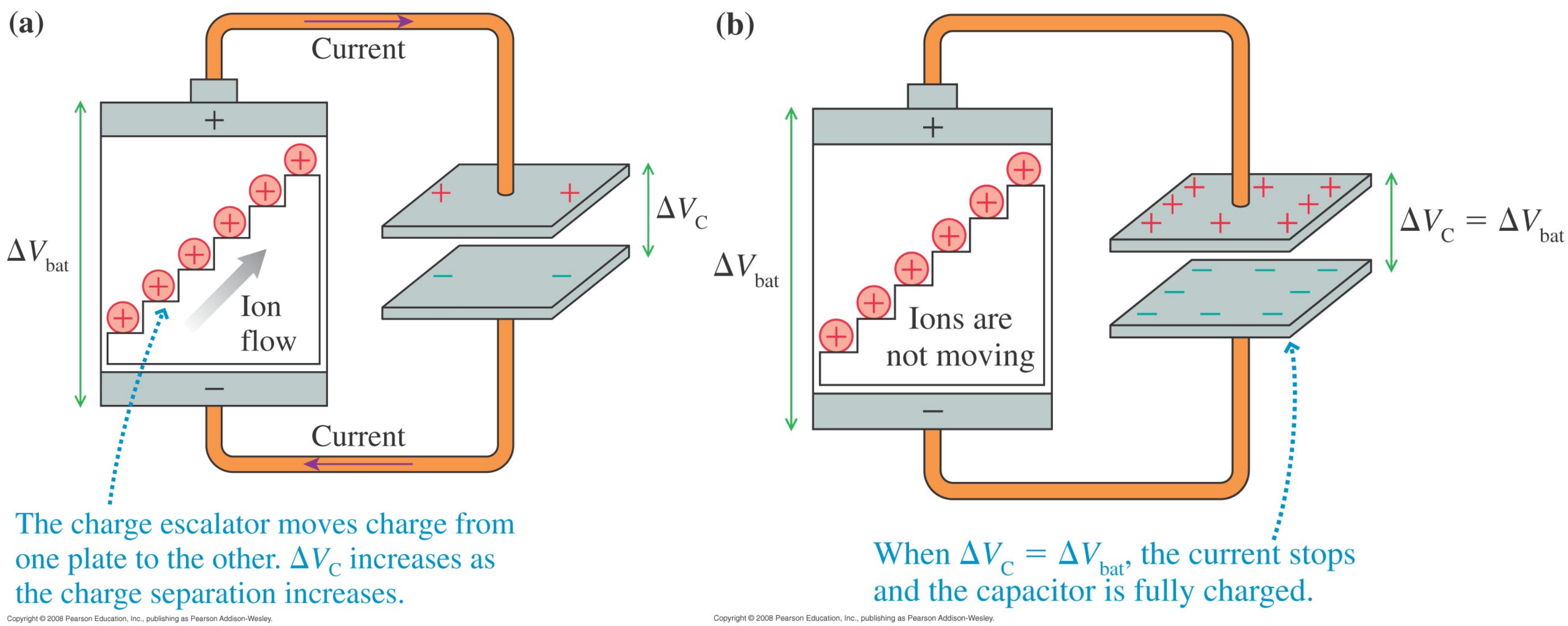
Stuff you asked about:

- “They never said what capacitance is or what its used for”
- “The charged plate questions were the most difficult, especially with the green conductor plates in the middle. I don't know how to account for that. ”
- “What is the difference between capacitors and batteries, although they both stores energy, can we say that capacitors are a kind a batteries or vice versa?”





Capacitors vs. Batteries



- A battery uses an electrochemical reaction to separate charges. ΔV between terminals is called "EMF", symbol: $\mathcal{E}$, $\mathcal{E}$ or $\mathscr{E}$.
- A Capacitor stores charge on its plates.

Some capacitors

Our Comments

WE BELIEVE THERE ARE ONLY THREE THINGS YOU NEED TO KNOW TO DO ALL OF HOMEWORK!

1. $E = 0$ within the material of a conductor: (at static equilibrium)

Charges move inside a conductor in order to cancel out the fields that would be there in the absence of the conductor. This principle determines the induced charge densities on the surfaces of conductors.

2. Gauss' Law:

$$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

If charge distributions have sufficient symmetry (spherical, cylindrical, planar), then Gauss' law can be used to determine the electric field everywhere.

3. Definition of Potential:

$$\Delta V_{a \to b} \equiv \frac{\Delta U_{a \to b}}{q} = -\int_a^b \vec{E} \cdot d\vec{l}$$

CONCEPTS DETERMINE THE CALCULATION !

Conductors

The Main Points

- Charges free to move
- $E = 0$ in a conductor
- Surface = Equipotential
- E at surface perpendicular to surface

CheckPoint: Two Spherical Conductors 1

Two spherical conductors are separated by a large distance. They each carry the same positive charge Q. Conductor A has a larger radius than conductor B.

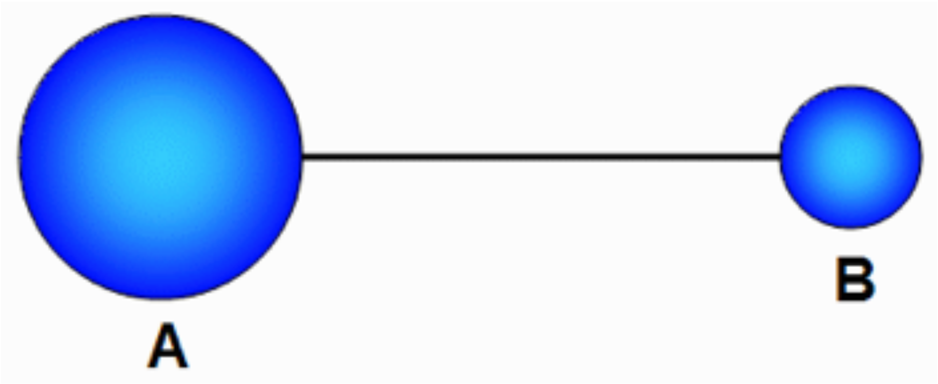
Compare the potential at the surface of conductor A with the potential at the surface of conductor B.

A. $V_A > V_B$
B. $V_A = V_B$
C. $V_A < V_B$

"The potential is the same as from a point charge at the center of A or B. So, following V=kQ/r, A would have a smaller potential "

CheckPoint: Two Spherical Conductors 2

Two spherical conductors are separated by a large distance. They each carry the same positive charge Q. Conductor A has a larger radius than conductor B.



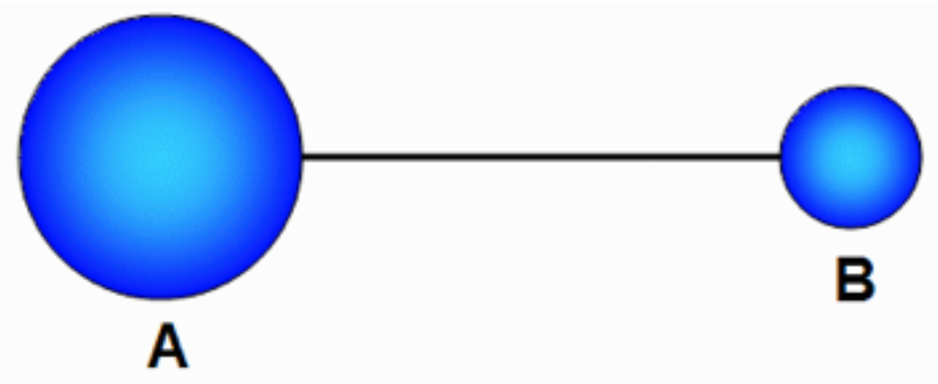
The two conductors are now connected by a wire. How do the potentials at the conductor surfaces compare now?

A. $V_A > V_B$
B. $V_A = V_B$
C. $V_A < V_B$

"The potentials will become equal since the charges will want to go to places of lower potential, until it balances out. "

CheckPoint: Two Spherical Conductors 3

Two spherical conductors are separated by a large distance. They each carry the same positive charge Q. Conductor A has a larger radius than conductor B.



What happens to the charge on conductor A after it is connected to conductor B by the wire?

A. Q_A increases
B. Q_A decreases
C. Q_A does not change

"Since B initially has a higher potential, charges move from B to A. "

Example 23-15 Two Charged Spherical Conductors

Two uncharged spherical conductors of radius $R_1 = 6.0$ cm and $R_2 = 2.0$ cm (Figure 23-26) and separated by a distance much greater than 6.0 cm are connected by a long, very thin conducting wire. A total charge $Q = +80$ nC is placed on one of the spheres and the system is allowed to reach electrostatic equilibrium. (*a*) What is the charge on each sphere? (*b*) What is the electric field strength at the surface of each sphere? (*c*) What is the electric potential of each sphere? (Assume that the charge on the connecting wire is negligible.)

(*a*) 1. Conservation of charge gives us one relation between the charges Q_1 and Q_2:

$$Q_1 + Q_2 = Q$$

2. Equating the potential of the spheres gives us a second relation for the charges Q_1 and Q_2:

$$\frac{kQ_1}{R_1} = \frac{kQ_2}{R_2} \quad \Rightarrow \quad Q_2 = \frac{R_2}{R_1} Q_1$$

3. Combine the results from steps 1 and 2 and solve for Q_1 and Q_2:

$$Q_1 + \frac{R_1}{R_2} Q_1 = Q \quad \text{so}$$

$$Q_1 = \frac{R_1}{R_1 + R_2} Q = \frac{6.0 \text{ cm}}{8.0 \text{ cm}} (80 \text{ nC}) = \boxed{60 \text{ nC}}$$

$$Q_2 = Q - Q_1 = \boxed{20 \text{ nC}}$$

(*b*) Use these results to calculate the electric field strengths at the surface of the spheres:

$$E_1 = \frac{kQ_1}{R_1^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(60 \times 10^{-9} \text{ C})}{(0.060 \text{ m})^2}$$

$$= \boxed{150 \text{ kN/C}}$$

$$E_2 = \frac{kQ_2}{R_2^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(20 \times 10^{-9} \text{ C})}{(0.020 \text{ m})^2}$$

$$= \boxed{450 \text{ kN/C}}$$

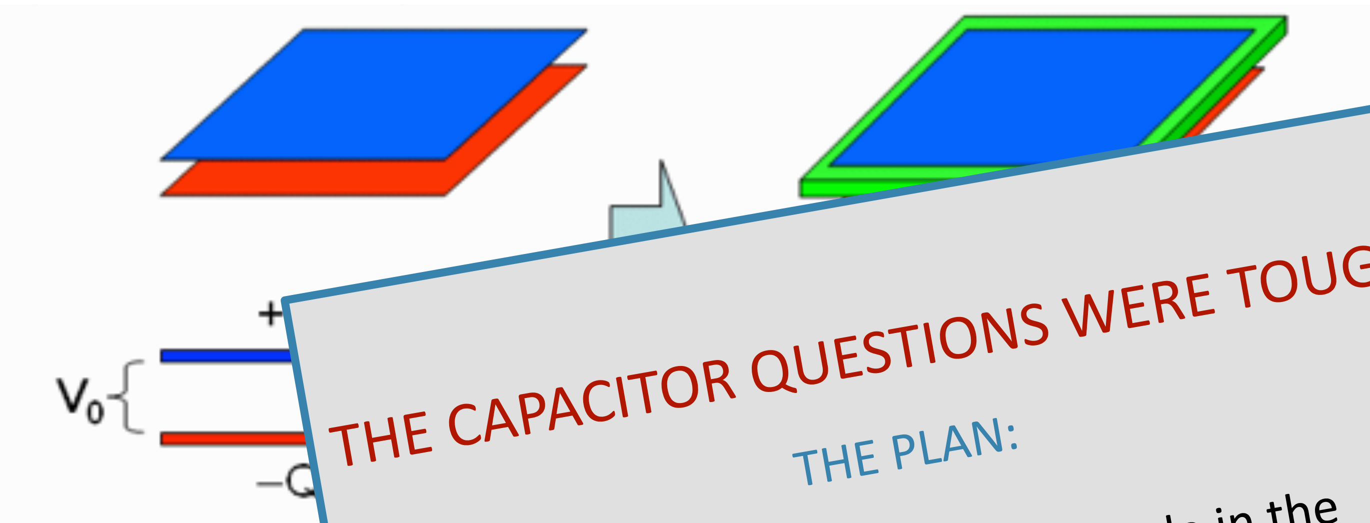
(*c*) Calculate the common potential from kQ/R for either sphere:

$$V_1 = \frac{kQ_1}{R_1} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(60 \times 10^{-9} \text{ C})}{0.060 \text{ m}}$$

$$= \boxed{9.0 \text{ kV}}$$

CheckPoint: Charged Parallel Plates 1

Two parallel plates of equal area carry equal and opposite charge $\mathbf{Q_0}$. The potential difference between the two plates is measured to be $\mathbf{V_0}$. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value $\mathbf{Q_1}$ such that the potential difference between the plates remains the same.



Compare $\mathbf{Q_1}$ and $\mathbf{Q_0}$.

$\mathbf{Q_1 < Q_0}$

$\mathbf{Q_1 = Q_0}$

$\mathbf{Q_1 > Q_0}$

THE CAPACITOR QUESTIONS WERE TOUGH!

THE PLAN:

We'll work through the example in the PreLecture and then do the preflight questions.

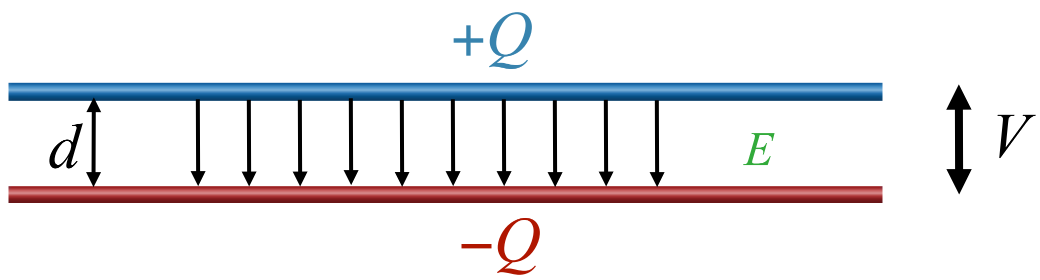
Capacitance

Capacitance is defined for any pair of spatially separated conductors.

$$C \equiv \frac{Q}{V}$$

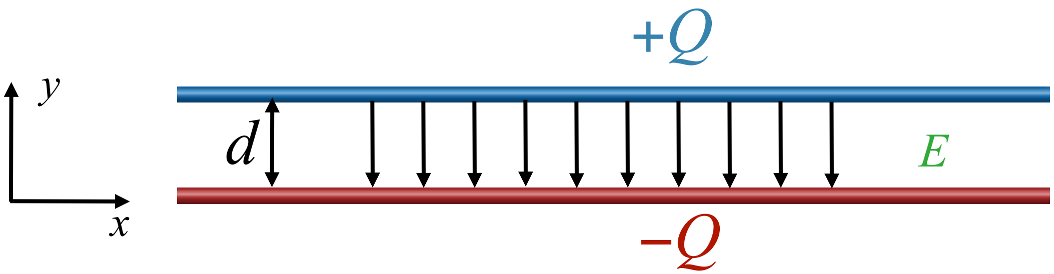
How do we understand this definition ?

- Consider two conductors, one with excess charge = $+Q$ and the other with excess charge = $-Q$



- These charges create an electric field in the space between them
- We can integrate the electric field between them to find the potential difference between the conductor
- This potential difference should be proportional to Q !
  - The ratio of Q to the potential difference is the capacitance and only depends on the geometry of the conductors

Example (done in Prelecture 7)



What is σ ?

$$E = \frac{\sigma}{\varepsilon_o} \qquad \sigma = \frac{Q}{A}$$

A = area of plate

Second, integrate E to find the potential difference V

$$V = -\int_0^d \vec{E}\cdot d\vec{y} \longrightarrow V = -\int_0^d(-E\,dy) = E\int_0^d dy = \frac{Q}{\varepsilon_o A}d$$

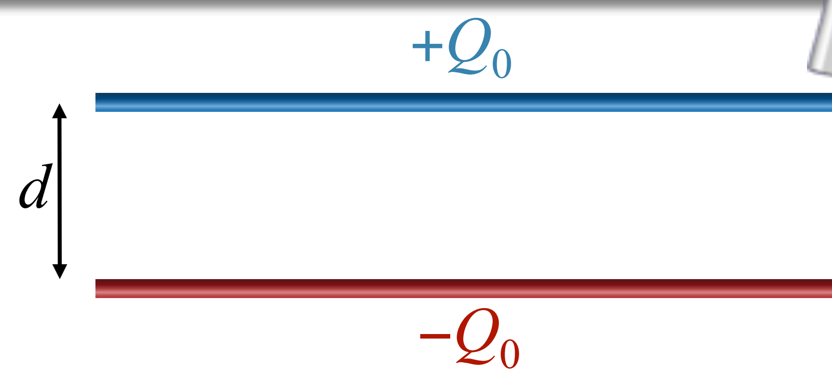
As promised, V is proportional to Q !

$$C \equiv \frac{Q}{V} = \frac{Q}{Qd/\varepsilon_o A} \longrightarrow C = \frac{\varepsilon_0 A}{d}$$

C determined by geometry !

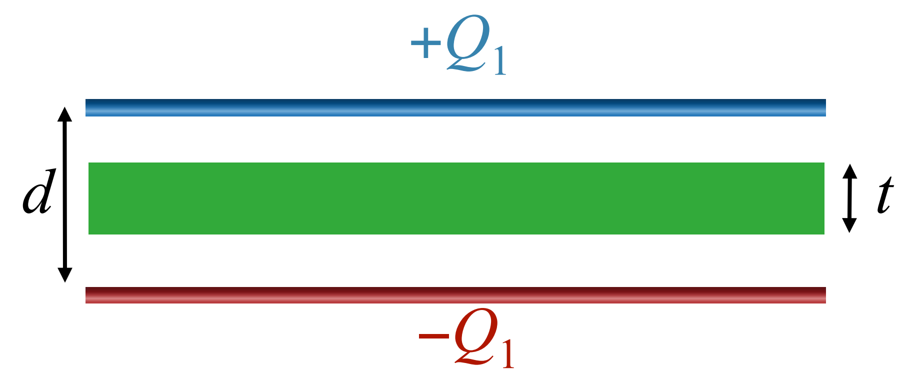
Clicker Question Related to CheckPoint

Initial charge on capacitor $= Q_0$



Insert uncharged conductor

Charge on capacitor now $= Q_1$



How is Q_1 related to Q_0 ?

A) $Q_1 < Q_0$

B) $Q_1 = Q_0$

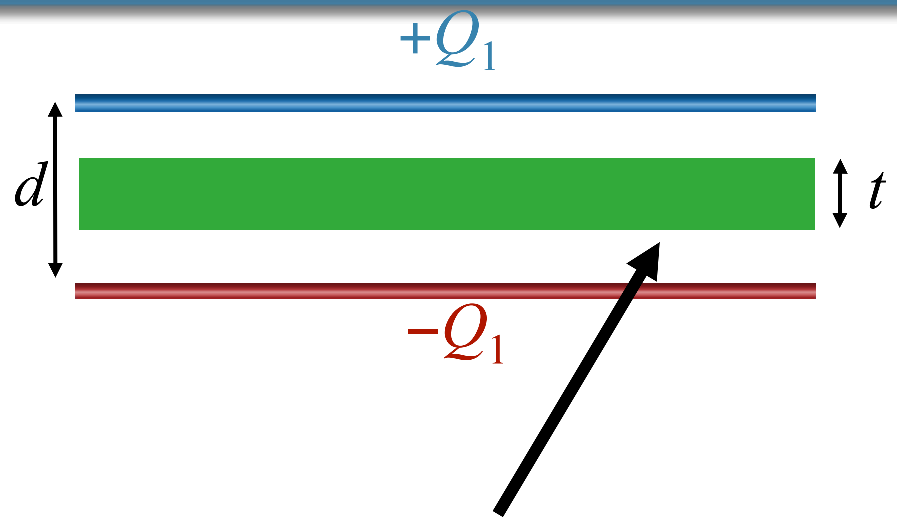
C) $Q_1 > Q_0$

Plates not connected to anything

↓

CHARGE CANNOT CHANGE !

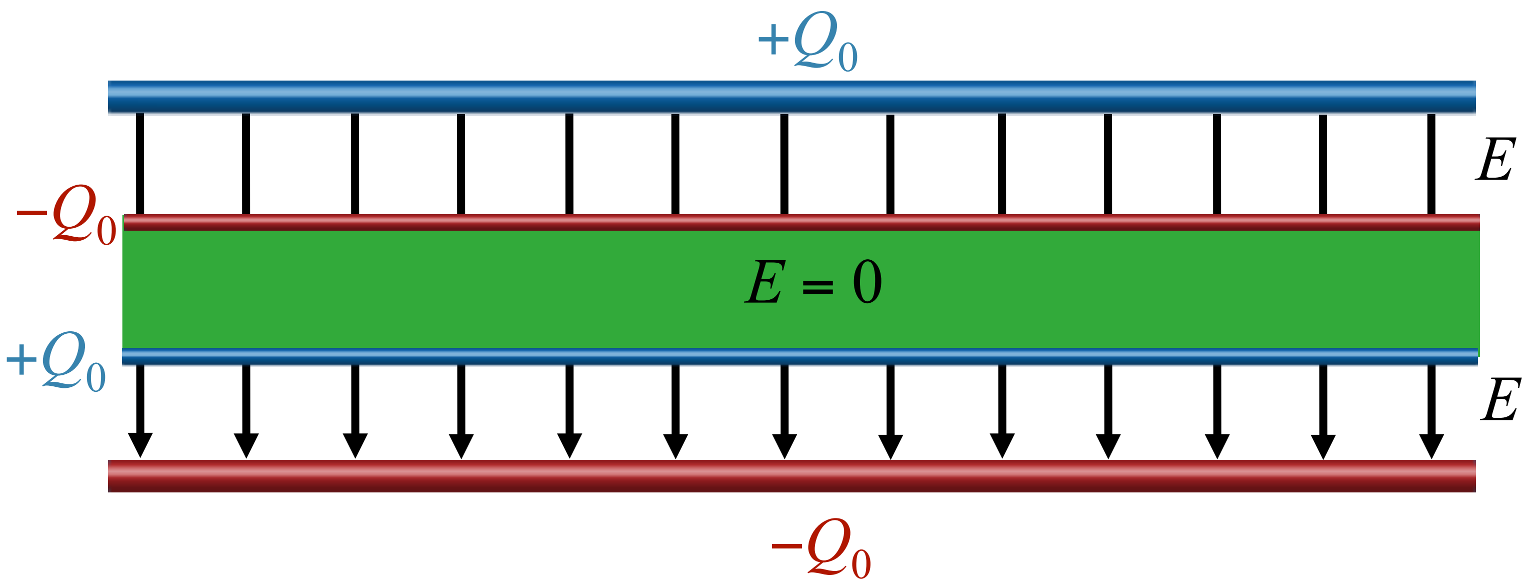
Where to Start ?



What is the total charge induced on the bottom surface of the conductor?

A) $+Q_1$
B) $-Q_1$
C) 0
D) Positive but the magnitude unknown
E) Negative but the magnitude unknown

Why ?



WHAT DO WE KNOW ?

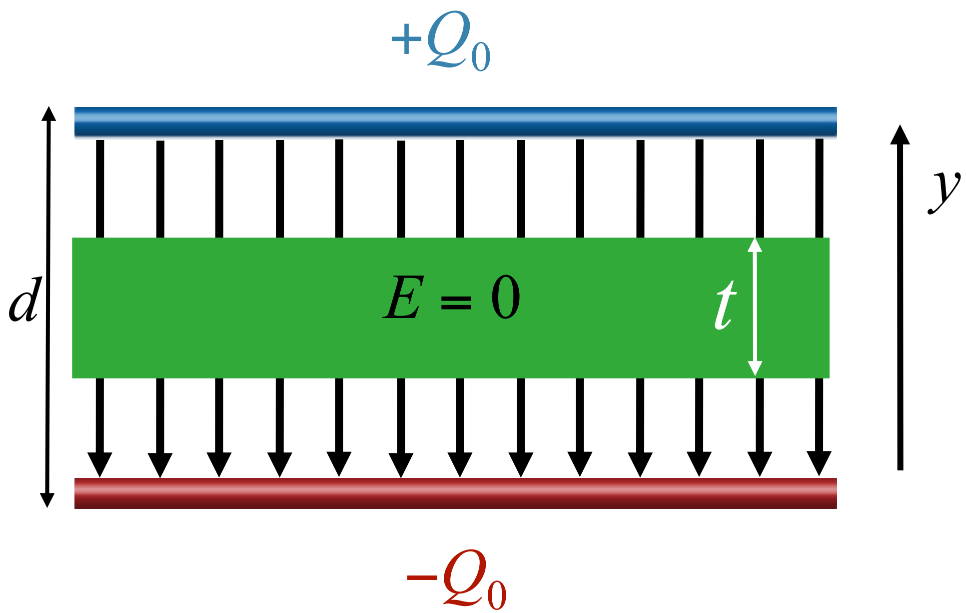
E must be = 0 in conductor !

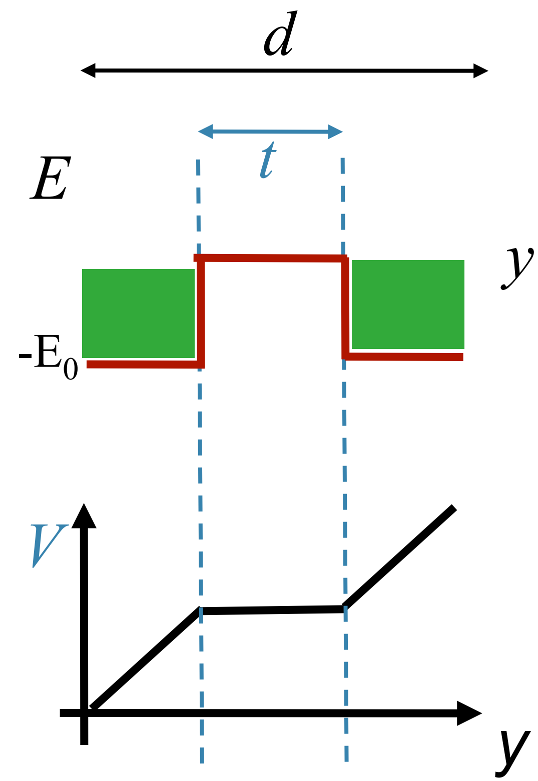
Charges inside conductor move to cancel E field from top & bottom plates.

Calculate V

Now calculate V as a function of distance from the bottom conductor.

$$V(y) = -\int_0^y \vec{E}\cdot d\vec{y}$$





What is $\Delta V = V(d)$?

A) $\Delta V = E_0 d$

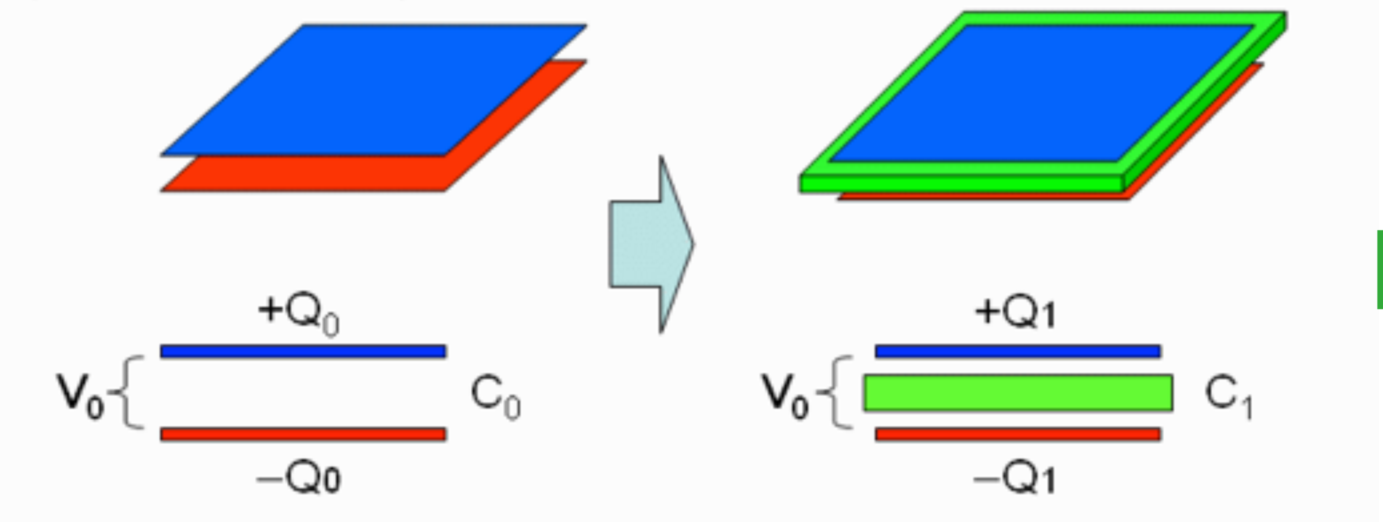
B) $\Delta V = E_0(d - t)$

C) $\Delta V = E_0(d + t)$

The integral = area under the curve

CheckPoint Results: Charged Parallel Plates 1

Two parallel plates of equal area carry equal and opposite charge $\mathbf{Q_0}$. The potential difference between the two plates is measured to be $\mathbf{V_0}$. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value $\mathbf{Q_1}$ such that the potential difference between the plates remains the same.



Compare $\mathbf{Q_1}$ and $\mathbf{Q_0}$.

A. $\mathbf{Q_1 < Q_0}$

B. $\mathbf{Q_1 = Q_0}$

C. $\mathbf{Q_1 > Q_0}$

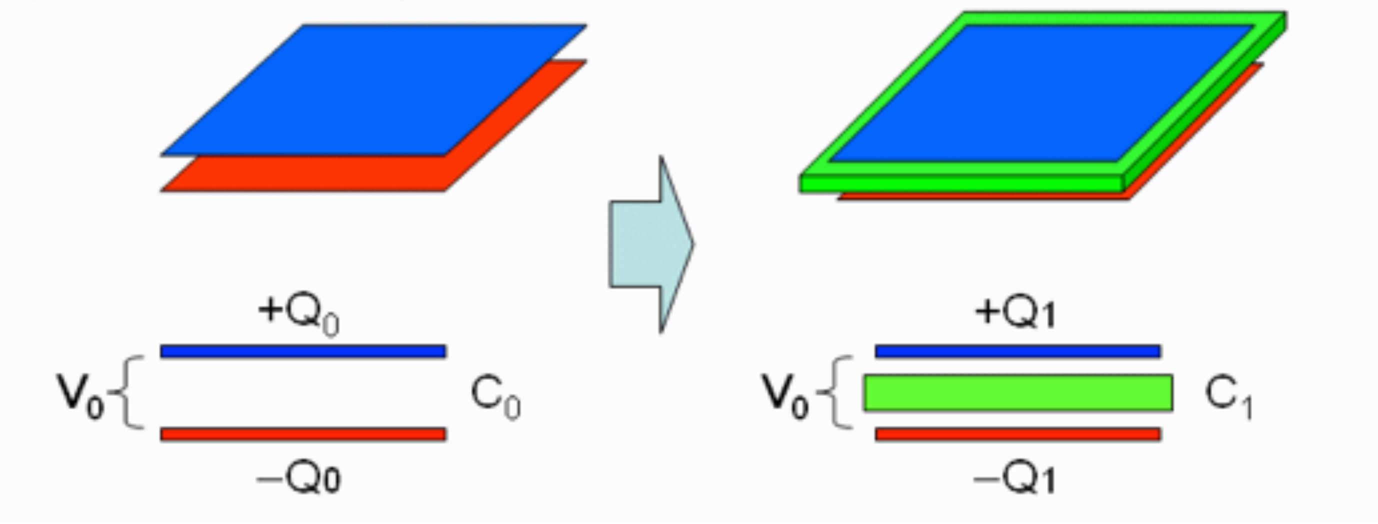
"The distance for Q1 is smaller therefore the charge must decrease to compensate for the change in distance "

"Since the potential remains the same, there is no change in the charge of Q. "

"the field through the conductor is zero, so it has constant potential. Because of this it must have greater charge so the total V is that same. "

CheckPoint Results: Charged Parallel Plates 2

Two parallel plates of equal area carry equal and opposite charge $\mathbf{Q_0}$. The potential difference between the two plates is measured to be $\mathbf{V_0}$. An uncharged conducting plate (the green thing in the picture below) is slipped into the space between the plates without touching either one. The charge on the plates is adjusted to a new value $\mathbf{Q_1}$ such that the potential difference between the plates remains the same.



Compare the capacitance of the two configurations in the above problem.

A. $C_1 > C_0$

B. $C_1 = C_0$

C. $C_1 < C_0$

"The distance for Q1 is smaller therefore the charge must decrease to compensate for the change in distance "

"Since the potential remains the same, there is no change in the charge of Q. "

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CheckPoint Results: Charged Parallel Plates 2

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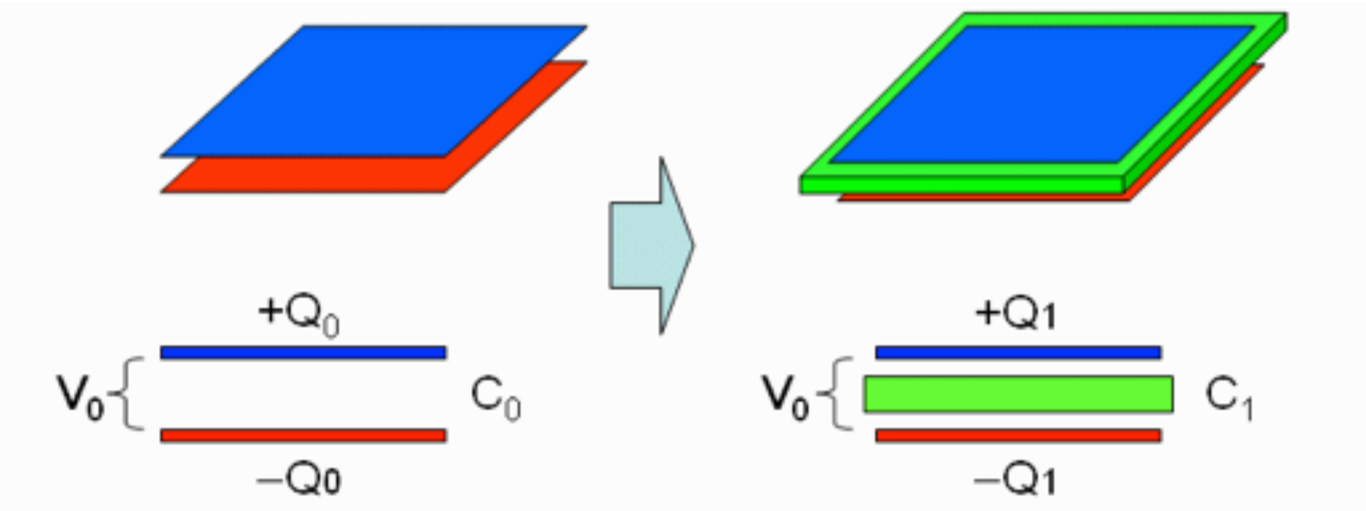


Compare the capacitance of the two configurations in the above problem.

A. $C_1 > C_0$
B. $C_1 = C_0$
C. $C_1 < C_0$

We can determine C from either case

same V (preflight)

same Q (lecture)

C depends only on geometry !

$$E_0 = Q_0/\varepsilon_0 A$$

Same Q:

$$V_0 = E_0 d \qquad C_0 = Q_0/E_0 d \qquad C_0 = \varepsilon_0 A/d$$

$$V_1 = E_0(d-t) \qquad C_1 = Q_0/[E_0(d-t)] \qquad C_1 = \varepsilon_0 A/(d-t)$$

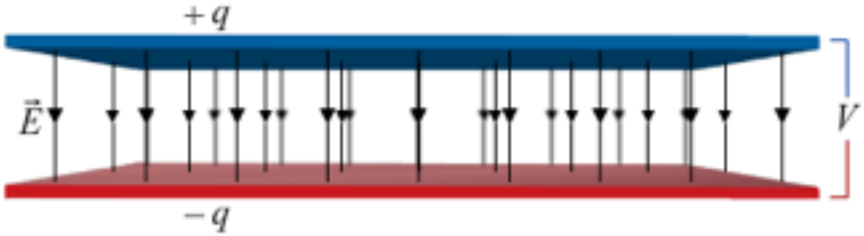
Energy in Capacitors

Energy Stored in Capacitors

$$U = \frac{1}{2}QV$$ or $$U = \frac{1}{2}\frac{Q^2}{C}$$ or $$U = \frac{1}{2}CV^2$$

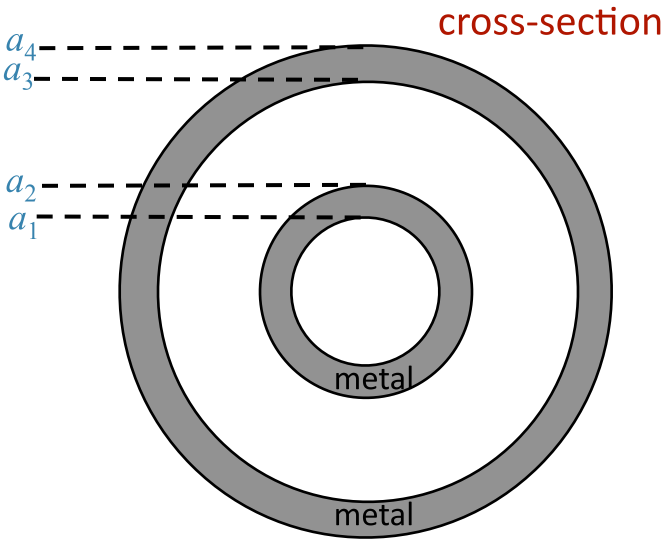
Energy Density

$$u = \frac{1}{2}\varepsilon_o E^2$$



Demo: BANG

Calculation



A capacitor is constructed from two conducting **cylindrical** shells of radii a_1, a_2, a_3, and a_4 and length L $(L >> a_i)$.

What is the capacitance C of this capacitor ?

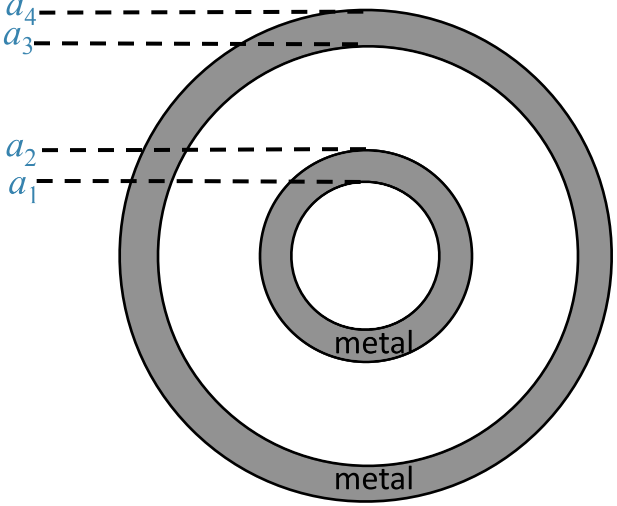
- Conceptual Analysis:

$$C \equiv \frac{Q}{V}$$

But what is Q and what is V ? They are not given?

- Important Point: C is a property of the object! (concentric cylinders here)
    - Assume some Q (i.e., $+Q$ on one conductor and $-Q$ on the other)
    - These charges create E field in region between conductors
    - This E field determines a potential difference V between the conductors
    - V should be proportional to Q; the ratio Q/V is the capacitance.

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

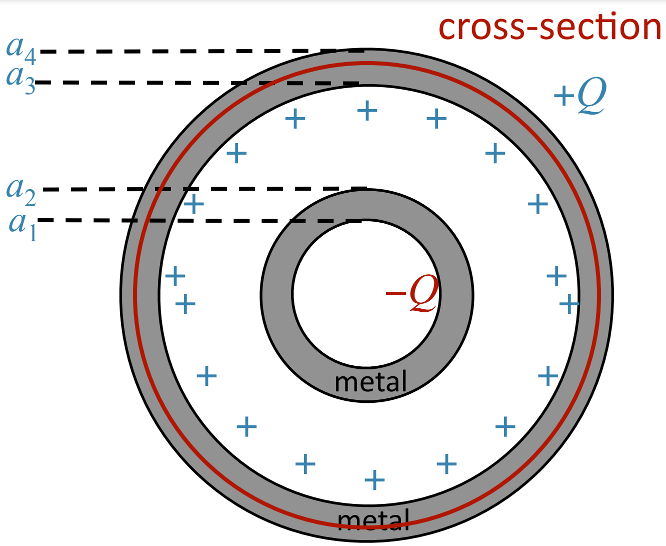
What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

➢ Strategic Analysis:

- Put $+Q$ on outer shell and $-Q$ on inner shell
- Cylindrical symmetry: Use Gauss' Law to calculate E everywhere
- Integrate E to get V
- Take ratio Q/V: should get expression only using geometric parameters (a_i, L)

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is $+Q$ on outer conductor located?

A) at $r = a_4$ B) at $r = a_3$ C) both surfaces D) throughout shell

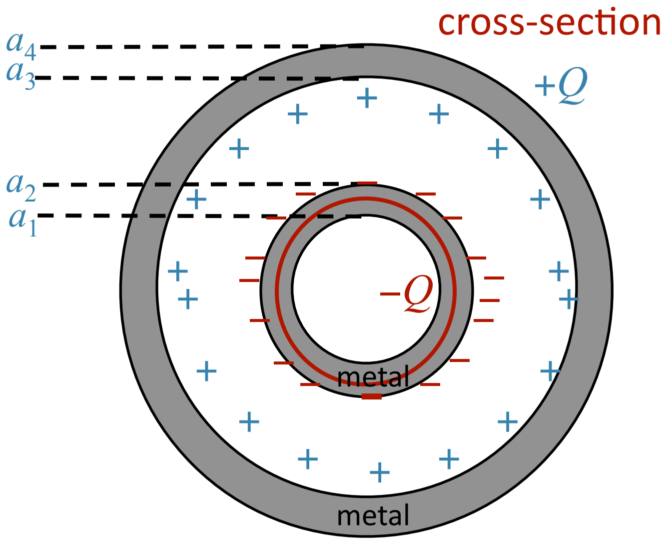
Why?

Gauss' law: $$\oint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

We know that $E = 0$ in conductor (between a_3 and a_4) → $Q_{enclosed} = 0$

$Q_{enclosed} = 0$ → $+Q$ must be on inside surface (a_3), so that $Q_{enclosed} = +Q - Q = 0$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

Where is $-Q$ on inner conductor located?

A) at $r = a_2$ B) at $r = a_1$ C) both surfaces D) throughout shell

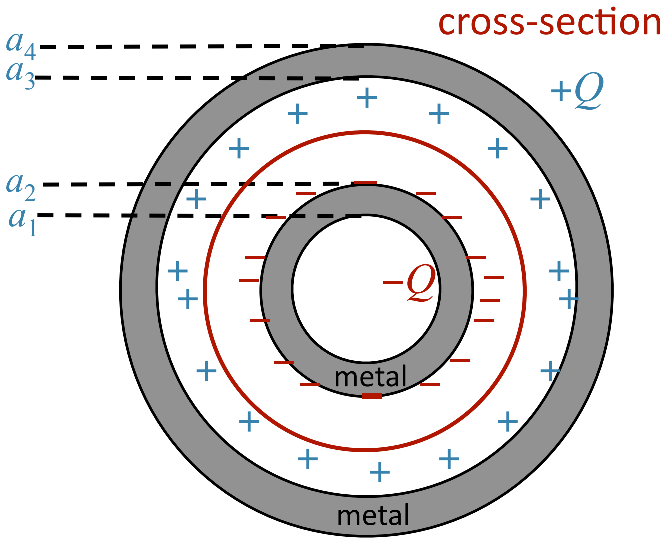
Why?

Gauss' law: $$\oint_{surface} \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

We know that $E = 0$ in conductor (between a_1 and a_2) → $Q_{enclosed} = 0$

$Q_{enclosed} = 0$ → $+Q$ must be on outer surface (a_2), so that $Q_{enclosed} = 0$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V}$$

$a_2 < r < a_3$: What is $E(r)$?

A) 0

B) $\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$

C) $\frac{1}{2\pi\varepsilon_o}\frac{Q}{Lr}$

D) $\frac{1}{2\pi\varepsilon_o}\frac{2Q}{Lr}$

E) $\frac{1}{4\pi\varepsilon_o}\frac{2Q}{r^2}$

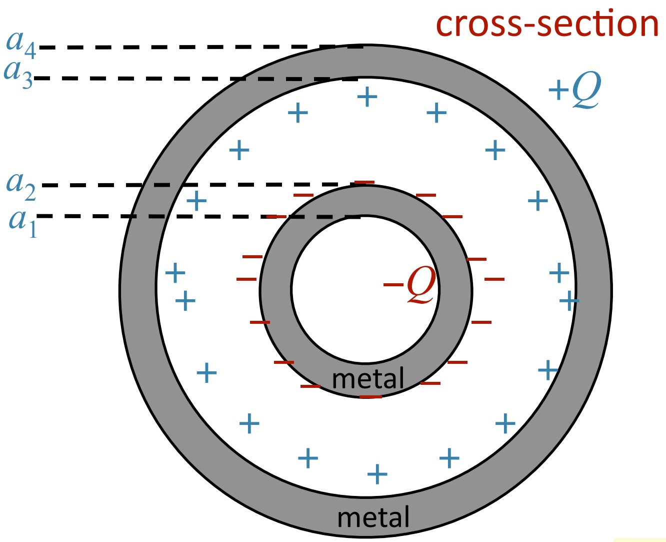
Why?

Gauss' law:

$$\oint_{\text{surface}} \vec{E}\cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0} \rightarrow E(2\pi r L) = \frac{Q}{\epsilon_0} \rightarrow E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$$

Direction: Radially in

Calculation



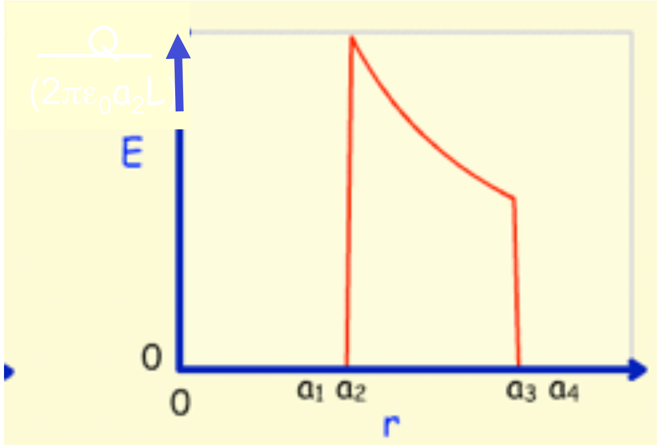
A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V} \qquad a_2 < r < a_3: \quad E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$$

$r < a_2$: $E(r) = 0$

since $Q_{enclosed} = 0$



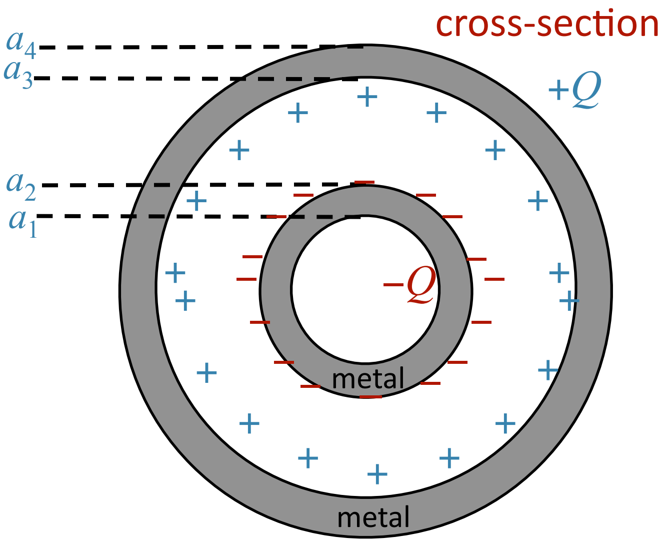
What is V ?

The potential difference between the conductors.

What is the sign of $V = V_{outer} - V_{inner}$?

A) $V_{outer} - V_{inner} < 0$ B) $V_{outer} - V_{inner} = 0$ C) $V_{outer} - V_{inner} > 0$

Calculation



A capacitor is constructed from two conducting cylindrical shells of radii a_1, a_2, a_3, and a_4 and length L ($L >> a_i$).

What is the capacitance C of this capacitor ?

$$C \equiv \frac{Q}{V} \qquad a_2 < r < a_3: \quad E = \frac{1}{2\pi\varepsilon_0}\frac{Q}{Lr}$$

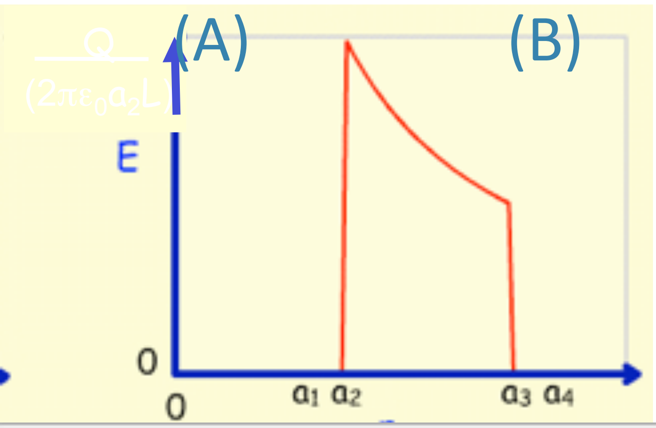
What is $V \equiv V_{outer} - V_{inner}$?

(A) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_1}{a_4}$

(B) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_4}{a_1}$

(C) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_3}{a_2}$

(D) $\dfrac{Q}{2\pi\varepsilon_o L}\ln\dfrac{a_2}{a_3}$



$$V = -\int_{a_2}^{a_3}\frac{-Q}{2\pi\varepsilon_o L}\frac{dr}{r} \rightarrow V = \frac{Q}{2\pi\varepsilon_o L}\int_{a_2}^{a_3}\frac{dr}{r} \rightarrow V = \frac{Q}{2\pi\varepsilon_0 L}\ln\frac{a_3}{a_2}$$

V proportional to Q, as promised

$$\rightarrow C \equiv \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{\ln(a_3/a_2)}$$