

Electricity & Magnetism

Lecture 11

Today's Concept:

RC Circuits

Kirchhoff's Rules example (if there's time)

Some of your comments

Two parallel plates, each having area $A = 2683 \text{ cm}^2$ are connected to the terminals of a battery of voltage $V_b = 6 \text{ V}$. The plates are separated by a distance $d = 0.58 \text{ cm}$. What is Q on the charged plate?

➤ $[Q = (\epsilon_0 A/d)V = 2.46 \times 10^{-10} \text{ C}]$

In the circuit in the prelecture why does current decrease as time goes on? Where does the charge flow to?

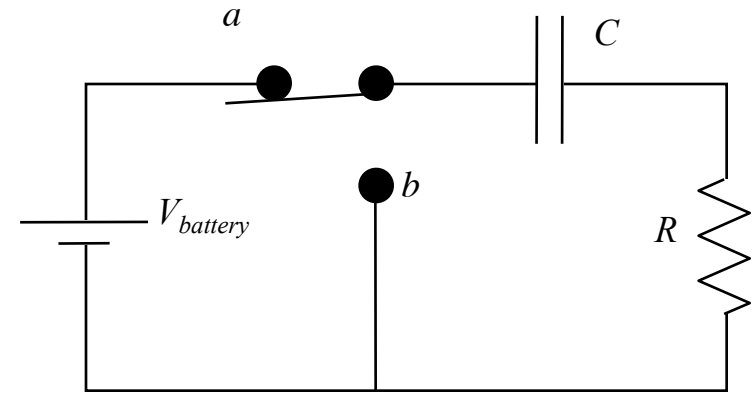
To the other plate

Can you please go over time constant. My brain can't take it anymore.

The time constant is a like a half-life only longer.

RC Circuit (Charging)

Capacitor uncharged, Switch is moved to position “a”



Kirchhoff's Voltage Rule

$$-V_{battery} + \frac{q}{C} + IR = 0$$

Short Term ($q = q_0 = 0$)

$$-V_{battery} + 0 + I_0 R = 0$$

$$I_0 = \frac{V_{battery}}{R}$$

Long Term ($I_c = 0$)

$$-V_{battery} + \frac{q_\infty}{C} + 0 \times R = 0$$

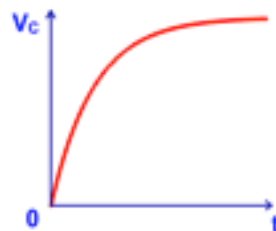
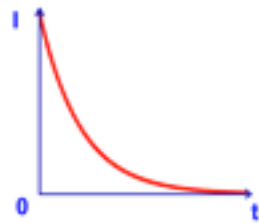
$$q_\infty = CV_{battery}$$

Intermediate

$$-V_{battery} + \frac{q}{C} + \frac{dq}{dt} R = 0$$

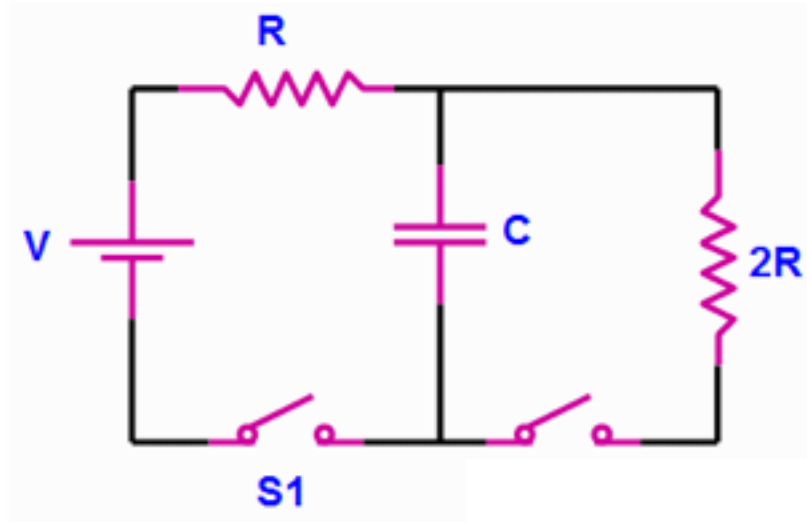
$$q(t) = q_\infty (1 - e^{-t/RC})$$

$$I(t) = I_0 e^{-t/RC}$$



CheckPoints 2 & 4

A circuit is wired up as shown below. The capacitor is initially uncharged and switches S1 and S2 are initially open.



Close $S1$,

V_1 = voltage across C immediately after

V_2 = voltage across C a long time after

s1 A) $V_1 = V$ $V_2 = V$

B) $V_1 = 0$ $V_2 = V$

C) $V_1 = 0$ $V_2 = 0$

D) $V_1 = V$ $V_2 = 0$

RC Circuit (Discharging)

Capacitor has $q_0 = CV$, Switch is moved to position “b”

Kirchhoff's Voltage Rule

$$+\frac{q}{C} + IR = 0$$

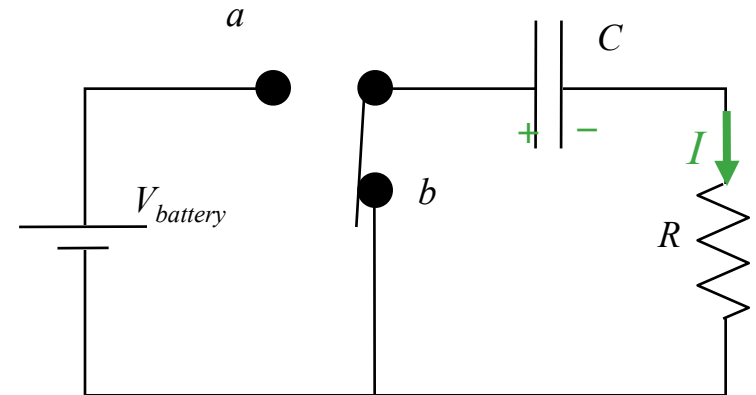
Short Term ($q = q_0$)

$$V_{\text{battery}} + IR = 0$$

$$I_0 = \frac{-V_{\text{battery}}}{R}$$

Long Term ($I_c = 0$)

$$\frac{q_\infty}{C} + 0 \times R = 0$$
$$q_\infty = 0$$

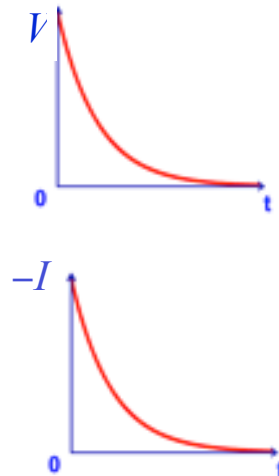


Intermediate

$$+\frac{q}{C} + \frac{dq}{dt} R = 0$$

$$q(t) = q_0 e^{-t/RC}$$

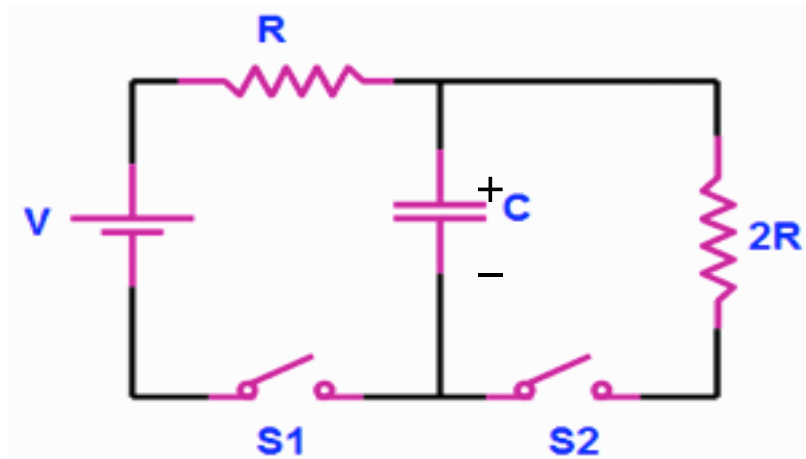
$$I(t) = I_0 e^{-t/RC}$$



CheckPoint 6



A circuit is wired up as shown below. The capacitor is initially uncharged and switches S1 and S2 are initially open.



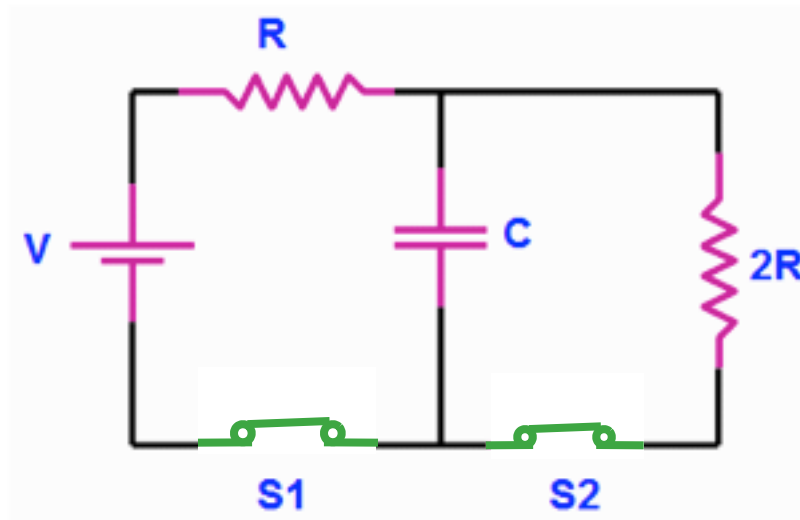
6) After being closed a long time, switch 1 is opened and switch 2 is closed. What is the current through the right resistor immediately after the switch 2 is closed?

- A $I_R = 0$
- B $I_R = V/3R$
- C $I_R = V/2R$
- D $I_R = V/R$

CheckPoint 8



A circuit is wired up as shown below. The capacitor is initially uncharged and switches S1 and S2 are initially open.



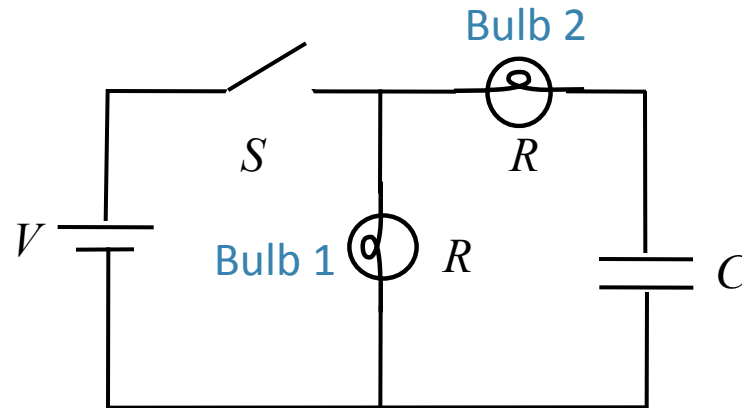
Now suppose both switches are closed. What is the voltage across the capacitor after we wait for a very long time.?

A) $V_c = 0$

B) $V_c = V$

C) $V_c = 2V/3$

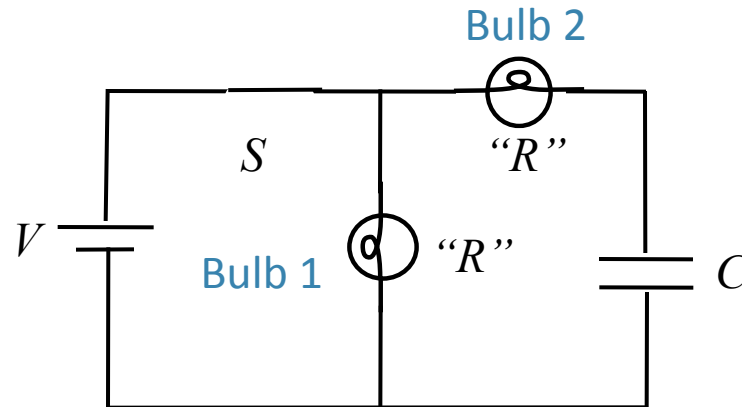
DEMO - Clicker Question 1



What will happen after I close the switch?

- A) Both bulbs come on and stay on.
- B) Both bulbs come on but then bulb 2 fades out.
- C) Both bulbs come on but then bulb 1 fades out.
- D) Both bulbs come on and then both fade out.

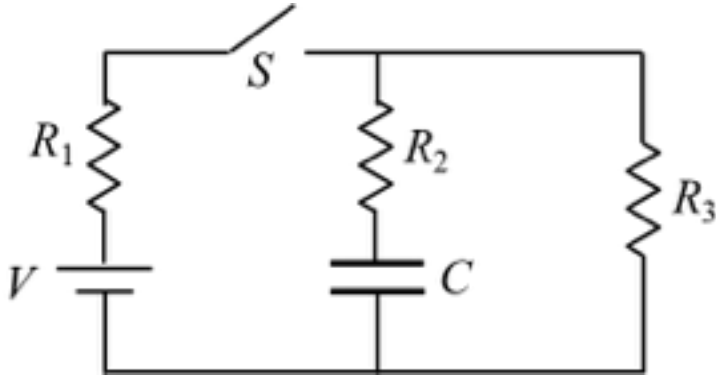
DEMO - Clicker Question 2



Suppose the switch has been closed a long time.
Now what will happen after open the switch?

- A) Both bulbs come on and stay on.
- B) Both bulbs come on but then bulb 2 fades out.
- C) Both bulbs come on but then bulb 1 fades out.
- D) Both bulbs come on and then both fade out.

Calculation



In this circuit, assume V , C , and R_i are known.
 C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Conceptual Analysis:

Circuit behavior described by Kirchhoff's Rules:

$$\Sigma V_{drops} = 0$$

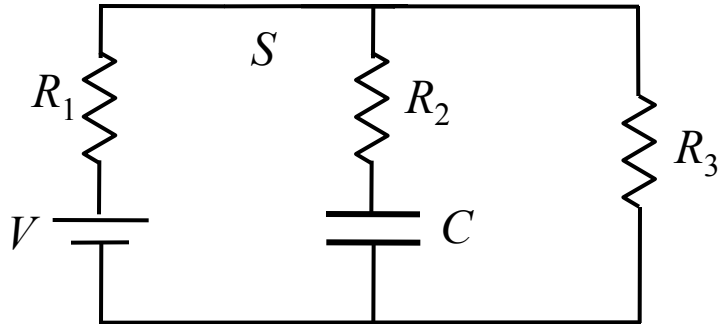
$$\Sigma I_{in} = \Sigma I_{out}$$

S closed and C charges to some voltage with some time constant

Strategic Analysis

Determine currents and voltages in circuit a long time after S closed

Calculation



In this circuit, assume V , C , and R_i are known.

C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Immediately after S is closed:

what is I_2 , the current through C

what is V_C , the voltage across C ?

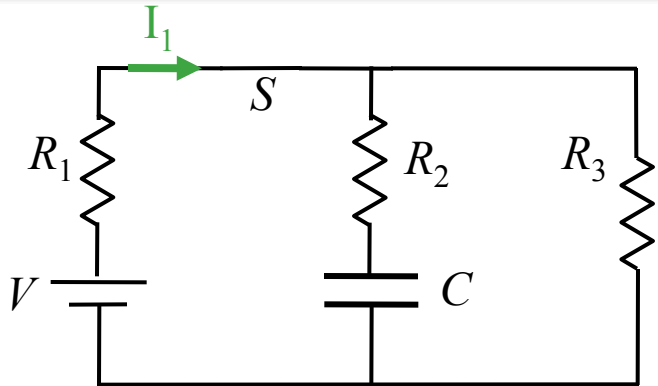
A) Only $I_2 = 0$ B) Only $V_C = 0$ C) Both I_2 and $V_C = 0$ D) Neither I_2 nor $V_C = 0$

Why?

We are told that C is initially uncharged ($V = Q/C$)

I_2 cannot be zero because charge must flow in order to charge C

Calculation



In this circuit, assume V , C , and R_i are known.

C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Immediately after S is closed, what is I_1 , the current through R_1 ?

$$\frac{V}{R_1}$$

A

$$\frac{V}{R_1 + R_3}$$

B

$$\frac{V}{R_1 + R_2 + R_3}$$

C

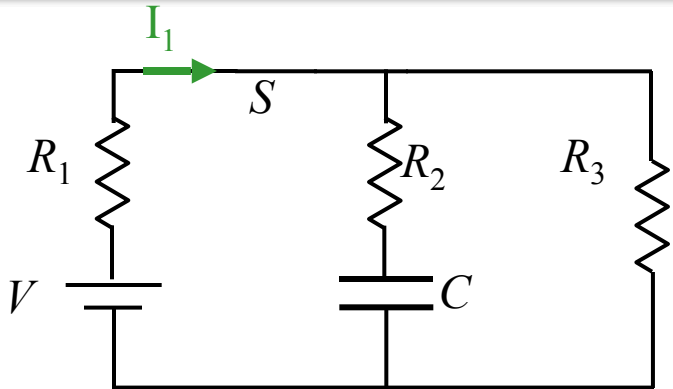
$$\frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

D

$$V \frac{R_1 + R_2 + R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

E

Calculation



In this circuit, assume V , C , and R_i are known.

C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

Immediately after S is closed, what is I_1 , the current through R_1 ?

$$\frac{V}{R_1}$$

A

$$\frac{V}{R_1 + R_3}$$

B

$$\frac{V}{R_1 + R_2 + R_3}$$

C

$$\frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

D

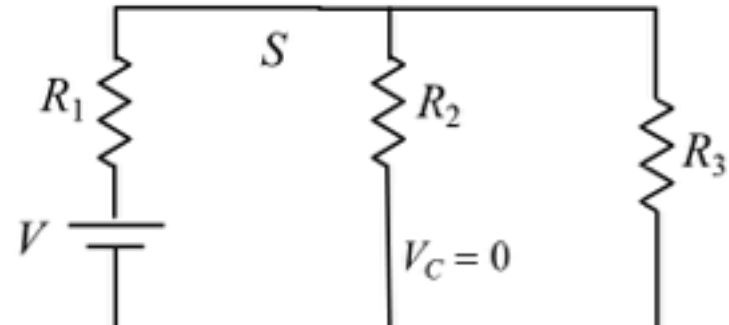
$$V \frac{R_1 + R_2 + R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

E

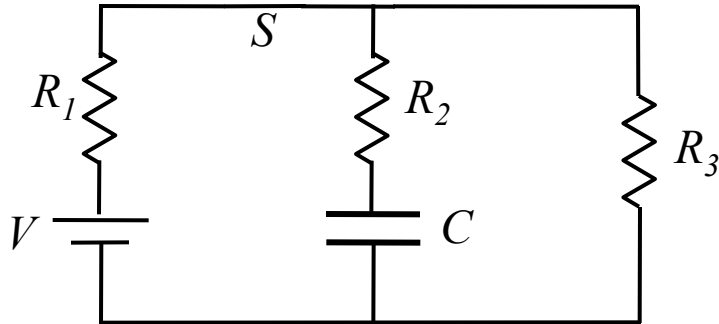
Why?

Draw circuit just after S closed (knowing $V_C = 0$)

R_1 is in series with the parallel combination of R_2 and R_3



Calculation



In this circuit, assume V , C , and R_i are known.
 C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

After S has been closed “for a long time”, what is I_C , the current through C ?

$$\frac{V}{R_2}$$

A

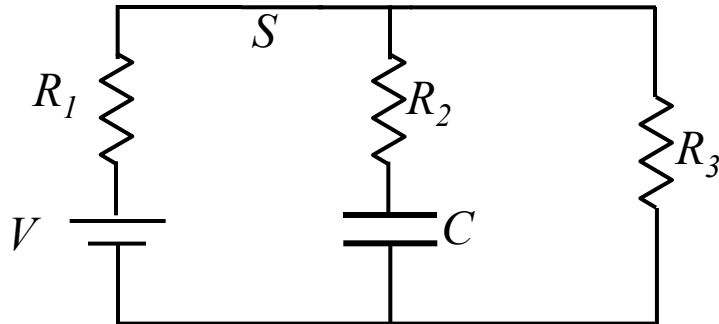
$$\frac{V}{R_1}$$

B

0

C

Calculation



In this circuit, assume V , C , and R_i are known.
 C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

After S has been closed “for a long time”, what is I_C , the current through C ?

$$\frac{V}{R_2}$$

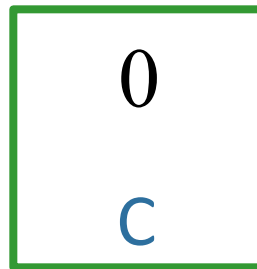
A

$$\frac{V}{R_1}$$

B

0

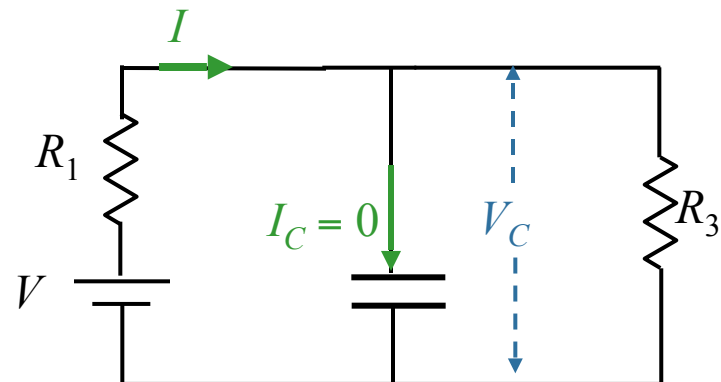
C



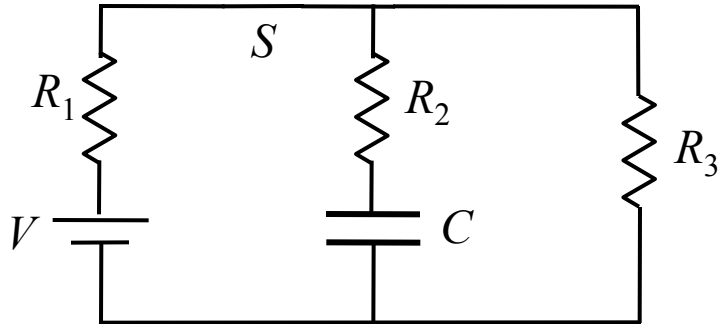
Why?

After a long time in a static circuit, the current through any capacitor approaches 0 !

This means we Redraw circuit with open circuit in middle leg



Calculation



In this circuit, assume V , C , and R_i are known.
 C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

After S has been closed “for a long time”, what is V_C , the voltage across C ?

$$V \frac{R_3}{R_1 + R_3}$$

A

$$V \frac{R_2}{R_1 + R_2}$$

B

$$V$$

C

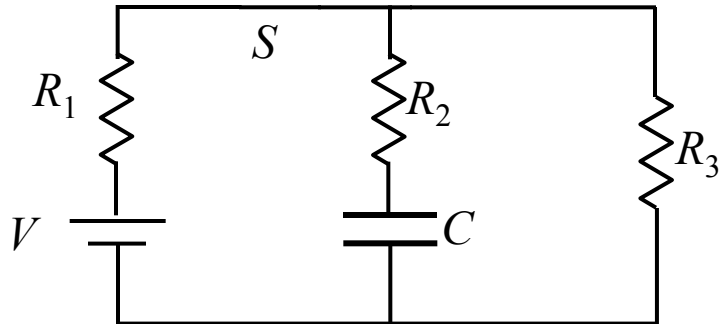
$$V \frac{R_2}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

D

$$0$$

E

Calculation



In this circuit, assume V , C , and R_i are known.
 C initially uncharged and then switch S is closed.

What is the voltage across the capacitor after a long time ?

After S has been closed “for a long time”, what is V_C , the voltage across C ?

$$V \frac{R_3}{R_1 + R_3}$$

A

$$V \frac{R_2}{R_1 + R_2}$$

B

$$V$$

C

$$V \frac{R_2}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

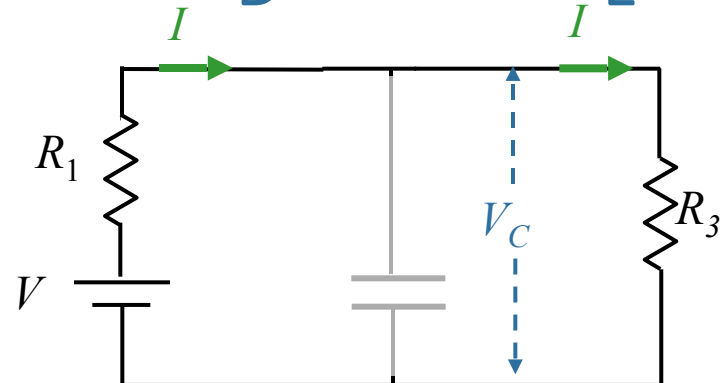
D

$$0$$

E

Why?

$$- V_C = V_3 = IR_3 = (V/(R_1 + R_3))R_3$$

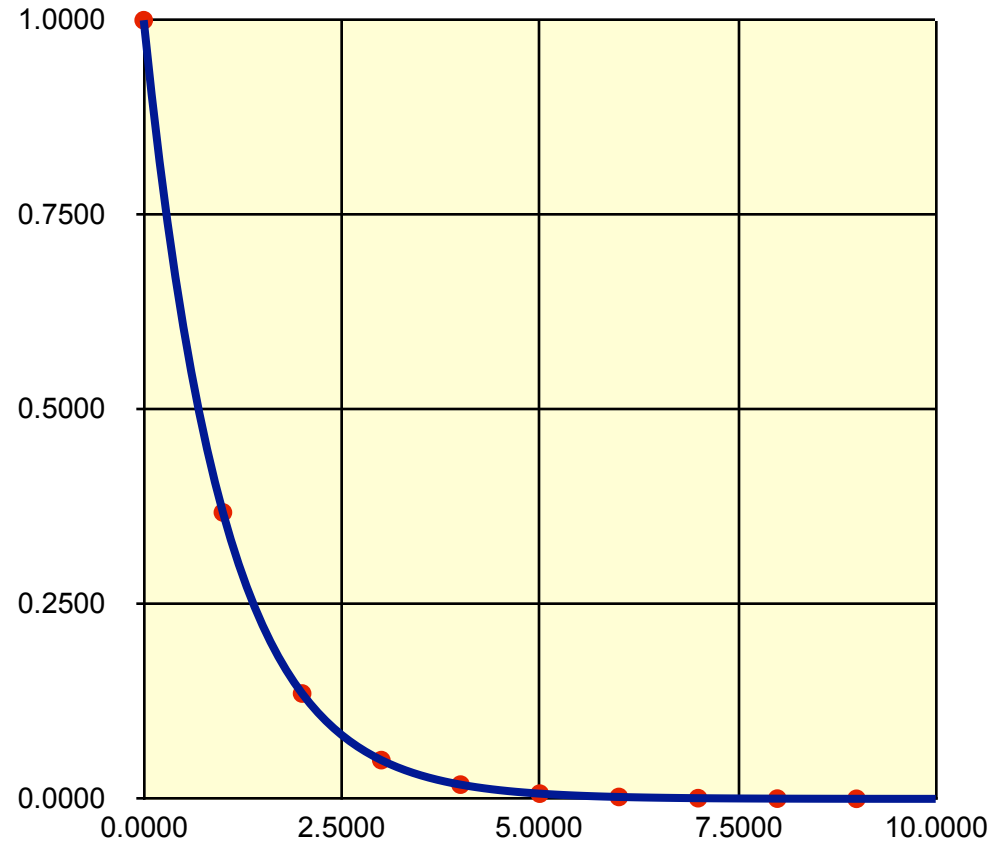


How do Exponentials Work?

$$Q(t) = Q_0 e^{-t/RC}$$

$$\frac{Q(t)}{Q_0}$$

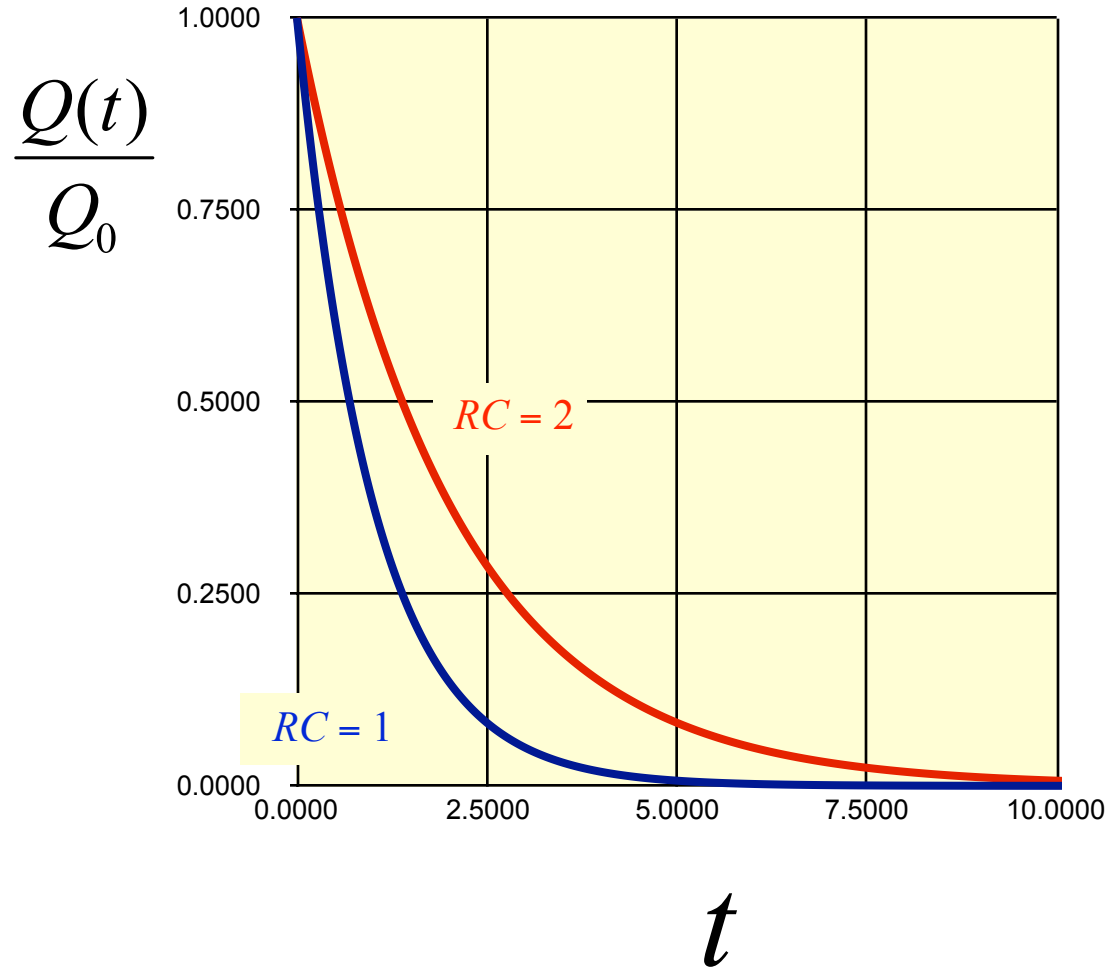
“Fraction of initial charge that remains”



“How many time constants worth of time that have elapsed”

$$\frac{t}{RC}$$

$$Q(t) = Q_0 e^{-t/RC}$$



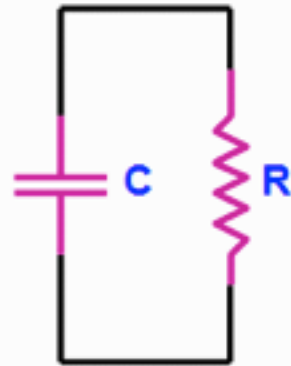
Time constant:

$$\tau = RC$$

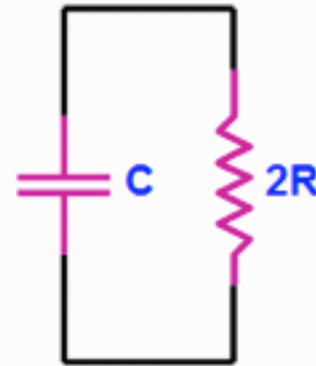
The bigger τ is,
the longer it takes to get
the same change...

Checkpoint 10

The two circuits shown below contain identical capacitors that hold the same charge at $t = 0$. Circuit 2 has twice as much resistance as circuit 1.



Circuit 1



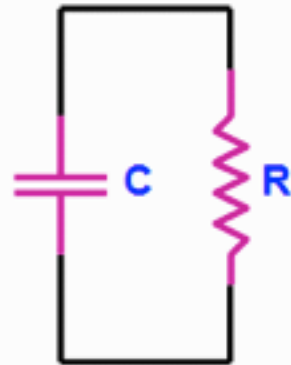
Circuit 2

Which circuit has the largest time constant?

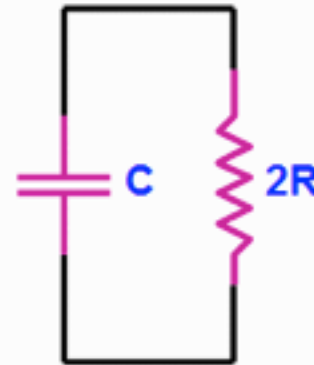
- A) Circuit 1
- ☒ B) Circuit 2
- C) Same

Checkpoint 12

The two circuits shown below contain identical capacitors that hold the same charge at $t = 0$. Circuit 2 has twice as much resistance as circuit 1.



Circuit 1



Circuit 2

Which of the following statements best describes the charge remaining on each of the two capacitors for any time after $t = 0$?

A $Q_1 < Q_2$

B $Q_1 > Q_2$

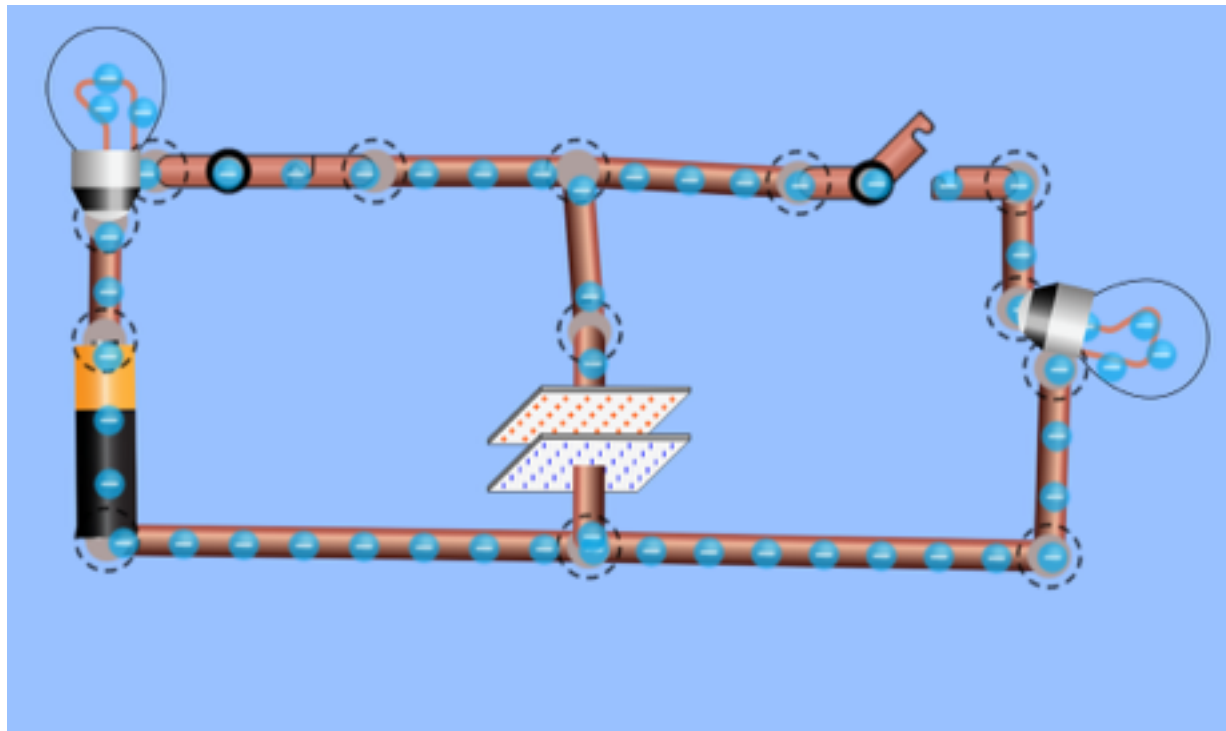
C $Q_1 = Q_2$

D $Q_1 < Q_2$ at first and then $Q_1 > Q_2$ after a long time

E $Q_1 > Q_2$ at first and then $Q_1 < Q_2$ after a long time

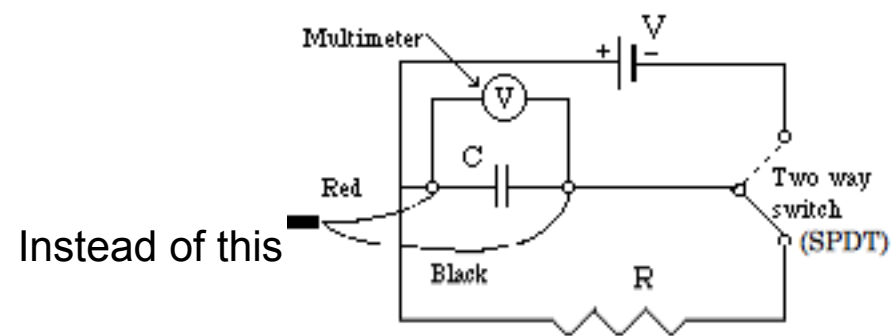
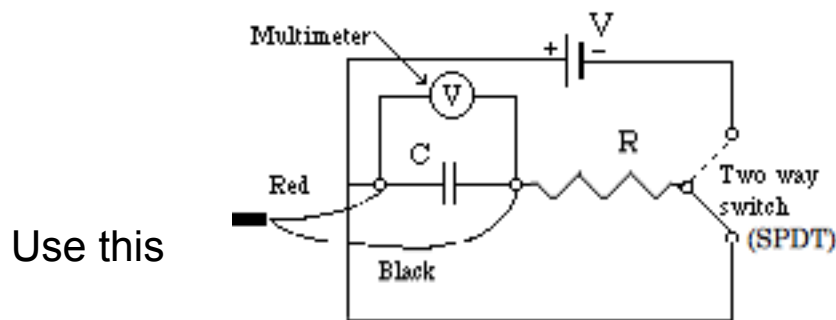
Simulation

https://phet.colorado.edu/sims/html/circuit-construction-kit-ac/latest/circuit-construction-kit-ac_en.html



There is no Session 3

Please record BOTH **Charge** and **Discharge** curves for your circuit. (Activity 24-8)



You can save time by fitting your data in Logger Pro.

Use the “Analyze” function under the menus.