

# *Electricity & Magnetism*

## *Lecture 12*

Today's Concept:

Magnetic Force on Moving Charges

$$\vec{F} = q\vec{v} \times \vec{B}$$

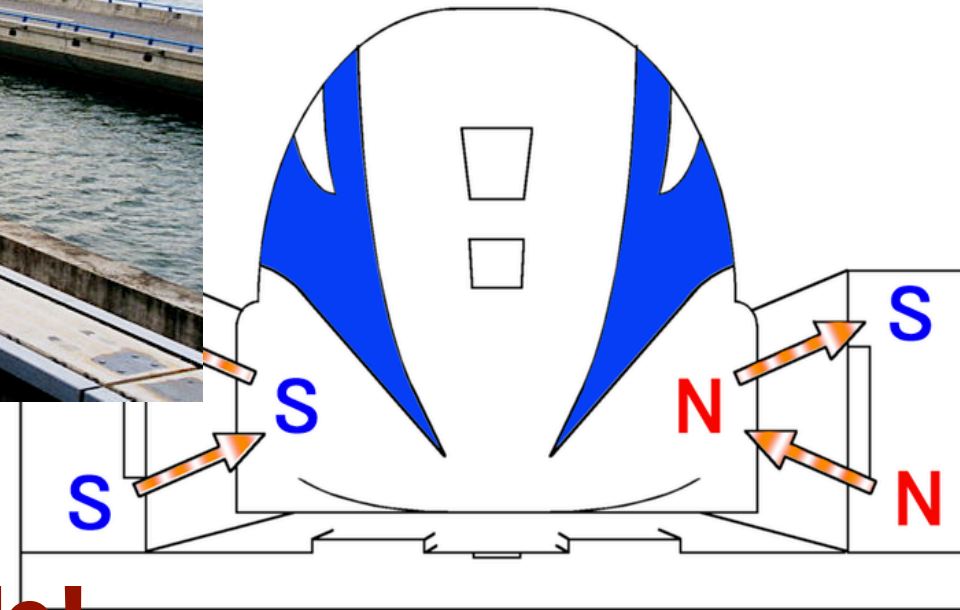
# Comments

this stuff gets fun now!

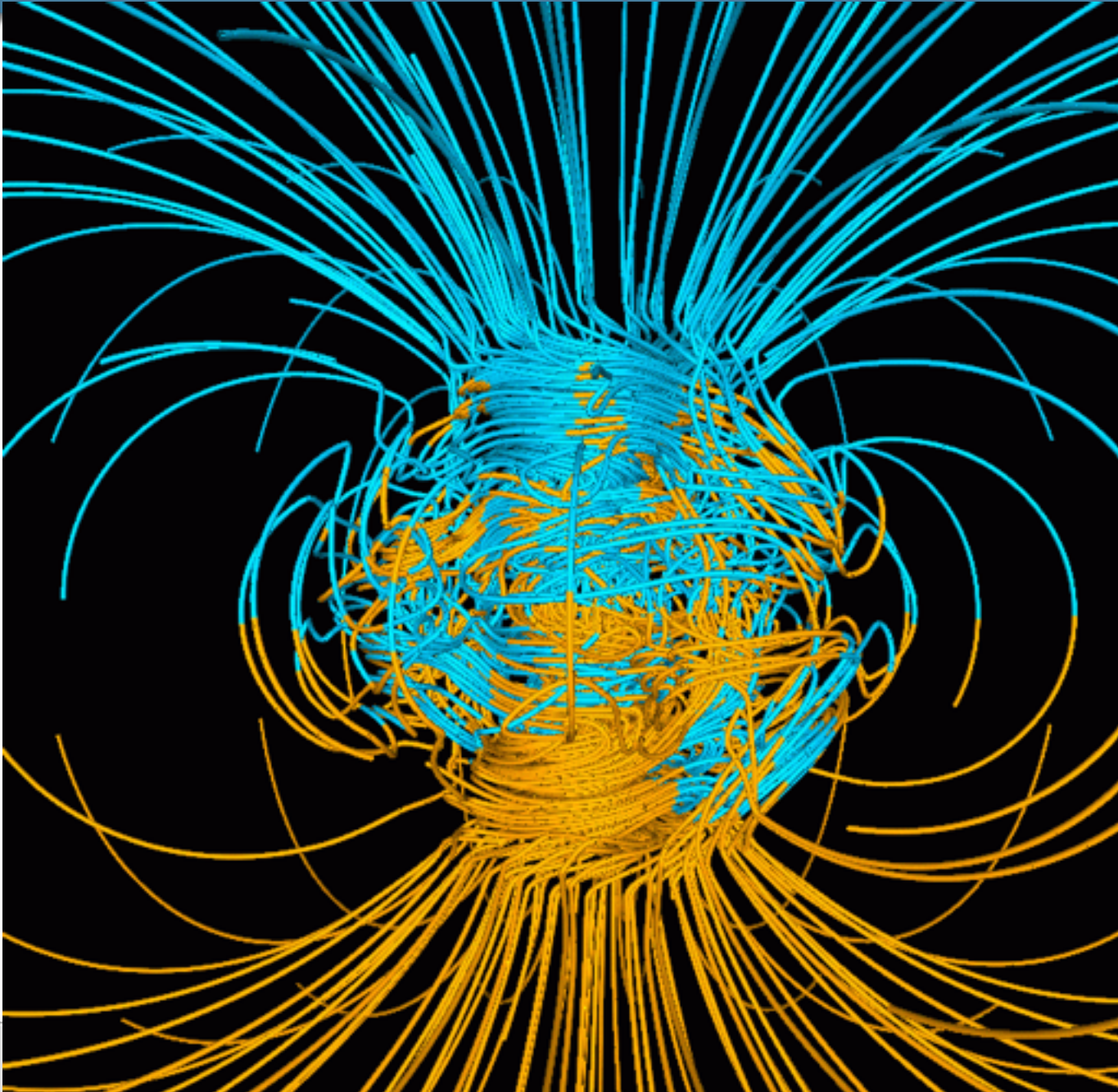


"see" on a flat screen,  
etc., etc., ...

**Will do!**



# *Earth's Magnetic Field*



# *Magnetic North Pole Location*

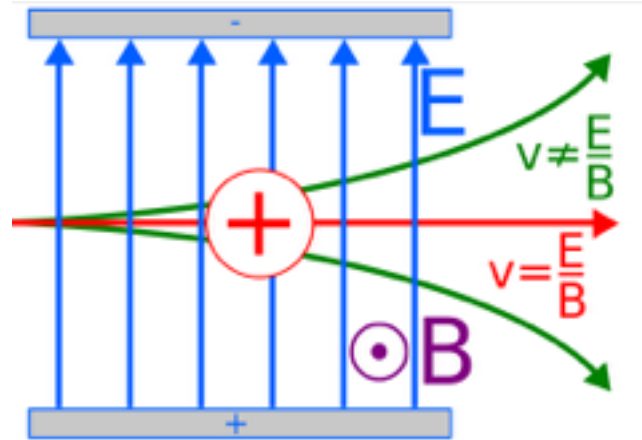
The North Magnetic Pole has moved rapidly in recent years away from Canada towards Russia.



# Today's Comments



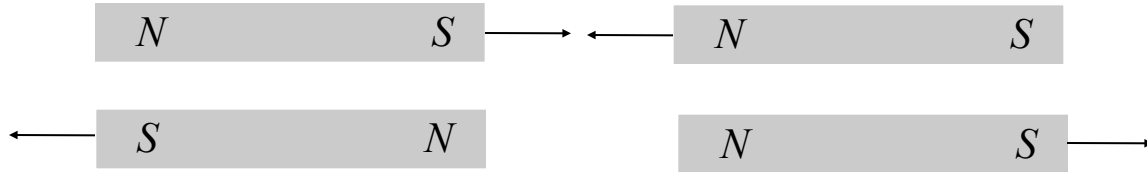
Can we discuss more on the cross product and why the magnetic force is the negative of the electric force? Thank you!



What is going on? Seems like electromagnetism will be a ride for sure!!!

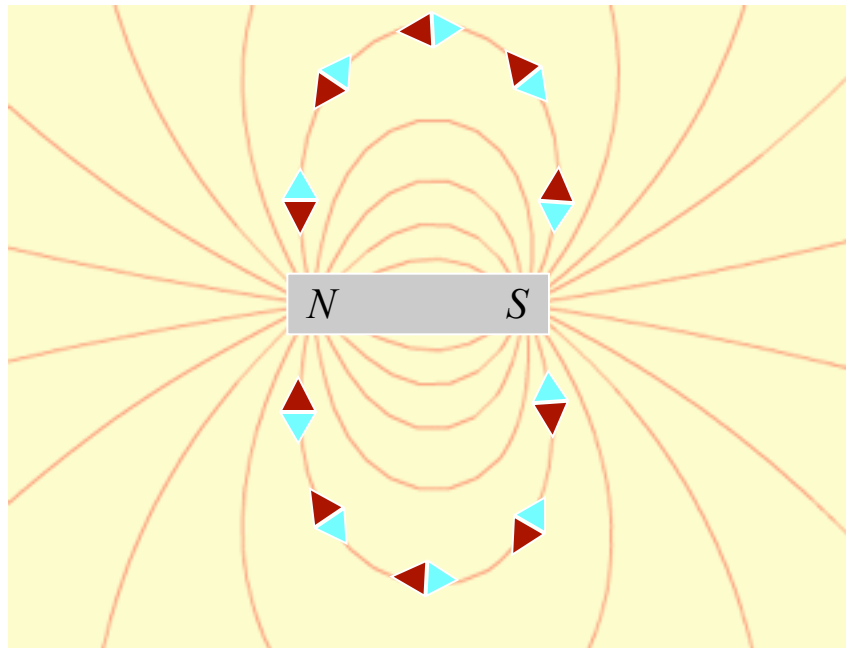
# Magnetic Observations

## Bar Magnets

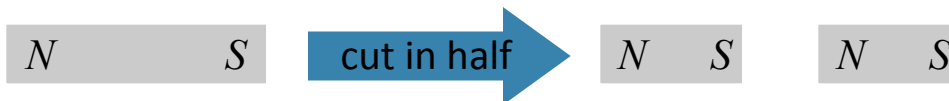


## Compass Needles

These are “magnetic dipoles”  
and behave similarly to  
“electric dipoles”

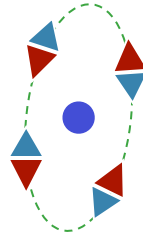


## Magnetic Charge?



# Magnetic Observations

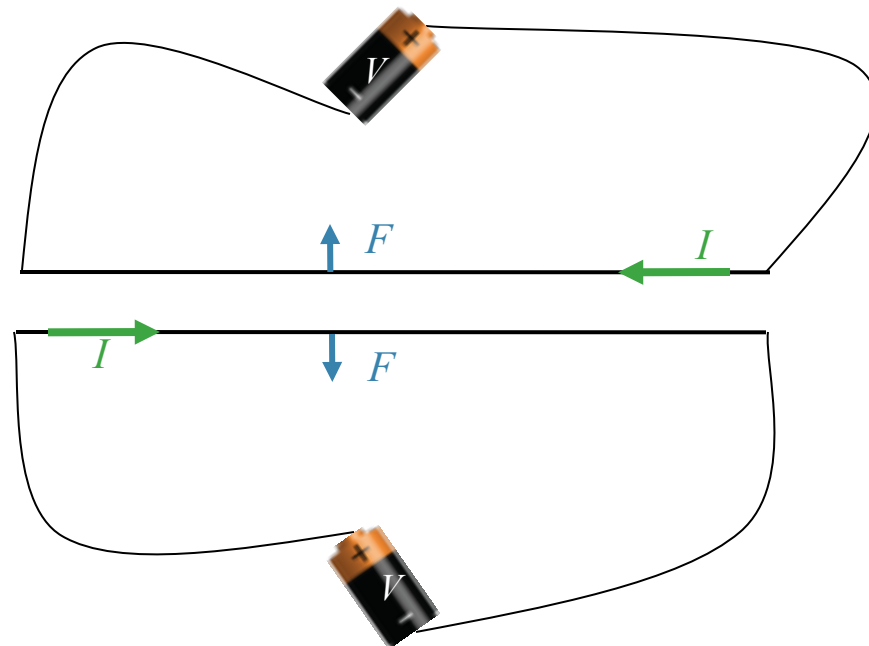
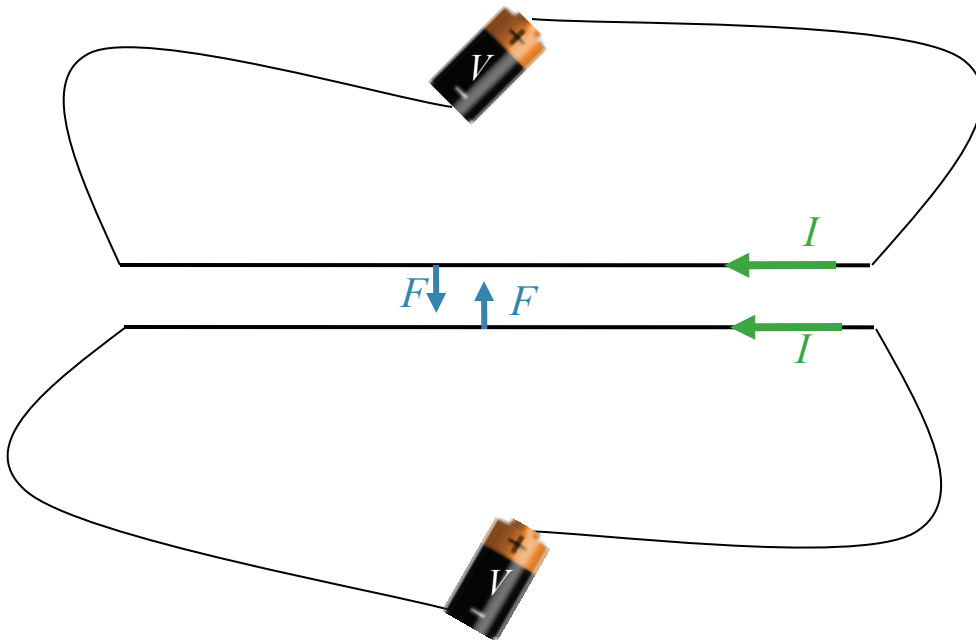
Compass needle deflected by electric current



$I$

Magnetic fields created by electric currents

Magnetic fields exert forces on electric currents (charges in motion)



# Magnetism & Moving Charges

All observations are explained by two equations:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Today

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

Next Week

# Cross Product Review

## Cross Product different from Dot Product

$\mathbf{A} \cdot \mathbf{B}$  is a scalar;  $\mathbf{A} \times \mathbf{B}$  is a vector

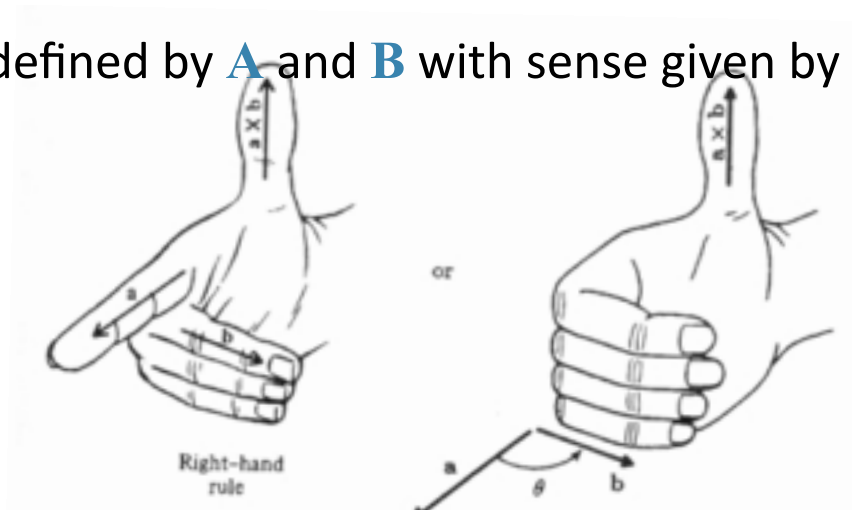
$\mathbf{A} \cdot \mathbf{B}$  proportional to the component of  $\mathbf{B}$  parallel to  $\mathbf{A}$

$\mathbf{A} \times \mathbf{B}$  proportional to the component of  $\mathbf{B}$  perpendicular to  $\mathbf{A}$   
is a scalar;  $\mathbf{A} \times \mathbf{B}$  is a vector

## Definition of $\mathbf{A} \times \mathbf{B}$

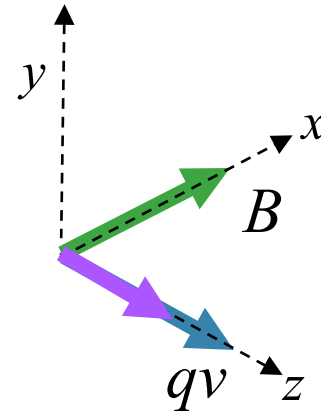
Magnitude:  $|\mathbf{A}||\mathbf{B}| \sin \theta$

Direction: perpendicular to plane defined by  $\mathbf{A}$  and  $\mathbf{B}$  with sense given by right-hand-rule



# Remembering Directions: The Right Hand Rule

$$\vec{F} = q\vec{v} \times \vec{B}$$



Which way does the force point?

A.

x

B.

-x

C.

y

D.

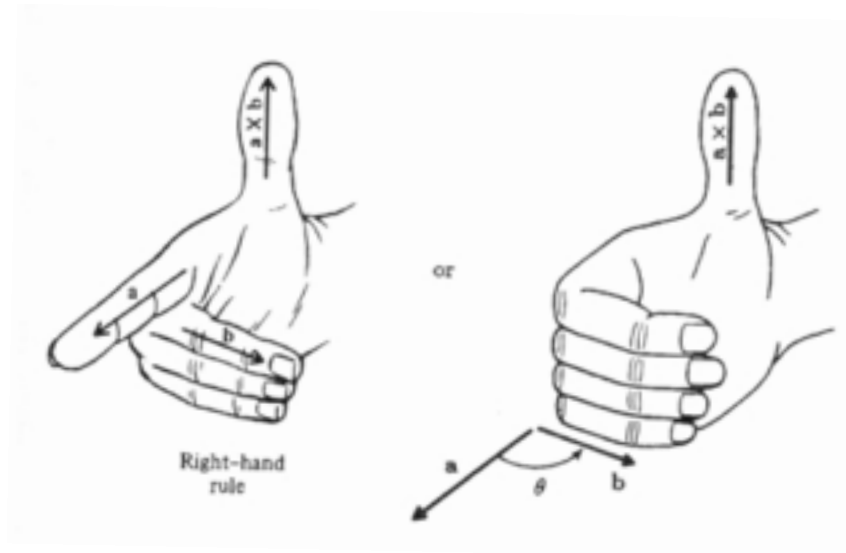
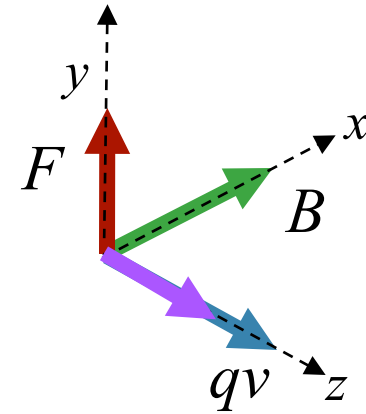
-y

E.

z

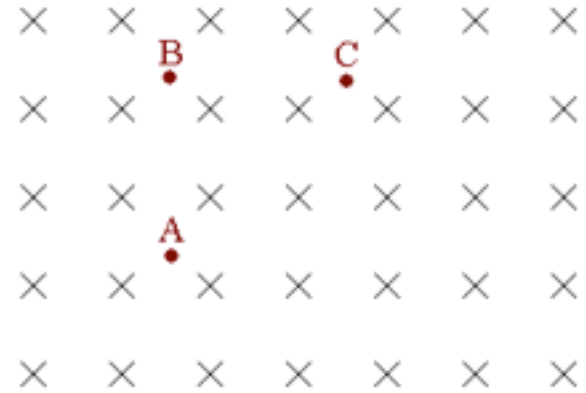
# Remembering Directions: The Right Hand Rule

$$\vec{F} = q\vec{v} \times \vec{B}$$



## CheckPoint 2

Three points are arranged in a uniform magnetic field. The B field points into the screen.



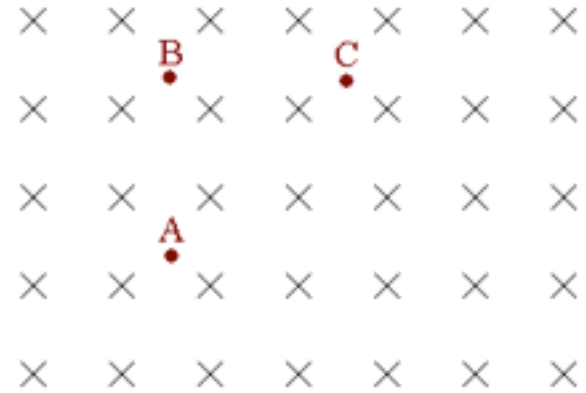
A positively charged particle is at point A and is stationary. The direction of the magnetic force on the particle is

- A. right
- B. left
- C. into screen
- D. out of screen
- E. zero

## CheckPoint 4



Three points are arranged in a uniform magnetic field. The **B** field points into the screen.



The positive charge moves from A to B. The direction of the magnetic force on it is

A. right

B. left

C. into screen

D. out of screen

E. zero

# Cross Product Practice



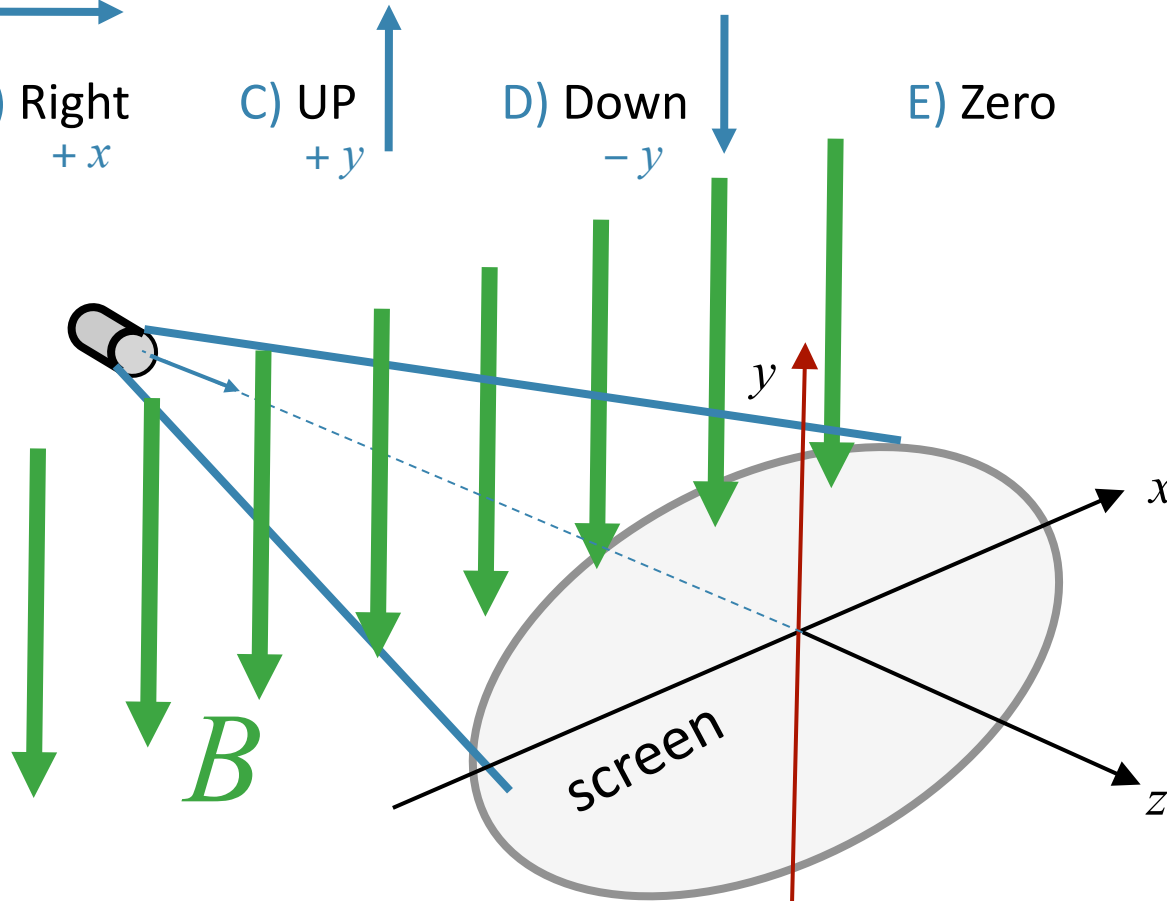
Protons (**positive charge**) coming out of screen

Magnetic field pointing down

What is direction of force on **POSITIVE** charge?

$$\vec{F} = q\vec{v} \times \vec{B}$$

- ←      →
- A) Left      B) Right      C) UP      D) Down      E) Zero
- $-x$        $+x$        $+y$        $-y$



# Motion of Charge $q$ in Uniform $B$ Field

Force is perpendicular to  $\mathbf{v}$

Speed does not change

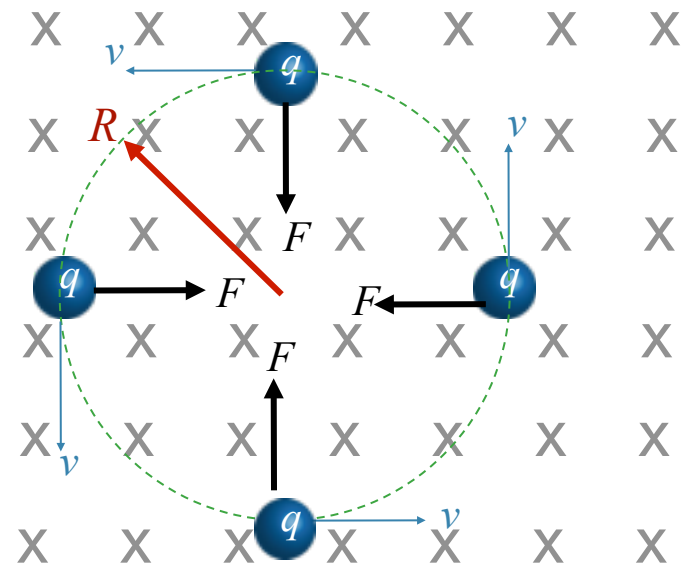
Uniform Circular Motion

Solve for  $R$ :

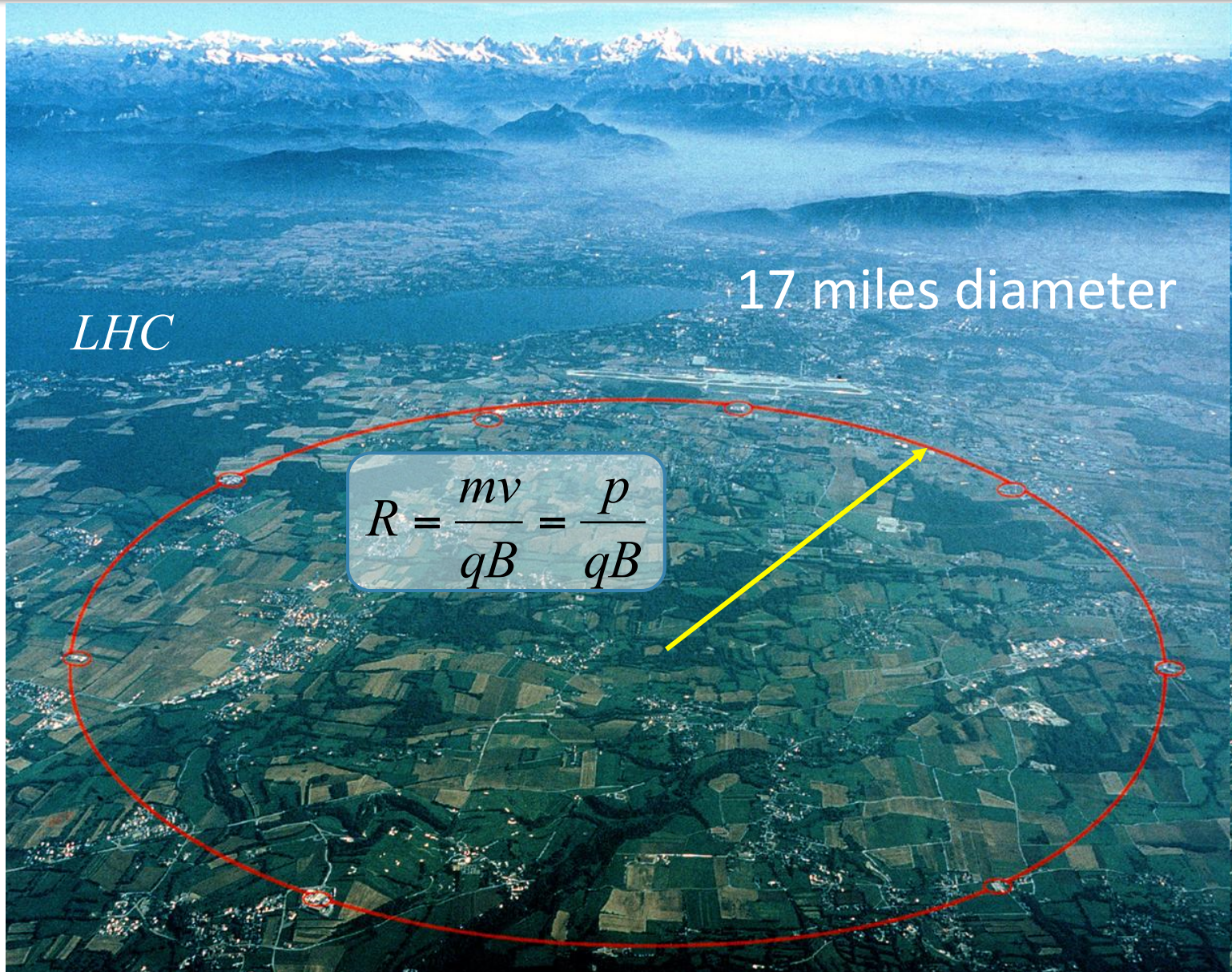
$$\vec{F} = q\vec{v} \times \vec{B} \Rightarrow F = qvB$$

$$a = \frac{v^2}{R}$$

$$qvB = m \frac{v^2}{R} \quad \longrightarrow \quad R = \frac{mv}{qB}$$



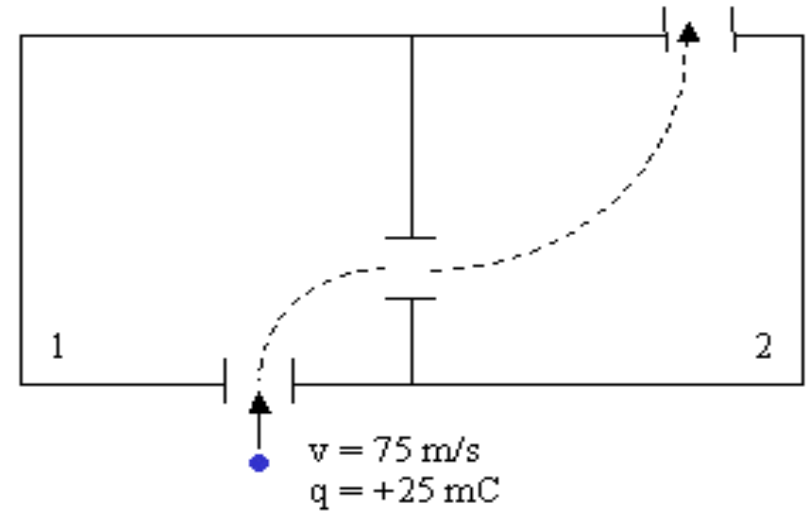
Uniform **B** into page



# CheckPoint 6



The drawing shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle is fired into chamber 1, and observed to follow the dashed path shown in the figure.



What is the direction of the magnetic field in chamber 1?

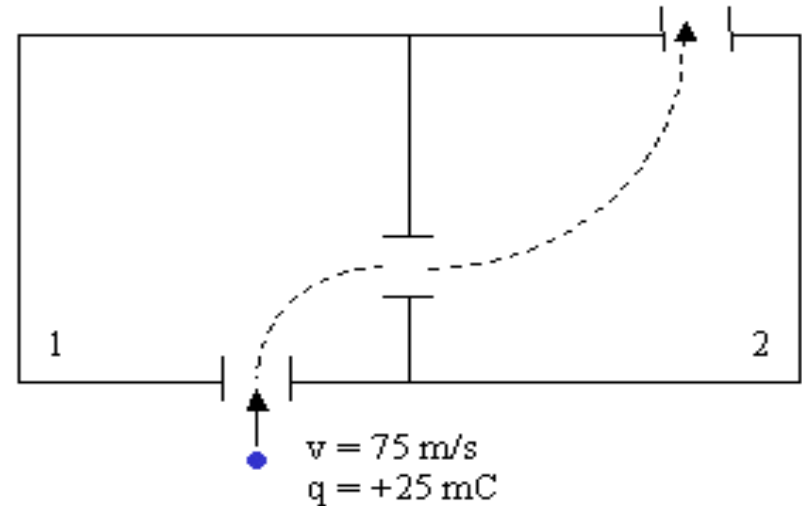
- A. up
- B. down
- C. into page
- D. out of page
- E. Zero

Confusion?

## CheckPoint 8

The drawing shows the top view of two interconnected chambers. Each chamber has a unique magnetic field. A positively charged particle is fired into chamber 1, and observed to follow the dashed path shown in the figure.

Compare the magnitude of the magnetic field in chamber 1 to the magnitude of the magnetic field in chamber 2.

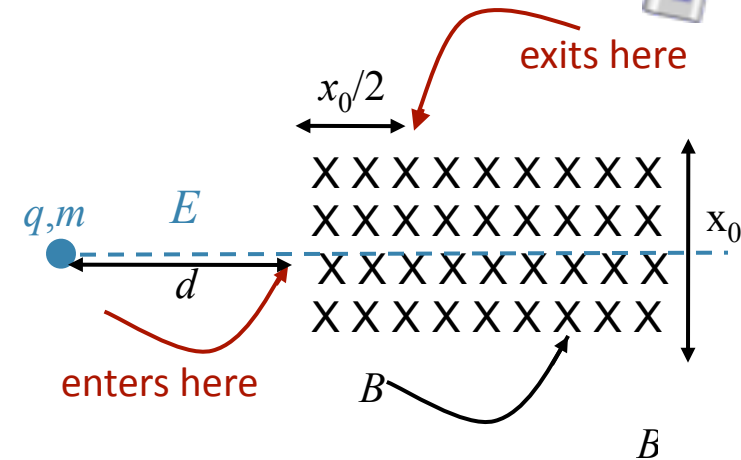


- A.  $|B_1| > |B_2|$
- B.  $|B_1| = |B_2|$
- C.  $|B_1| < |B_2|$

# Calculation

A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.

What is  $B$ ?



## Conceptual Analysis

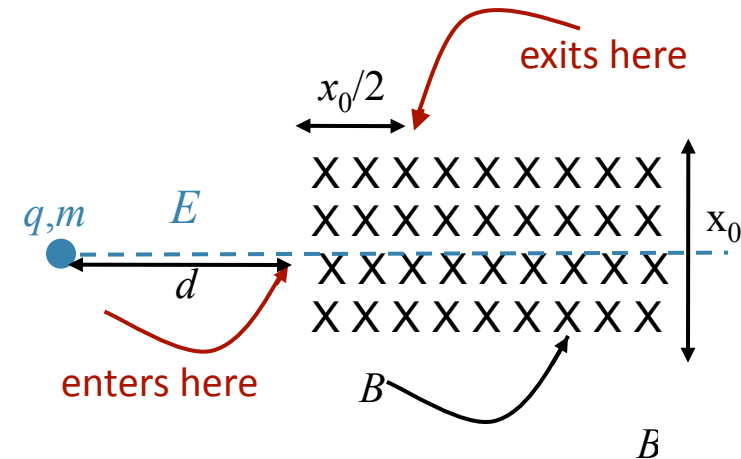
What do we need to know to solve this problem?

- A) Lorentz Force Law
- B)  $E$  field definition
- C)  $V$  definition
- D) Conservation of Energy/Newton's Laws
- E) All of the above

# Calculation

A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.

What is  $B$ ?



## Strategic Analysis

Calculate  $v$ , the velocity of the particle as it enters the magnetic field

Use Lorentz Force equation to determine the path in the field as a function of  $B$

Apply the entrance-exit information to determine  $B$

# Calculation

A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.

What is  $B$ ?

What is  $v_0$ , the speed of the particle as it enters the magnetic field ?

$$v_o = \sqrt{\frac{2E}{m}}$$

A

$$v_o = \sqrt{\frac{2qEd}{m}}$$

B

$$v_o = \sqrt{2ad}$$

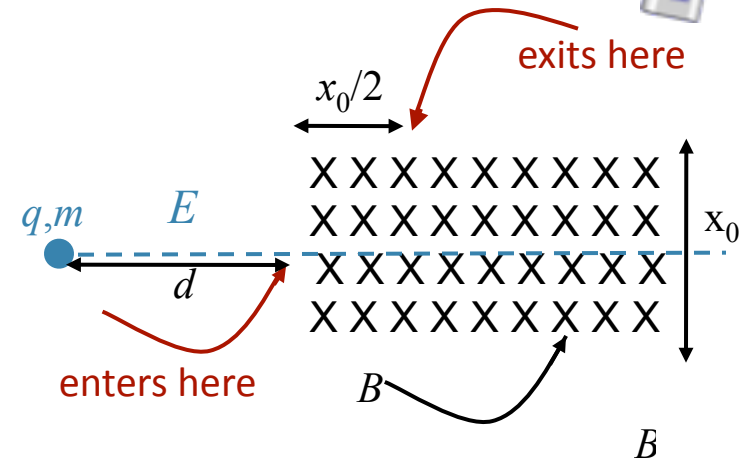
C

$$v_o = \sqrt{\frac{2qE}{md}}$$

D

$$v_o = \sqrt{\frac{qEd}{m}}$$

E



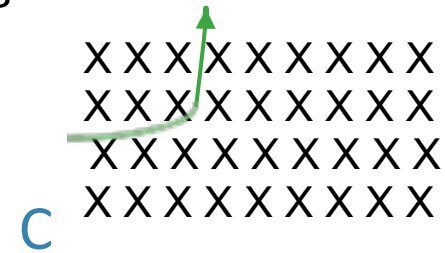
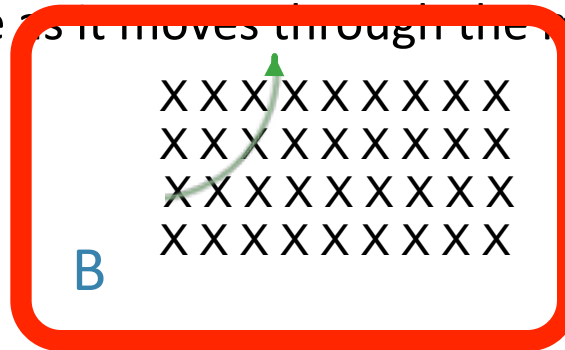
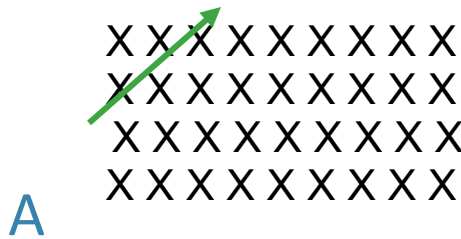
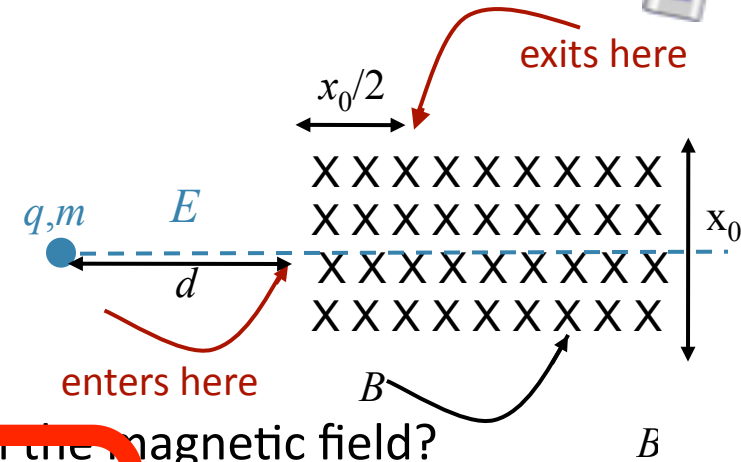
# Calculation



A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.

What is  $B$ ?  $v_o = \sqrt{\frac{2qEd}{m}}$

What is the path of the particle as it moves through the magnetic field?



# Calculation

A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.

What is  $B$ ?  $v_o = \sqrt{\frac{2qEd}{m}}$

What is the radius of path of particle?

$$R = x_o$$

A

$$R = 2x_o$$

B

$$R = \frac{1}{2}x_o$$

C

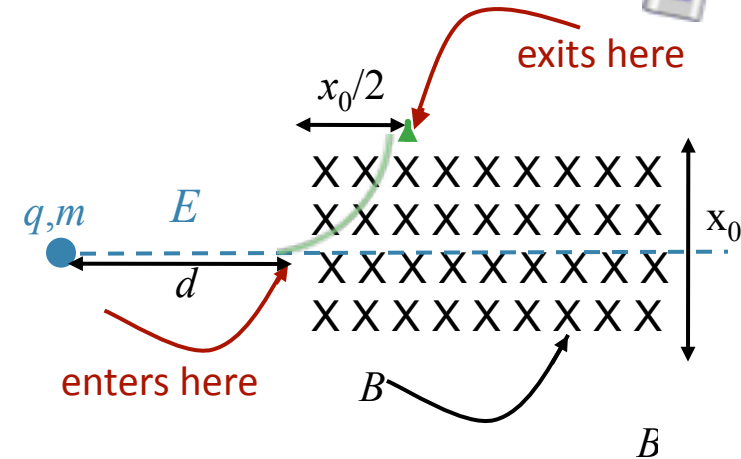
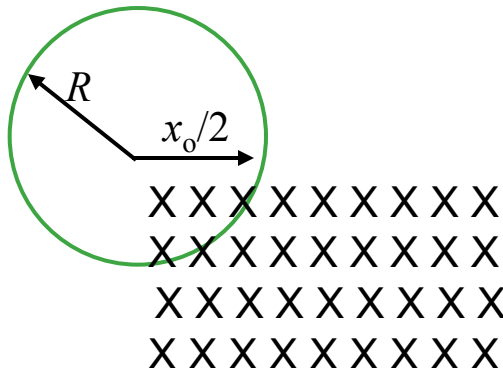
$$R = \frac{mv_o}{qB}$$

D

$$R = \frac{v_o^2}{a}$$

E

Why?



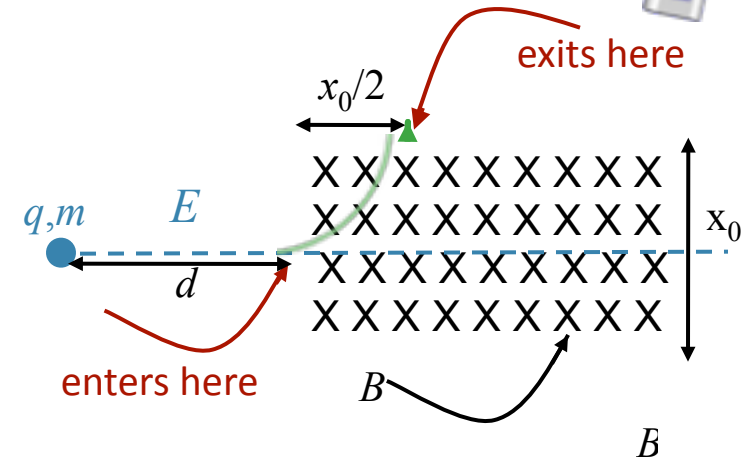
For D, we don't know  $B$  yet.

# Calculation

A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.

What is  $B$ ?

$$v_o = \sqrt{\frac{2qEd}{m}} \quad R = \frac{1}{2}x_0$$



$$B = \frac{2}{x_o} \sqrt{\frac{2mEd}{q}}$$

A

$$B = \frac{E}{v}$$

B

$$B = E \sqrt{\frac{m}{2qEd}}$$

C

$$B = \frac{1}{x_o} \sqrt{\frac{2mEd}{q}}$$

D

$$B = \frac{mv_o}{qx_o}$$

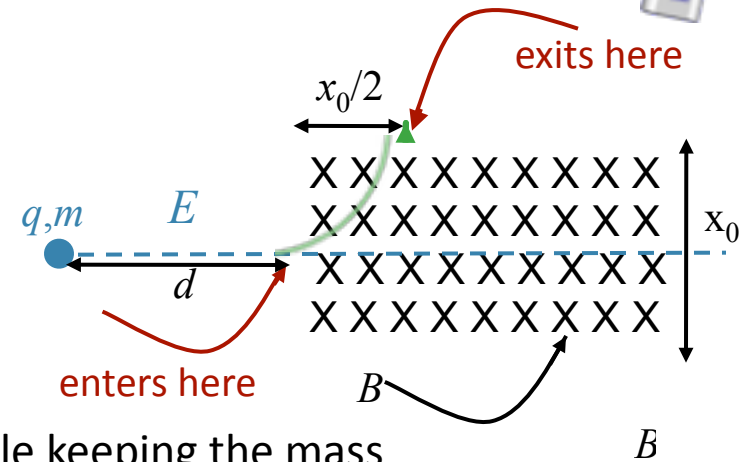
E

# Follow-Up

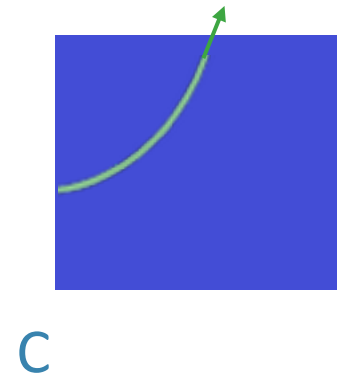
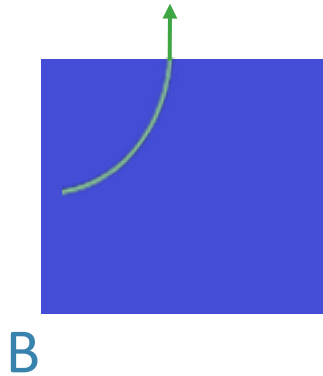
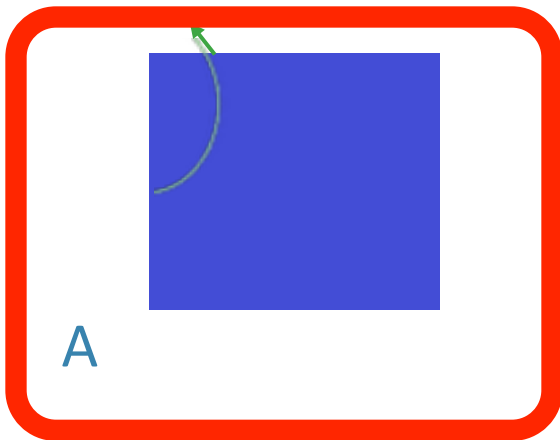
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What is  $B$ ?

$$B = \frac{2}{x_0} \sqrt{\frac{2mEd}{q}}$$



Suppose the charge of the particle is doubled ( $Q = 2q$ ), while keeping the mass constant. How does the path of the particle change?

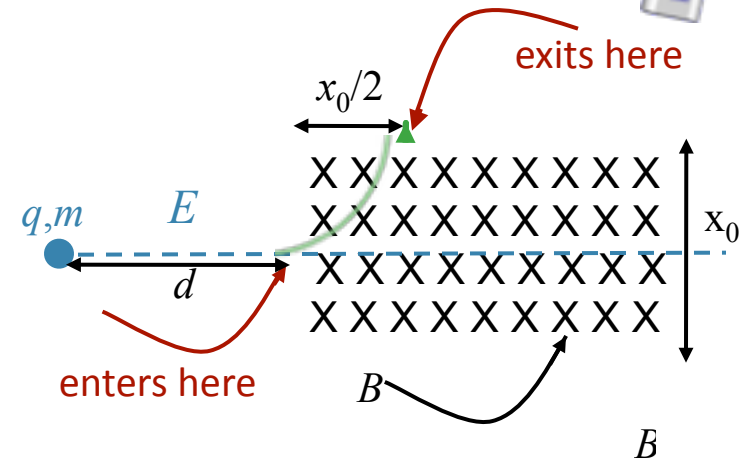


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What is  $B$ ?

$$B = \frac{2}{x_0} \sqrt{\frac{2mEd}{q}}$$



Suppose the charge of the particle is doubled ( $Q = 2q$ ), while keeping the mass constant. How does the path of the particle change?

How does  $v$ , the new velocity at the entrance, compare to the original velocity  $v_0$ ?

A  $v = \frac{v_0}{2}$

B  $v = \frac{v_0}{\sqrt{2}}$

C  $v = v_0$

D  $v = \sqrt{2}v_0$

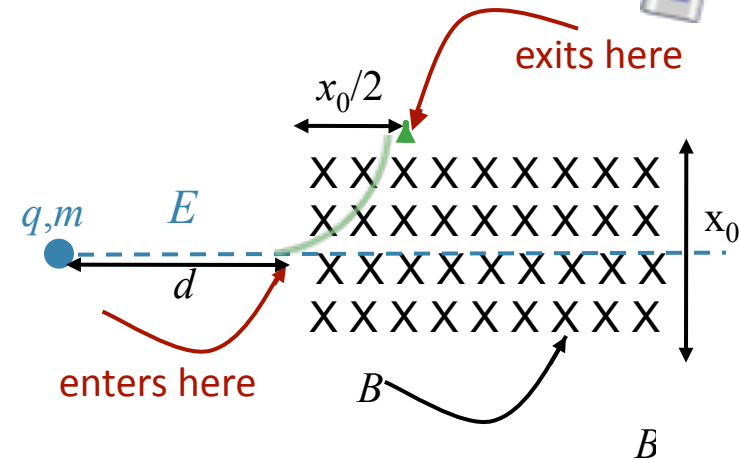
E  $v = 2v_0$

# Follow-Up

A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.

What is  $B$ ?

$$B = \frac{2}{x_0} \sqrt{\frac{2mEd}{q}}$$



Suppose the charge of the particle is doubled ( $Q = 2q$ ), while keeping the mass constant. How does the path of the particle change?

$$v = \sqrt{2}v_0$$

How does  $F$ , the magnitude of the new force at the entrance, compare to  $F_0$ , the magnitude of the original force?

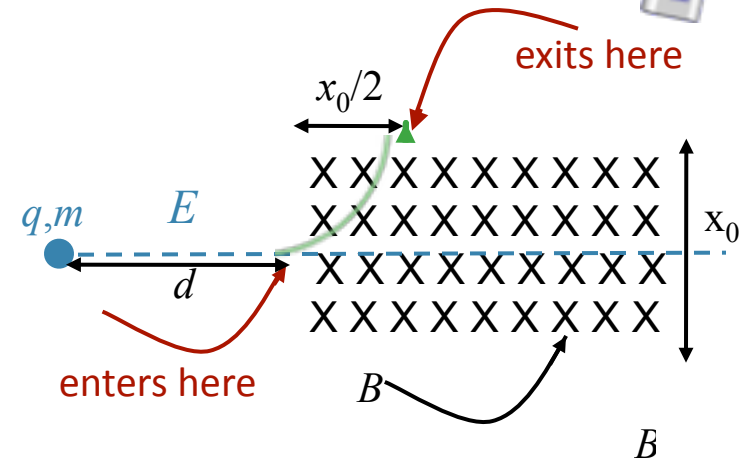
- A  $F = \frac{F_0}{\sqrt{2}}$ 
 B  $F = F_0$ 
 C  $F = \sqrt{2}F_0$ 
 D  $F = 2F_0$ 
 E  $F = 2\sqrt{2}F_0$

# Follow-Up

A particle of charge  $q$  and mass  $m$  is accelerated from rest by an electric field  $E$  through a distance  $d$  and enters and exits a region containing a constant magnetic field  $B$  at the points shown. Assume  $q, m, E, d$ , and  $x_0$  are known.

What is  $B$ ?

$$B = \frac{2}{x_0} \sqrt{\frac{2mEd}{q}}$$



Suppose the charge of the particle is doubled ( $Q = 2q$ ), while keeping the mass constant. How does the path of the particle change?

$v = \sqrt{2}v_o$      $F = 2\sqrt{2}F_o$

How does  $R$ , the radius of curvature of the path, compare to  $R_o$ , the radius of curvature of the original path?

A  $R = \frac{R_o}{2}$

B  $R = \frac{R_o}{\sqrt{2}}$

C  $R = R_o$

D  $R = \sqrt{2}R_o$

E  $R = 2R_o$

$$\frac{mv^2}{R} = F$$

$$R = \frac{mv^2}{F}$$

$$R = \frac{m2v_0^2}{2\sqrt{2}F_0} = \frac{mv_0^2}{\sqrt{2}F_0} = \frac{1}{\sqrt{2}}R_0$$

<https://www.geogebra.org/m/xpRMzPgc>