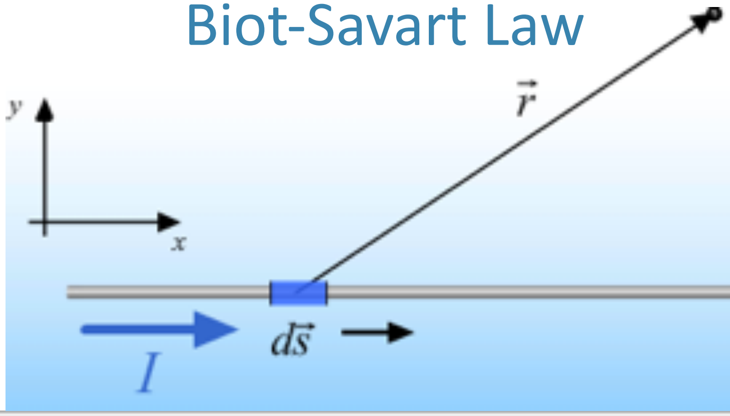
Electricity & Magnetism
Lecture 14

Today's Concept:

Biot-Savart Law



$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

Your comments are important to somebody

Could you summarize all the right hand rules? why are there like 17 different right hand rules lol im lost help plz

I think I dislocated my wrist while trying to apply the right hand rule.

Shouldn't all current in a wire flow on the outside surface, and therefore the magnetic field inside it be zero?

I am about to cry after looking at the Biot-Savart law.

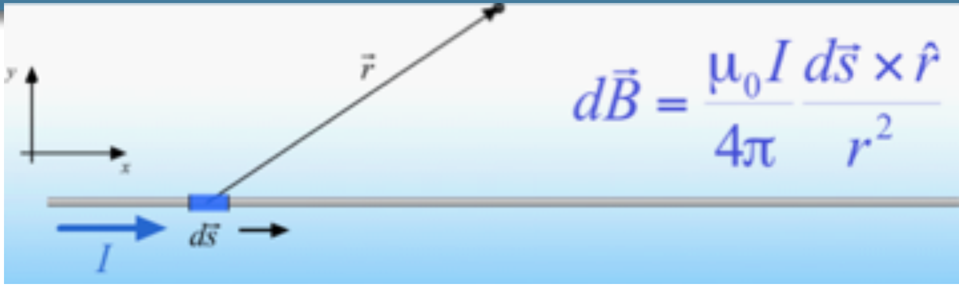
Magnetic video

https://www.youtube.com/watch?v=nRDVm5rn_2A

Biot-Savart Law:

What is it?

Fundamental law for determining the direction and magnitude of the magnetic field due to an element of current.

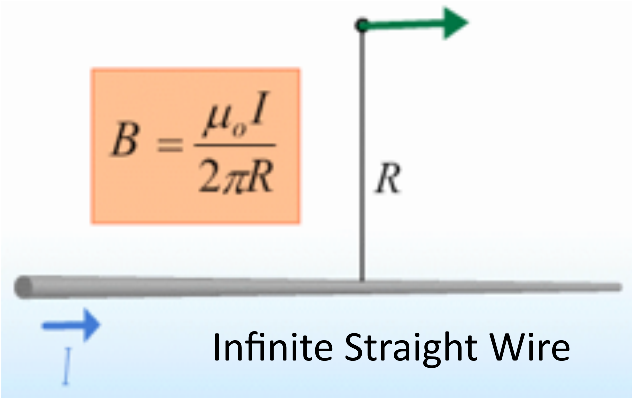


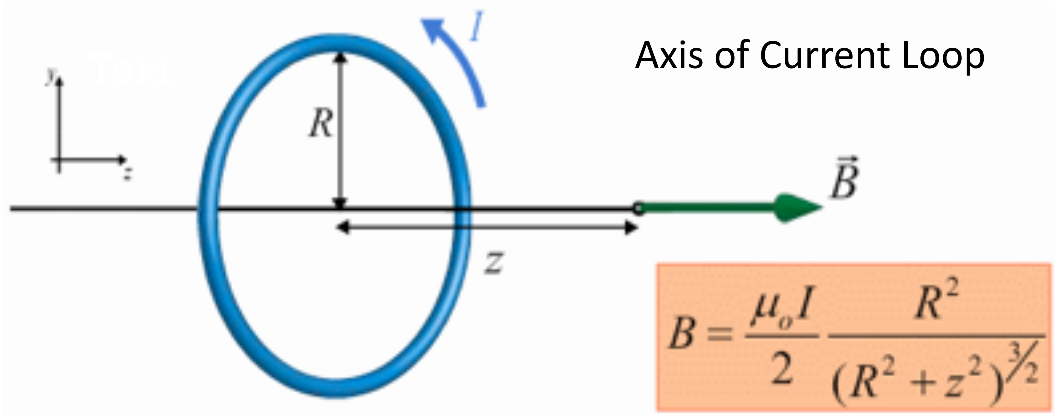
It is named for Jean-Baptiste Biot and Felix Savart who discovered this relationship in 1820.

We can use this law to calculate the magnetic field produced by ANY current distribution

BUT

Easy analytic calculations are possible only for a few distributions:





Plan for Today: Mainly use the results of these calculations!

GOOD NEWS: Remember Gauss' Law? Allowed us to calculate E for symmetrical charge distributions

→

NEXT TIME: Introduce **Ampère's Law** Allows us to calculate B for symmetrical current distributions

$$\vec{B} = \frac{\mu_o I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

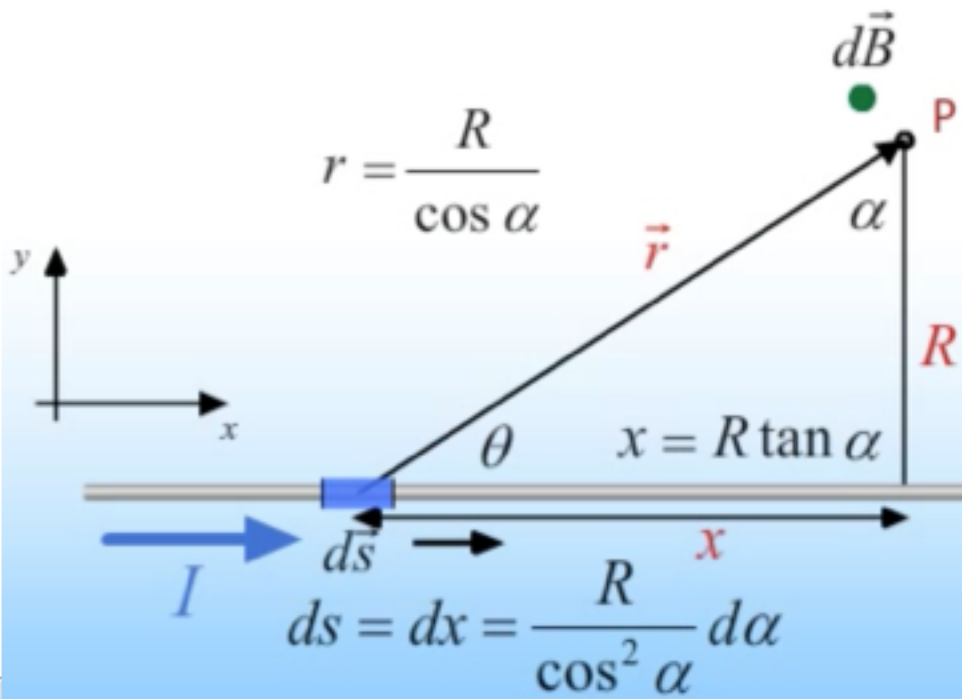
$$= \frac{\mu_o I}{4\pi} \int \frac{\sin\theta \, ds}{r^2} \hat{z}$$

$$= \frac{\mu_o I}{4\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\cos\alpha}{r^2} \frac{R}{\cos^2\alpha} d\alpha \, \hat{z}$$

$$= \frac{\mu_o I}{4\pi} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\cos\alpha}{\left(\frac{R}{\cos\alpha}\right)^2} \frac{R}{\cos^2\alpha} d\alpha \, \hat{z}$$

$$= \frac{\mu_o I}{4\pi R} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos\alpha \, d\alpha \, \hat{z}$$

$$= \frac{\mu_o I}{2\pi R} \hat{z}$$



B from Infinite Line of Current

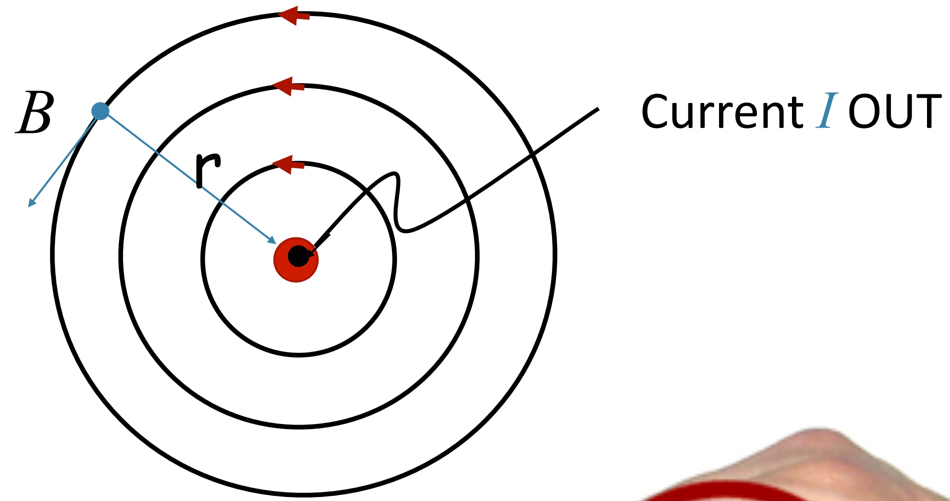
Integrating $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$ gives result

Magnitude:

$$B = \frac{\mu_0 I}{2\pi r}$$

$r =$ distance from wire

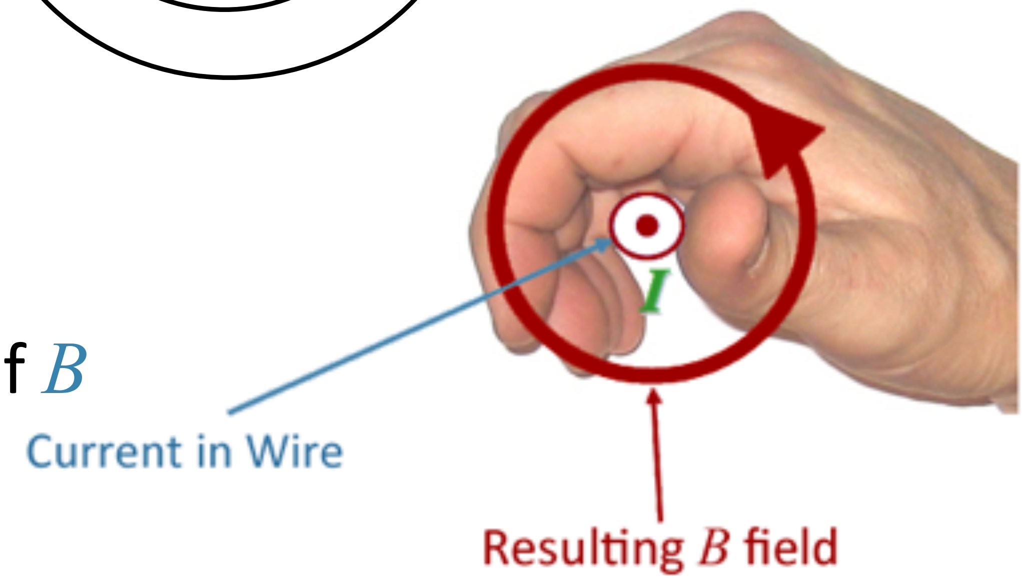
$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$



Direction:

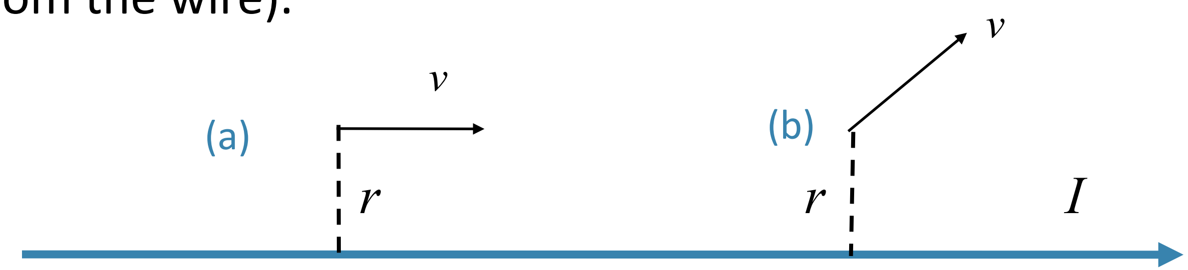
Thumb: on I

Fingers: curl in direction of B



Currents + Charges

A long straight wire is carrying current from left to right. Two identical charges are moving with equal speed. Compare the magnitude of the force on charge a moving directly to the right, to the magnitude of the force on charge b moving up and to the right at the instant shown (i.e. same distance from the wire).



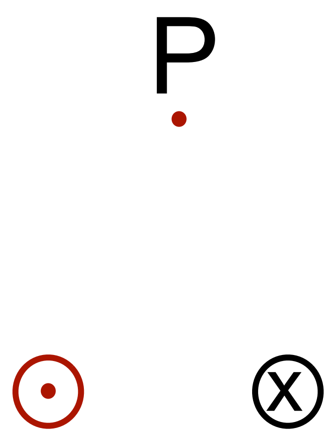
A) $|F_a| > |F_b|$

B) $|F_a| = |F_b|$

C) $|F_a| < |F_b|$

Adding Magnetic Fields

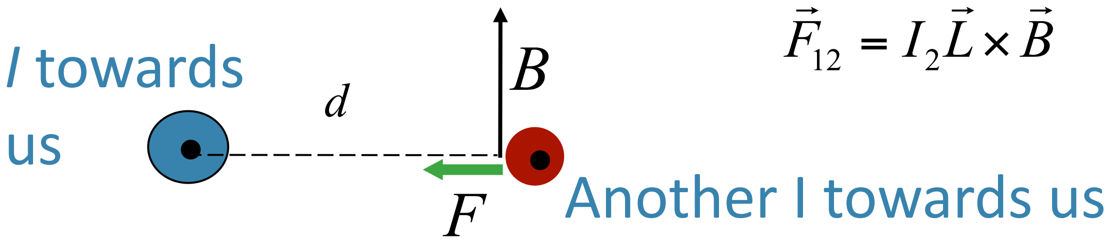
Two long wires carry opposite current



What is the direction of the magnetic field above, and midway between the two wires carrying current – at the point marked "P"?

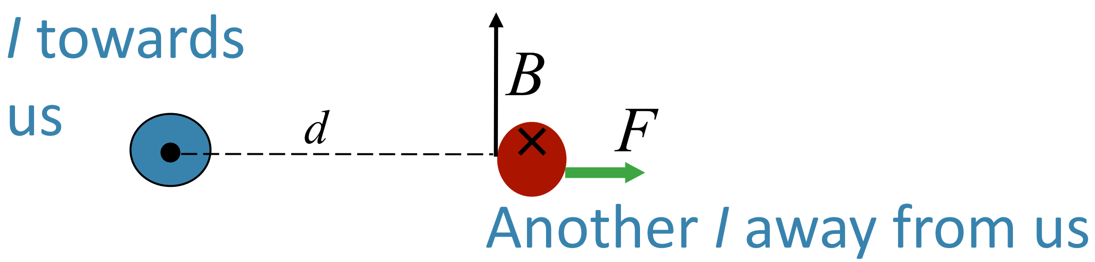
A) Left B) Right C) Up D) Down E) Zero

Force Between Current-Carrying Wires



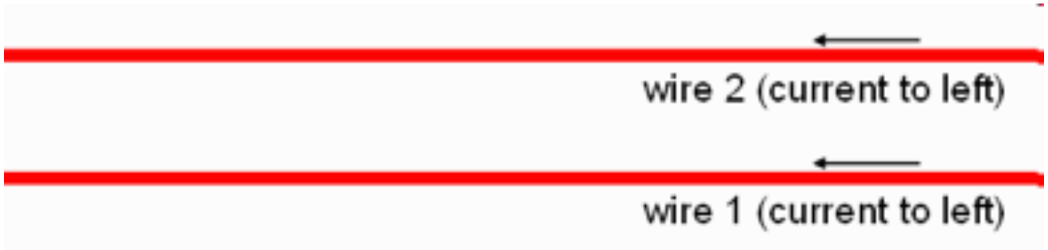
$$\vec{F}_{12} = I_2 \vec{L} \times \vec{B} \quad \Rightarrow \quad F_{12} = I_2 L \frac{\mu_0}{2\pi d} I_1$$

Conclusion: Currents in same direction attract!



Conclusion: Currents in opposite direction repel!

CheckPoint 2 & 4



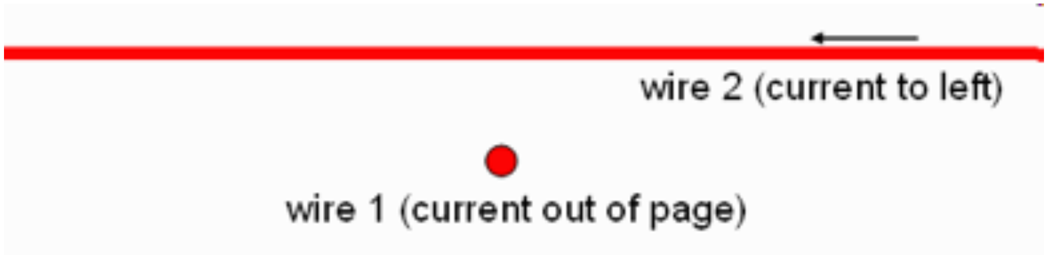
What is the direction of the force on wire 2 due to wire 1?

A) Up B) Down C) Into Screen D) Out of screen E) Zero

What is the direction of the torque on wire 2 due to wire 1?

A) Up B) Down C) Into Screen D) Out of screen E) Zero

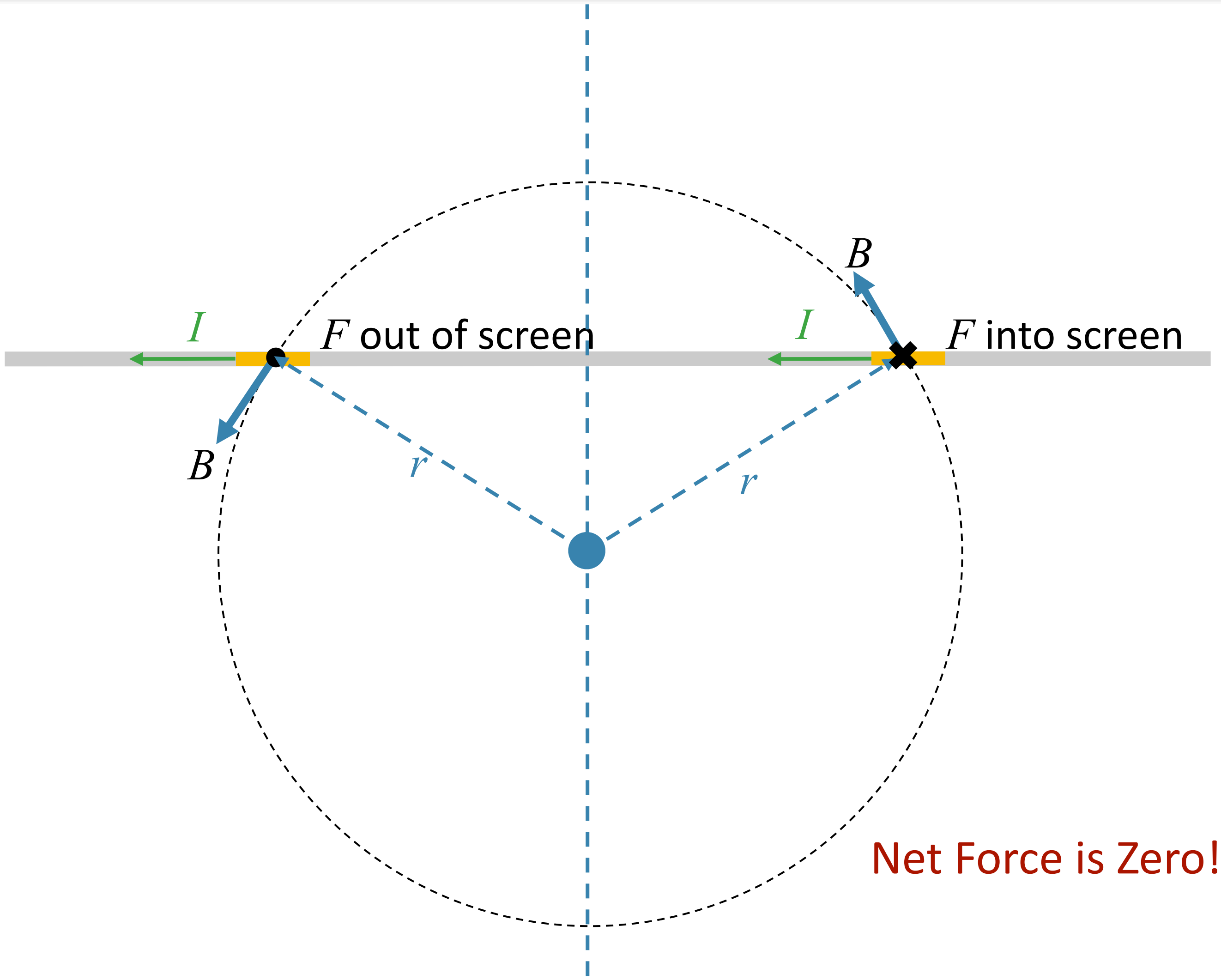
CheckPoint 7



What is the direction of the force on wire 2 due to wire 1?

A) Up B) Down C) Into Screen D) Out of scree E) Zero

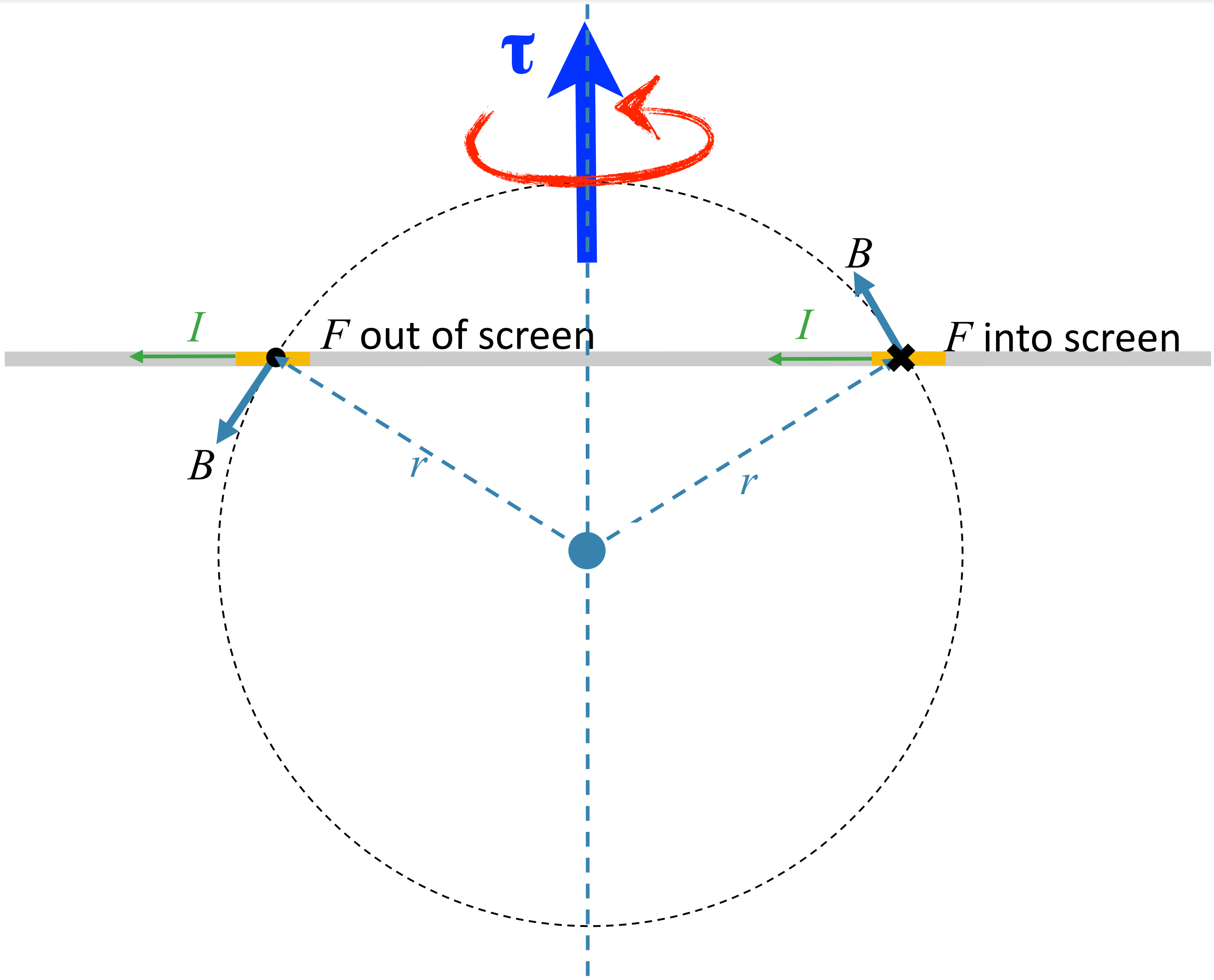
Consider Force on Symmetric Segments



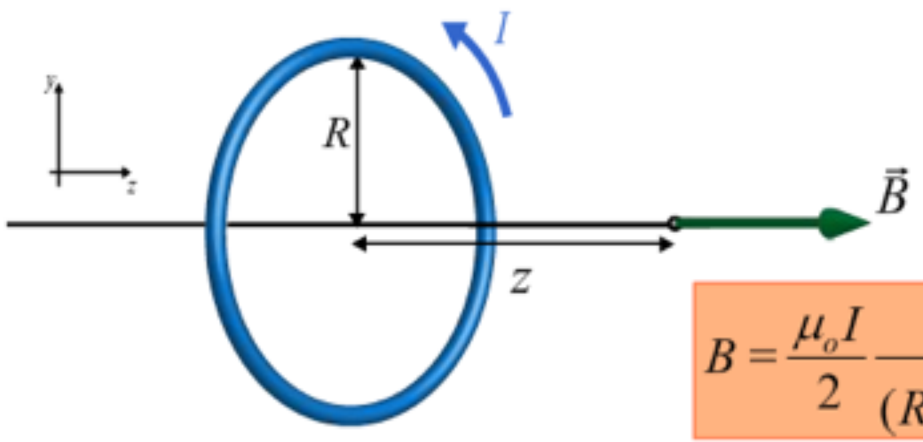
Net Force is Zero!

What about torque?

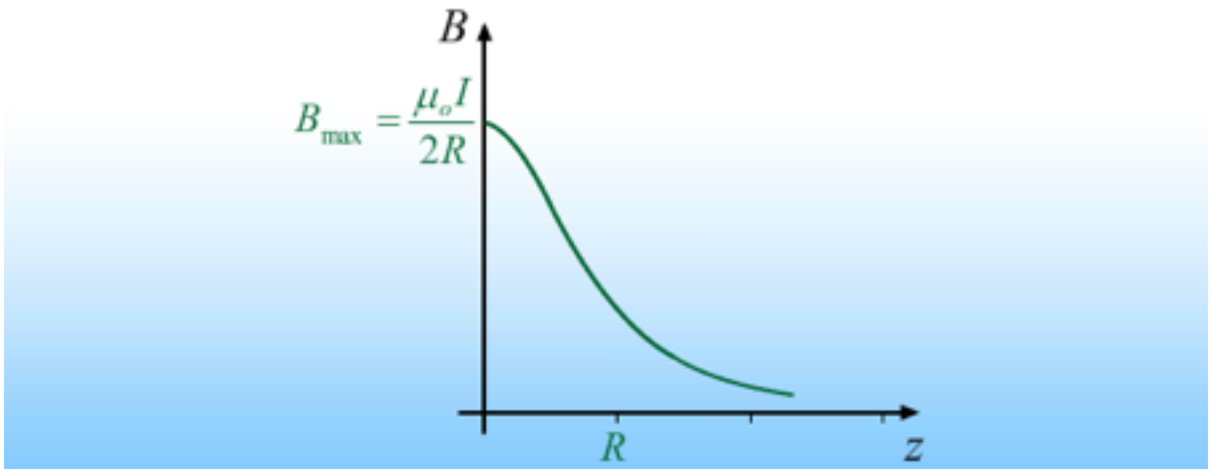
Consider Force on Symmetric Segments

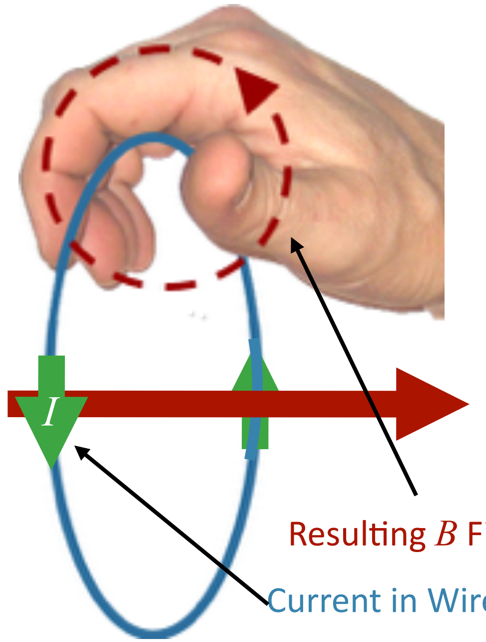


B on axis from Current Loop



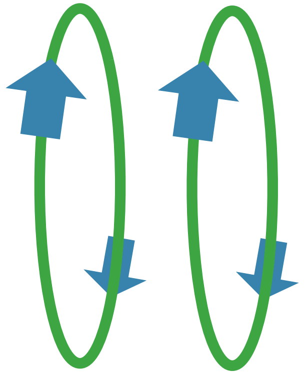
$$B = \frac{\mu_o I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}$$





Two Current Loops

Two identical loops are hung next to each other. Current flows in the same direction in both.



The loops will:

A) Attract each other

B) Repel each other

Question

how many right hand rules do we have to remember in this course

?

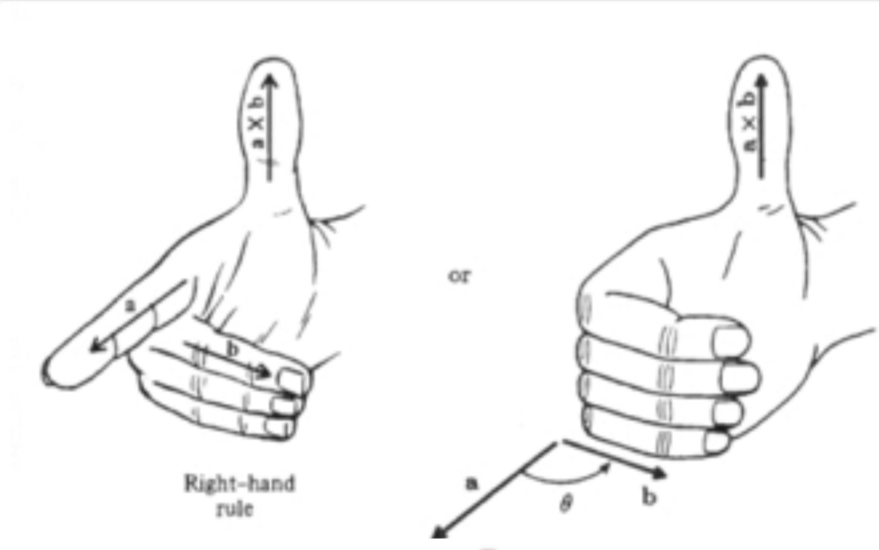
Right Hand Rule Review

1. ANY CROSS PRODUCT

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \vec{F} = I\vec{L} \times \vec{B}$$

$$\tau = \vec{r} \times \vec{F} \qquad \tau = \vec{\mu} \times \vec{B}$$

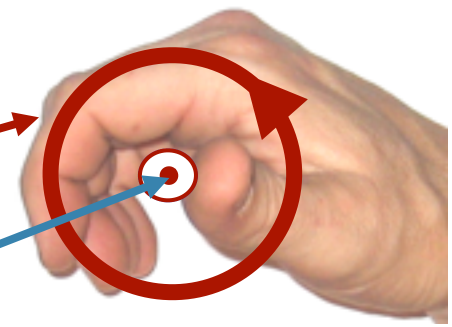
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$



2. Direction of Magnetic Moment

Fingers: Current in Loop

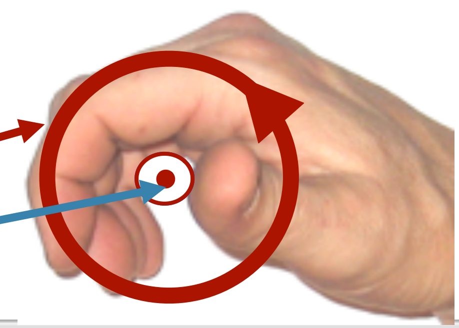
Thumb: Magnetic Moment



3. Direction of Magnetic Field from Wire

Fingers: Magnetic Field

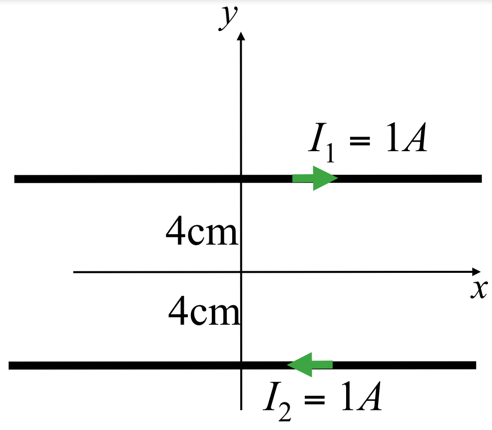
Thumb: Current



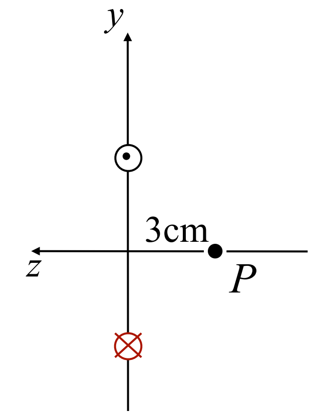
Calculation

Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?



Front view



Side view

Conceptual Analysis

Each wire creates a magnetic field at P

B from infinite wire: $B = \mu_0 I / 2\pi r$

Total magnetic field at P obtained from superposition

Strategic Analysis

Calculate B at P from each wire separately

Total B = vector sum of individual B fields

Calculation

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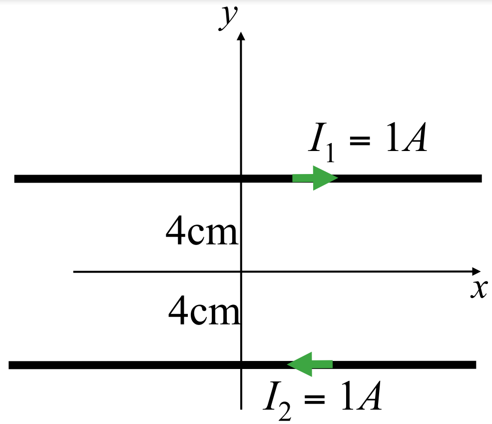
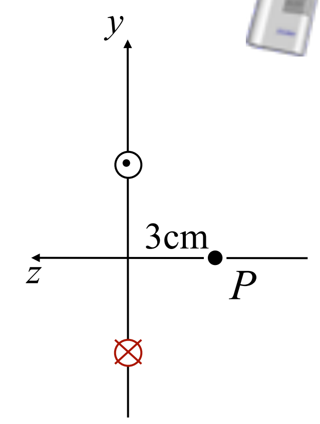
What is the B field at point P?

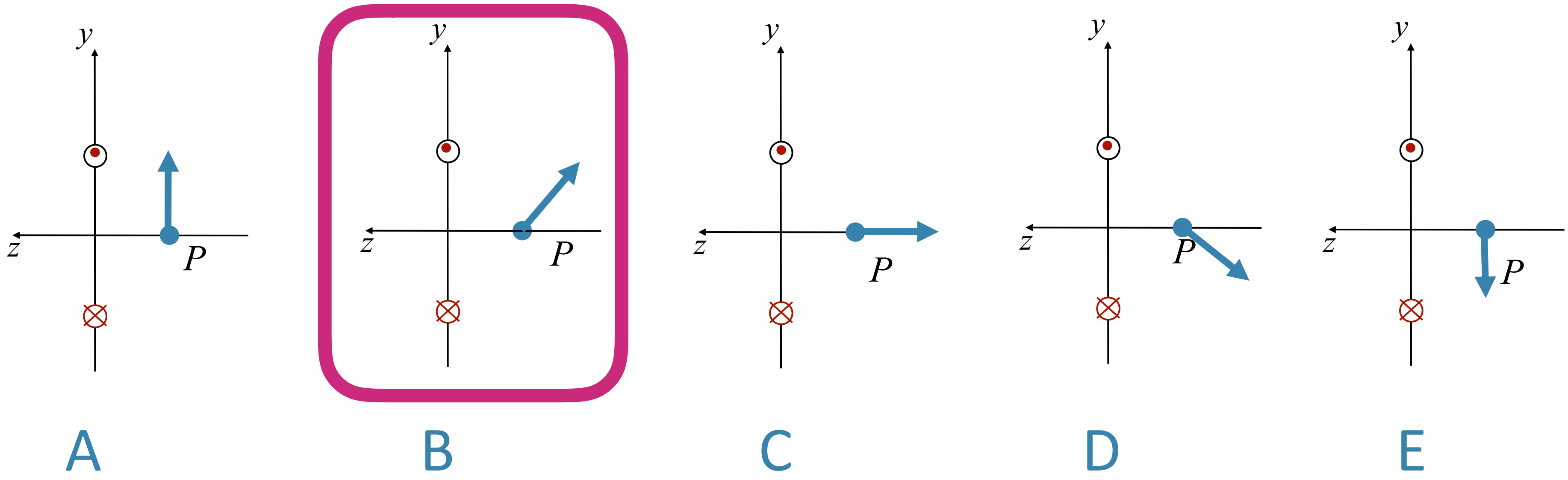


Front view



Side view

What is the direction of B at P produced by the **top** current I_1?



Calculation

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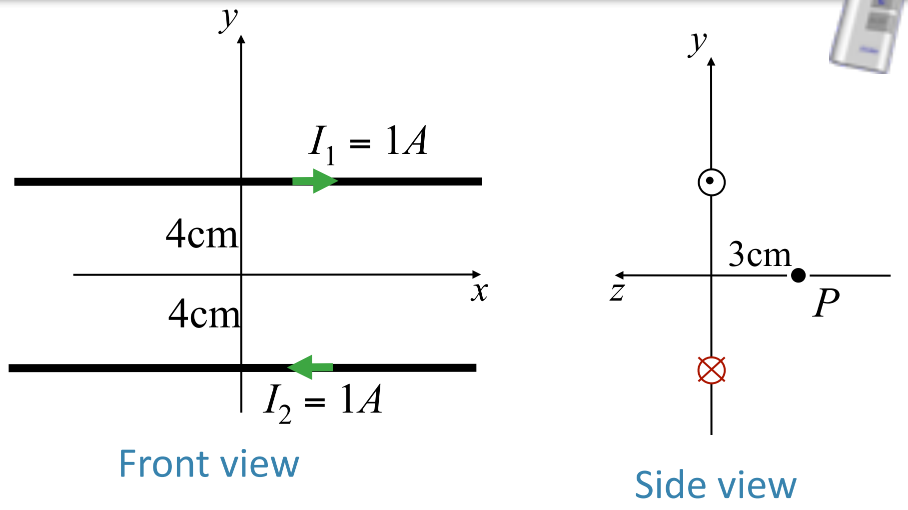
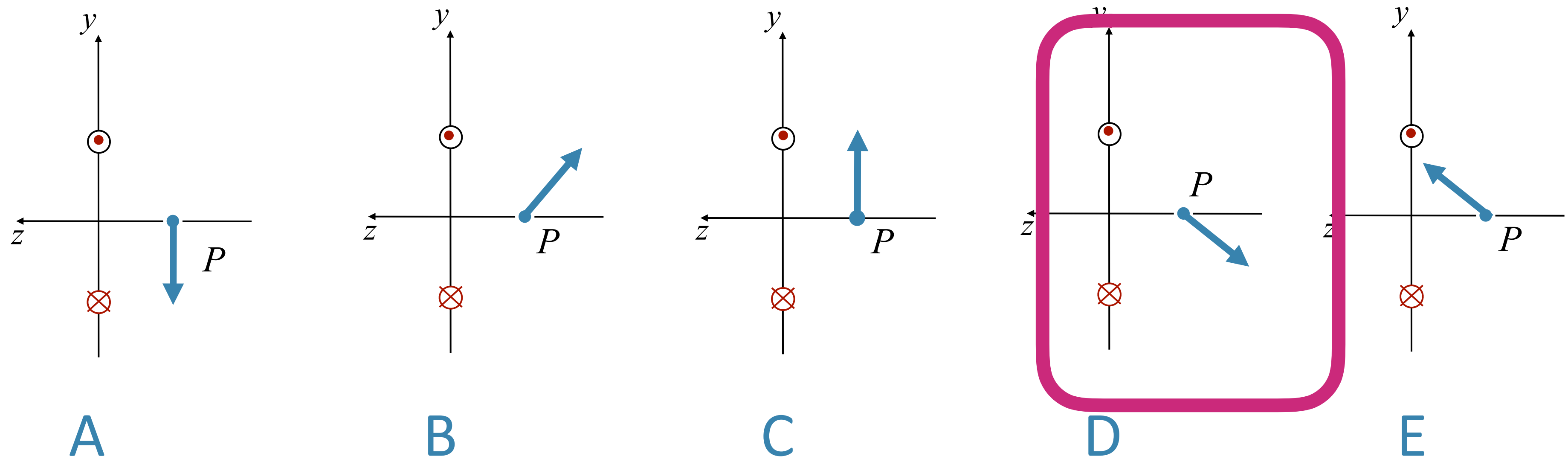
What is the B field at point P?



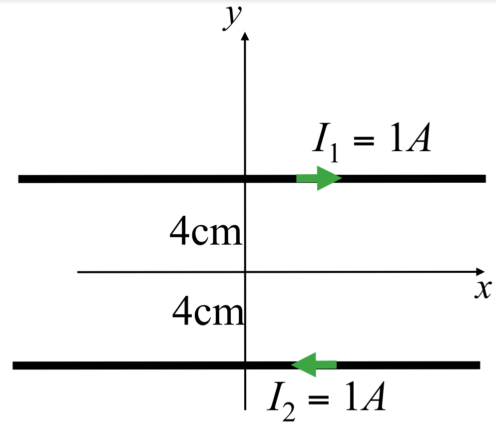
What is the direction of B at P produced by the **bottom** current I_2?



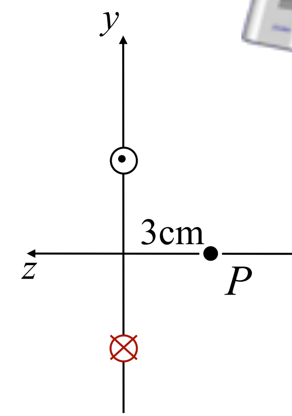
Calculation

Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?

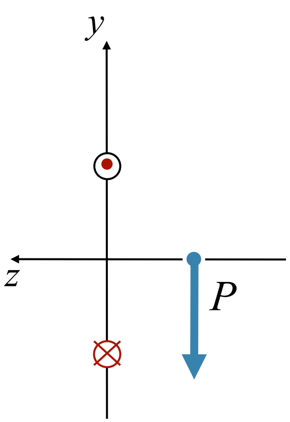


Front view

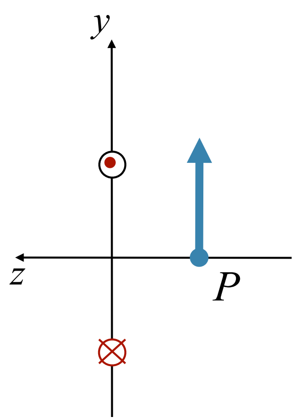


Side view

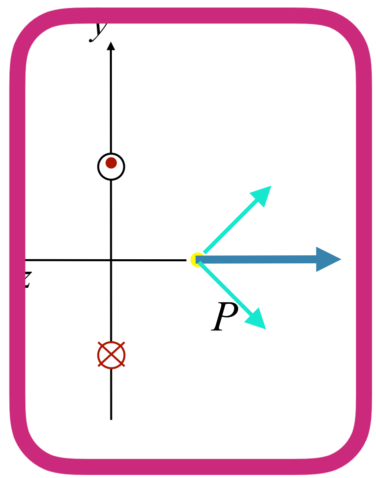
What is the direction of B at P?



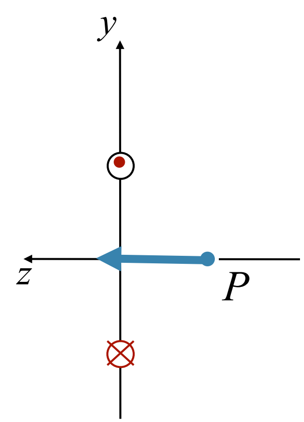
A



B



C



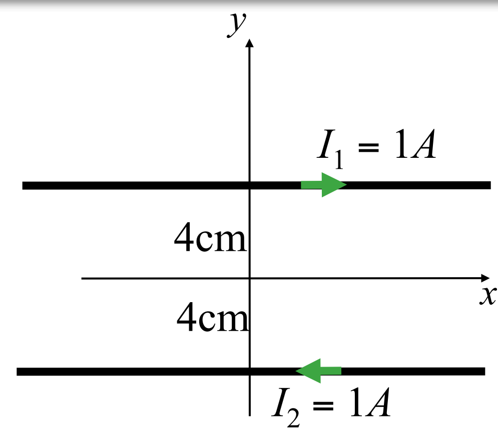
D

Calculation

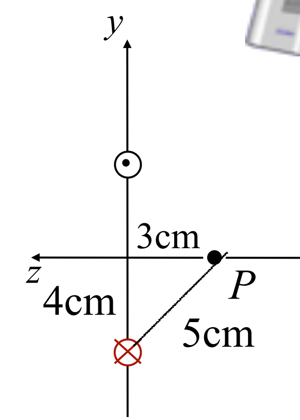
Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?

$$B = \frac{\mu_0 I}{2\pi r}$$



Front view



Side view

What is the magnitude of B at P produced by the top current I_1?

($\mu_0 = 4\pi \times 10^{-7}$ T·m/A)

A) 4.0×10^{-6} T B) 5.0×10^{-6} T C) 6.7×10^{-6} T

$$\frac{\mu_0}{2\pi} = 2 \times 10^{-7} \ \mathrm{T \cdot m/A}$$

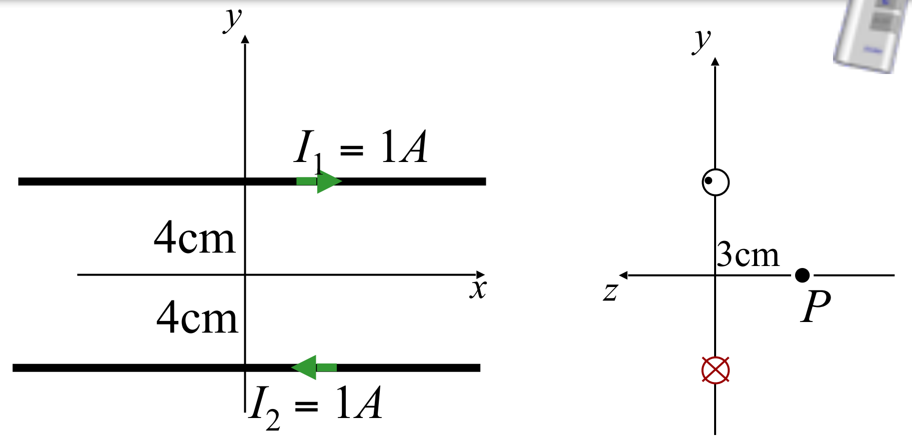
$$B = \frac{\mu_0 I}{2\pi r} = \frac{(2 \times 10^{-7})(1)}{5 \times 10^{-2}} = 4.0 \times 10^{-6} \ \mathrm{T}$$

Calculation

Two parallel horizontal wires are located in the vertical (x,y) plane as shown. Each wire carries a current of $I = 1A$ flowing in the directions shown.

What is the B field at point P?

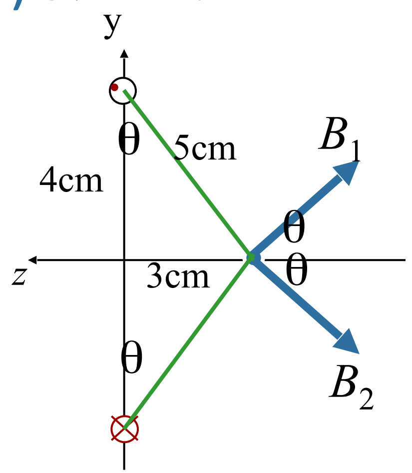
$B_{top} = 4 \times 10^{-6}\ T$



Front view

What is the magnitude of B at P? $(\mu_0 = 4\pi \times 10^{-7}\ T - m/A)$

A) $3.2 \times 10^{-6}\ T$ B) $4.8 \times 10^{-6}\ T$ C) $6.4 \times 10^{-6}\ T$ D) $8.0 \times 10^{-6}\ T$



$$B_{1x} = B_1 \cos\theta$$

$$B_{2x} = B_2 \cos\theta$$

$$\Rightarrow B_x = 2B_1 \cos\theta = 2 \times 4 \times 10^{-6} \times \left(\frac{4}{5}\right) = 6.4 \times 10^{-6}$$