Electricity & Magnetism
Lecture 15

Today's Concept:

Ampere's Law

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enclosed}$$

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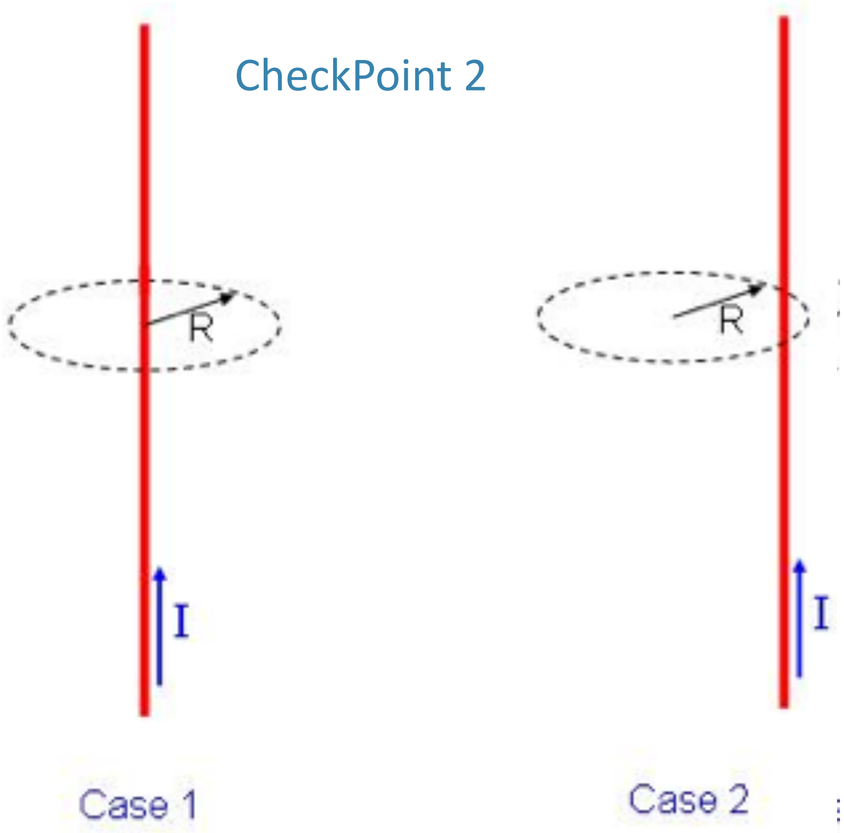
Practice on Enclosed Currents

CheckPoint 2



For which loop is $\oint \vec{B} \cdot d\vec{\ell}$ the greatest?

A. Case 1 B. Case 2 C. the same

Practice on Enclosed Currents

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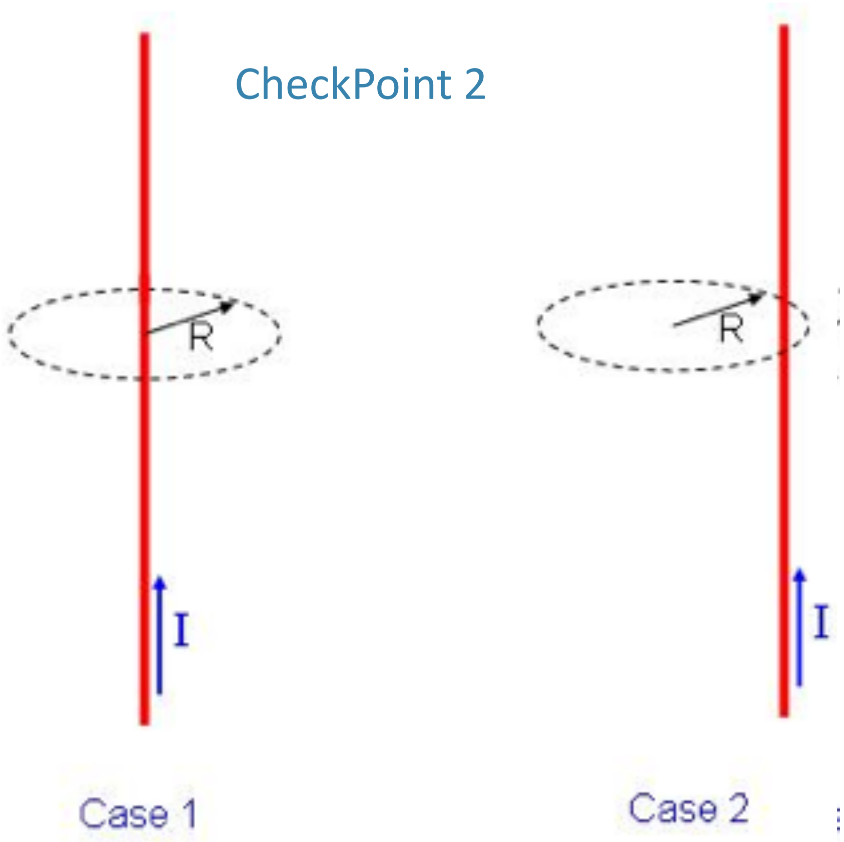

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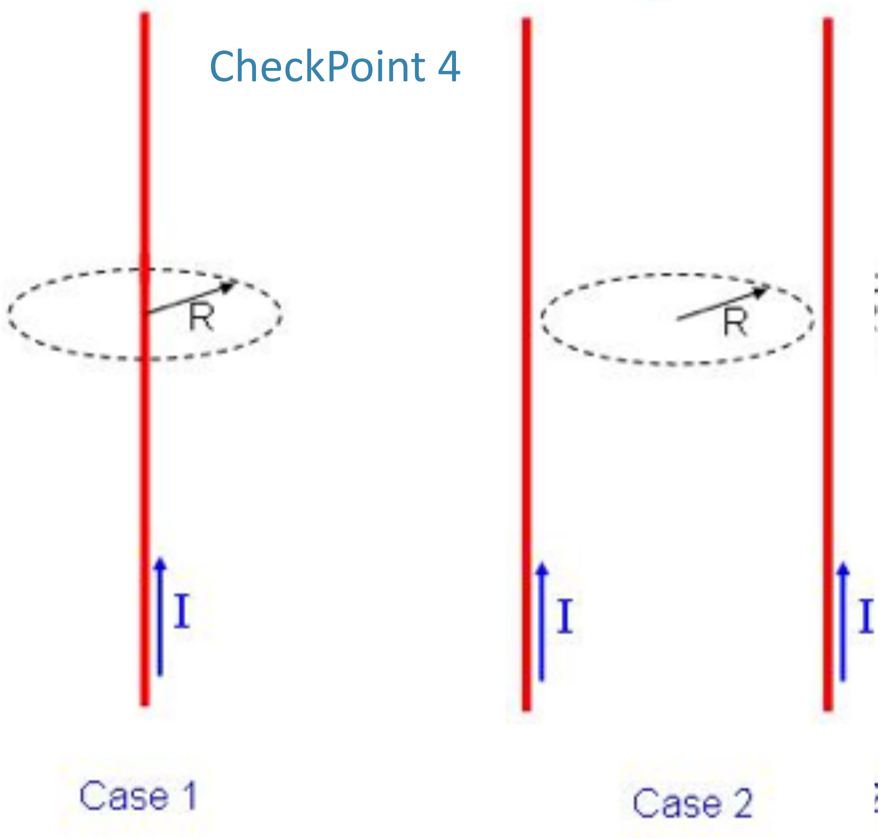

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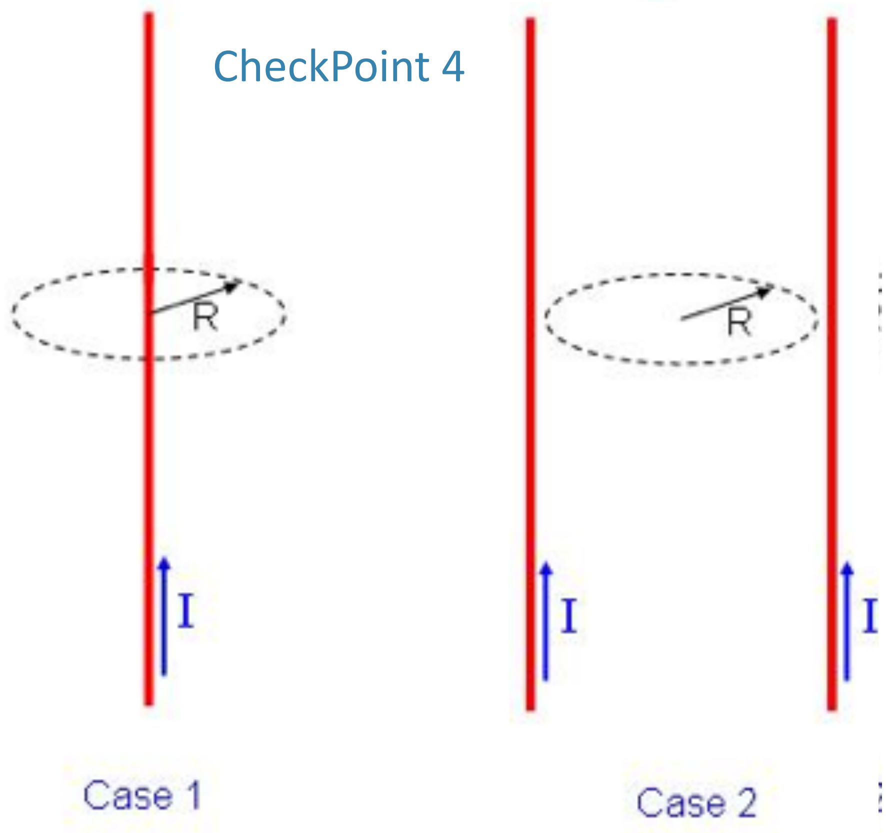

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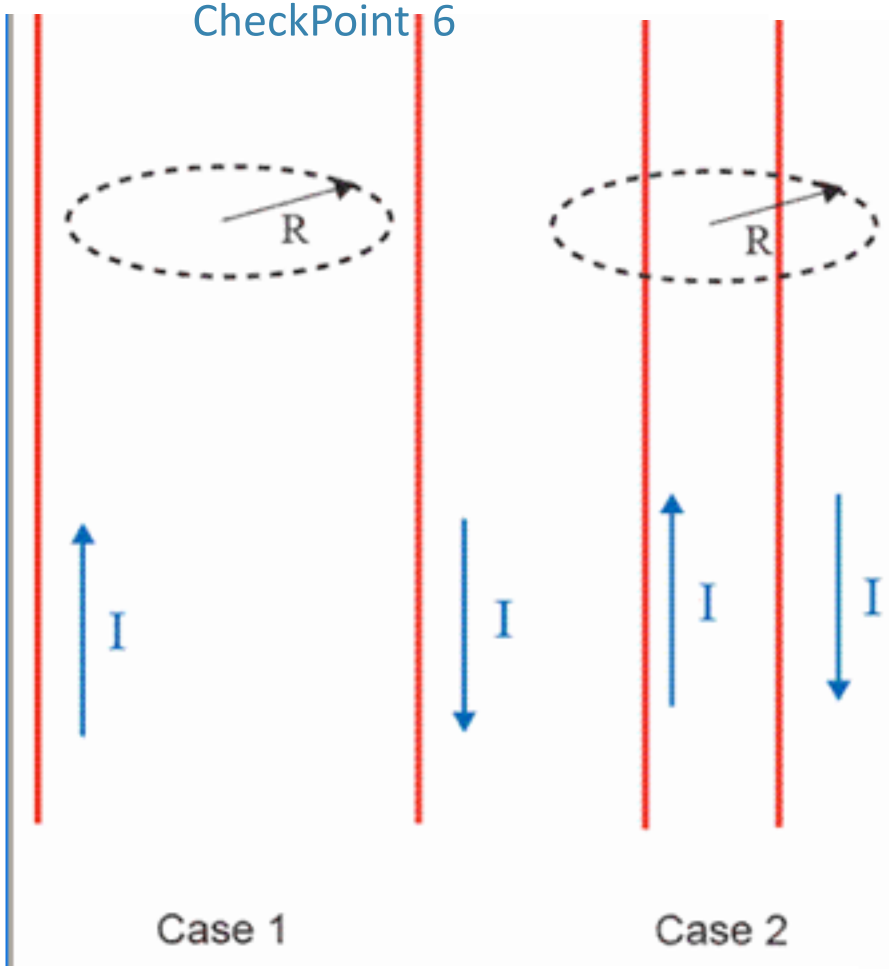
Practice on Enclosed Currents

CheckPoint 6



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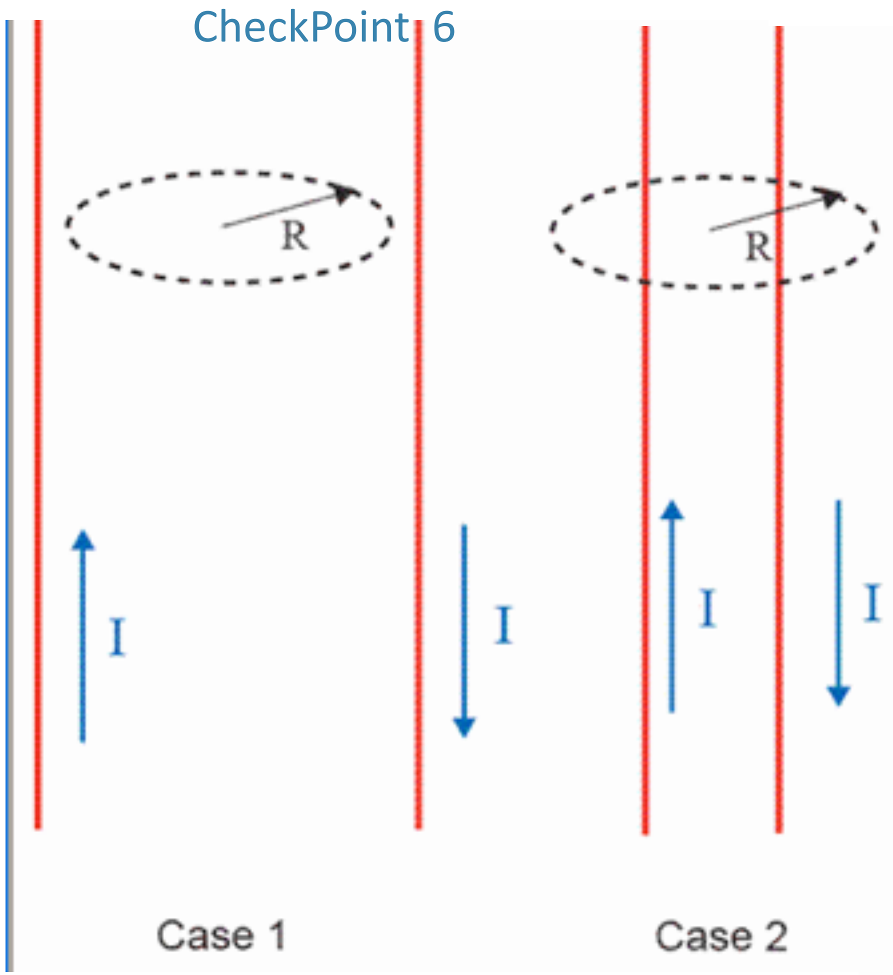
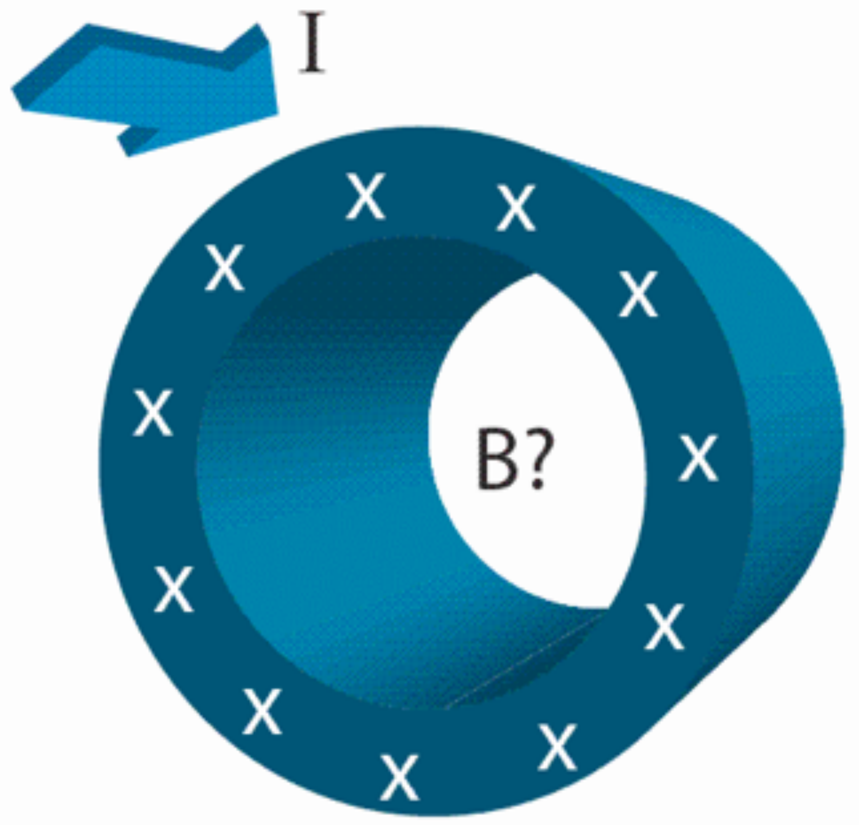


For which loop is $\oint \vec{B} \cdot d\vec{\ell}$ the greatest?

A. Case 1 B. Case 2 C. the same

CheckPoint 8

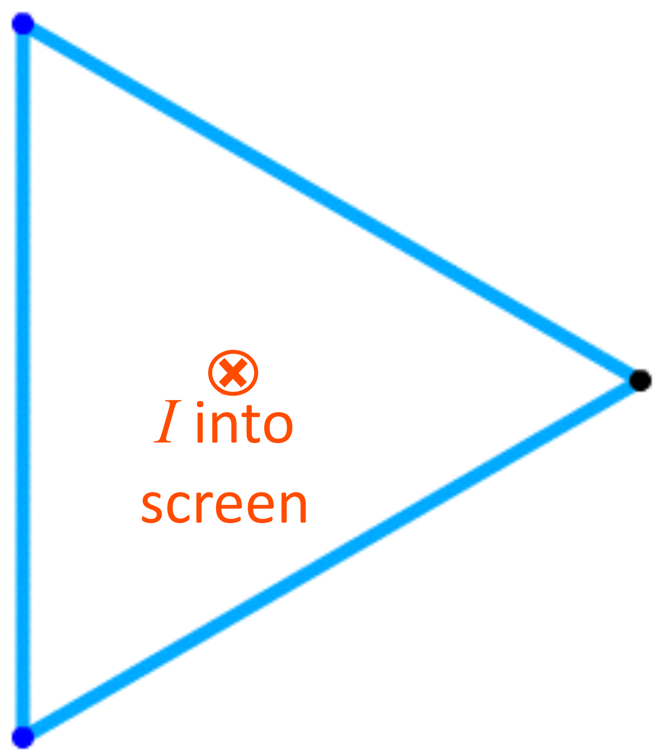
An infinitely long hollow conducting tube carries current *I* in the direction shown.

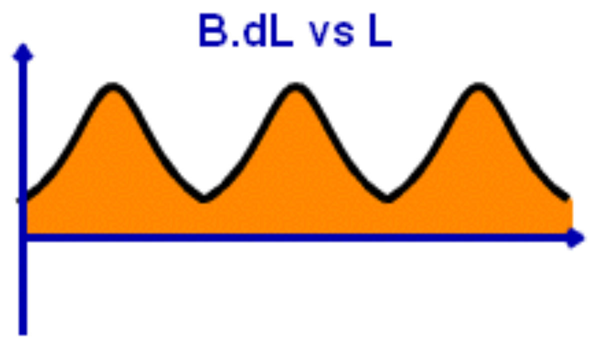


What is the direction of *B* inside the tube?

A) clockwise
B) counterclockwise
C) radially inward to the center
D) radially outward from the center
E) the magnetic field is zero

Ampere’s Law





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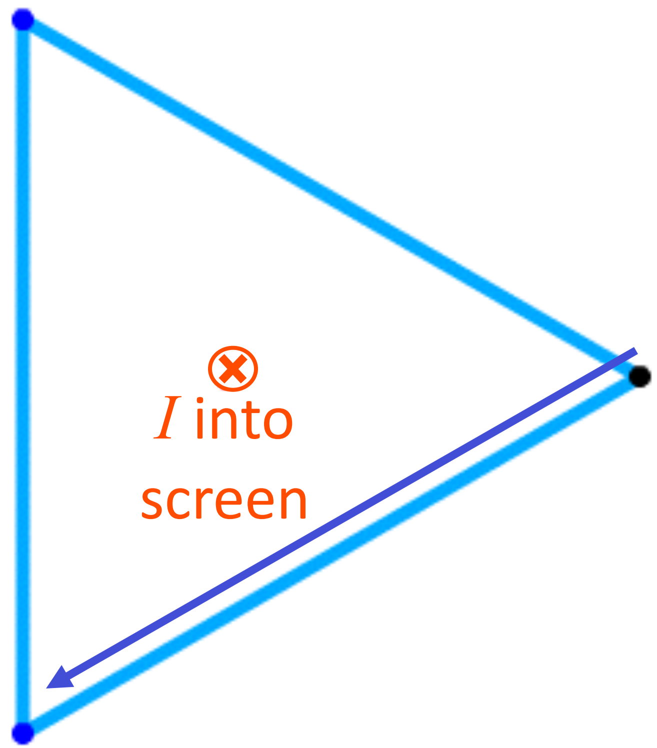

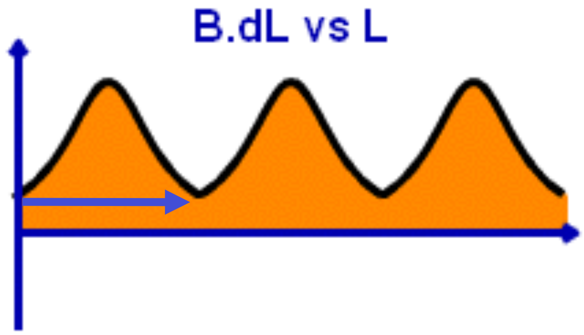

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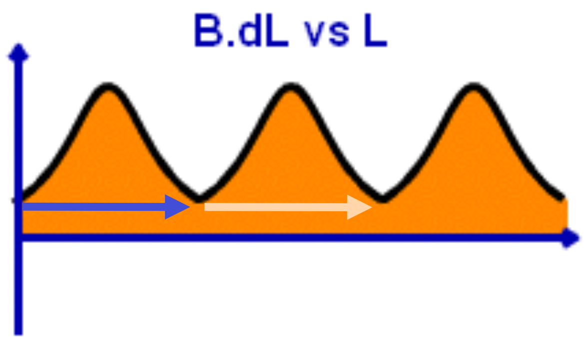

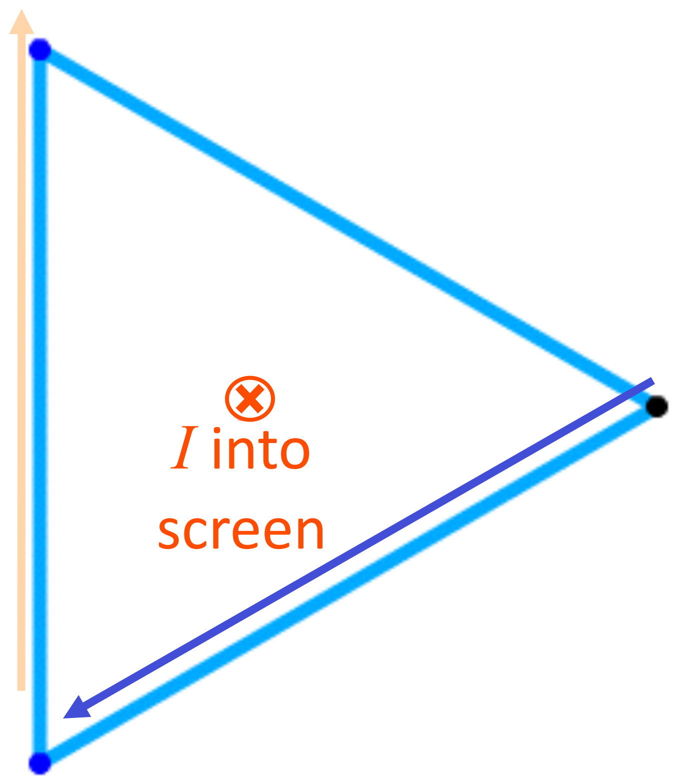

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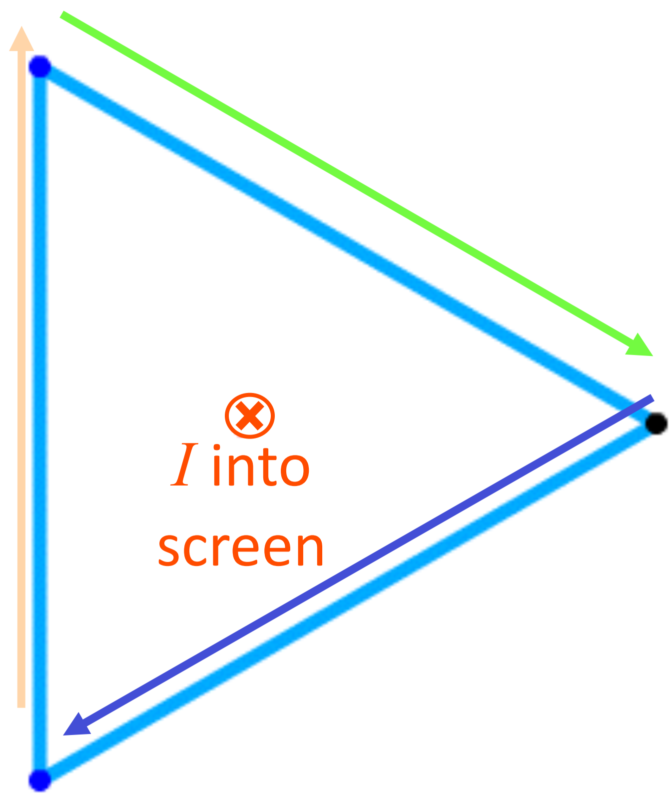

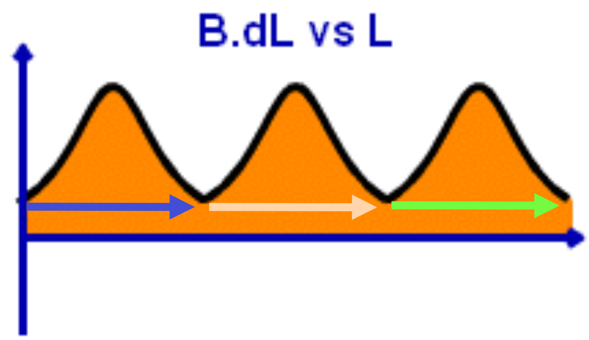

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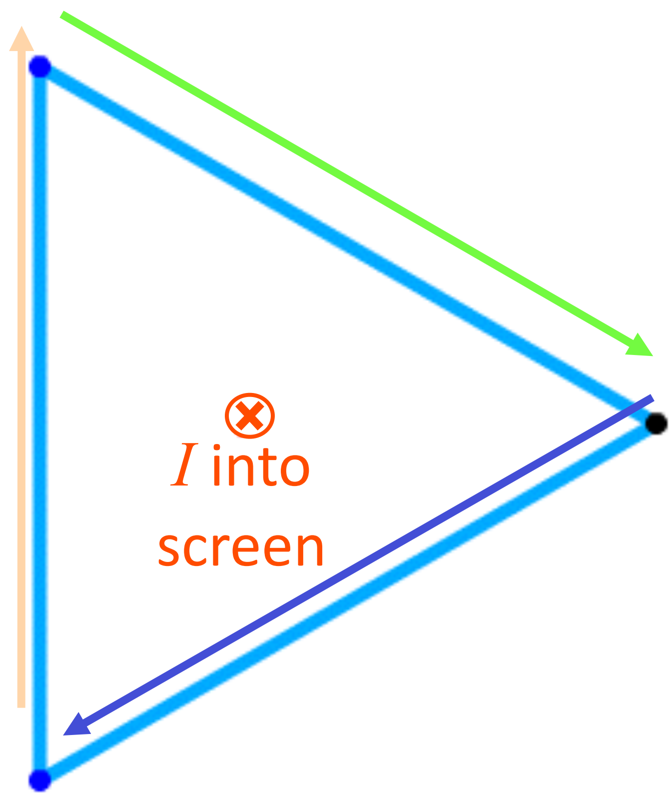
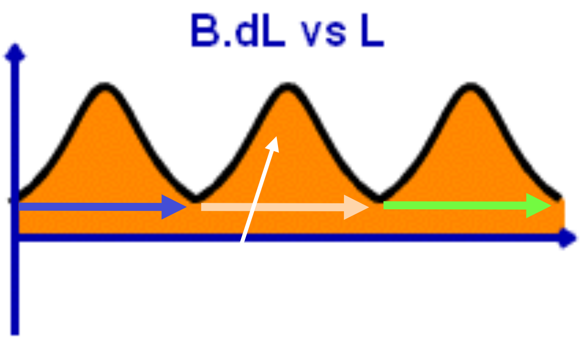
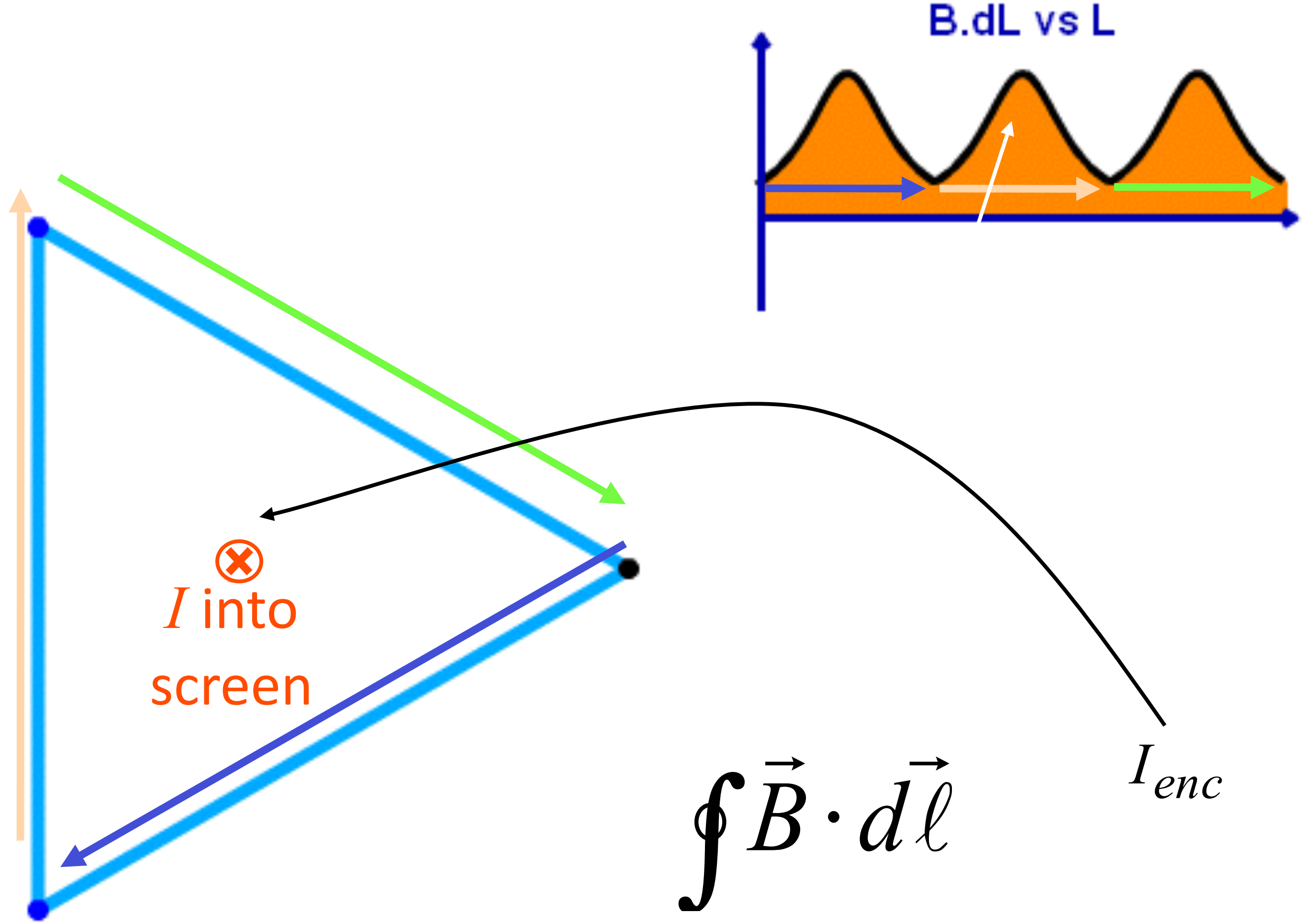




$$\oint \vec{B} \cdot d\vec{\ell}$$

Ampere's Law

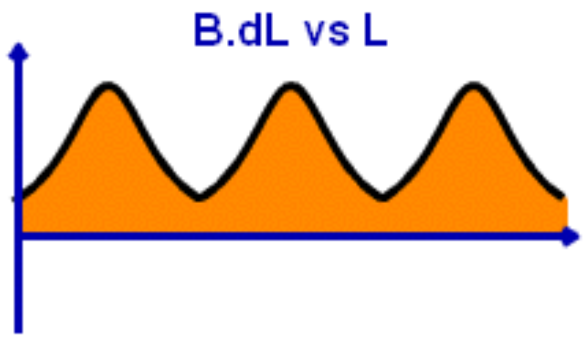


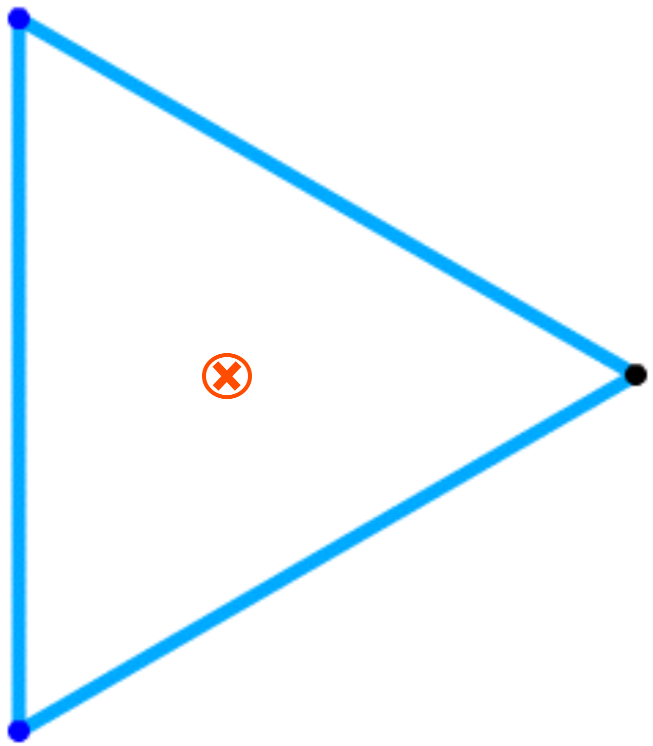
$$\oint \vec{B} \cdot d\vec{\ell}$$

$$I_{enc}$$

Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$





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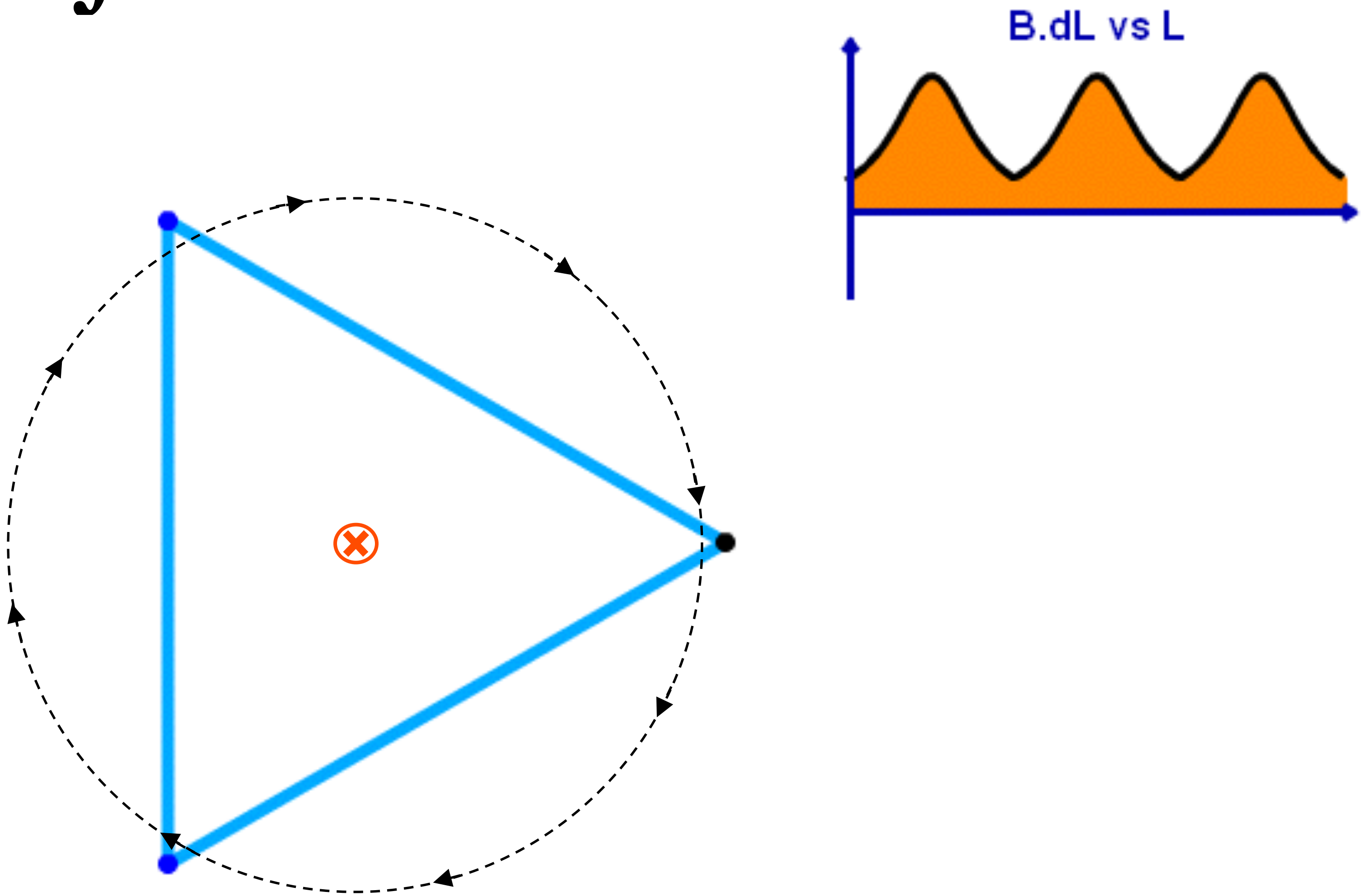

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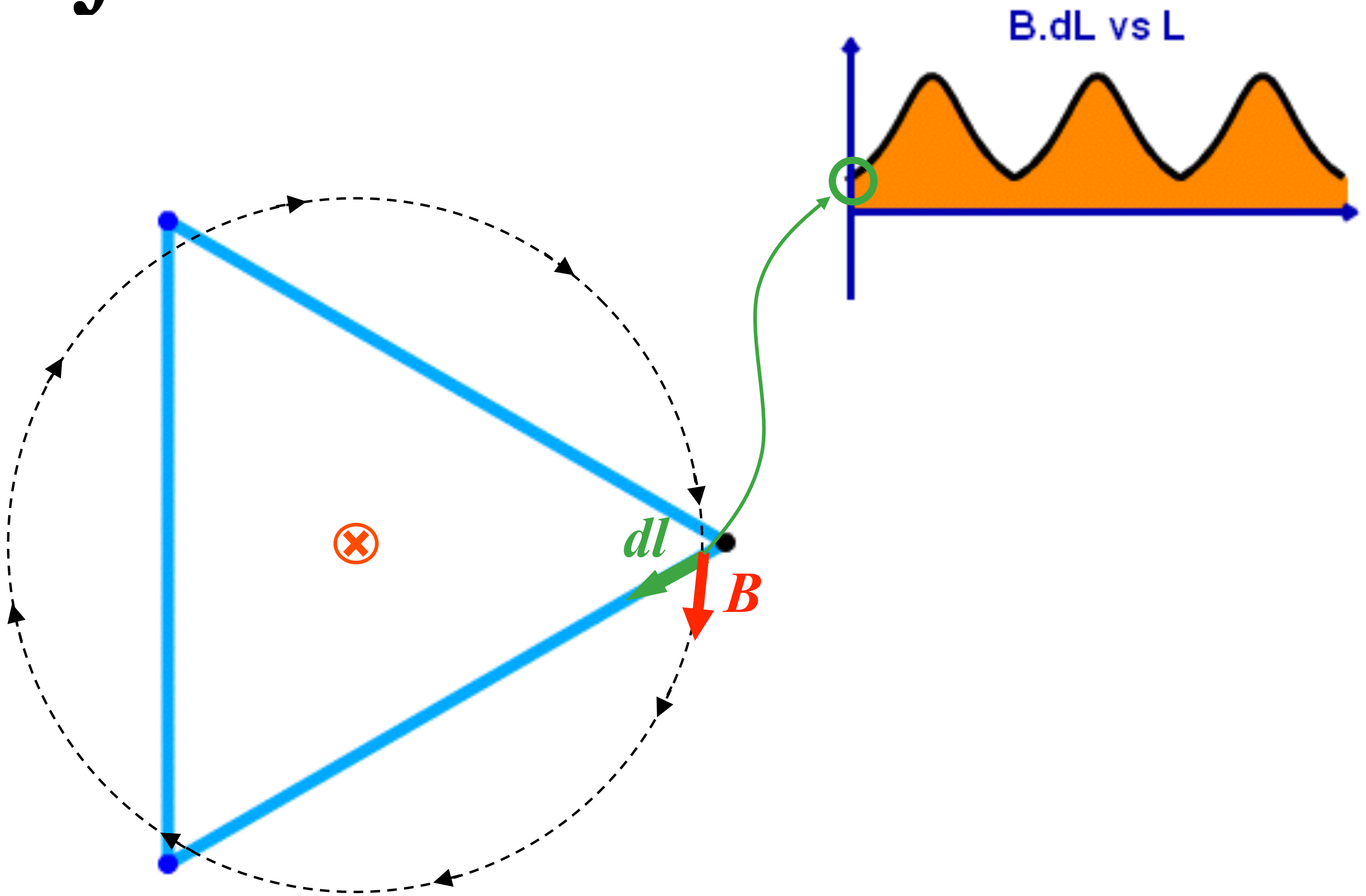

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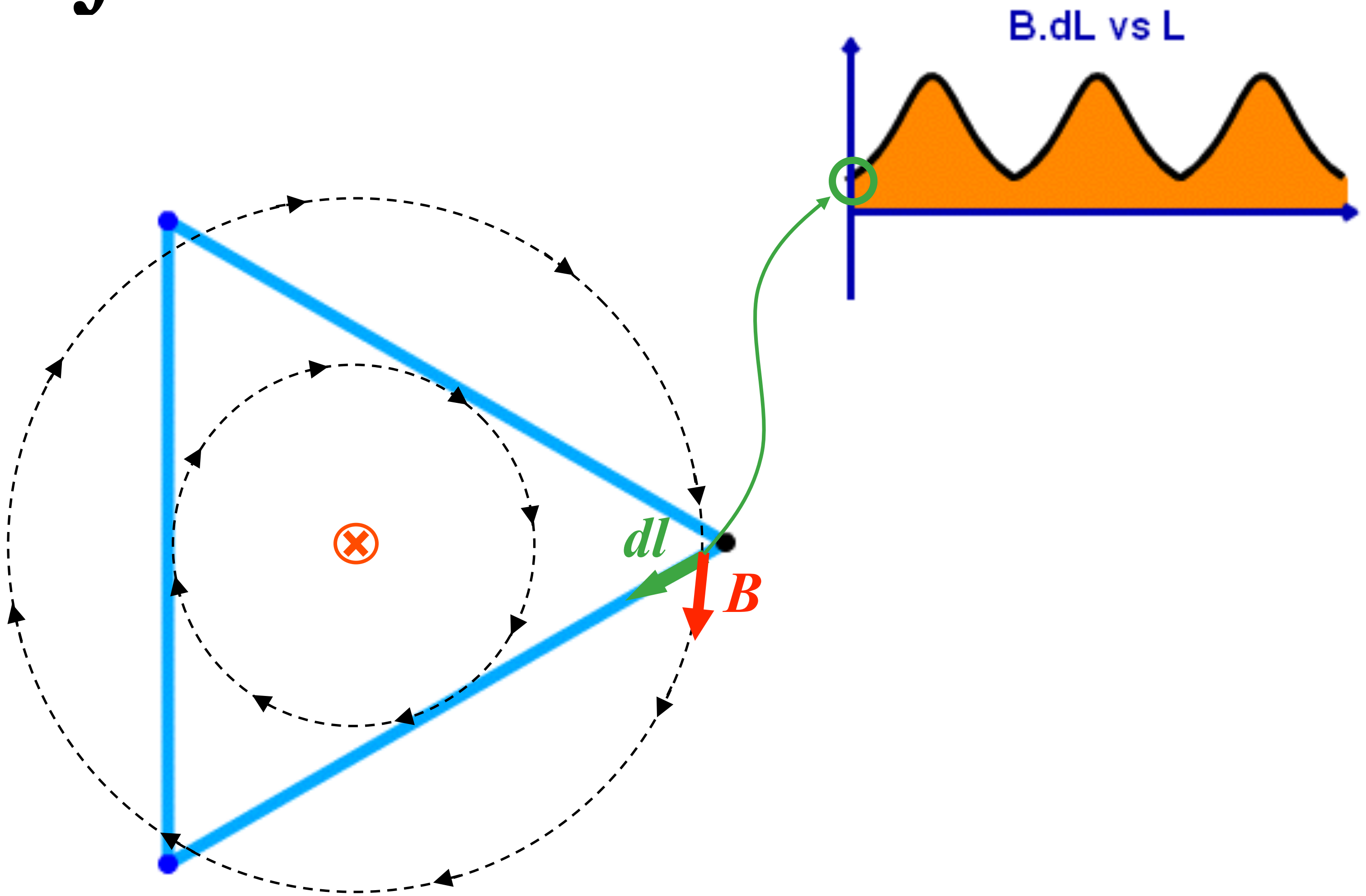

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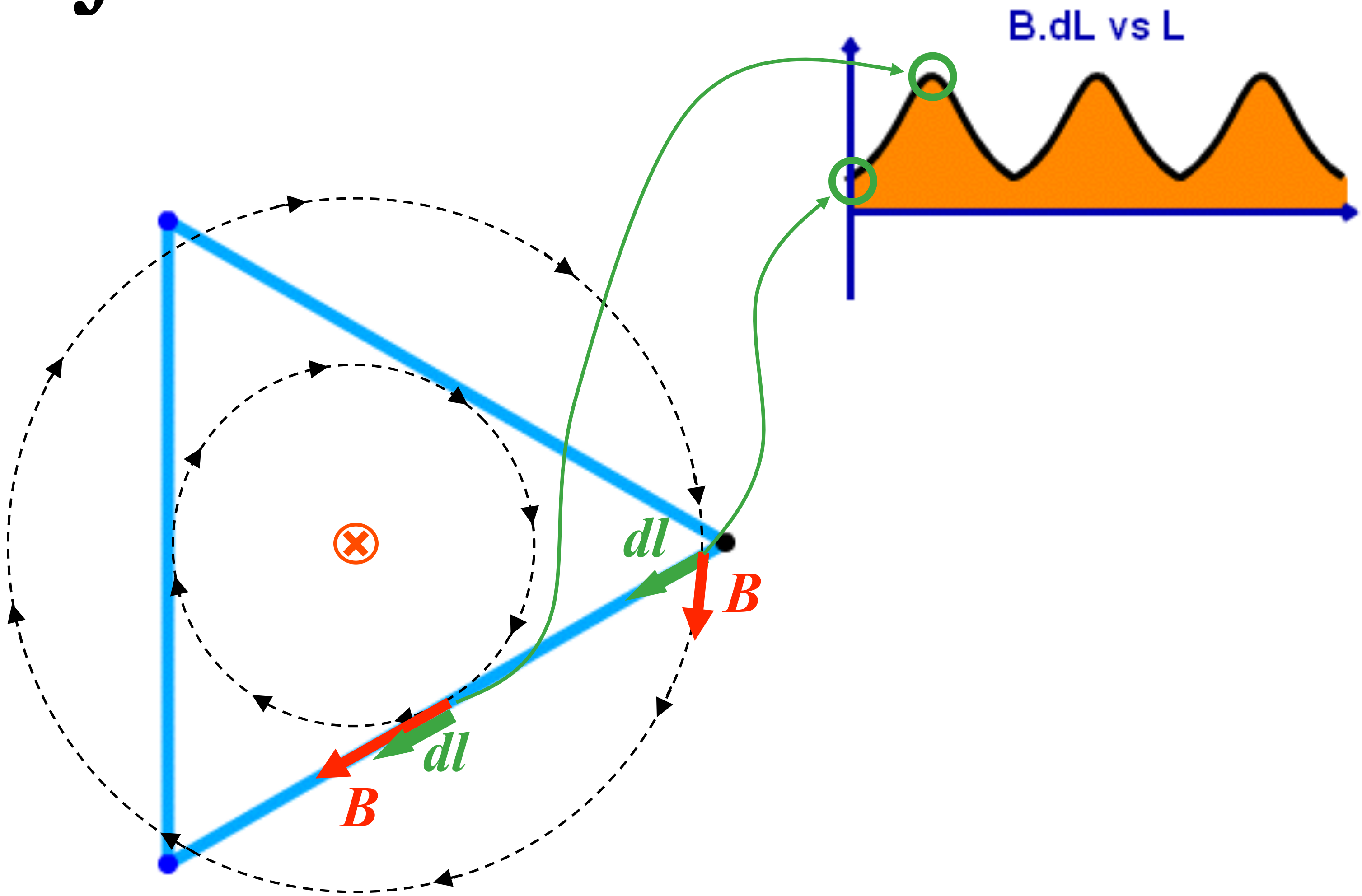

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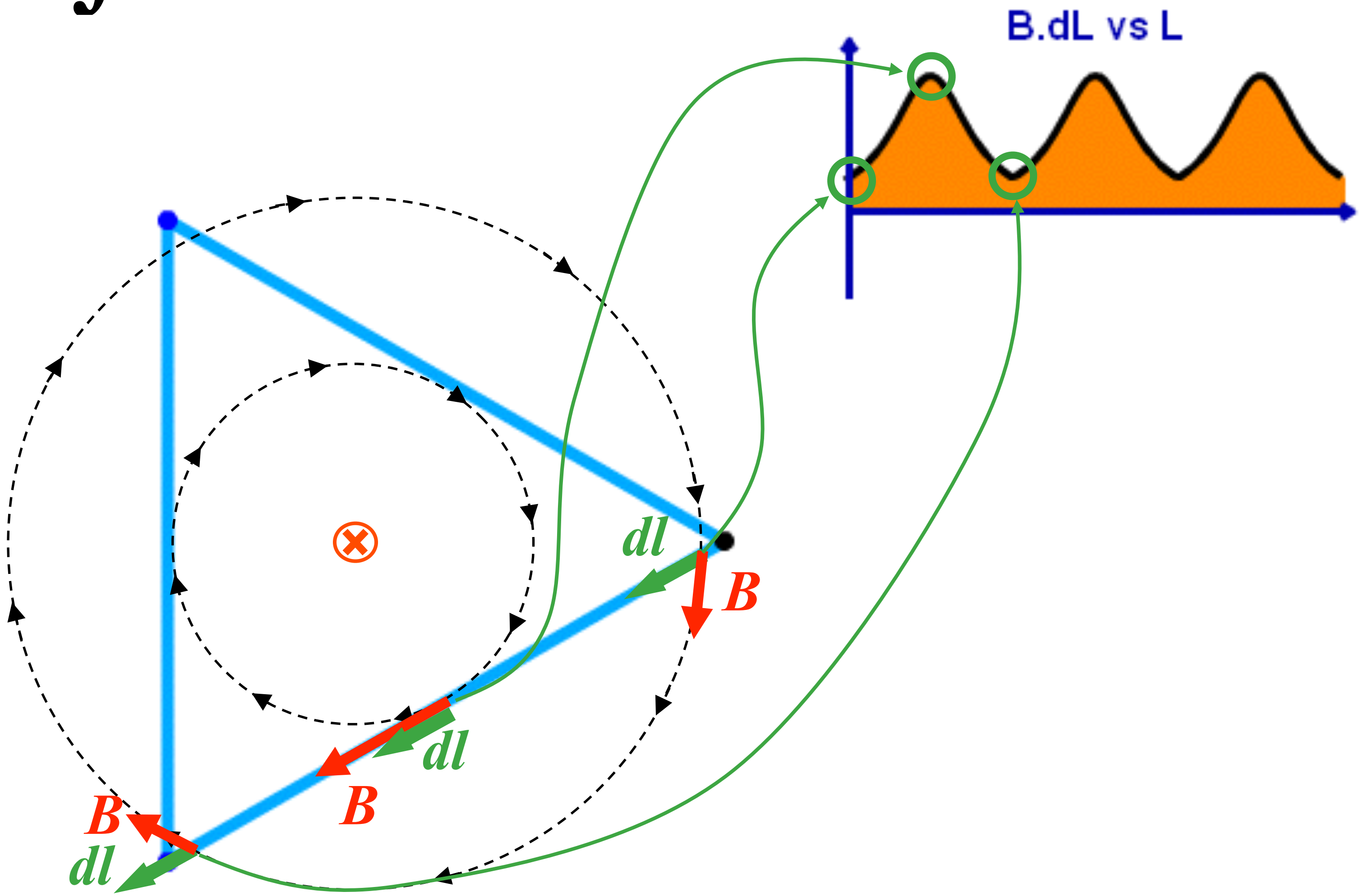

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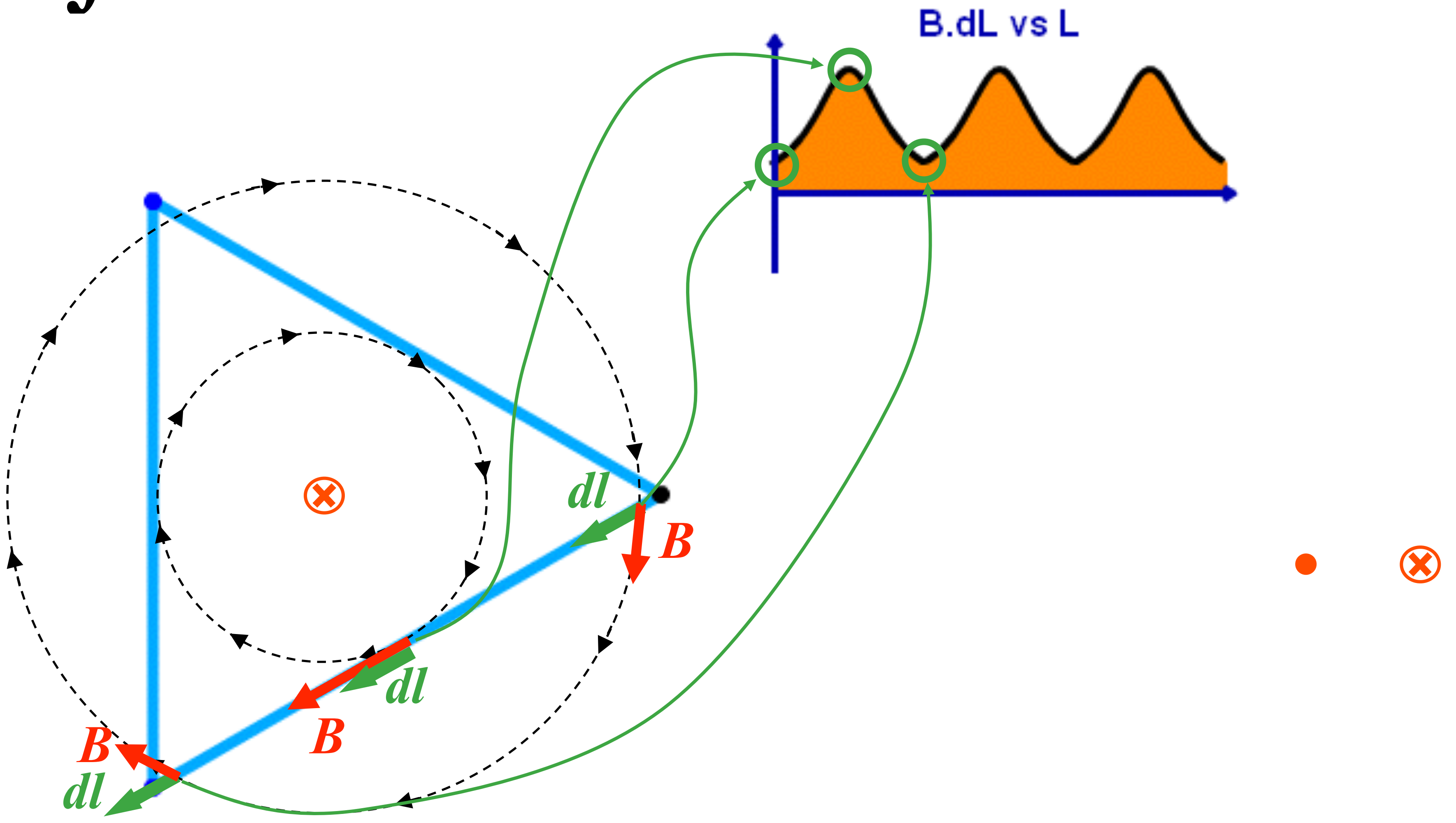

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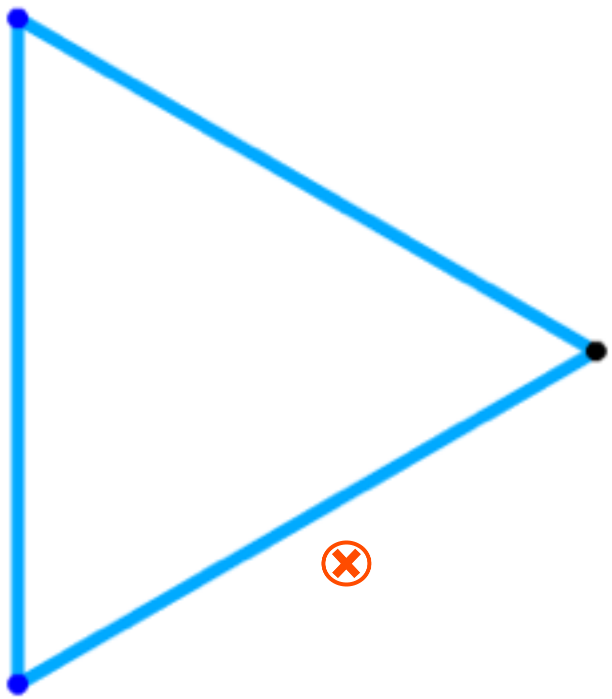

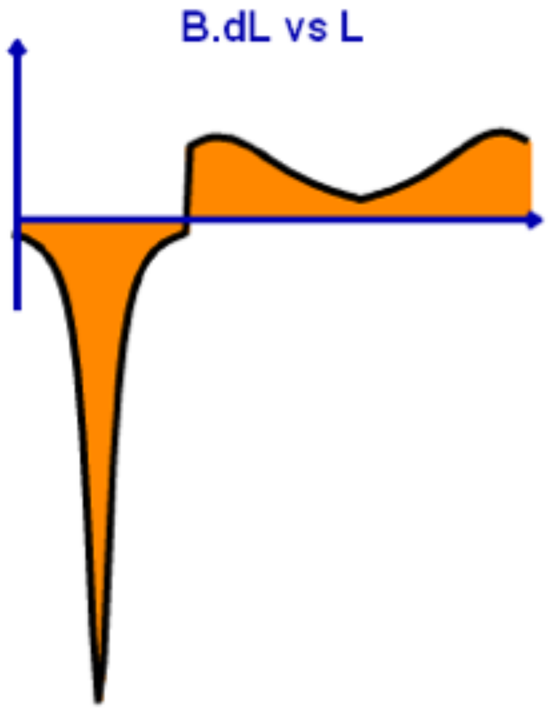

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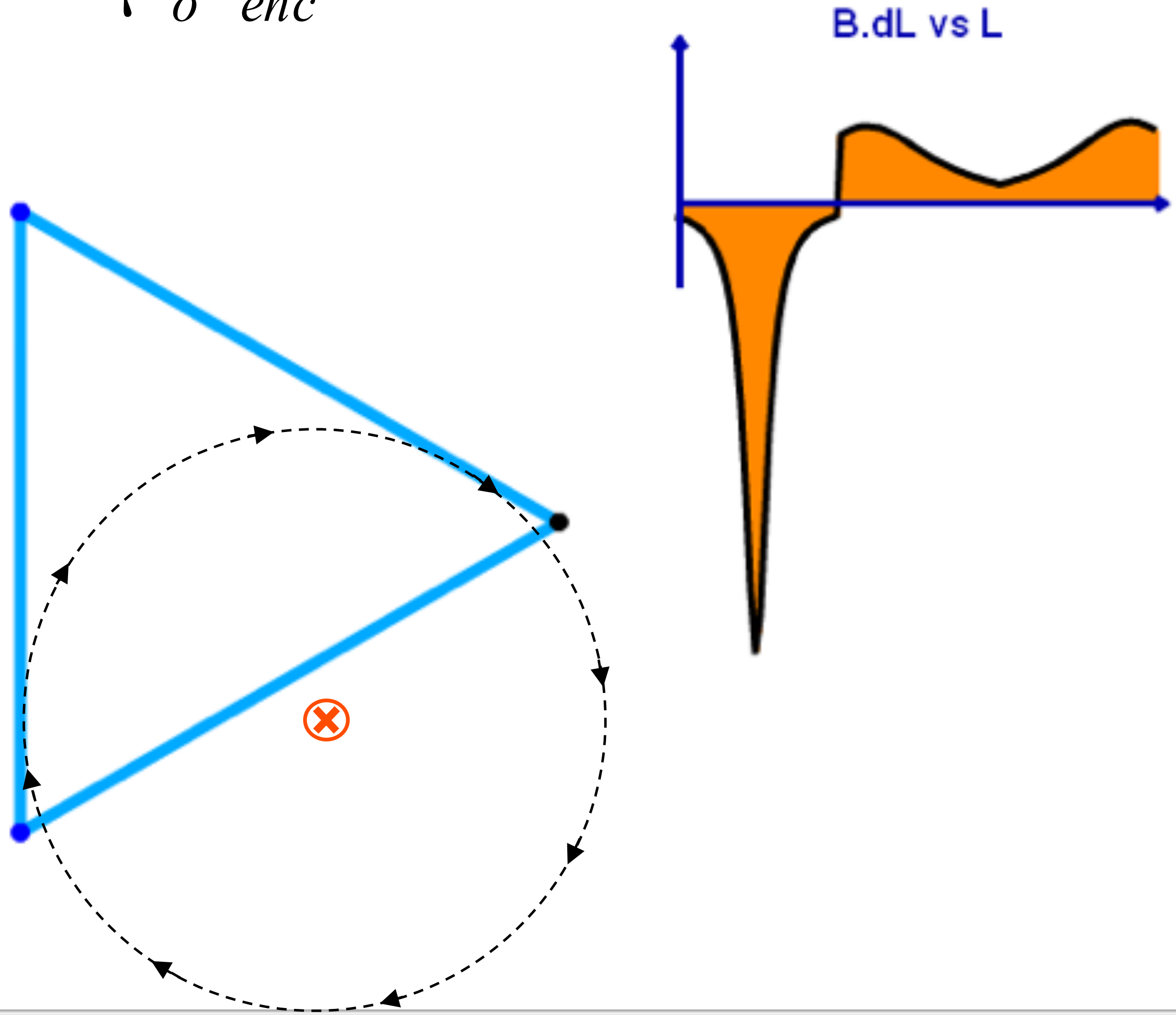

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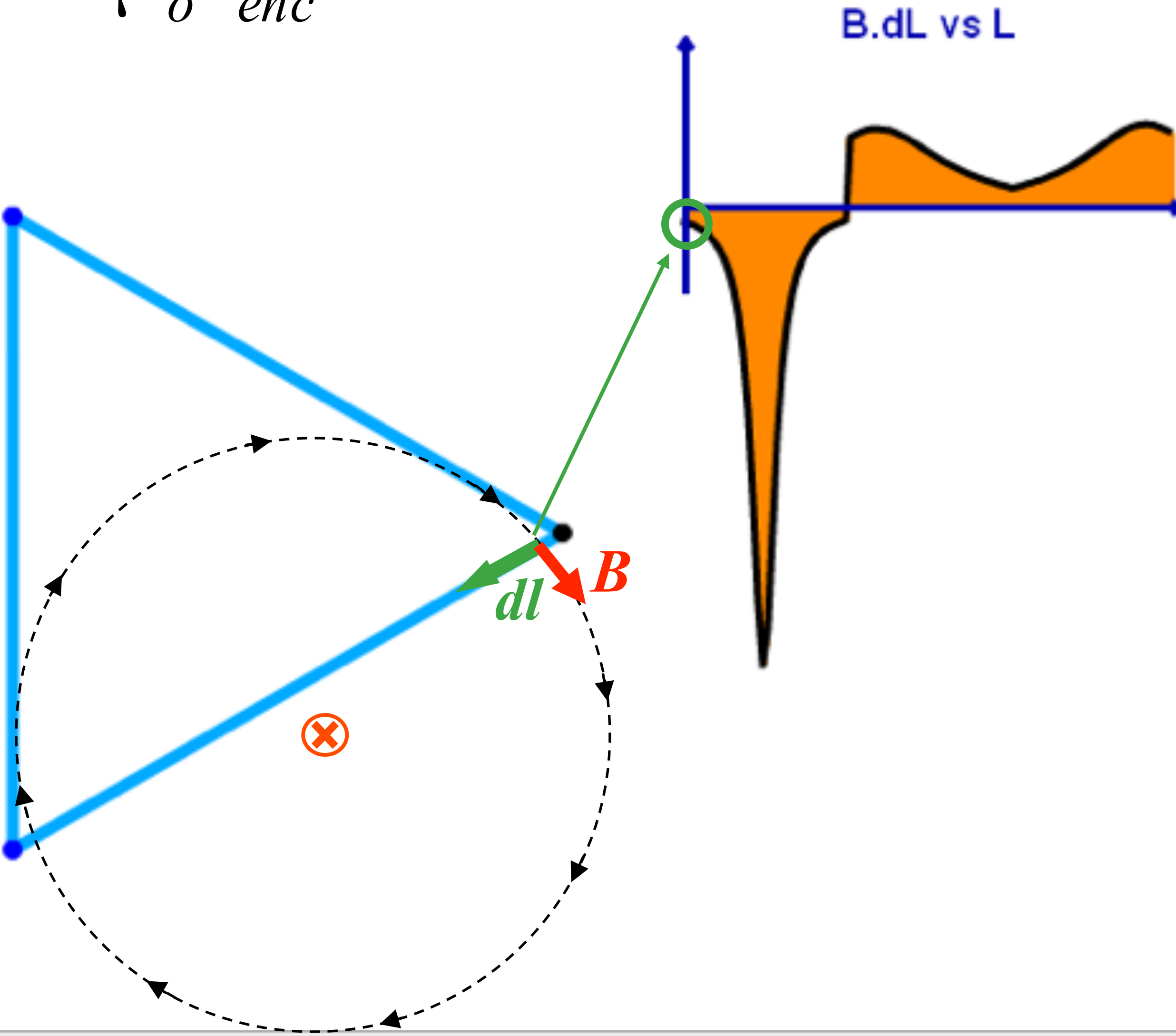

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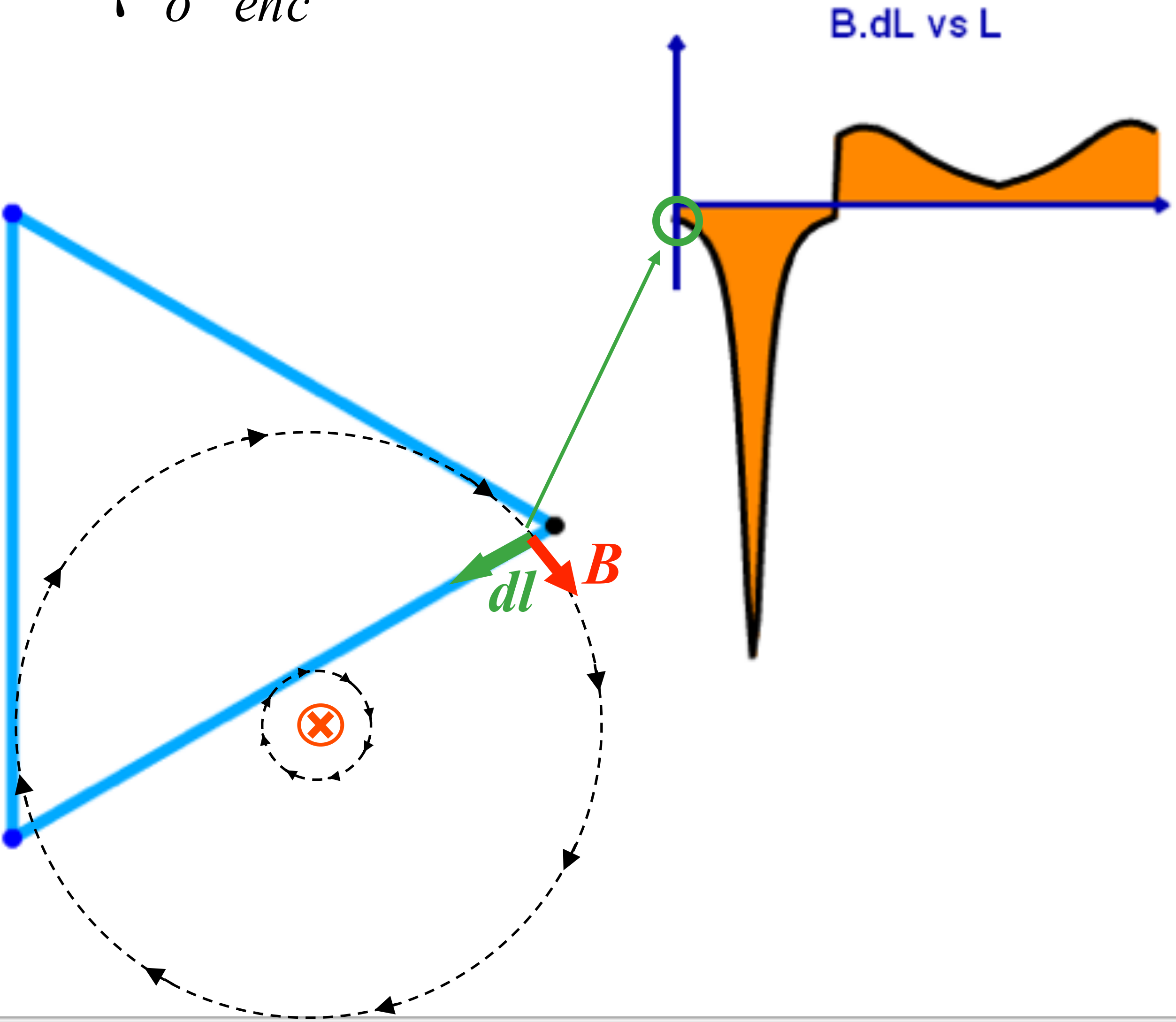

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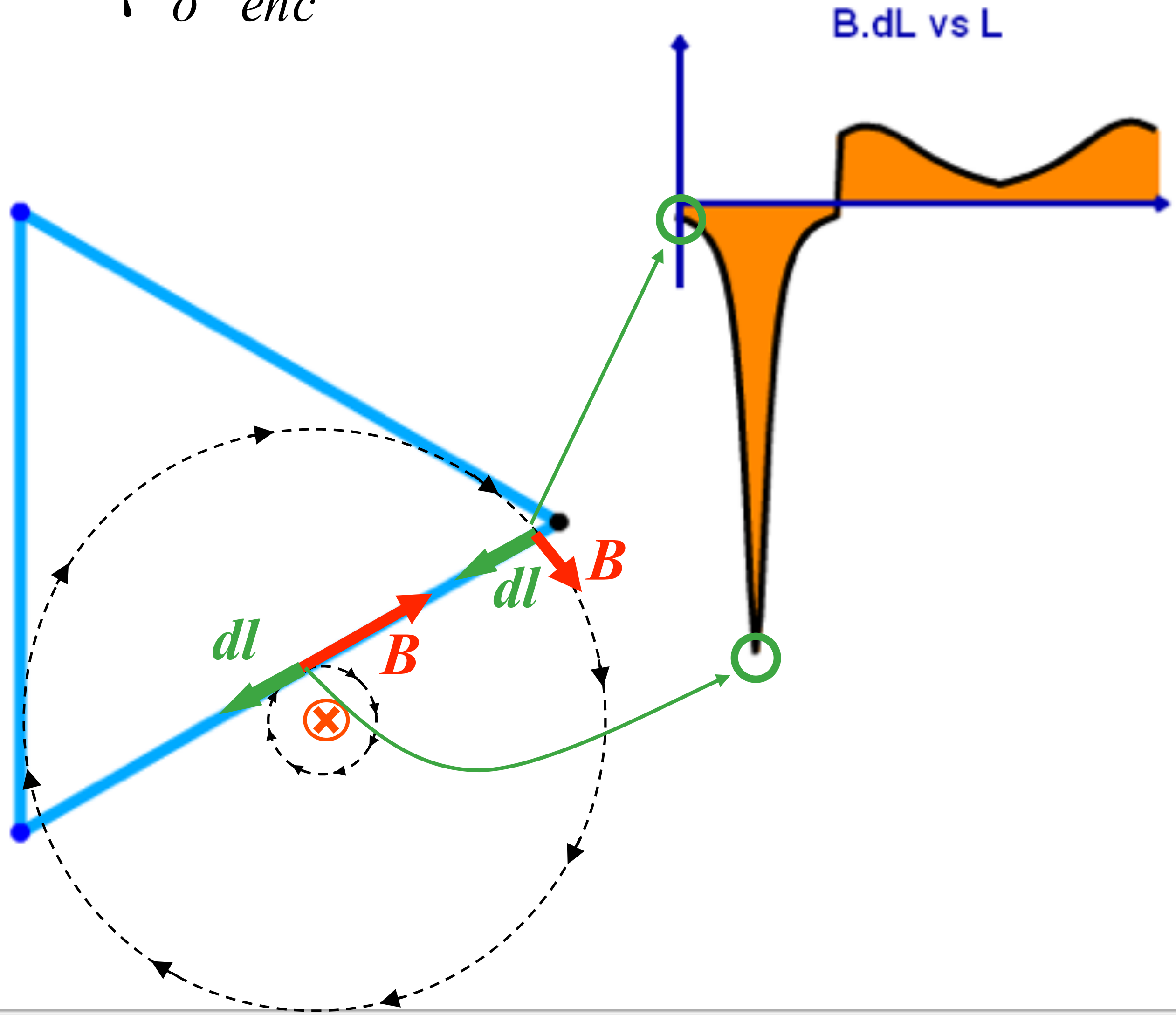

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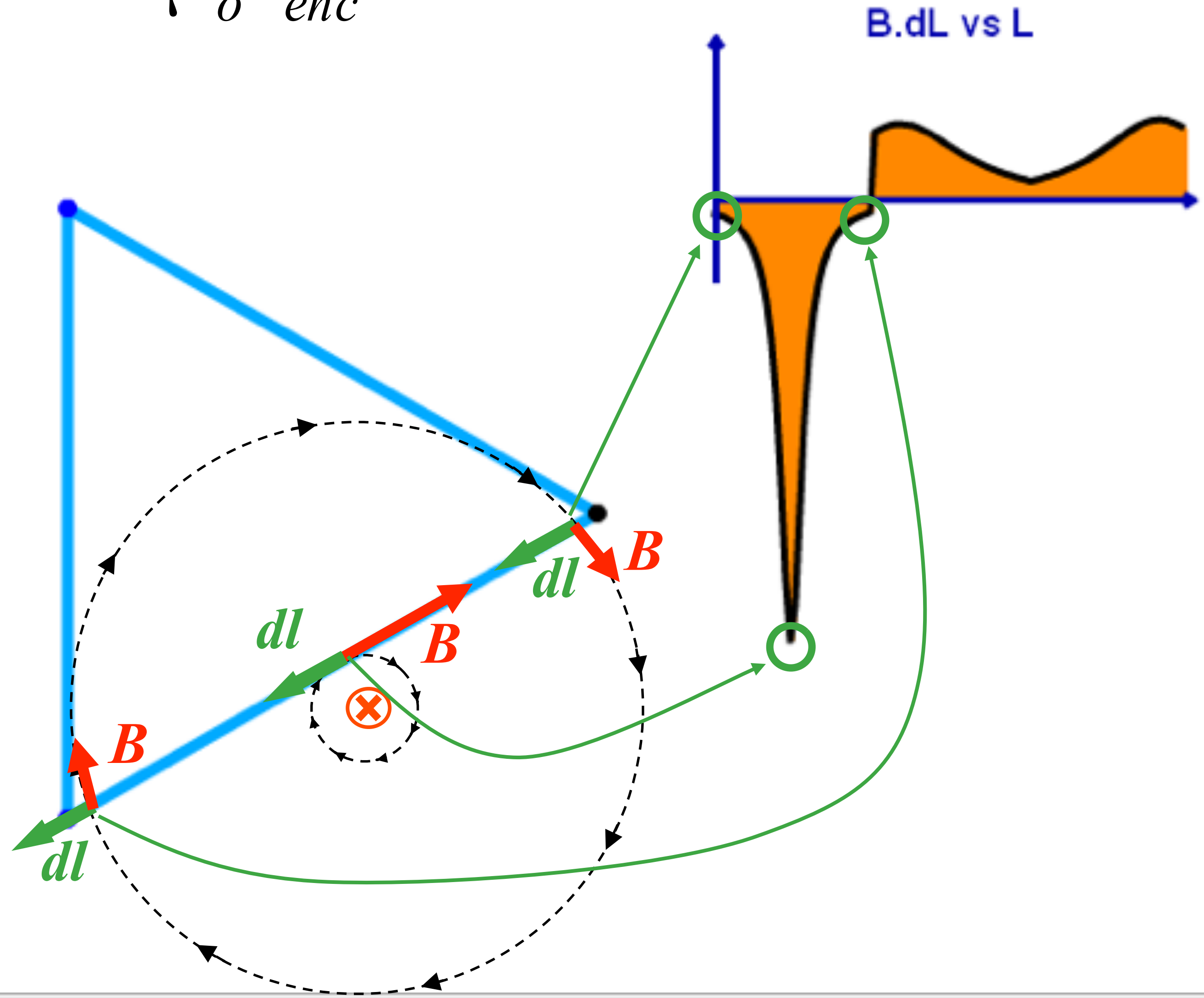

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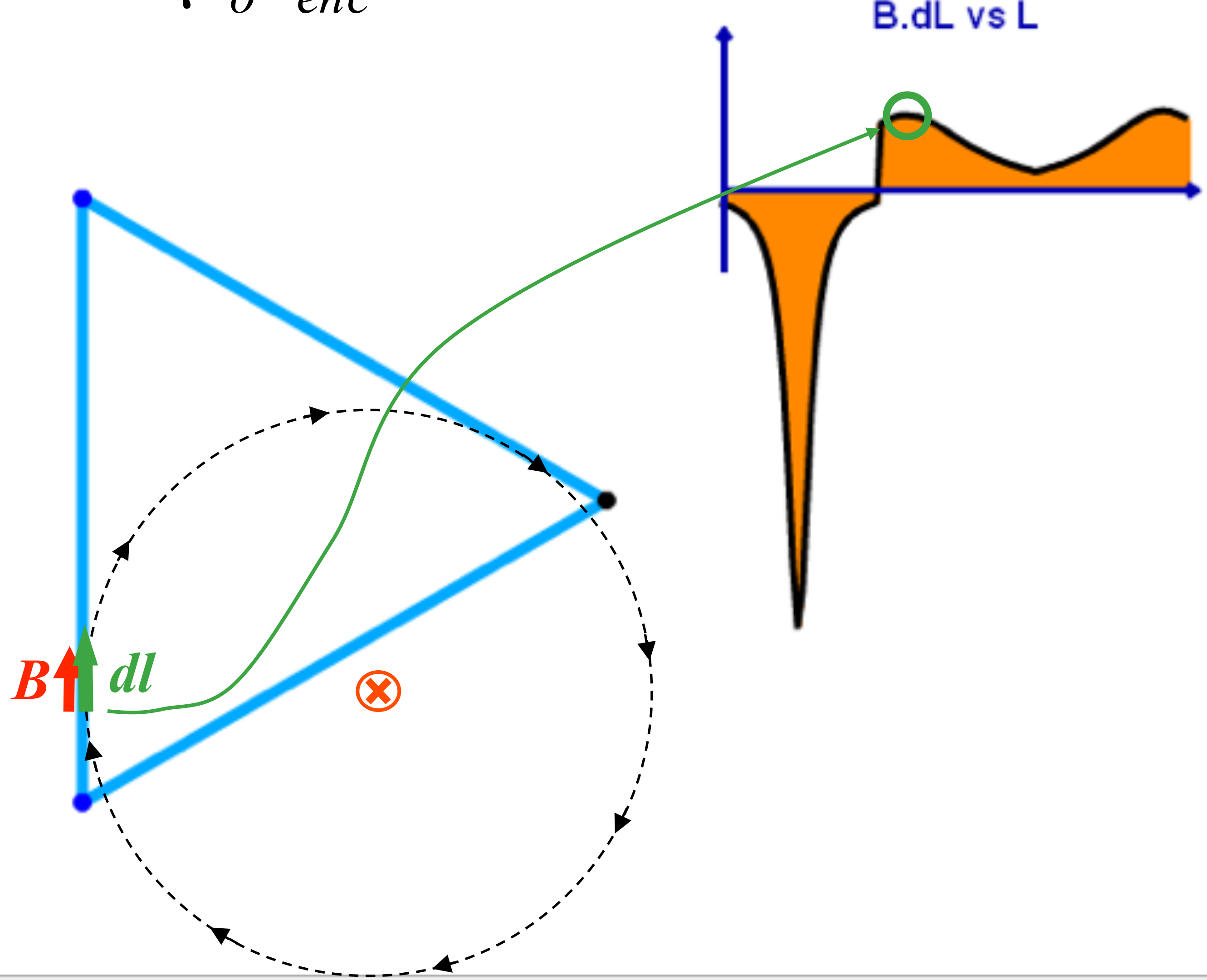

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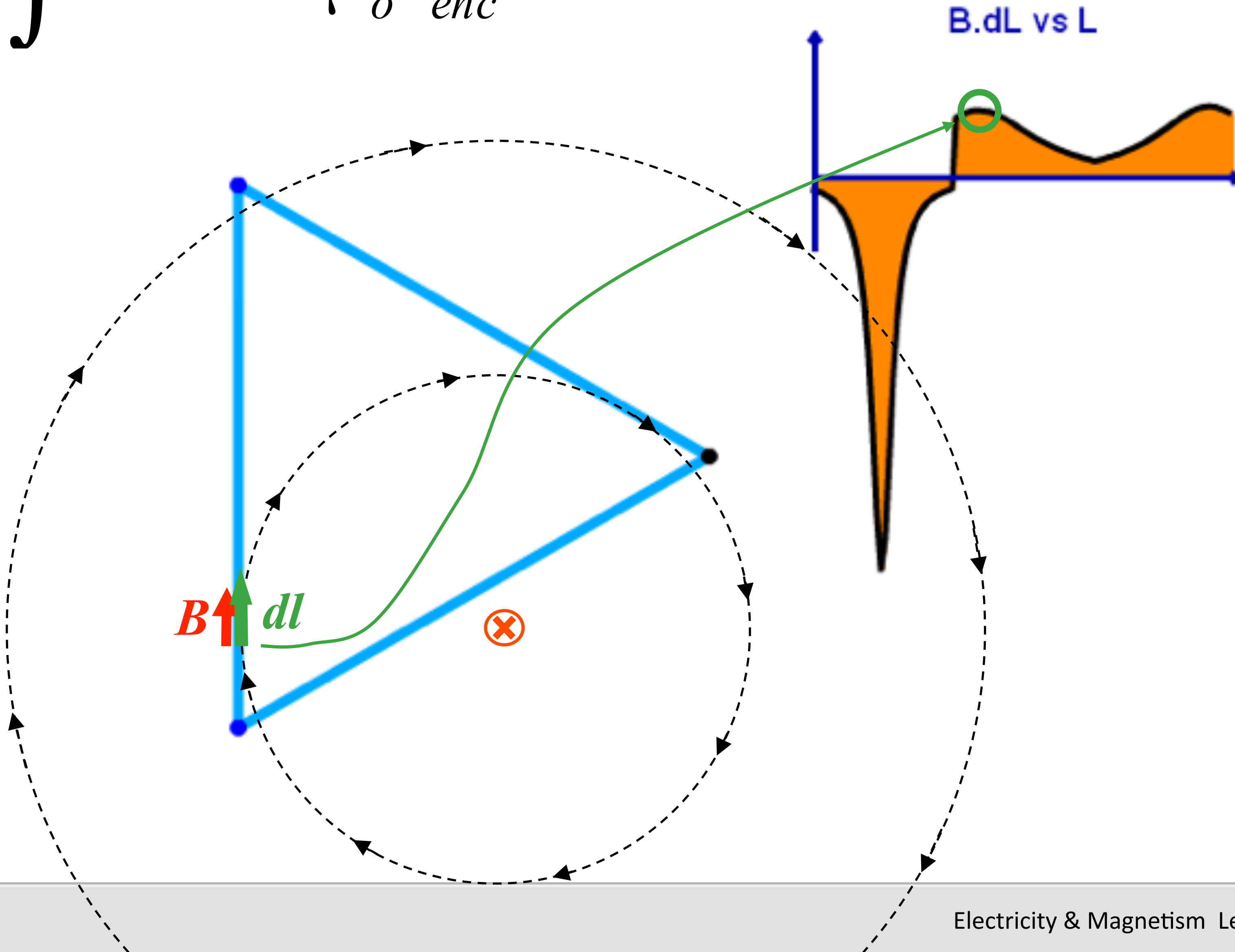

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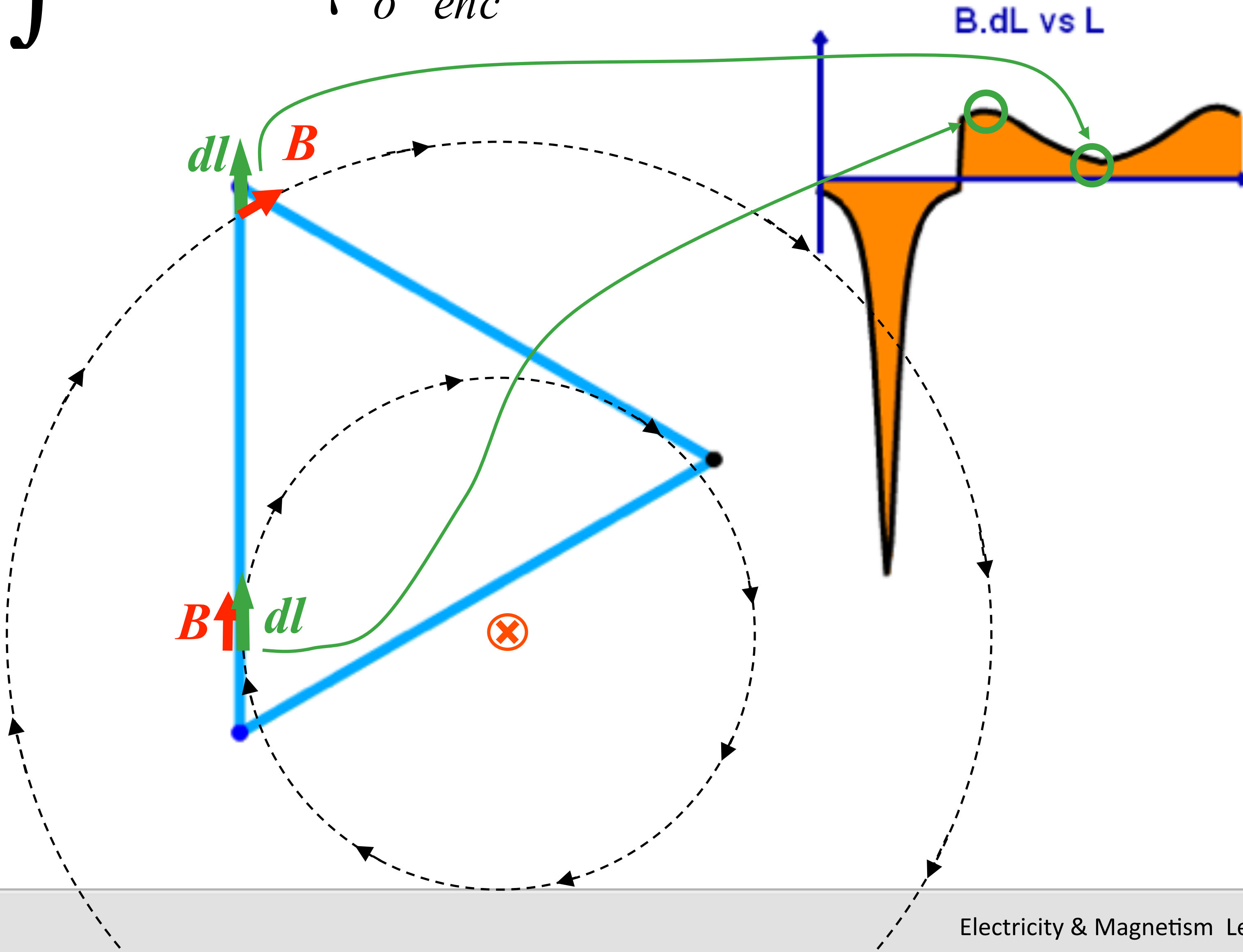

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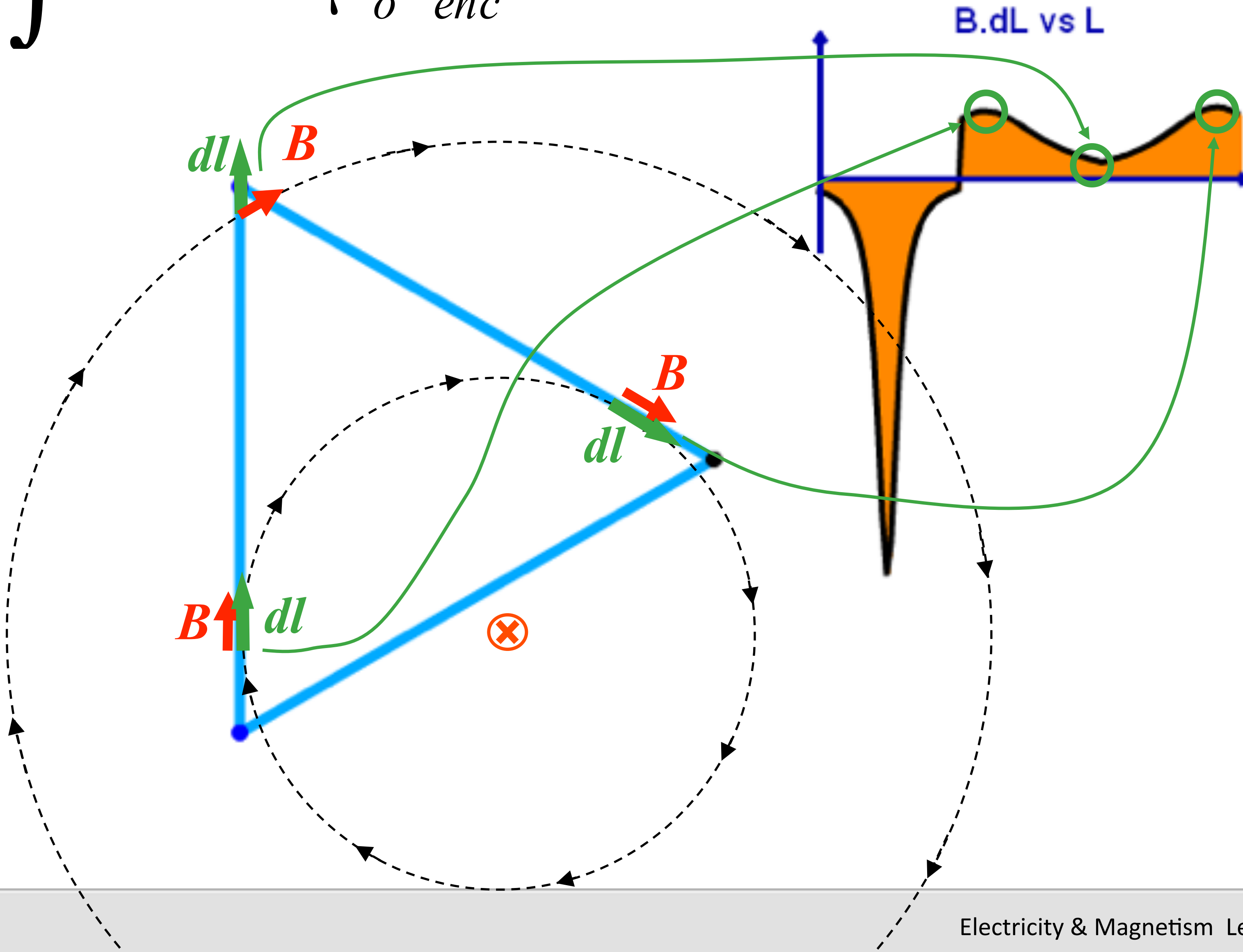
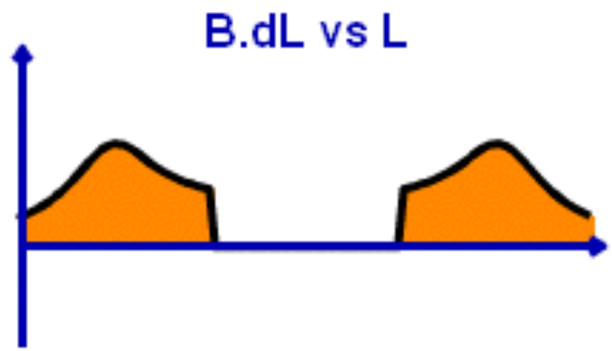
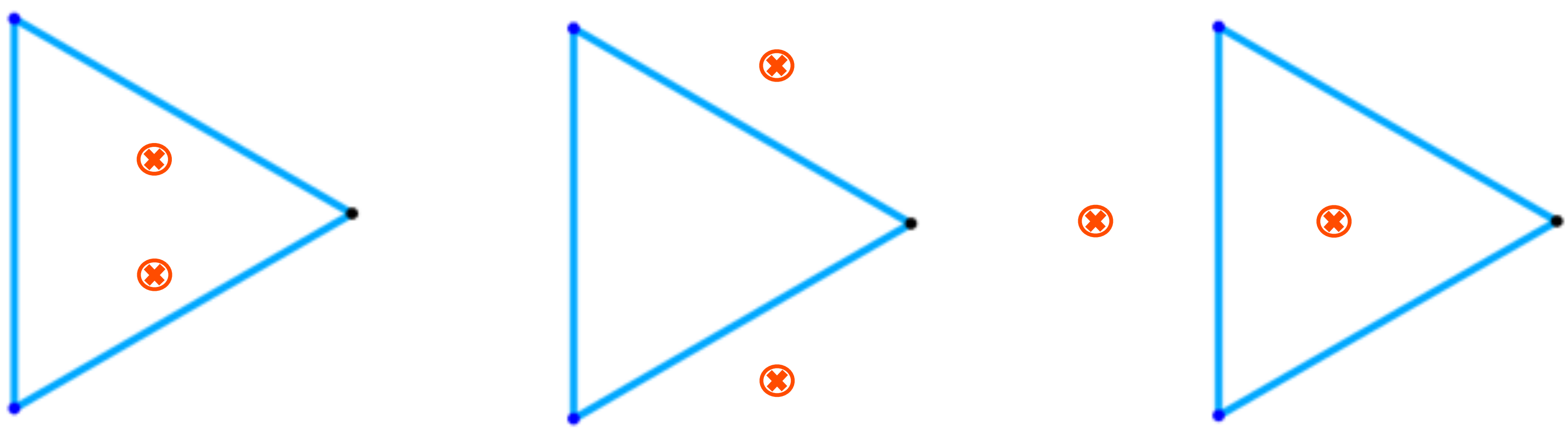


Which of the following current distributions would give rise to the $B \cdot dL$ distribution at the right?





A B C

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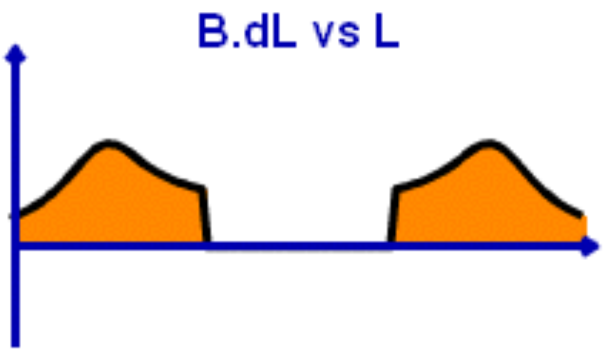
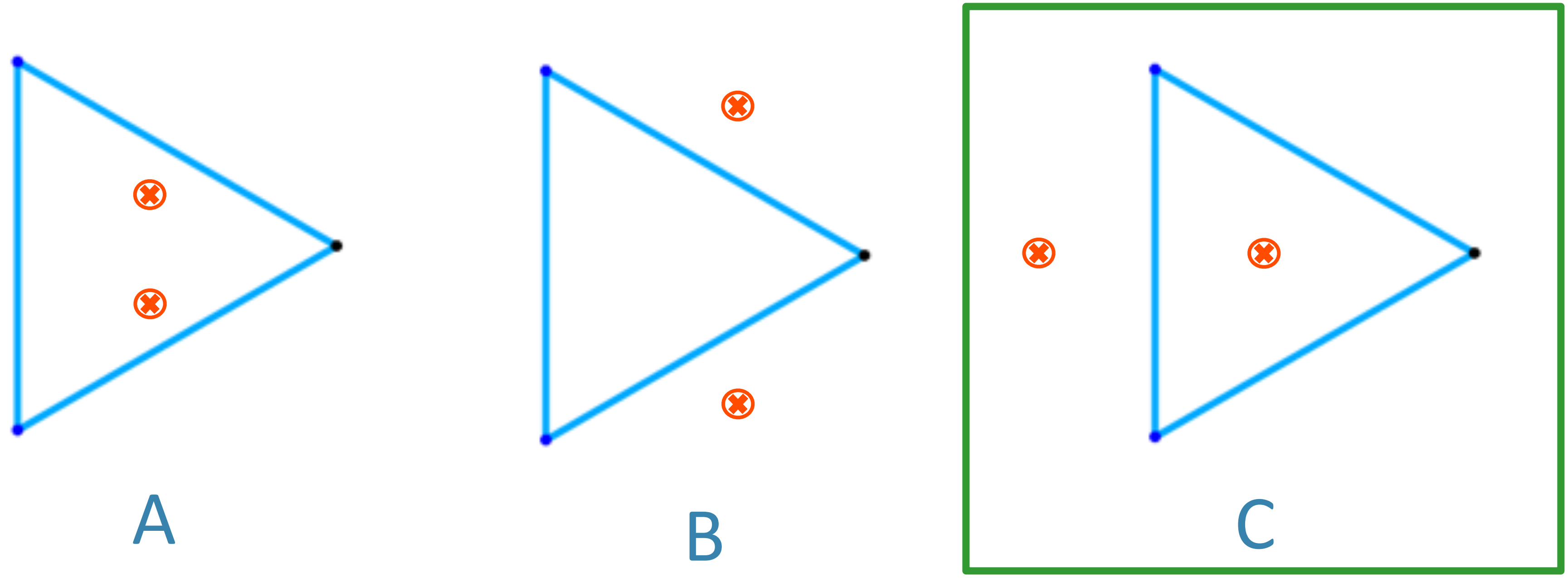
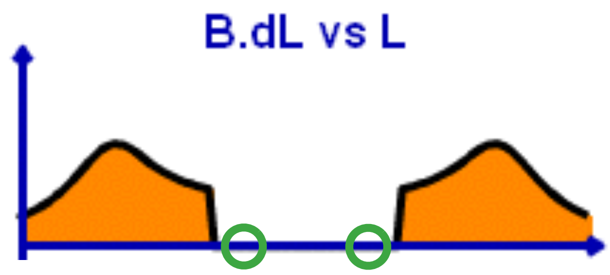
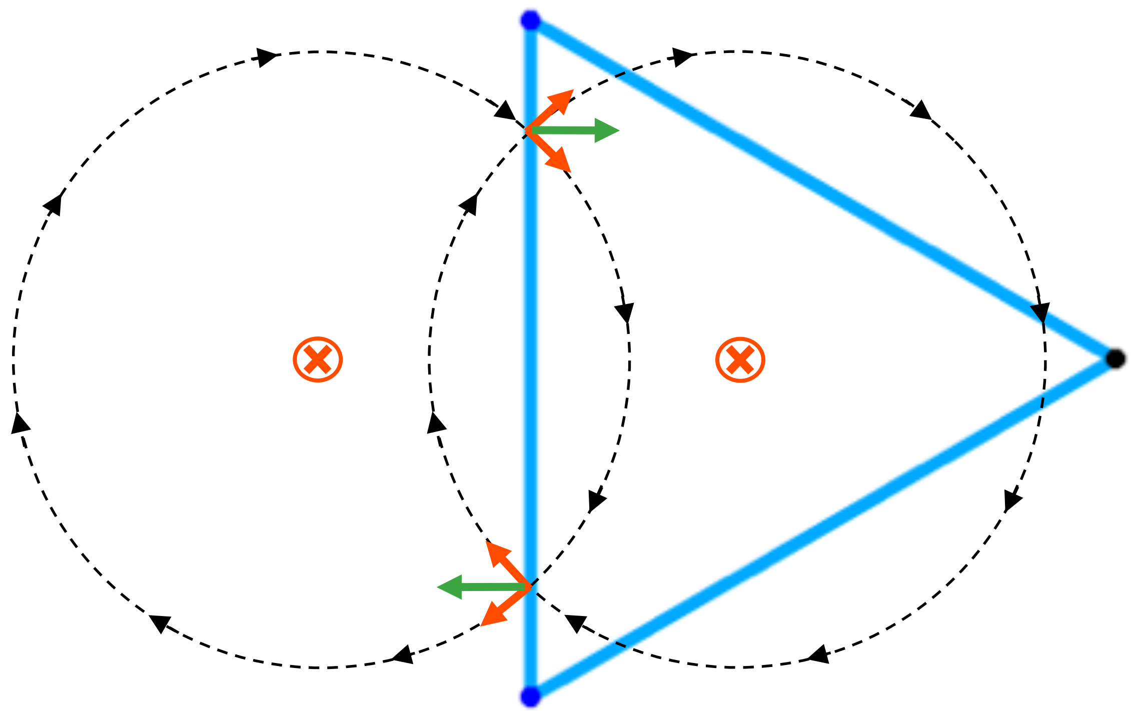
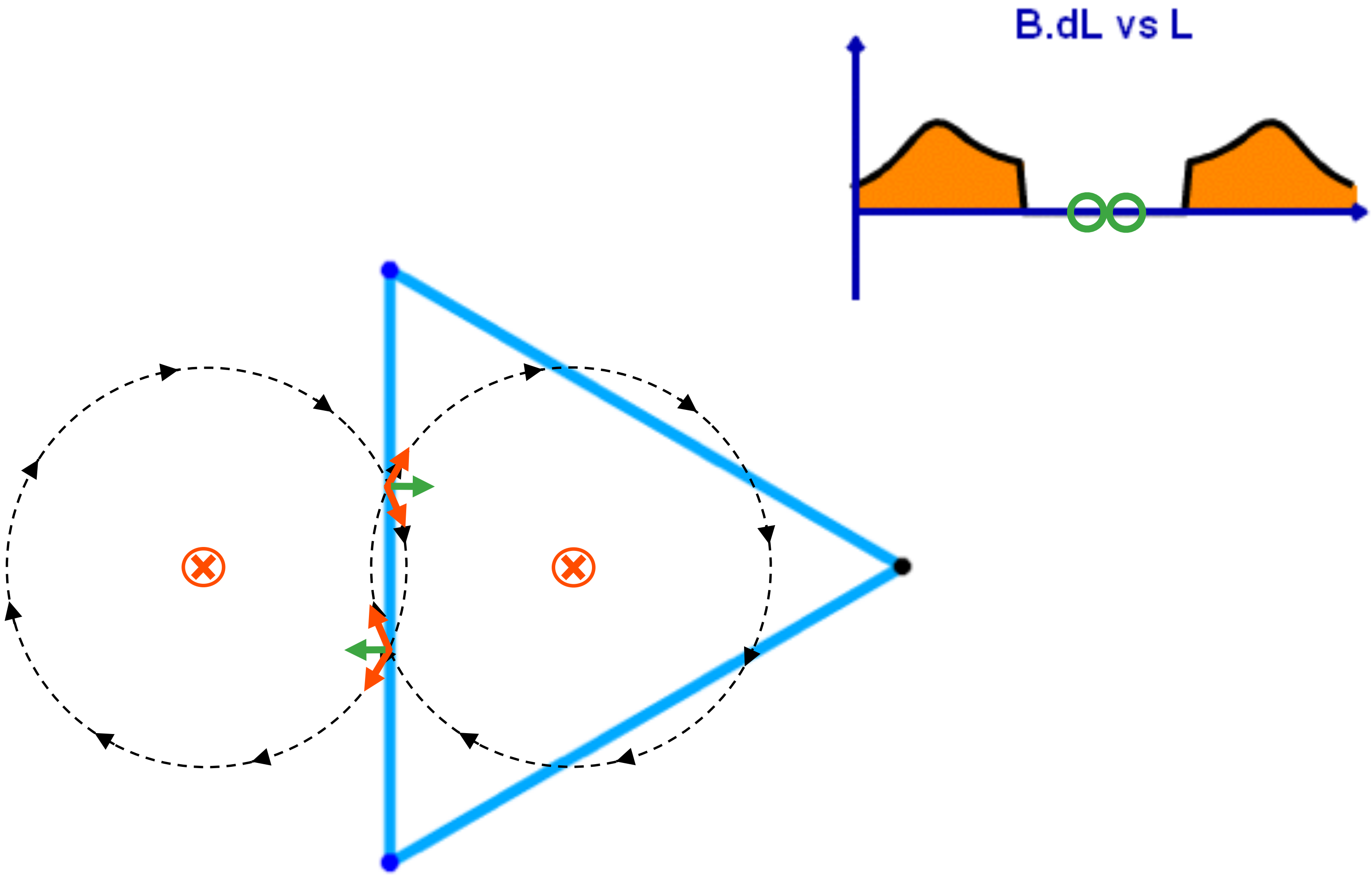





B.dL vs L

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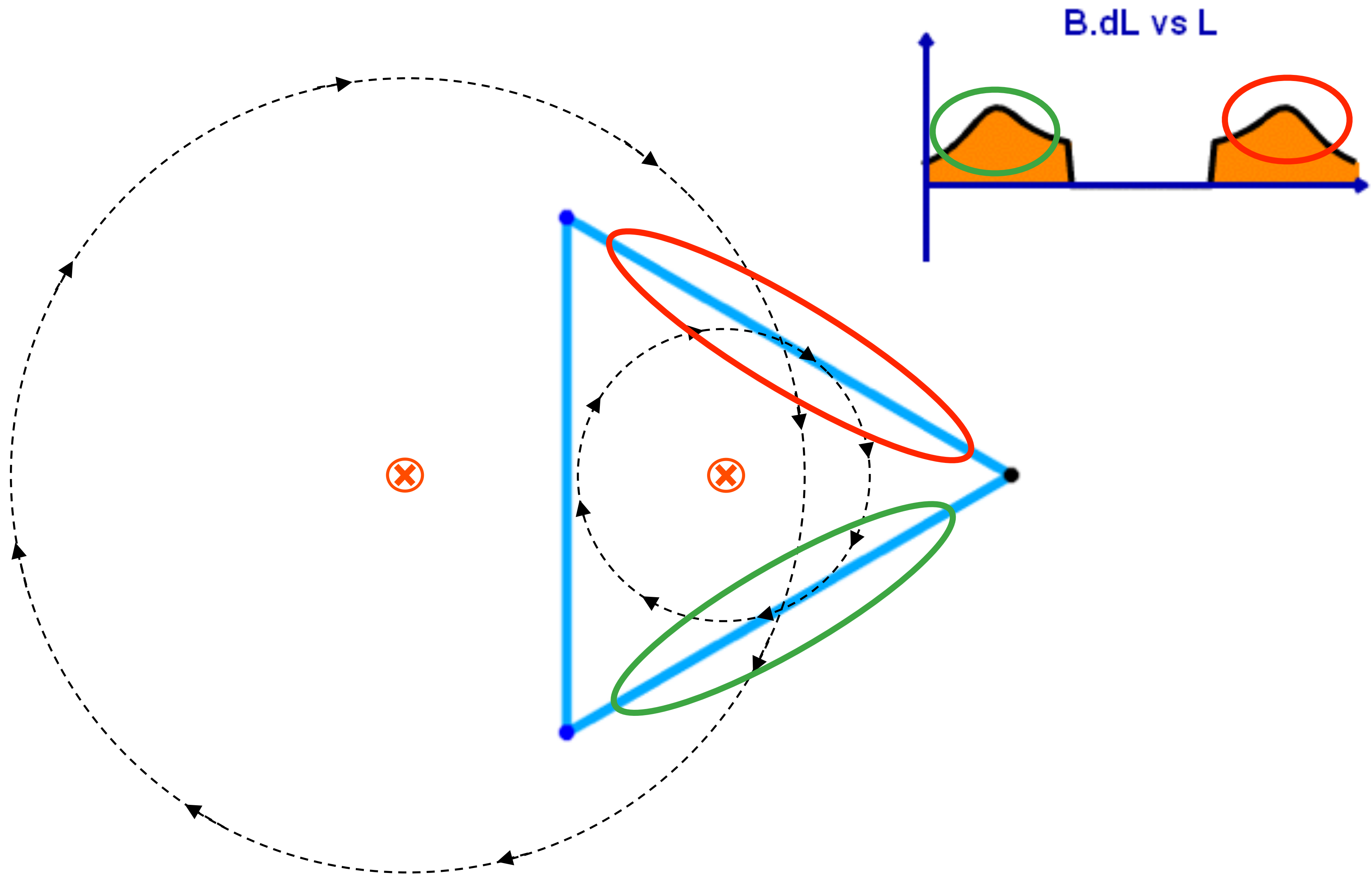
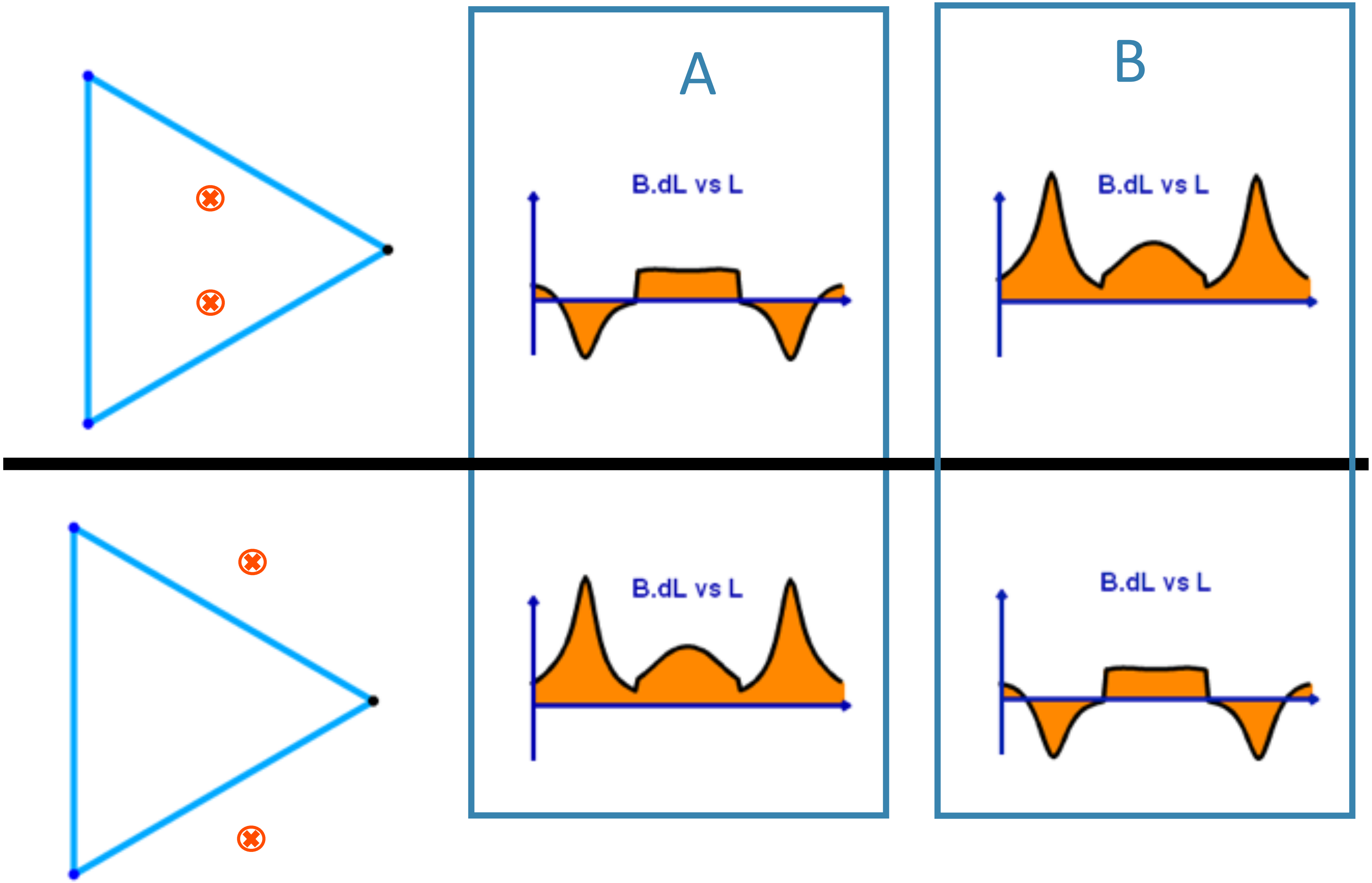

B.dL vs L

Match the other two:



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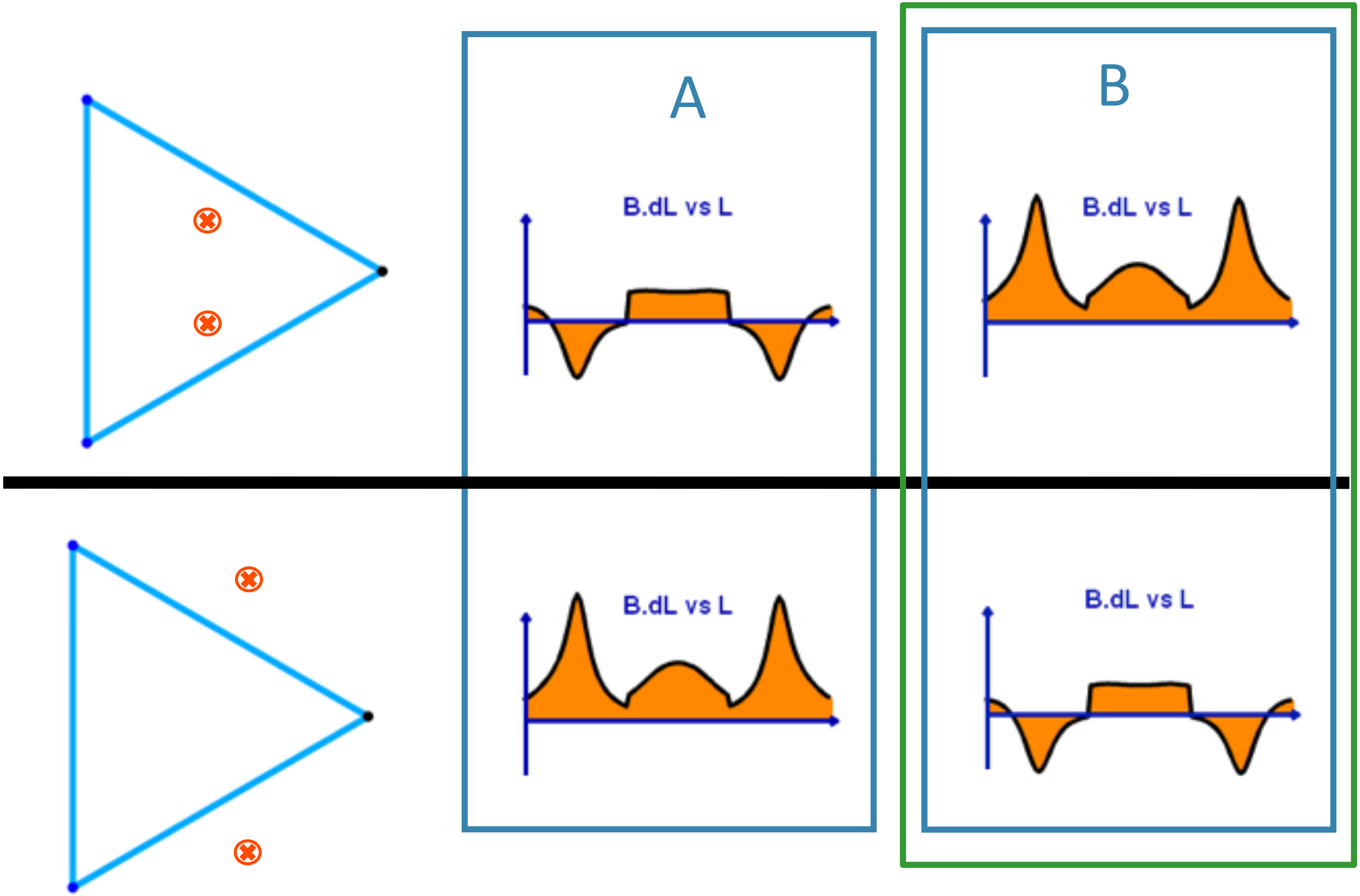
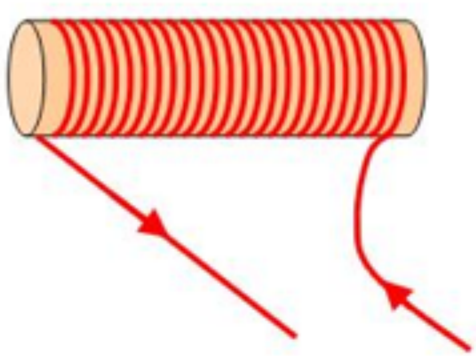


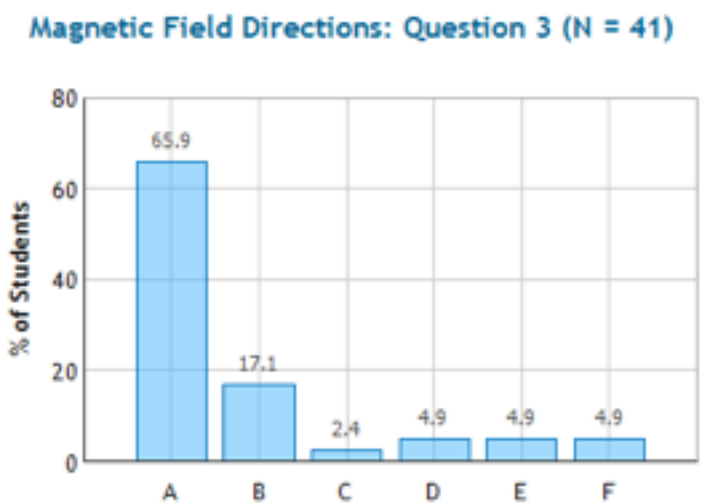
CheckPoint 10

A current carrying wire is wrapped around cardboard tube as shown below.



In which direction does the magnetic field point inside the tube?

A) left
B) right
C) up
D) down
E) out of the screen
F) into the screen



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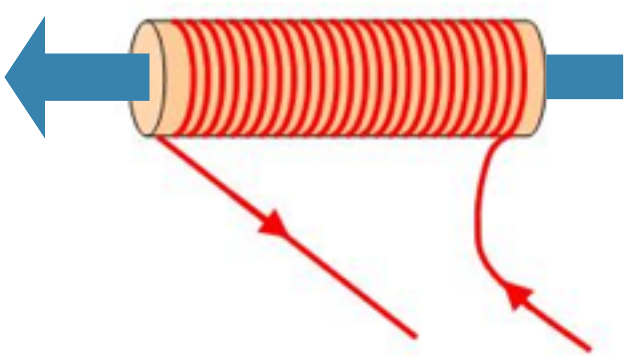

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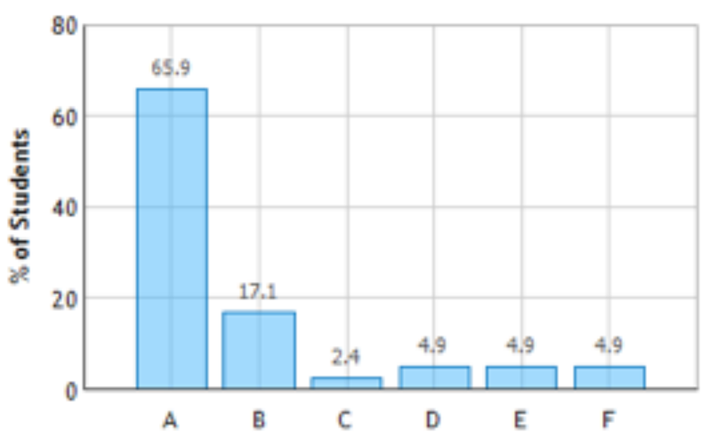

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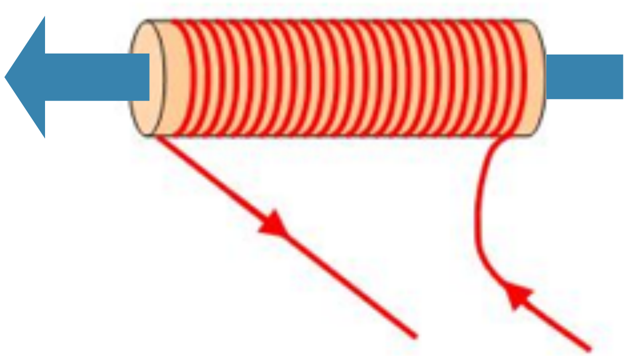

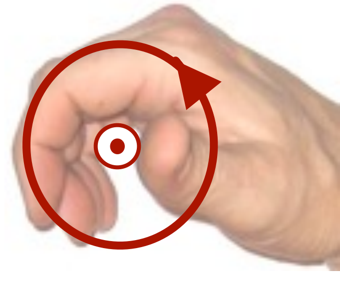

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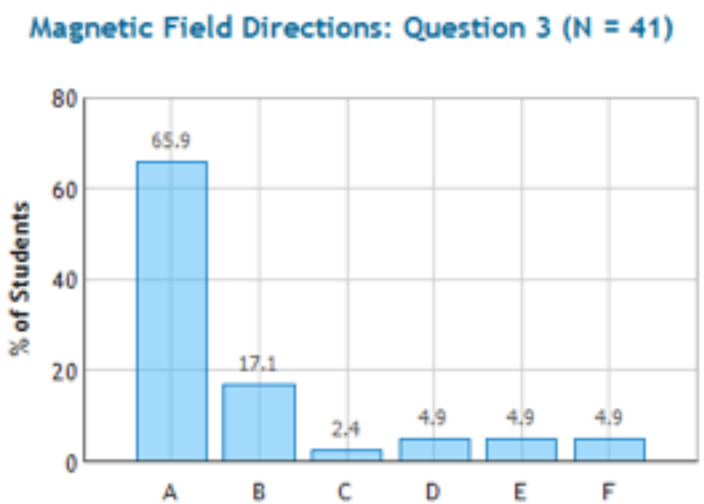
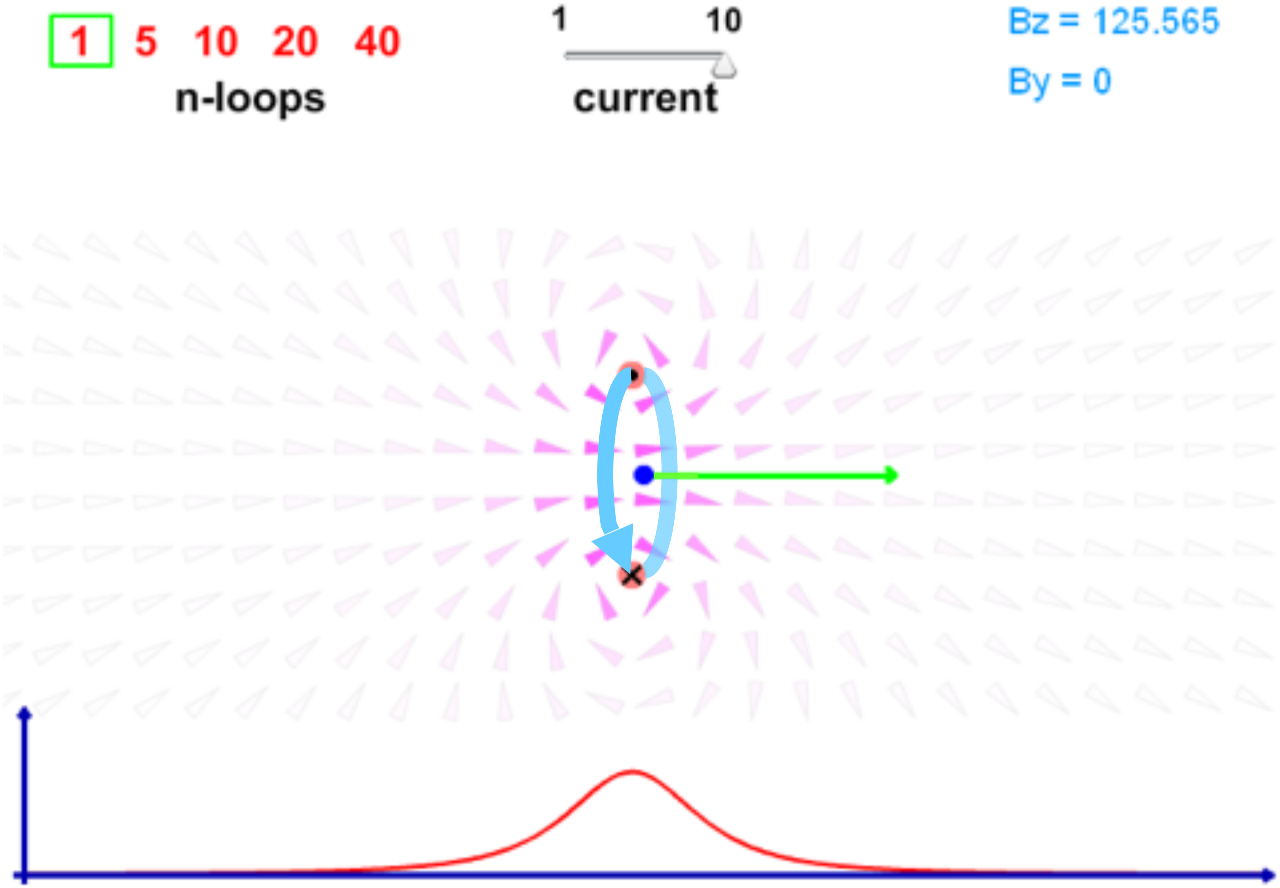
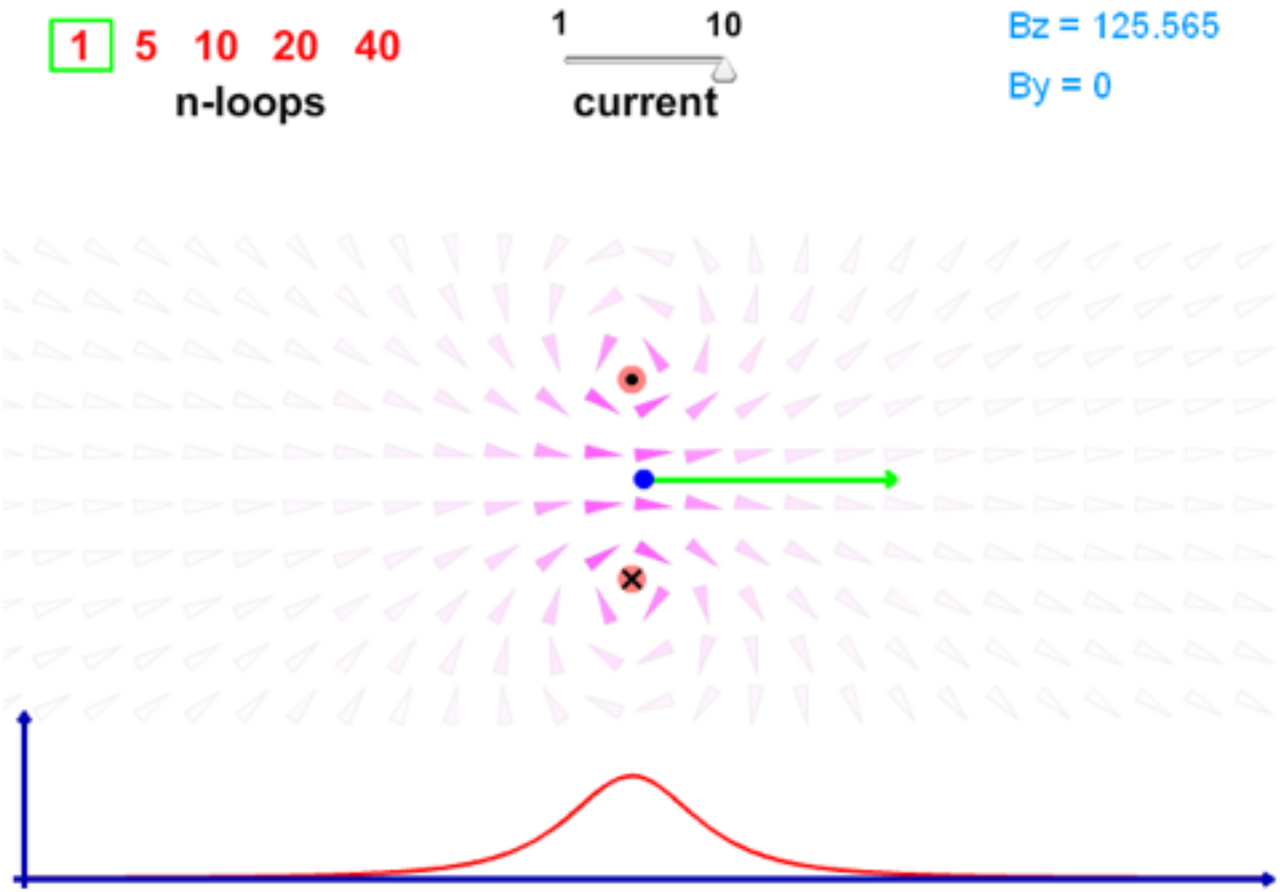




Click graph to play simulation



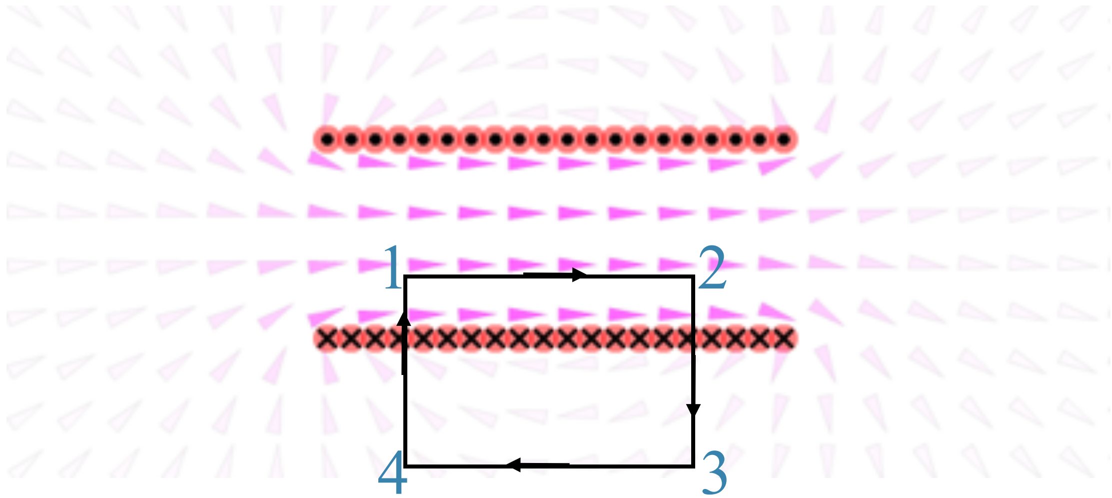
Click graph to play simulation

Simulation Applet

http://www.falstad.com/vector3dm/

Solenoid

Several loops packed tightly together form a uniform magnetic field inside, and nearly zero magnetic field outside.



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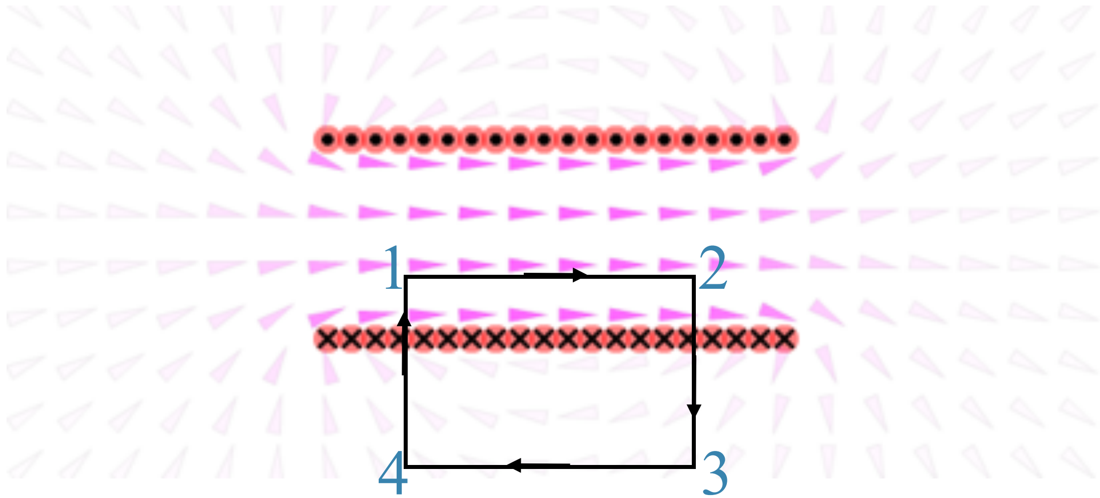

From this simulation, we can assume a constant field inside the solenoid and zero field outside the solenoid, and apply Ampere's law to find the magnitude of the constant field inside the solenoid!

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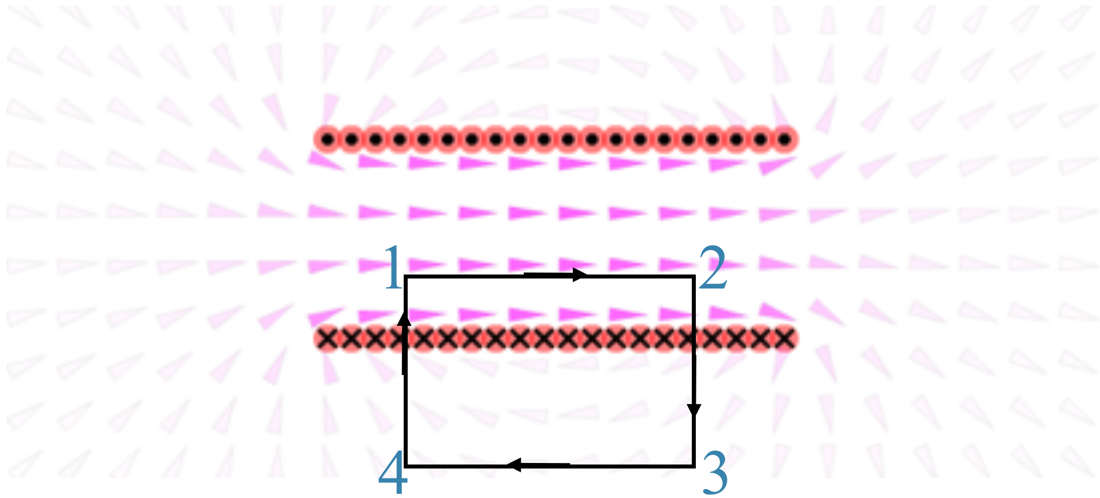

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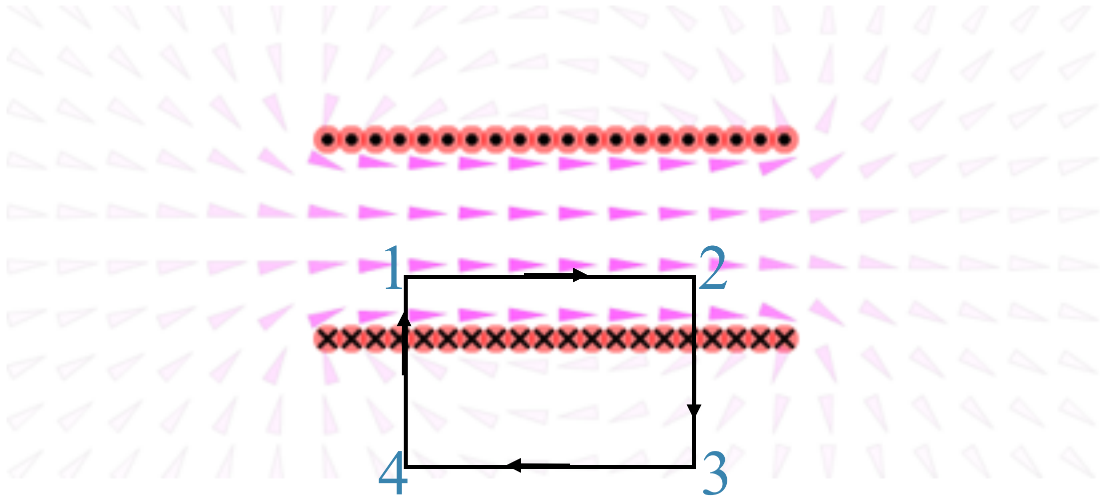

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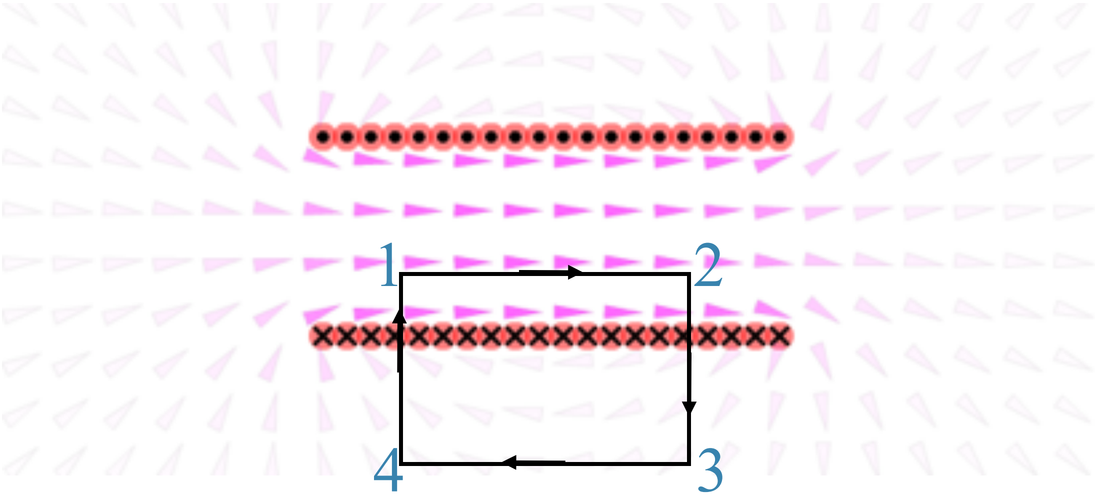
$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc} \longrightarrow \int_1^2 \vec{B} \bullet d\vec{\ell} + \int_2^3 \vec{B} \bullet d\vec{\ell} + \int_3^4 \vec{B} \bullet d\vec{\ell} + \int_4^1 \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

Solenoid

Several loops packed tightly together form a uniform magnetic field inside, and nearly zero magnetic field outside.



From this simulation, we can assume a constant field inside the solenoid and zero field outside the solenoid, and apply Ampere's law to find the magnitude of the constant field inside the solenoid!

$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc} \longrightarrow \int_1^2 \vec{B} \bullet d\vec{\ell} + \int_2^3 \vec{B} \bullet d\vec{\ell} + \int_3^4 \vec{B} \bullet d\vec{\ell} + \int_4^1 \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

$$BL + 0 + 0 + 0 = \mu_o I_{enc}$$

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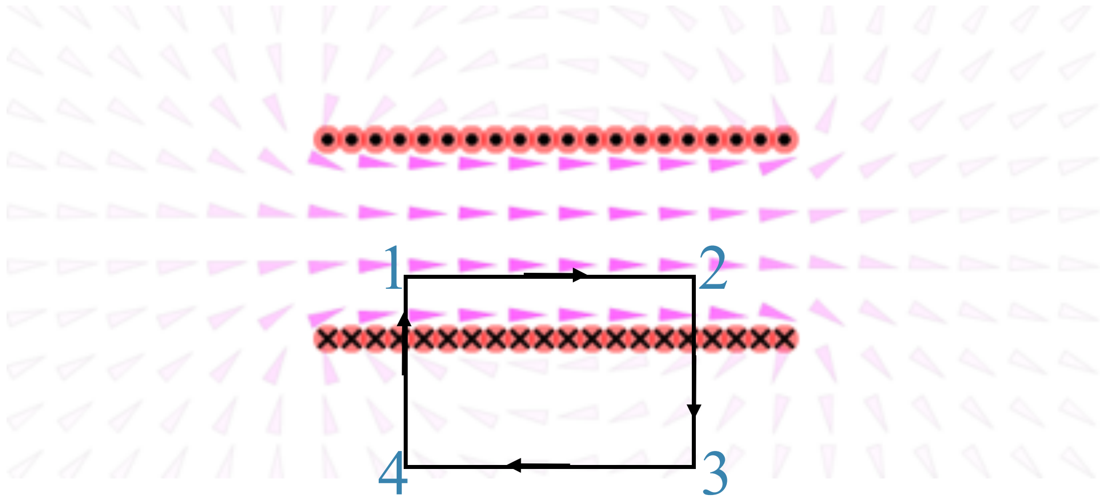

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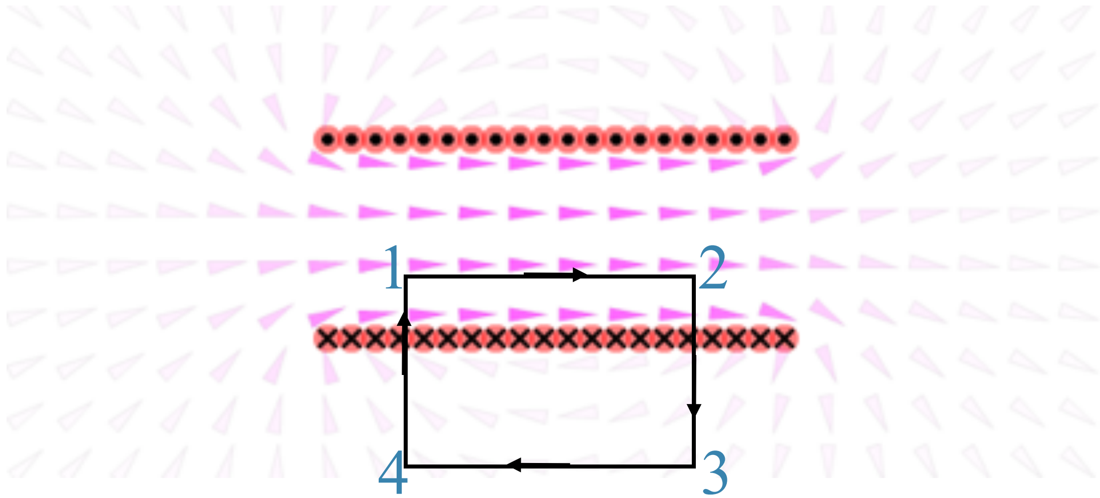

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$$BL + 0 + 0 + 0 = \mu_o I_{enc} \rightarrow BL = \mu_o nLI \rightarrow B = \mu_o nI$$

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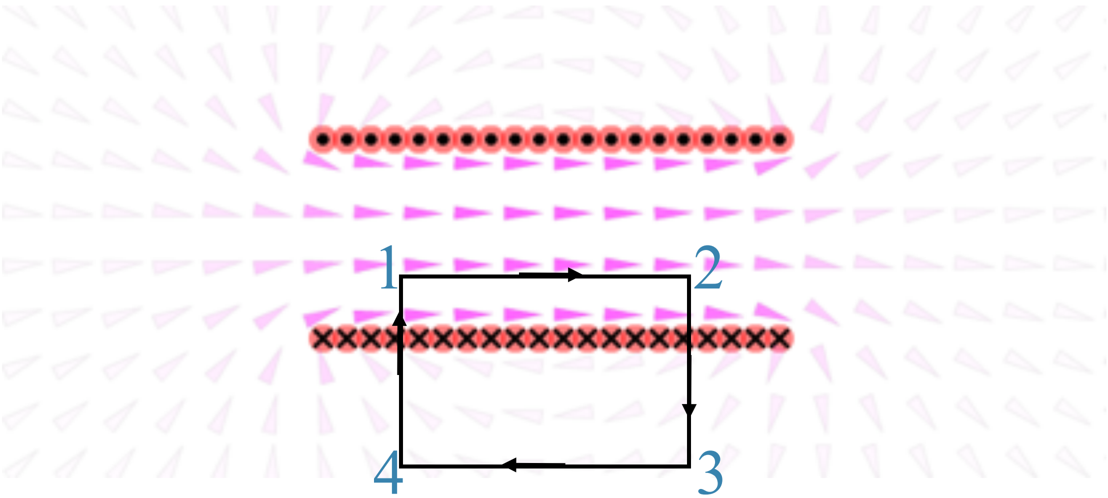

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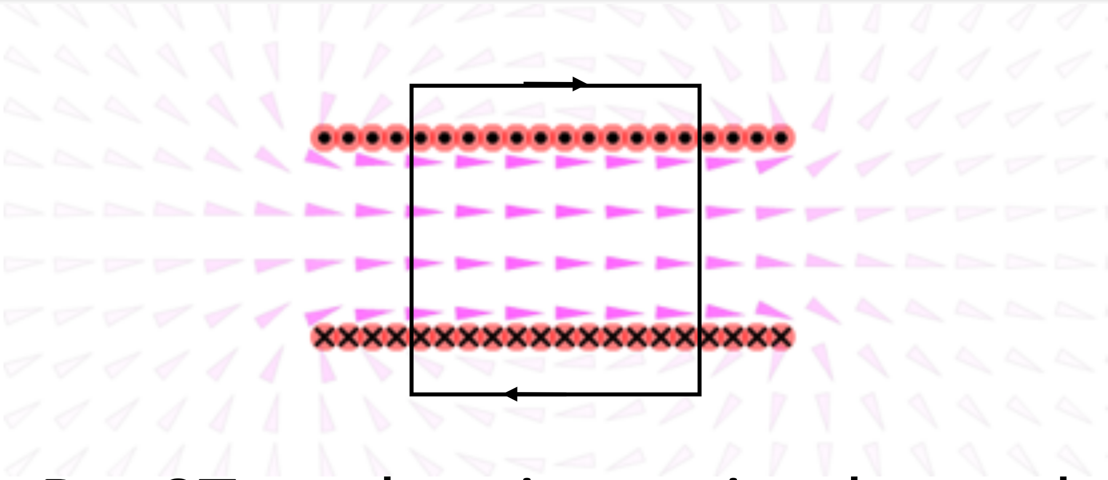
n = # turns/length

Question



“ Is not B = 0T, as drawing a circular path enclosed by the cardboard cylinder contains no current?”

In this case both paths are outside the tube

- Net I-enclosed is zero
- Integral along all edges is zero too

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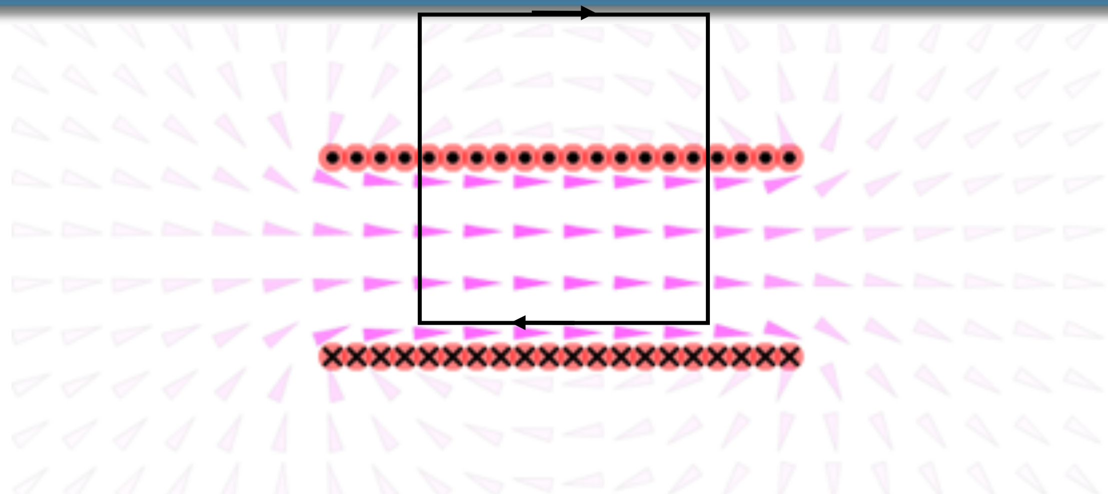

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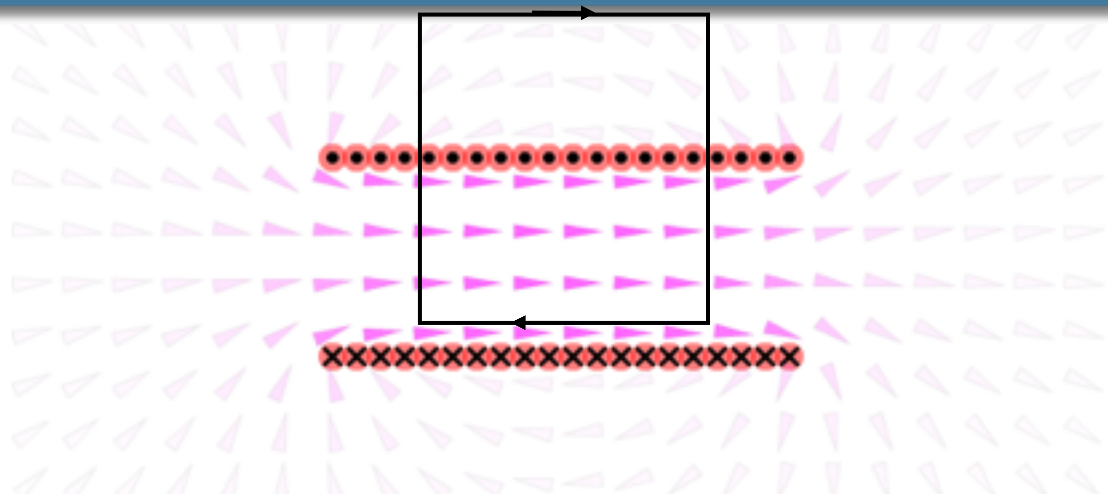


" Is not B = 0T, as drawing a circular path enclosed by the cardboard cylinder contains no current?"

In this case both paths are outside the tube

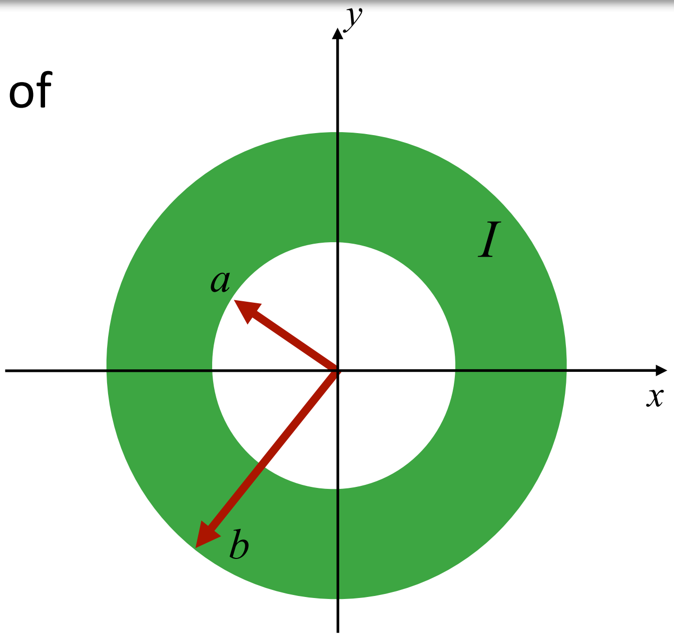
- Net I-enclosed is zero
- Integral along all edges is zero too

When one edge is inside then there is nonzero *I* enclosed. Integral on the side in the tube is nonzero.

Example Problem

An infinitely long cylindrical shell with inner radius a and outer radius b carries a uniformly distributed current I out of the screen.

Sketch $|B|$ as a function of r.



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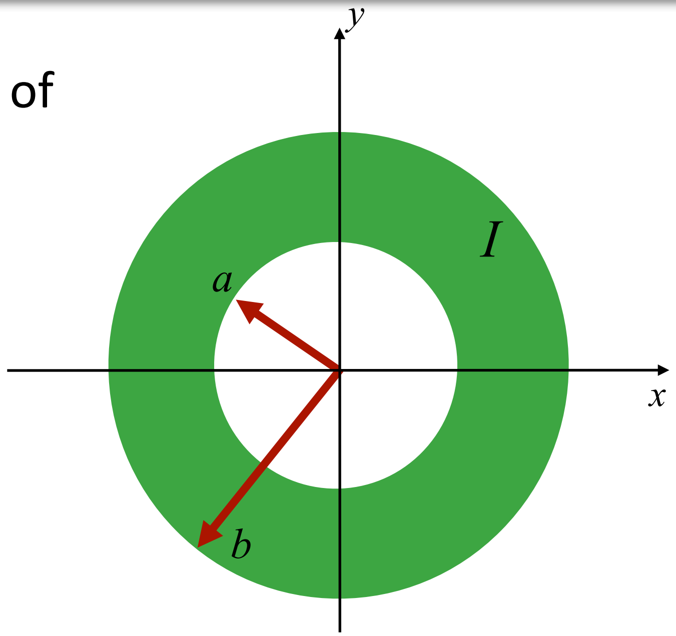

Conceptual Analysis

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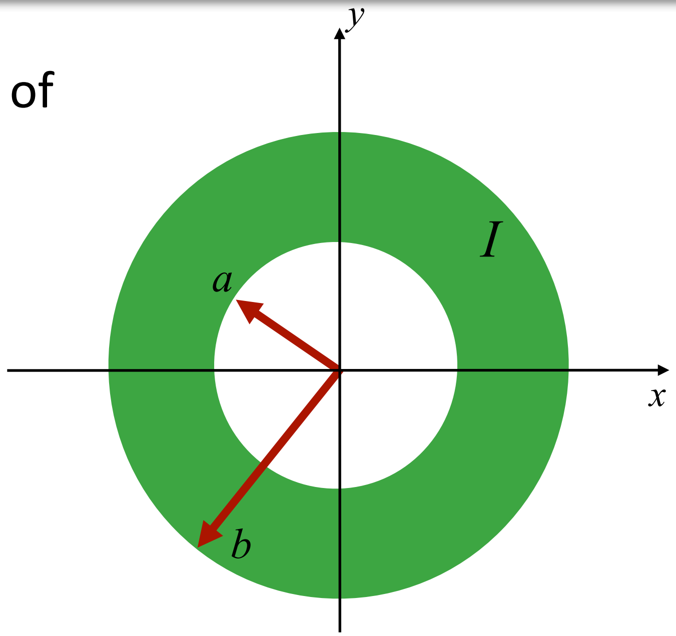

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Complete cylindrical symmetry (can only depend on r)
$\Rightarrow$ can use Ampere's law to calculate B

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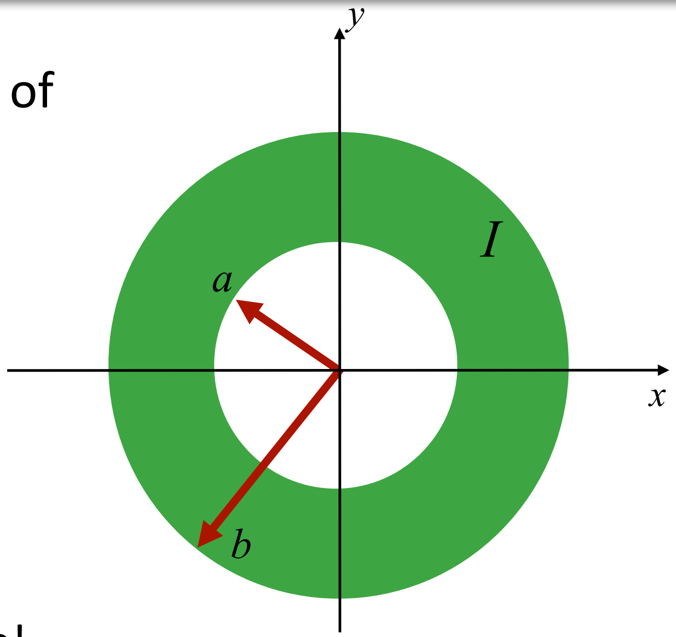

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B field can only be clockwise, counterclockwise or zero!

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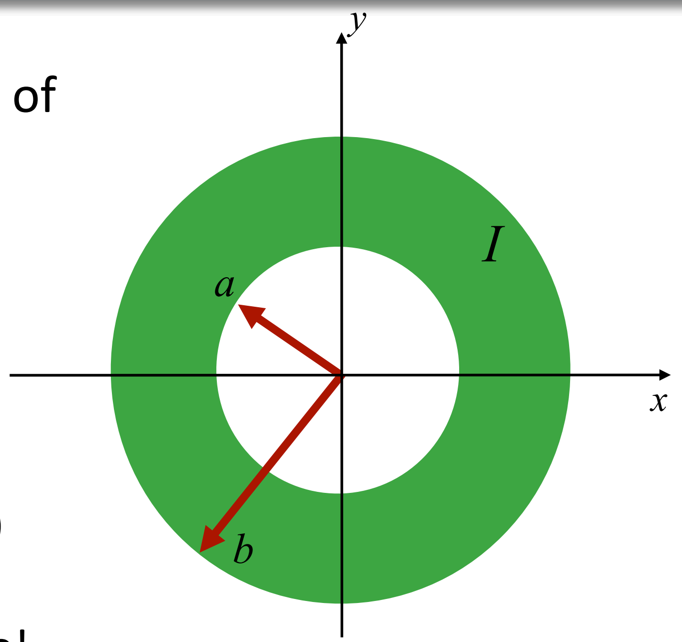

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$$\oint \vec{B} \bullet d\vec{\ell} = \mu_o I_{enc}$$

$$B\oint d\ell = \mu_o I_{enc}$$ For circular path concentric with shell.

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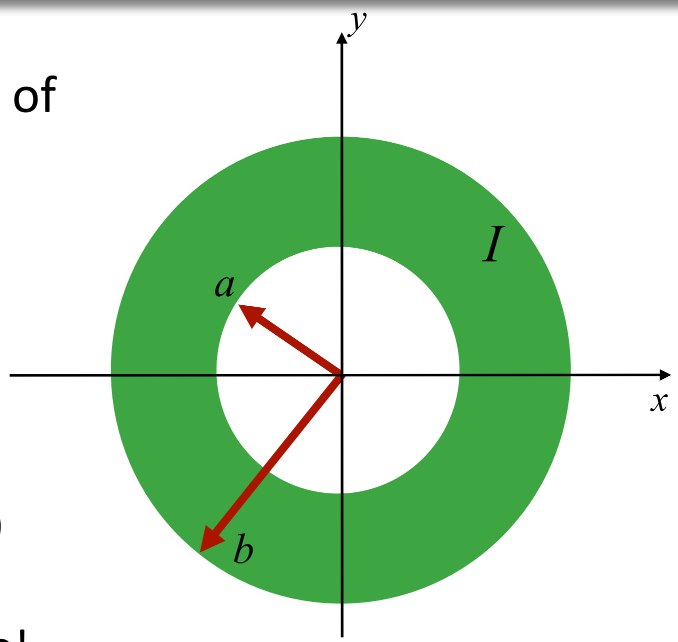

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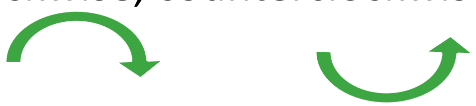

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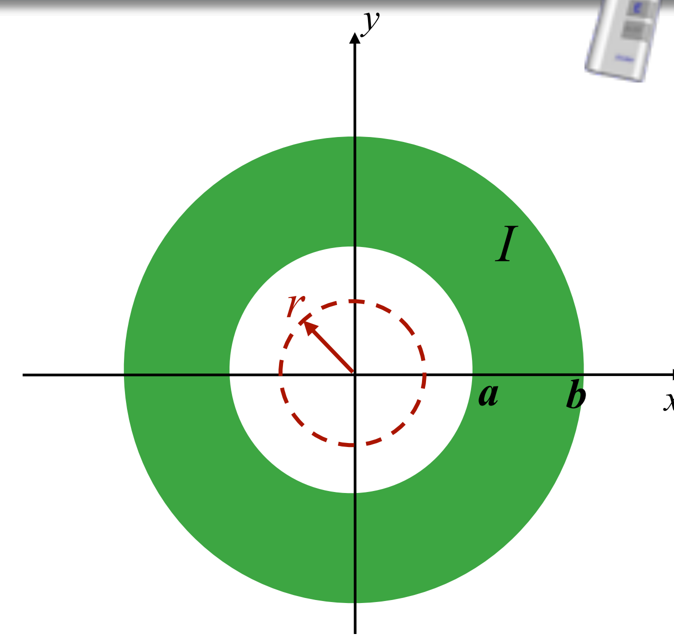
Strategic Analysis

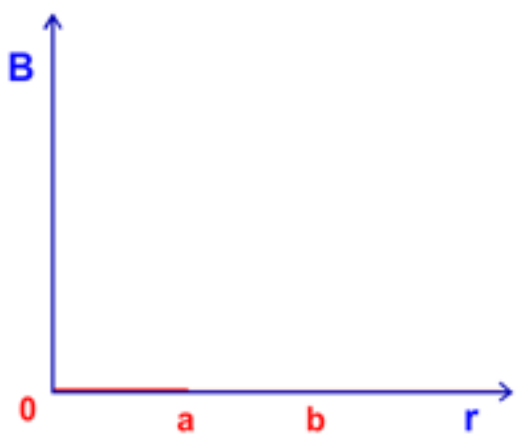
Calculate B for the three regions separately:

1) $r < a$
2) $a < r < b$
3) $r > b$

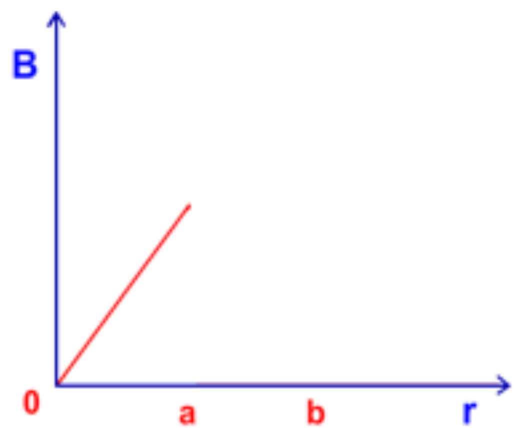
Example Problem

What does $|B|$ look like for $r < a$?





A



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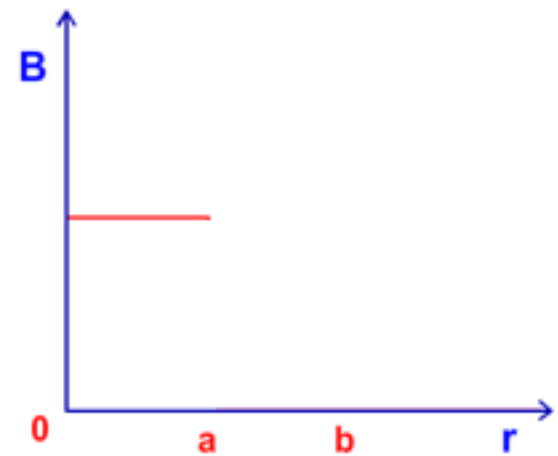
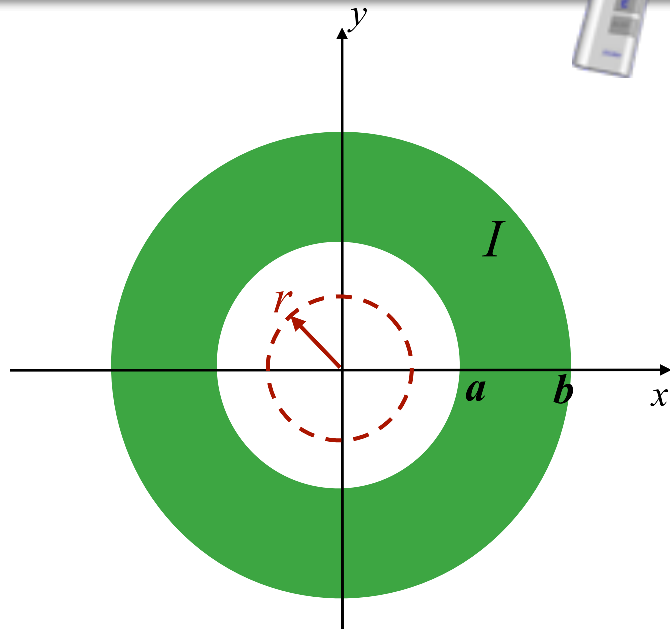


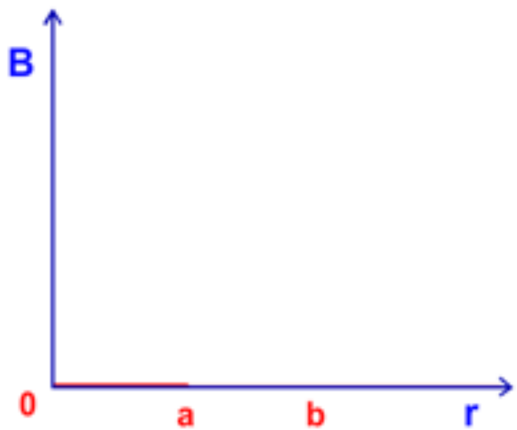
C

Example Problem

What does $|B|$ look like for $r < a$?



$$I_{\text{encl}} = 0$$



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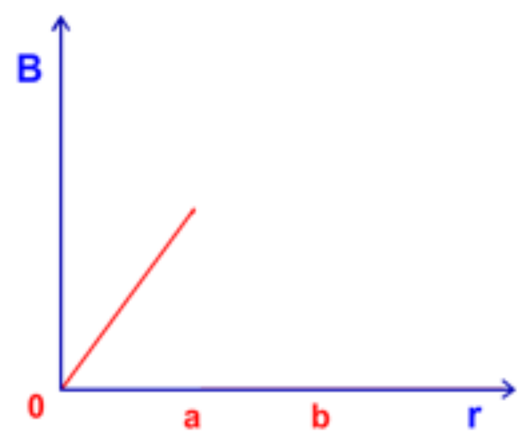

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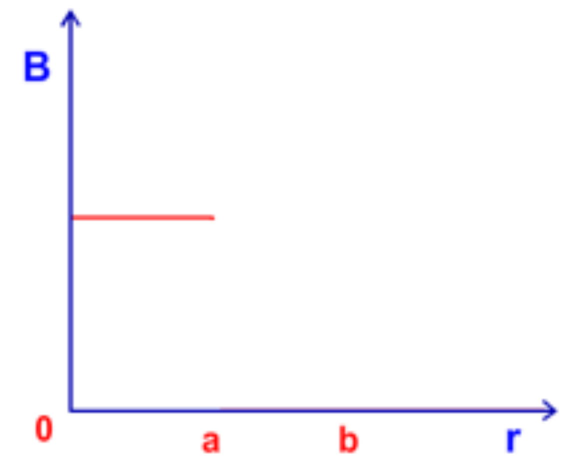

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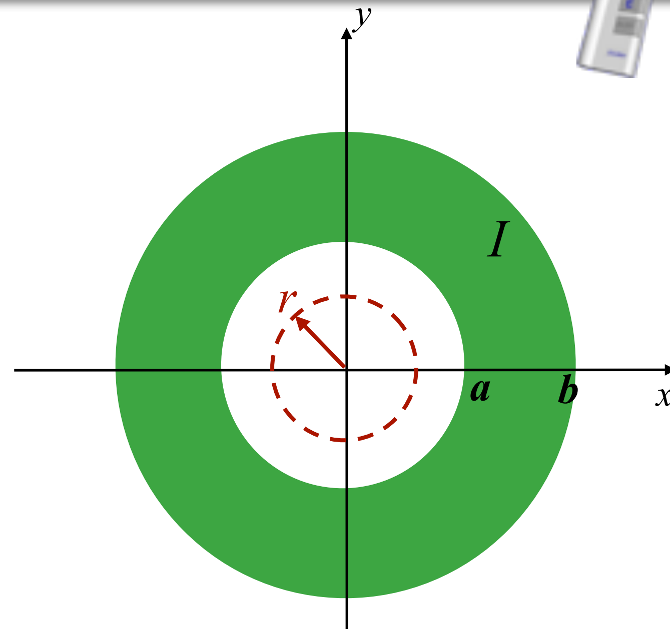

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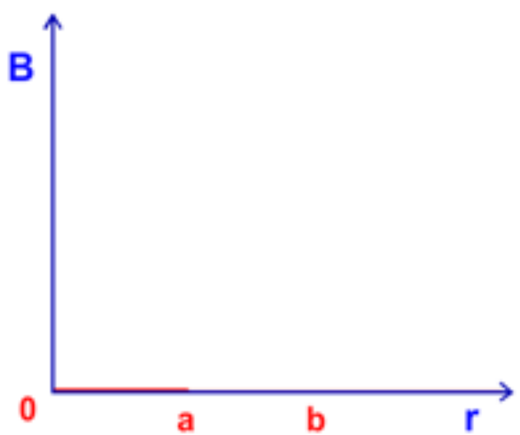

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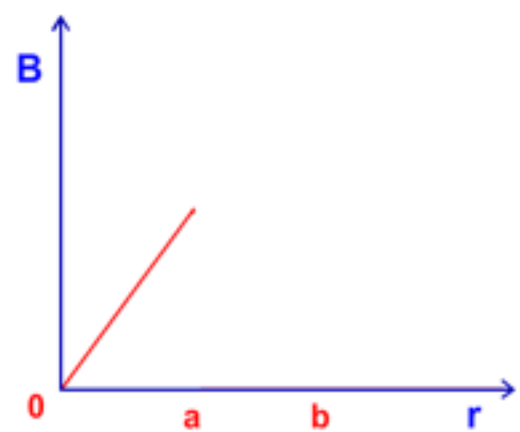

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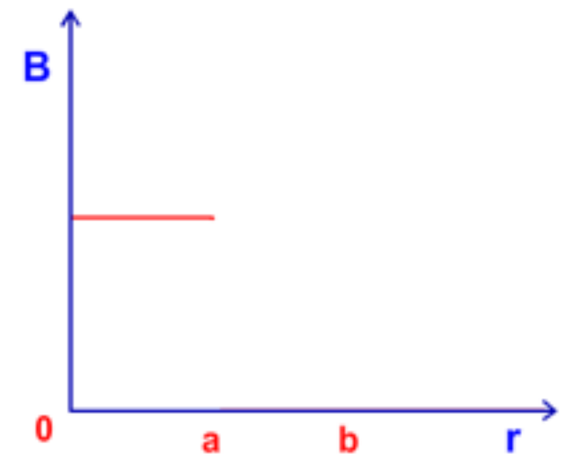

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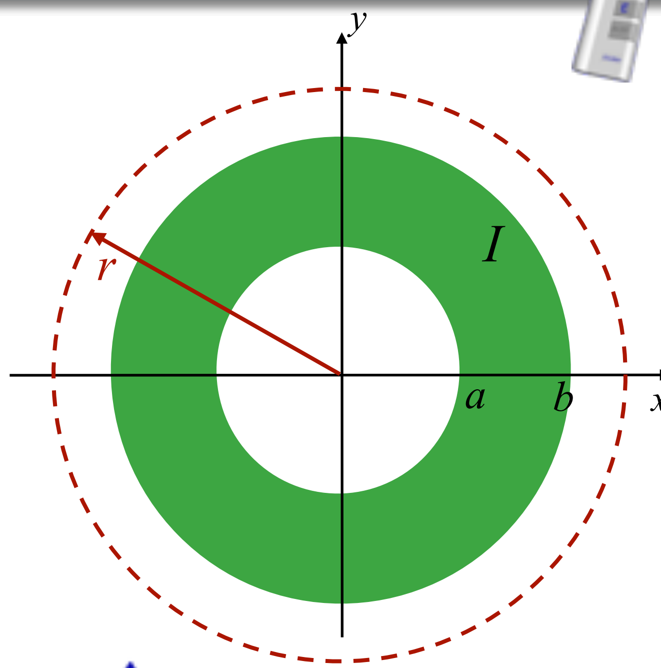

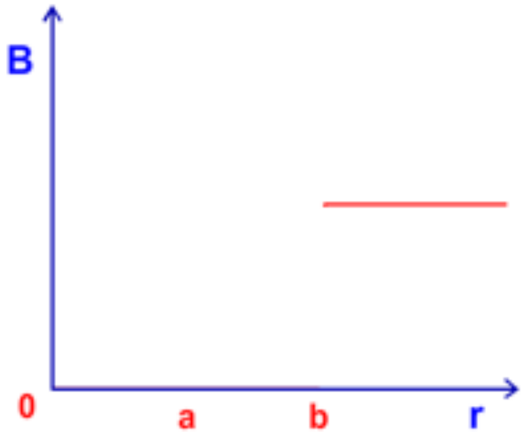

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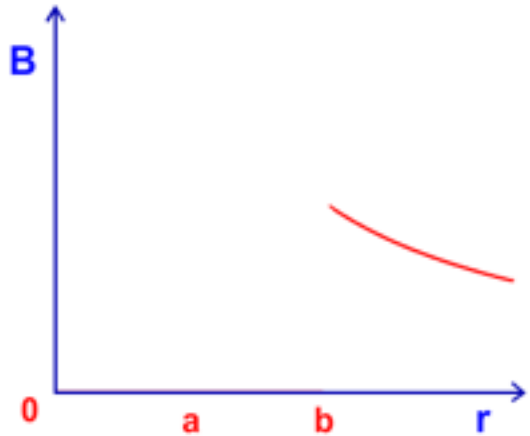

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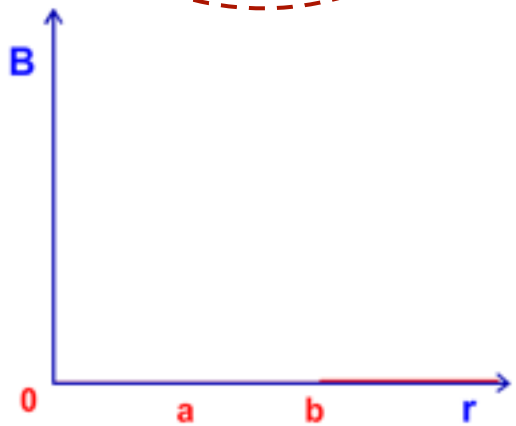

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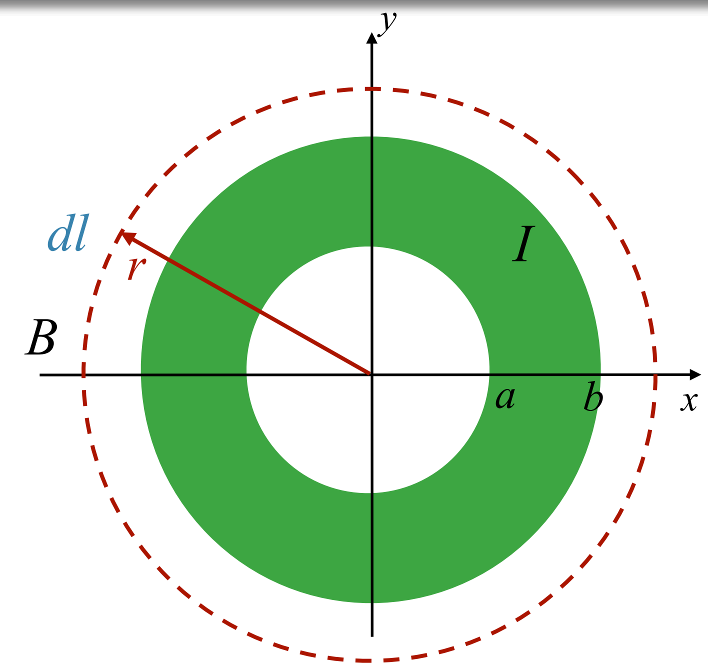

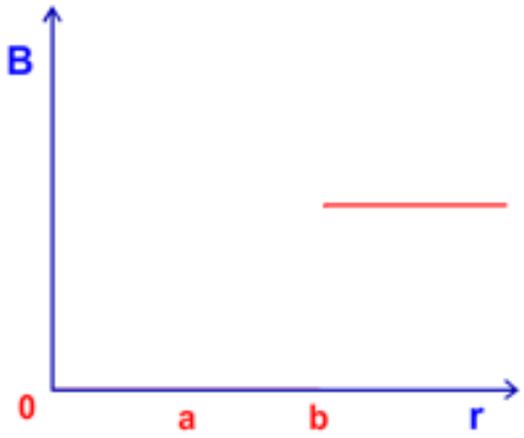

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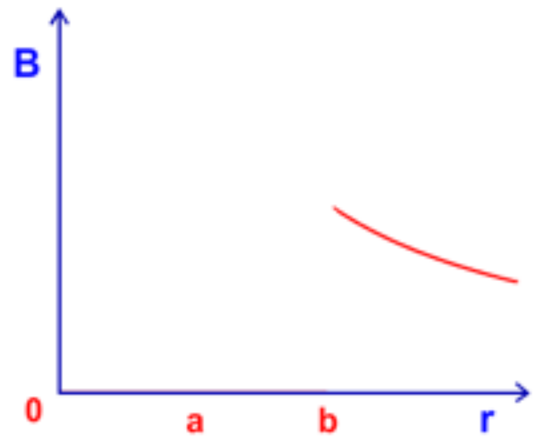

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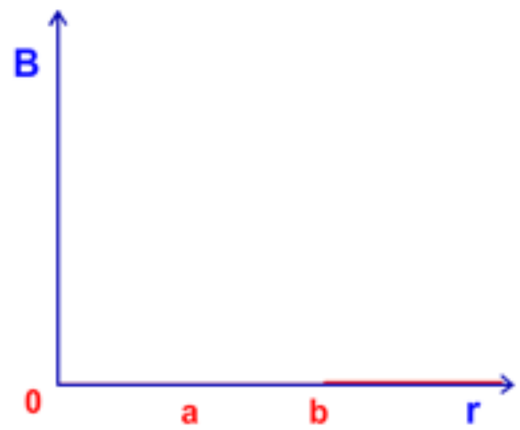

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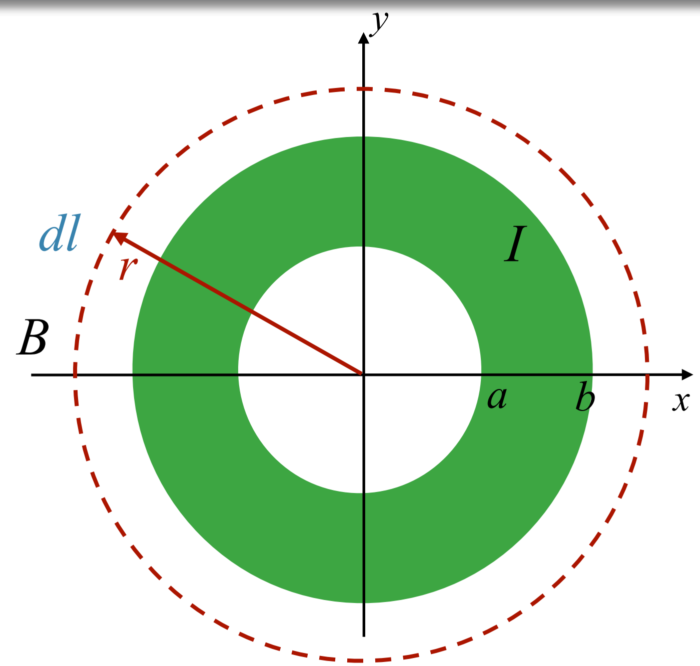

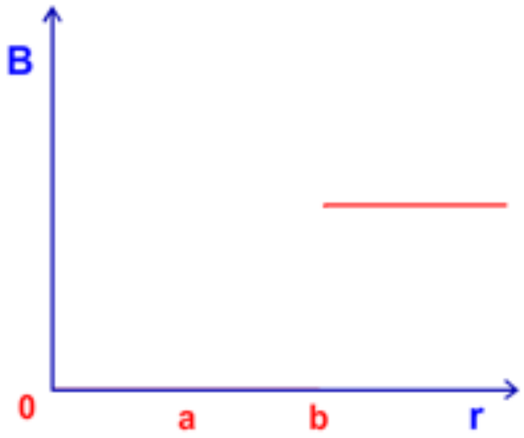

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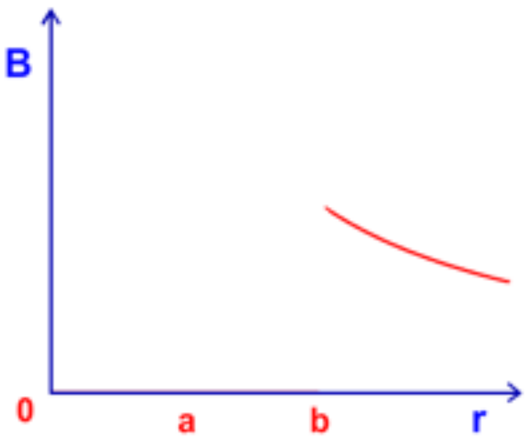

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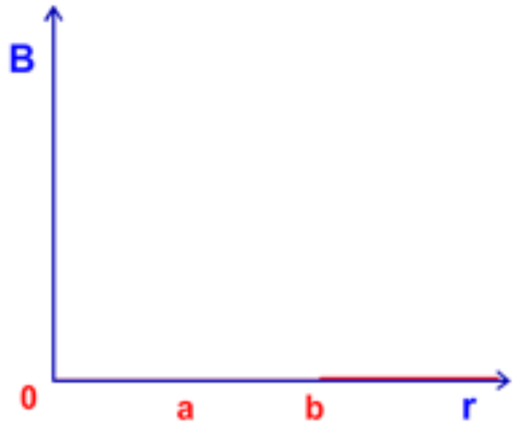

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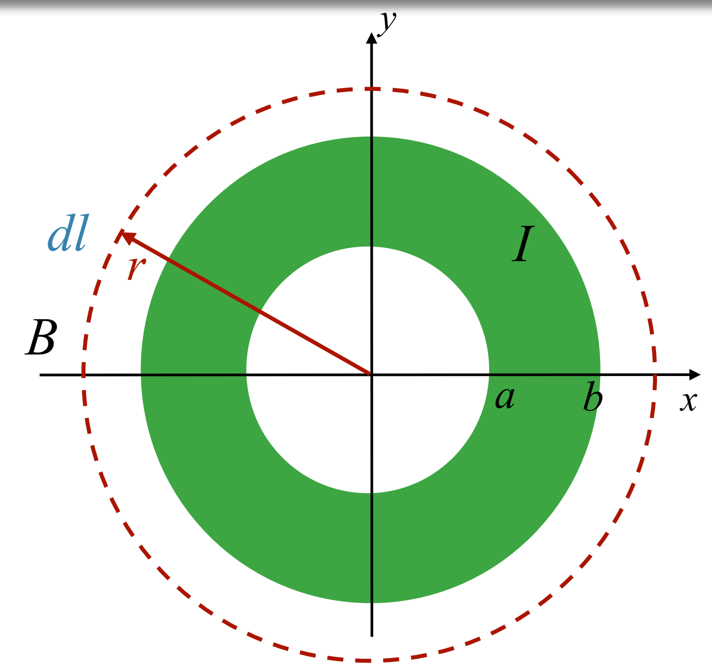

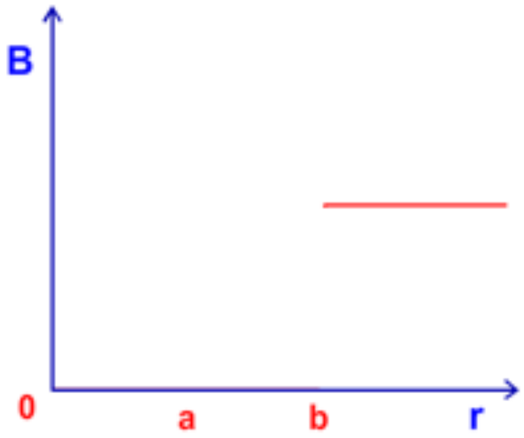

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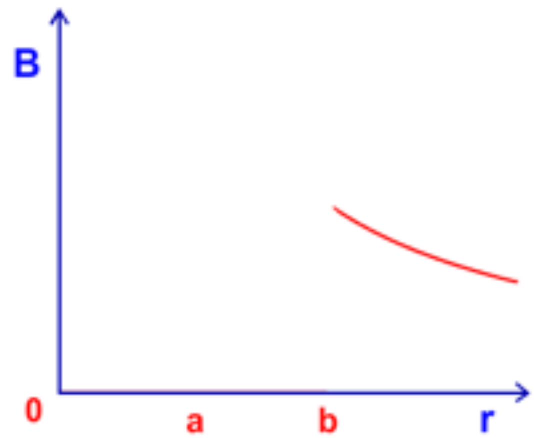

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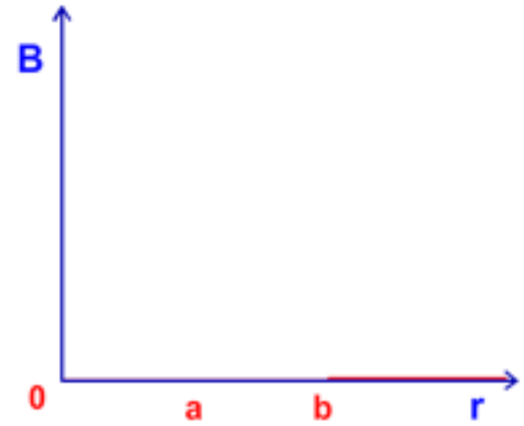

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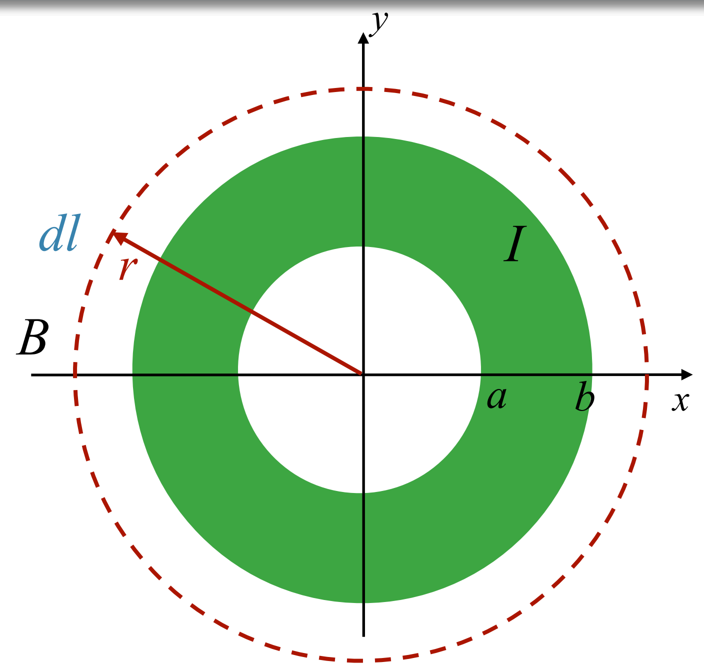
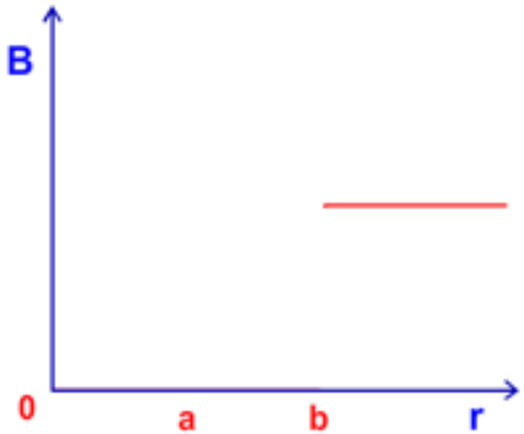
$\rightarrow \quad B = \dfrac{\mu_o I}{2\pi r}$





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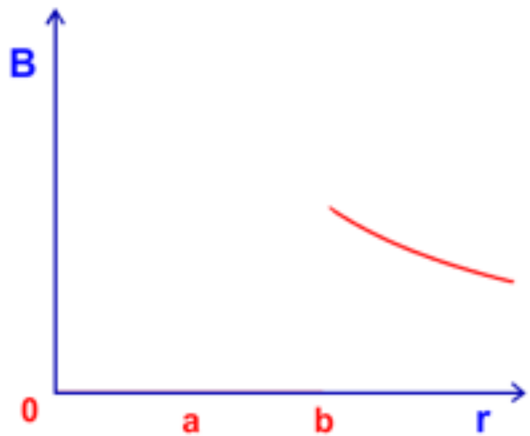

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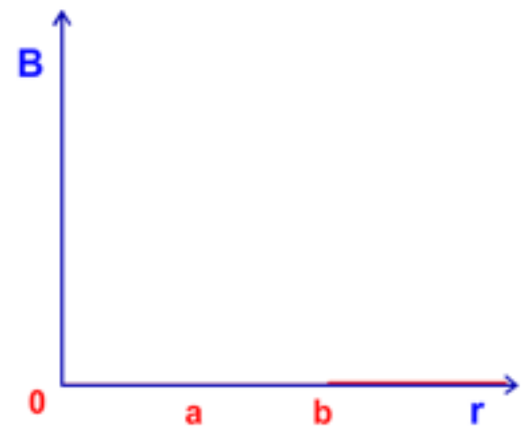

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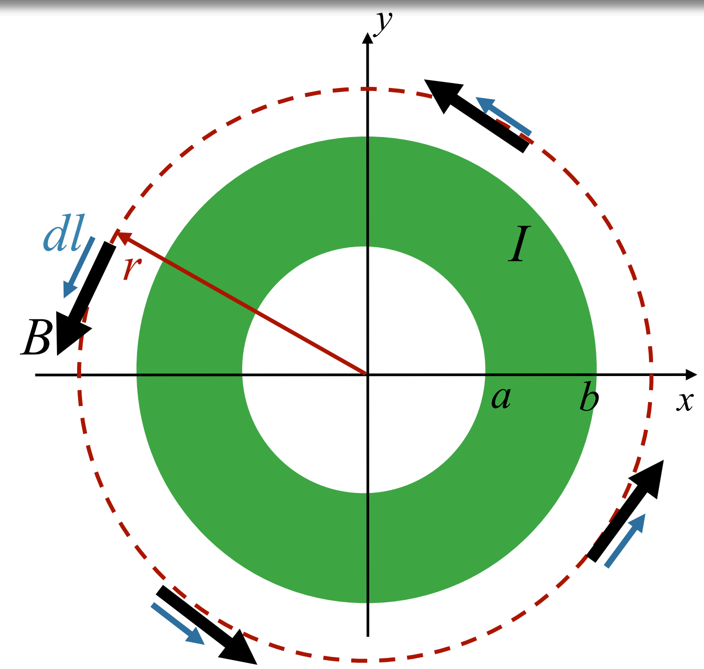

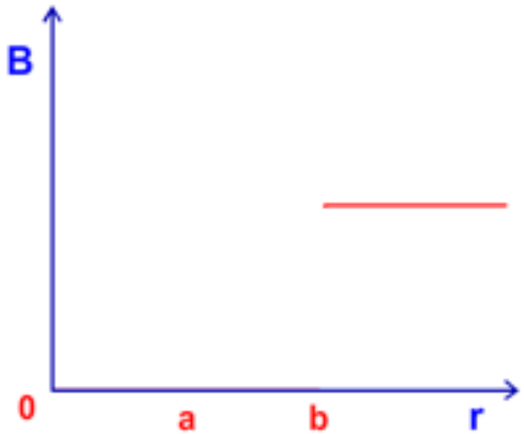

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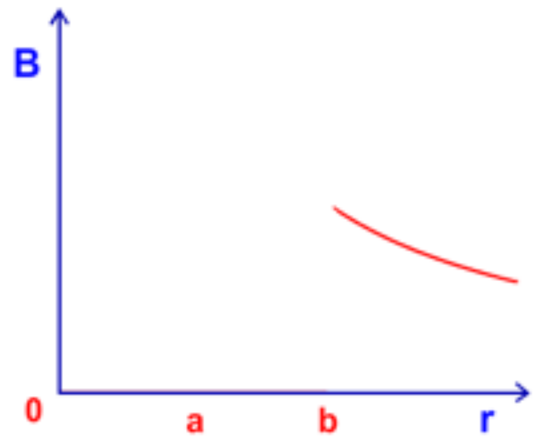

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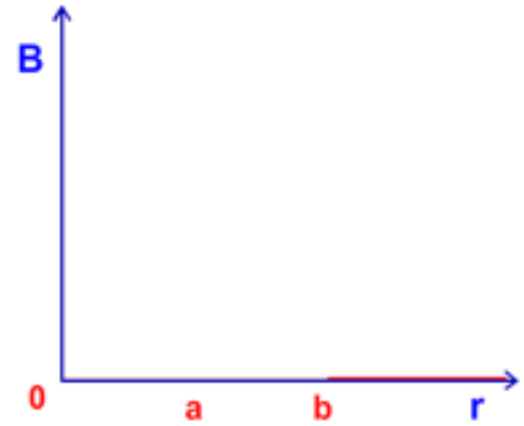

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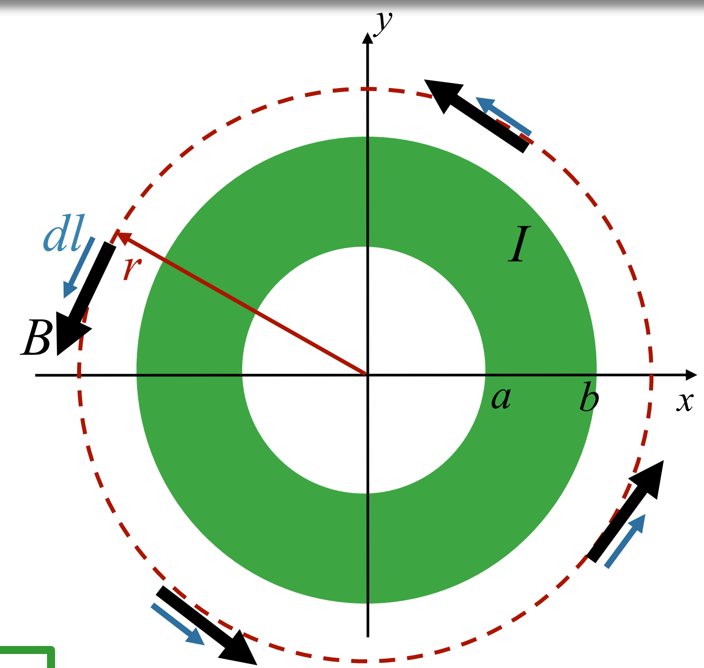

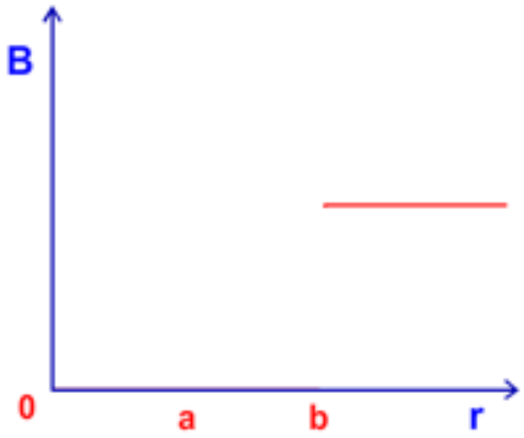

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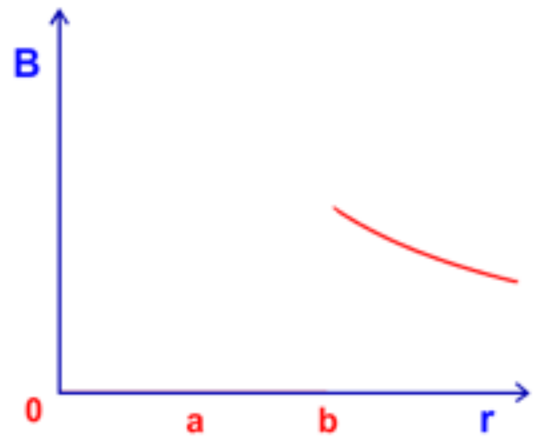

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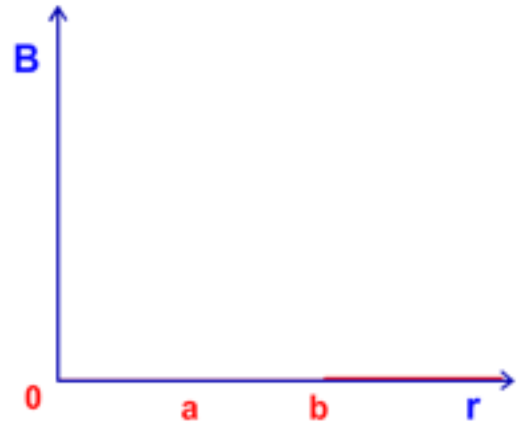
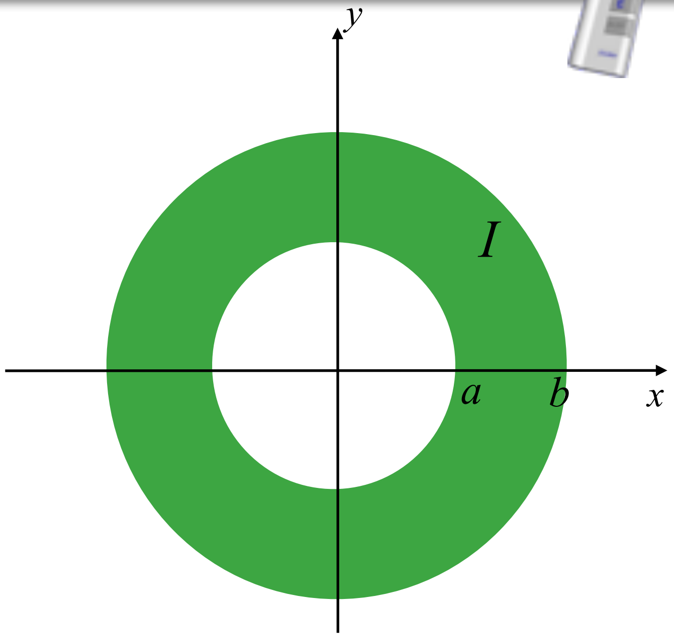


C

Example Problem

What is the current density j (Amp/m^2) in the conductor?



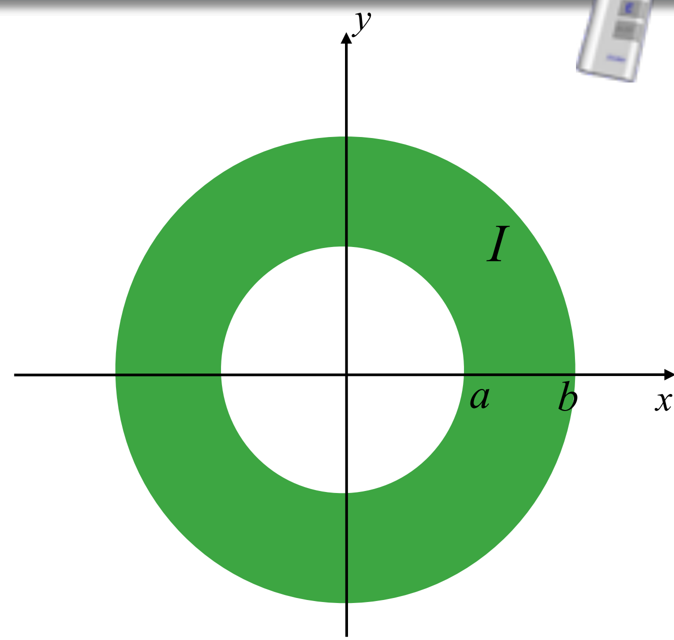
A) $j = \dfrac{I}{\pi b^2}$

B) $j = \dfrac{I}{\pi b^2 + \pi a^2}$

C) $j = \dfrac{I}{\pi b^2 - \pi a^2}$

Example Problem

What is the current density j (Amp/m^2) in the conductor?



A) $j = \dfrac{I}{\pi b^2}$

B) $j = \dfrac{I}{\pi b^2 + \pi a^2}$

C) $j = \dfrac{I}{\pi b^2 - \pi a^2}$

$$j = I / area$$

$$area = \pi b^2 - \pi a^2$$

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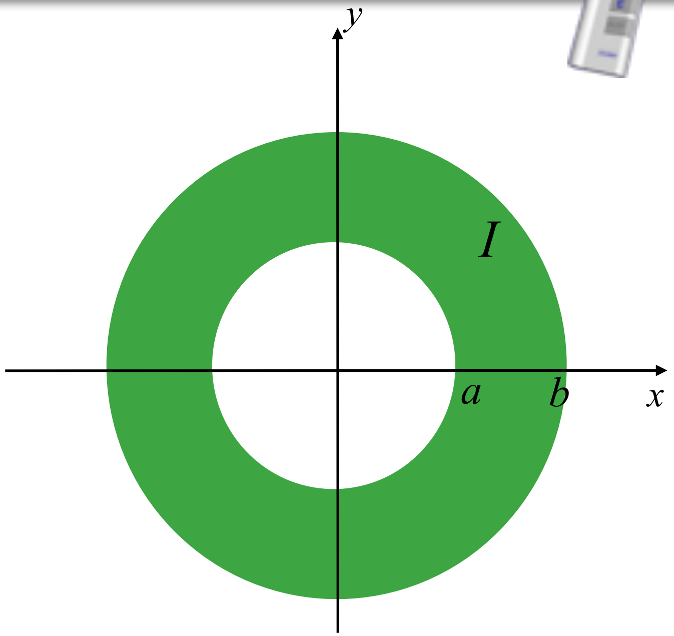

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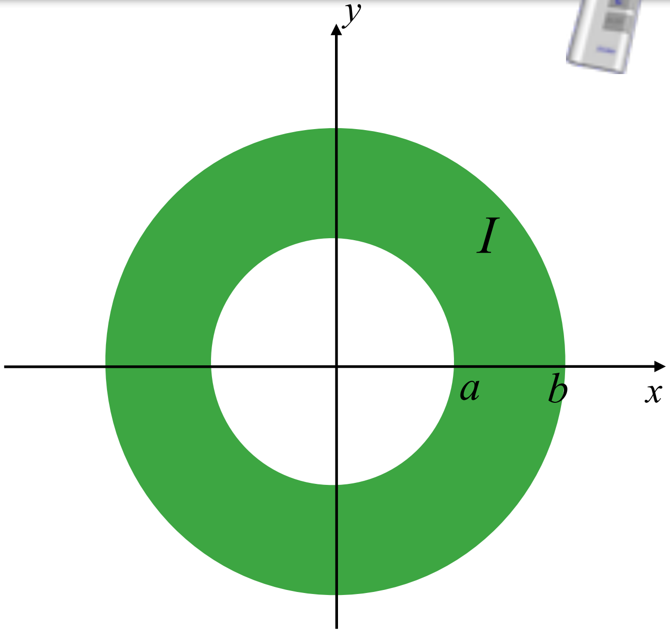

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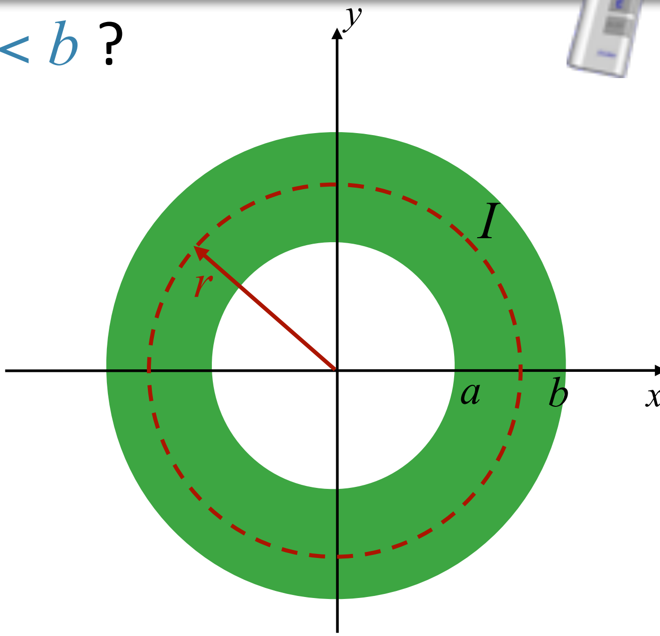
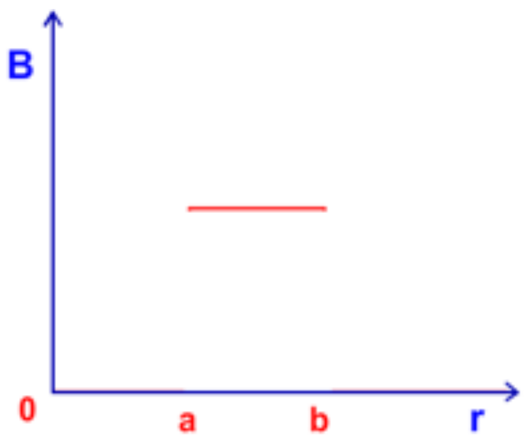
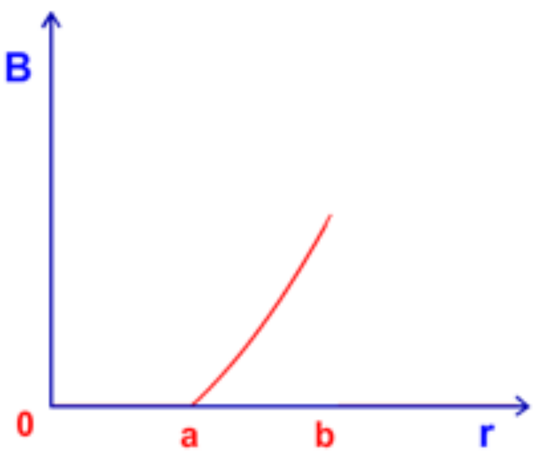
Example Problem

What does $|B|$ look like for $a < r < b$?

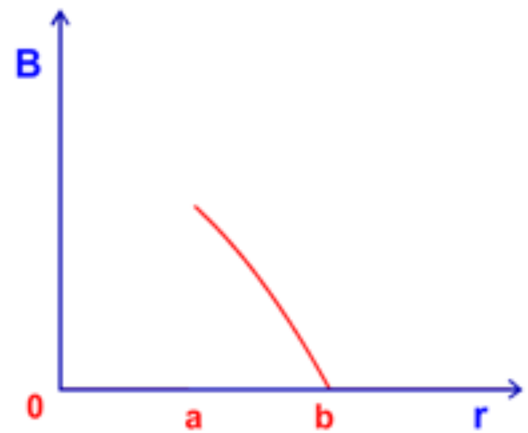




A



B



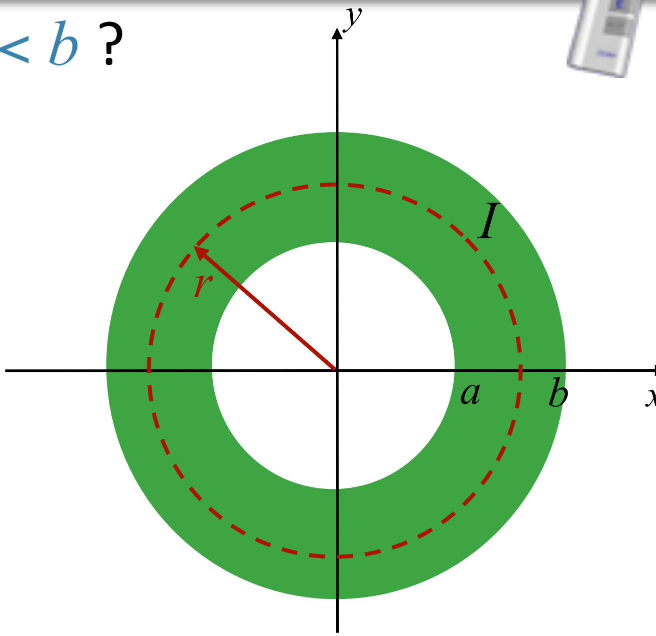
C

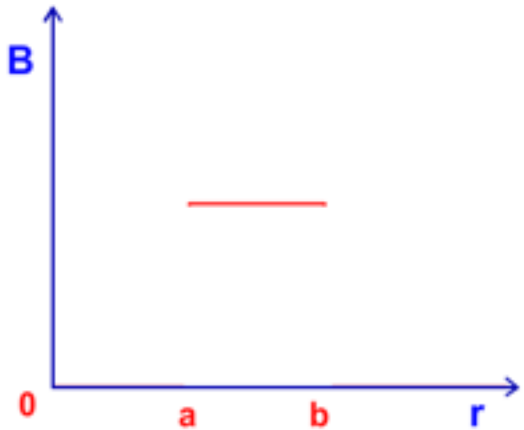
Example Problem

What does $|B|$ look like for $a < r < b$?

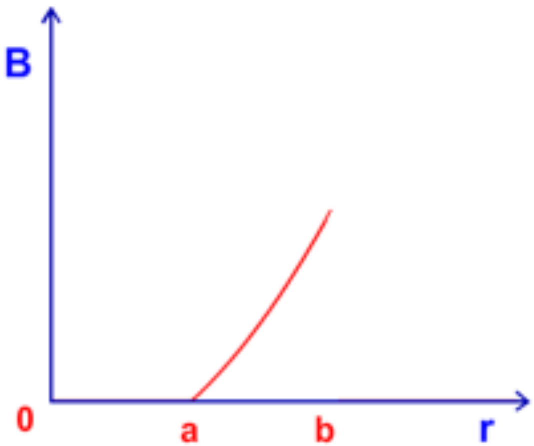
$$B \cdot 2\pi r = \mu_o \cdot j A_{enc}$$

$$B \cdot 2\pi r = \mu_o \cdot \frac{I}{\pi (b^2 - a^2)} \cdot \pi (r^2 - a^2)$$

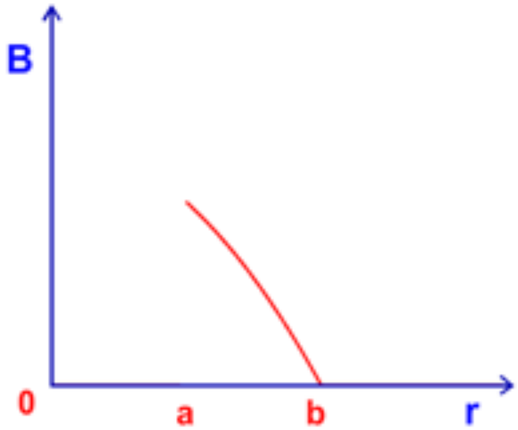




A



B



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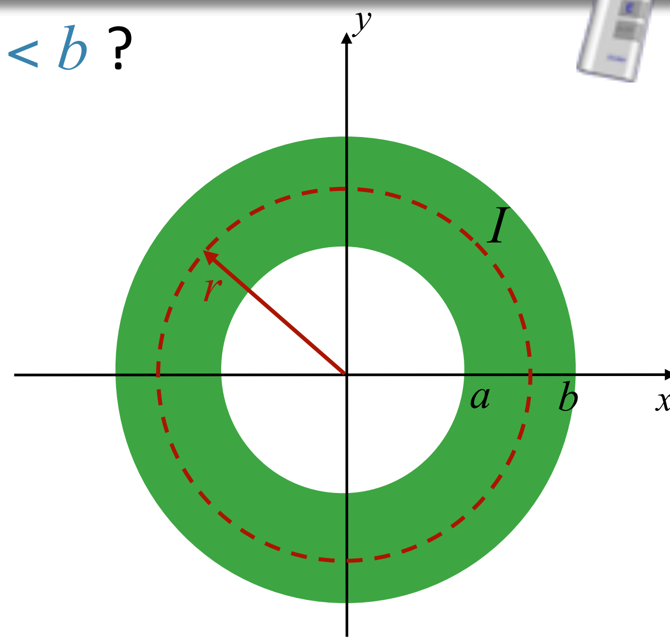

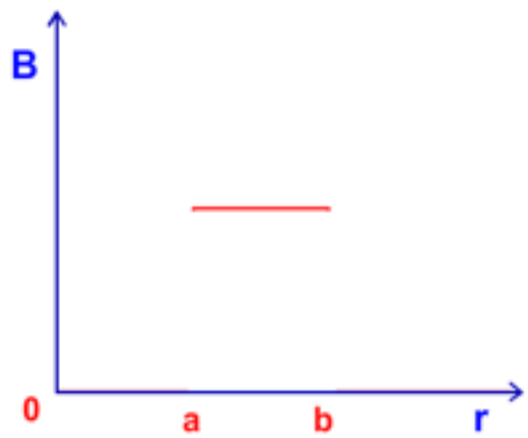

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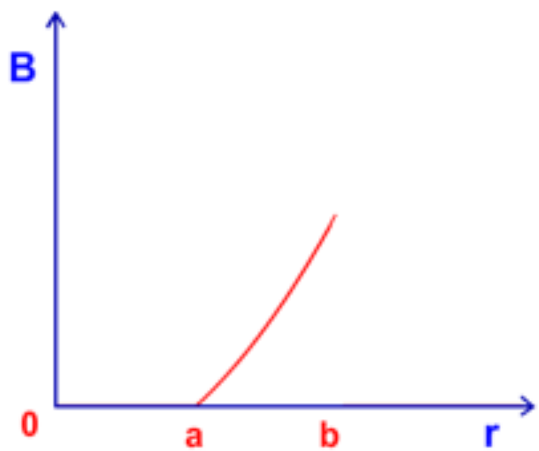
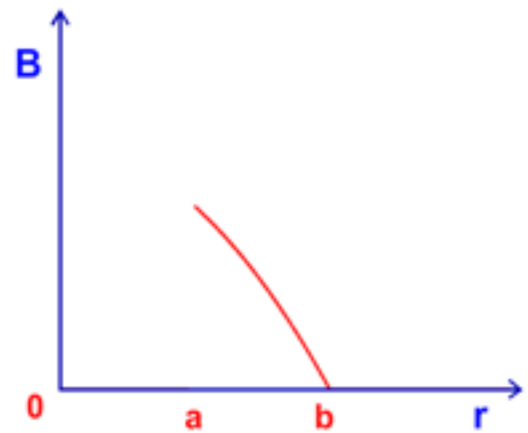


B



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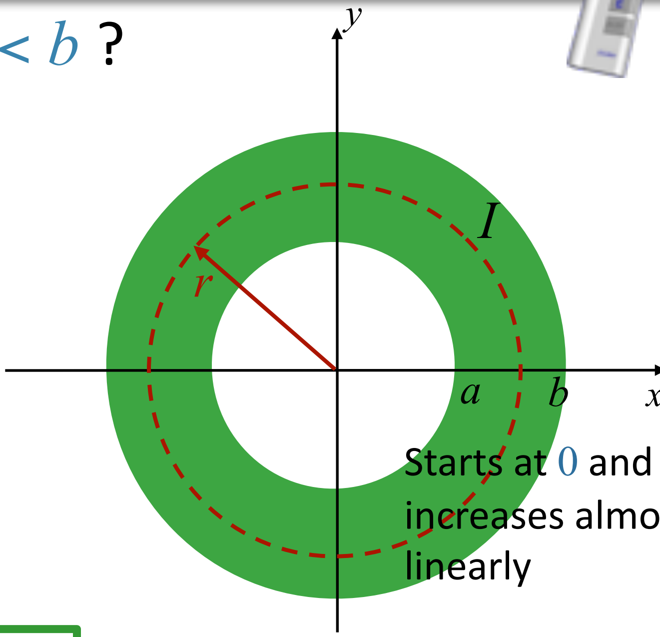
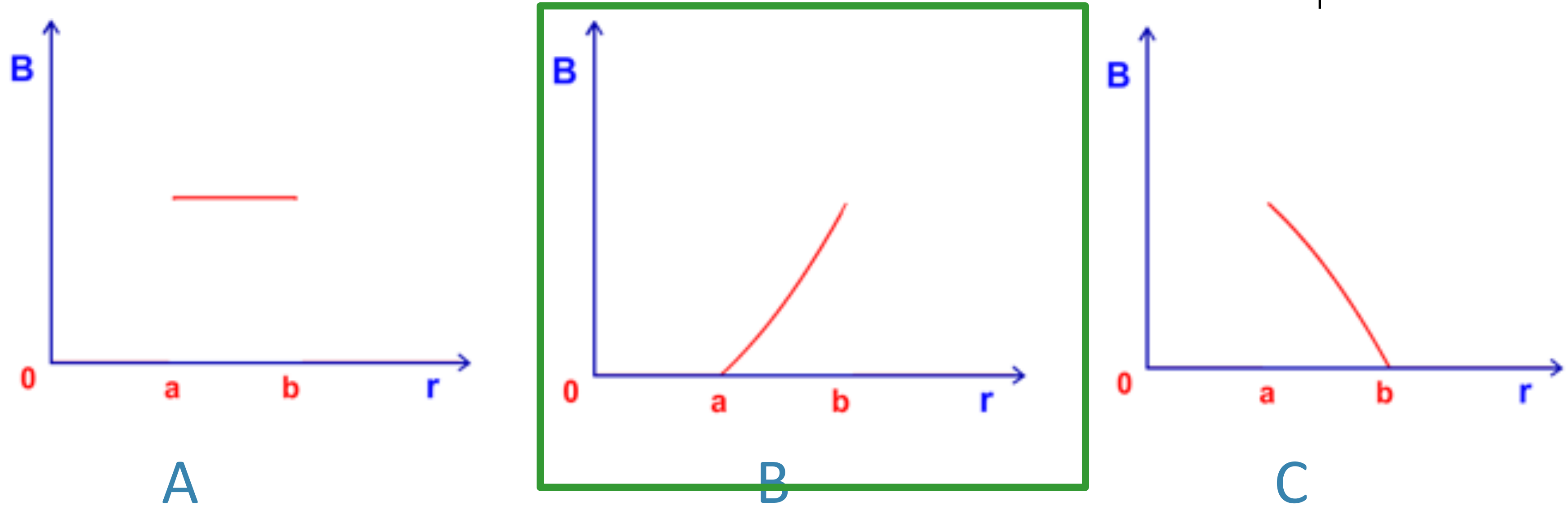
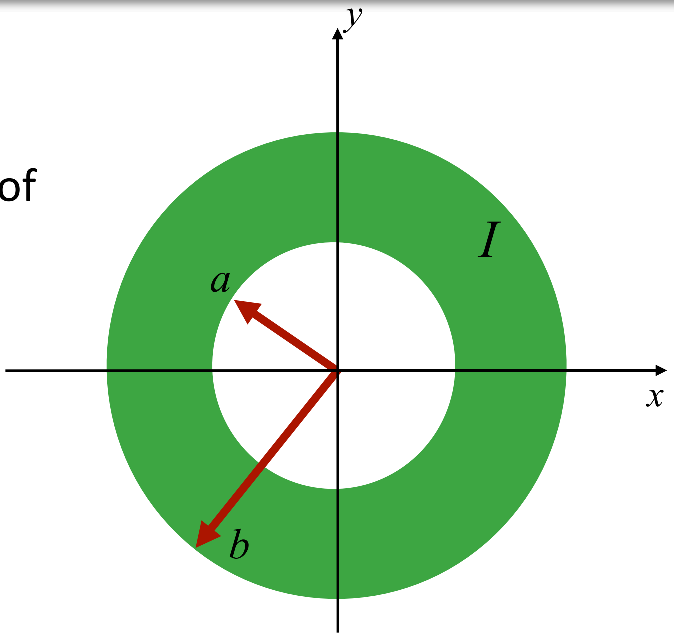


Starts at 0 and increases almost linearly



Example Problem

An infinitely long cylindrical shell with inner radius a and outer radius b carries a uniformly distributed current *I* out of the screen.



Sketch $|B|$ as a function of r.

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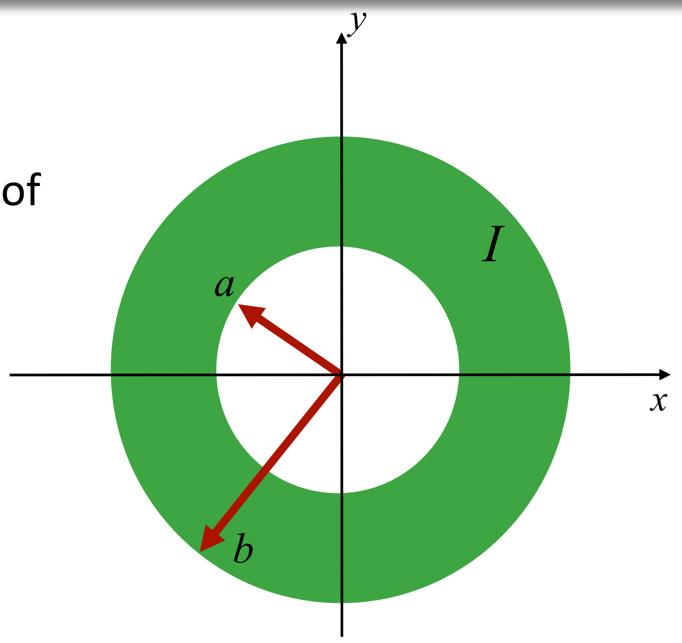

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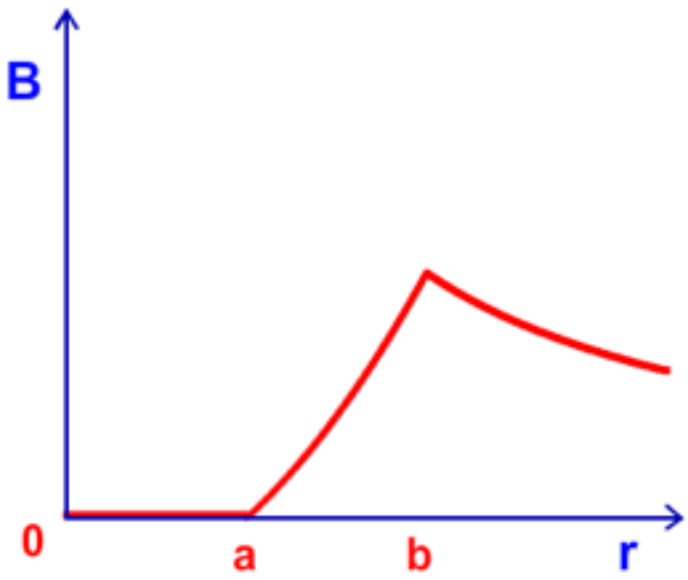
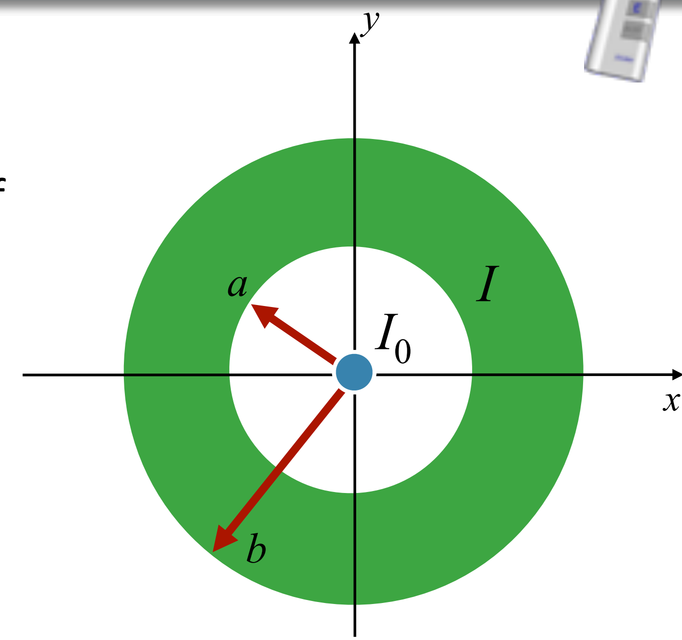


Follow-Up

Add an infinite wire along the z axis carrying current I_0.

What must be true about I_0 such that there is some value of r, $a < r < b$, such that $B(r) = 0$?



A) $|I_0| > |I|$ AND I_0 into screen

B) $|I_0| > |I|$ AND I_0 out of screen

C) $|I_0| < |I|$ AND I_0 into screen

D) $|I_0| < |I|$ AND I_0 out of screen

E) There is no current I_0 that can produce $B = 0$ there

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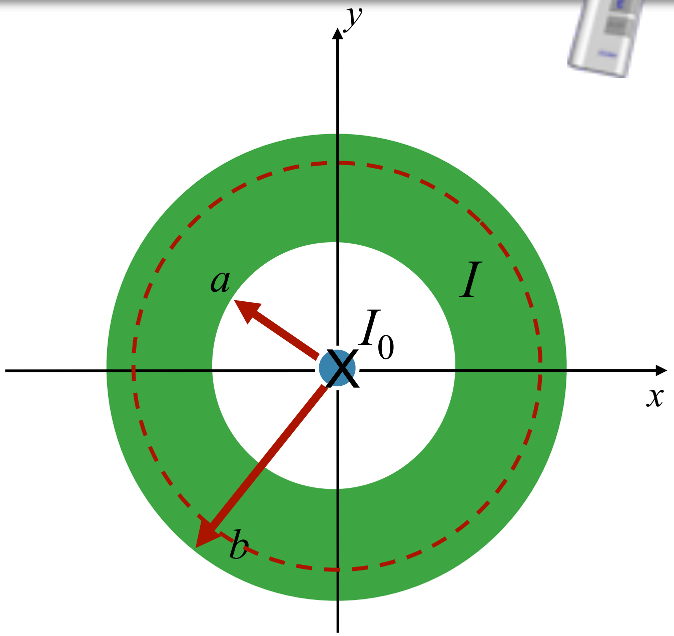

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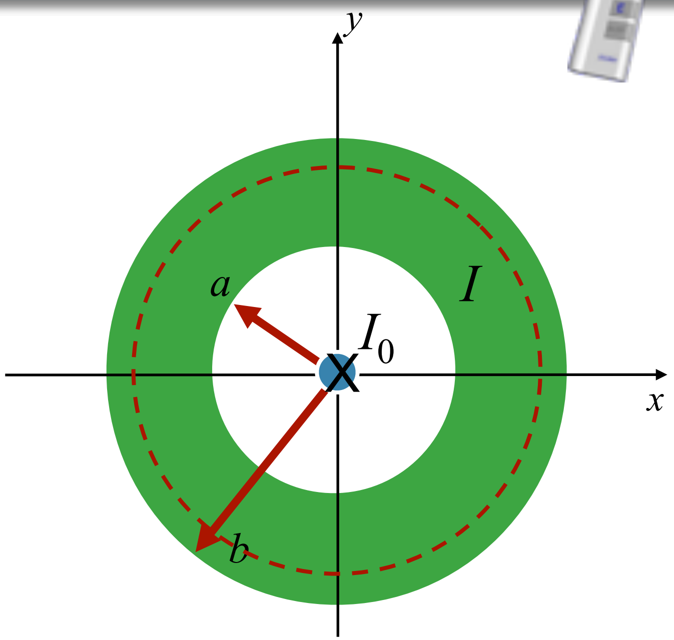

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