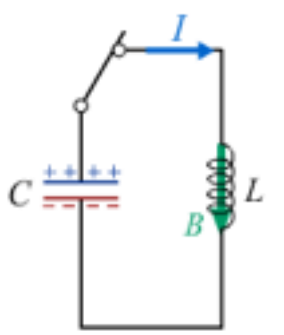
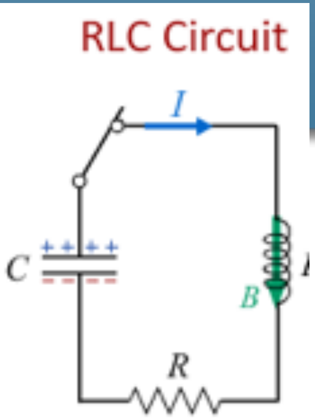
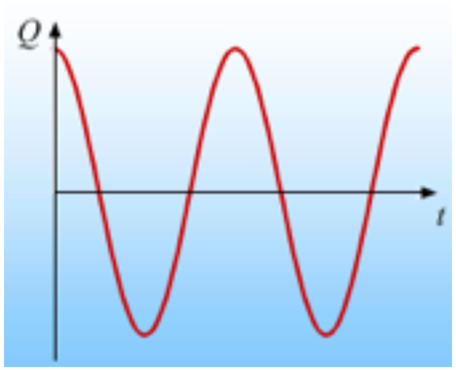
Electricity & Magnetism
Lecture 19





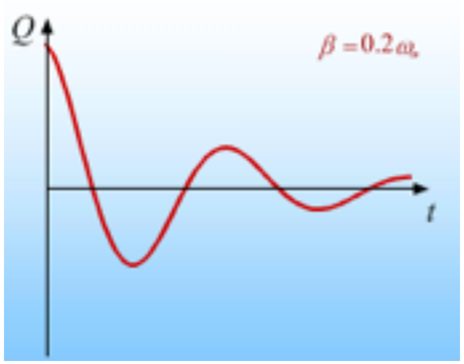
Today's Concepts:



A) Oscillation Frequency

B) Energy

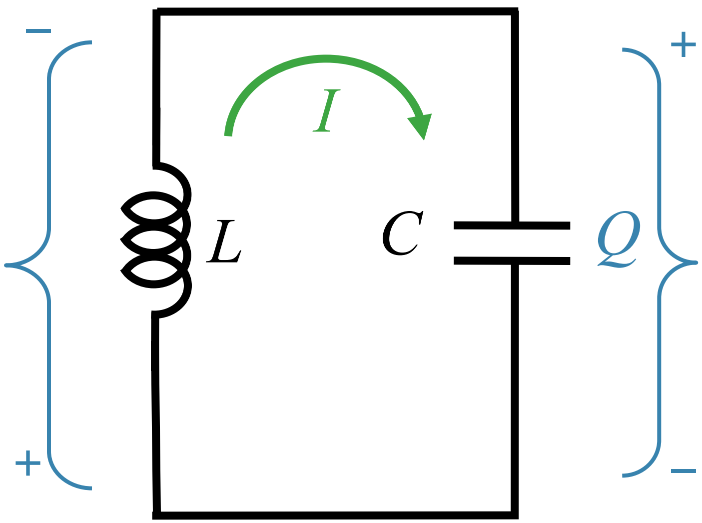
C) Damping



LC Circuit

“tank”

$$V_L = L\frac{dI}{dt}$$



$$V_C = \frac{Q}{C}$$

Circuit Equation: $\frac{Q}{C} + L\frac{dI}{dt} = 0$

$$I = \frac{dQ}{dt} \quad \Rightarrow \quad \frac{d^2Q}{dt^2} = -\frac{Q}{LC} \quad \Rightarrow \quad \frac{d^2Q}{dt^2} = -\omega^2 Q$$

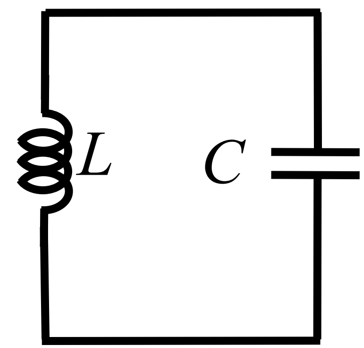
where

$$\omega = \frac{1}{\sqrt{LC}}$$

$$m \leftrightarrow L$$

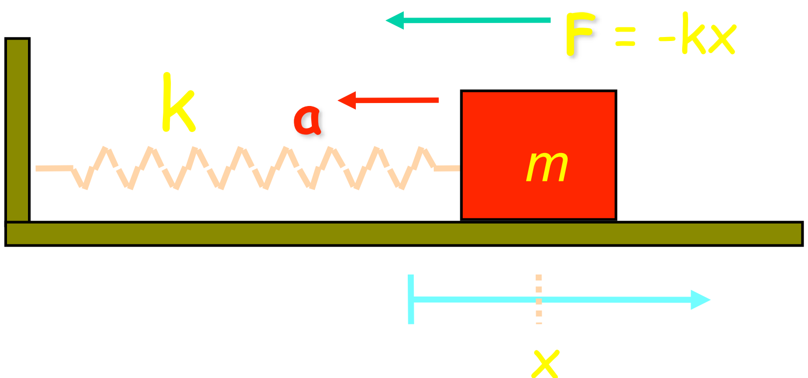
$$\frac{d^2Q}{dt^2} = -\omega^2 Q$$

$$\omega = \frac{1}{\sqrt{LC}}$$



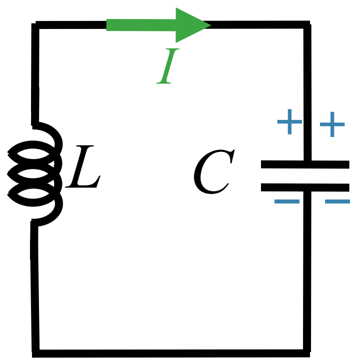
$$\frac{d^2x}{dt^2} = -\omega^2 x$$

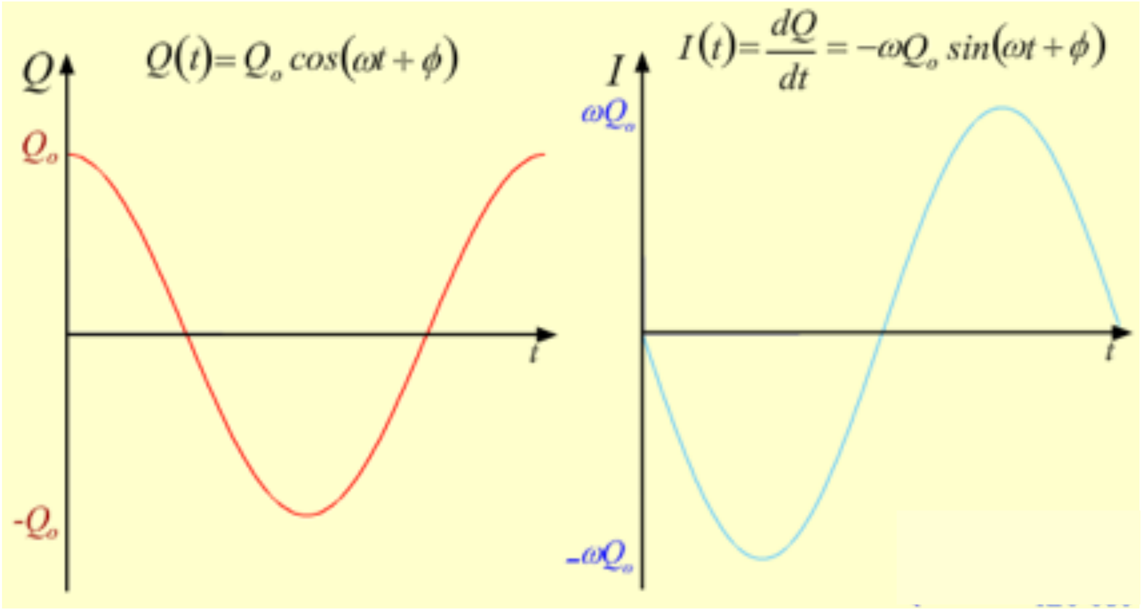
$$\omega = \sqrt{\frac{k}{m}}$$



Same thing if we notice that $k \leftrightarrow \frac{1}{C}$ and $m \leftrightarrow L$

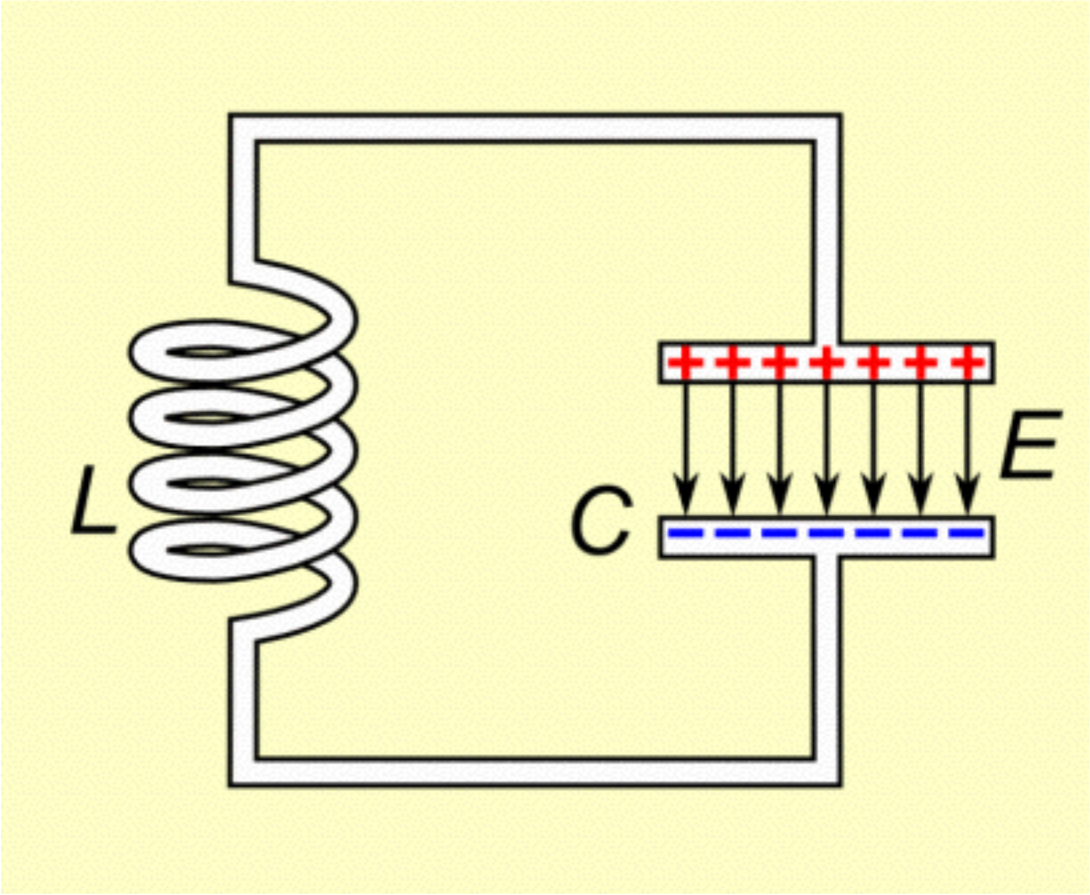
Time Dependence





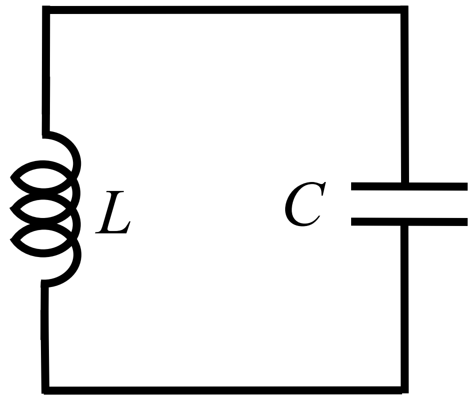
LC Circuits

click to play



CheckPoint 2

At time $t = 0$ the capacitor is fully charged with Q_{max} and the current through the circuit is 0.



What is the potential difference across the inductor at $t = 0$?

A) $V_L = 0$

B) $V_L = Q_{max}/C$ since $V_L = V_C$

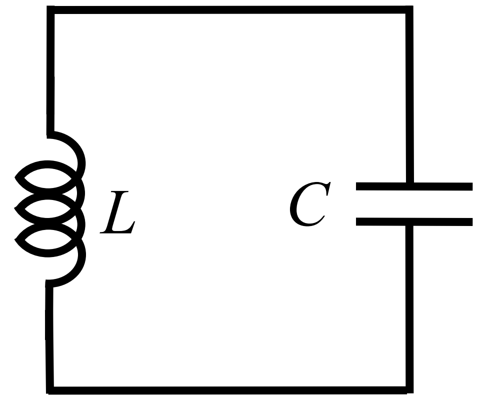
C) $V_L = Q_{max}/2C$

The voltage across the capacitor is Q_{max}/C Kirchhoff's Voltage Rule implies that must also be equal to the voltage across the inductor

Pendulum.

CheckPoint 4

At time $t = 0$ the capacitor is fully charged with Q_{max} and the current through the circuit is 0.



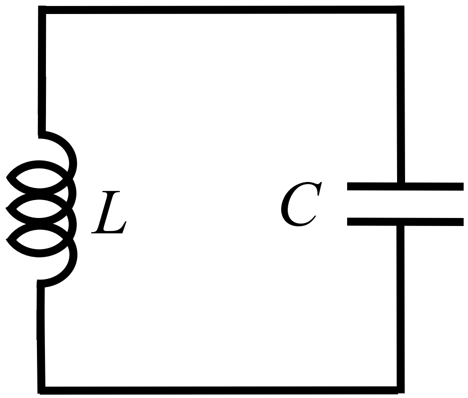
What is the potential difference across the inductor when the current is maximum?

A) $V_L = 0$
B) $V_L = Q_{max}/C$
C) $V_L = Q_{max}/2C$

dI/dt is zero when current is maximum

CheckPoint 6

At time $t = 0$ the capacitor is fully charged with Q_{max} and the current through the circuit is 0.



How much energy is stored in the capacitor when the current is a maximum ?

A) $U = Q_{max}^2/(2C)$
B) $U = Q_{max}^2/(4C)$
C) $U = 0$

Total Energy is constant!

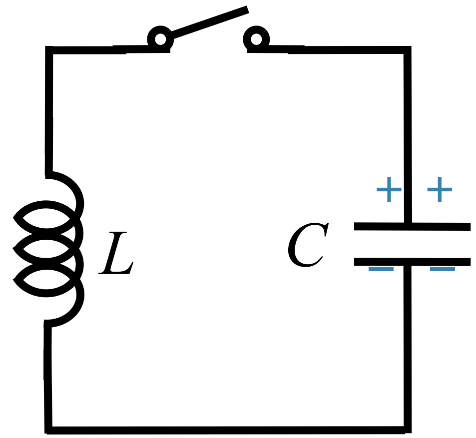
$U_{Lmax} = ½\, LI_{max}^2$

$U_{Cmax} = Q_{max}^2/2C$

$I = max$ when $Q = 0$

CheckPoint 8

The capacitor is charged such that the top plate has a charge $+Q_0$ and the bottom plate $-Q_0$. At time $t = 0$, the switch is closed and the circuit oscillates with frequency $\omega = 500$ radians/s.



$L = 4 \times 10^{-3}$ H

$\omega = 500$ rad/s

What is the value of the capacitor C?

A) $C = 1 \times 10^{-3}$ F

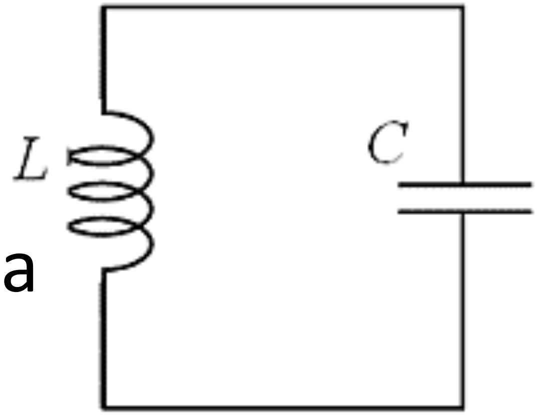
B) $C = 2 \times 10^{-3}$ F

C) $C = 4 \times 10^{-3}$ F

$$\omega = \frac{1}{\sqrt{LC}} \quad \rightarrow \quad C = \frac{1}{\omega^2 L} = \frac{1}{(25\times10^4)(4\times10^{-3})} = 10^{-3} \text{ F}$$

Prelecture

At $t = 0$ the capacitor is fully charged. Expressions for the charge and current as a function of time are shown to the right.



$$Q(t) = Q_{max}\cos(\omega t)$$

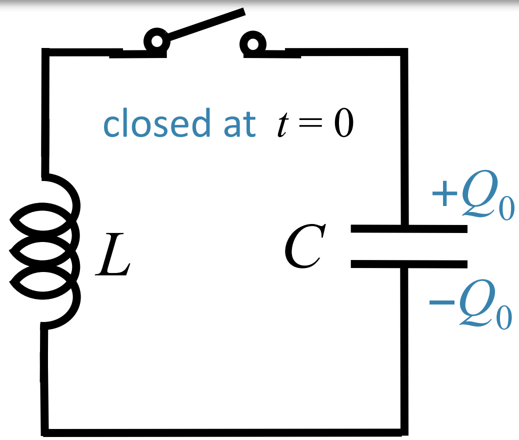
$$I(t) = I_{max}\sin(\omega t)$$

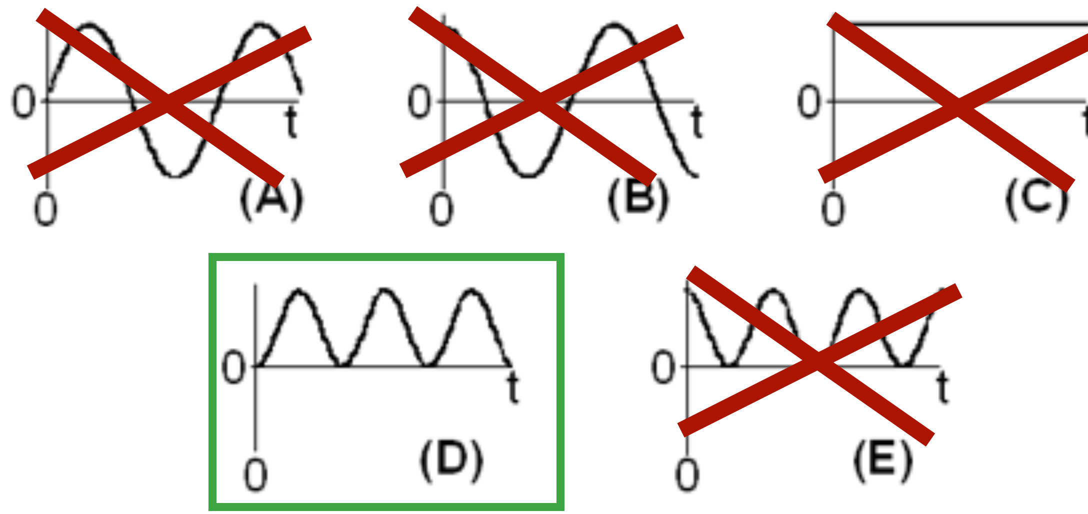
What is the current through the circuit, at the instant the charge on the capacitor is $Q_{max}/2$?

A) $I < I_{max}/2$ B) $I = I_{max}/2$ C) $I > I_{max}/2$

The total energy in the circuit is constant and is the sum of the energy in the capacitor (proporional to Q^2) and the energy in the inductor (proportional to I^2).
If Q = Qmax/2 then only 1/4 of the total energy is in the capacitor, so 3/4 of the energy must be in the inductor, which means that $I = \sqrt{(3/4)}\, I_{max}$, which is bigger than $I_{max}/2$.

CheckPoint 10





Which plot best represents the energy in the inductor as a function of time starting just after the switch is closed?

$$U_L = \frac{1}{2} L I^2$$

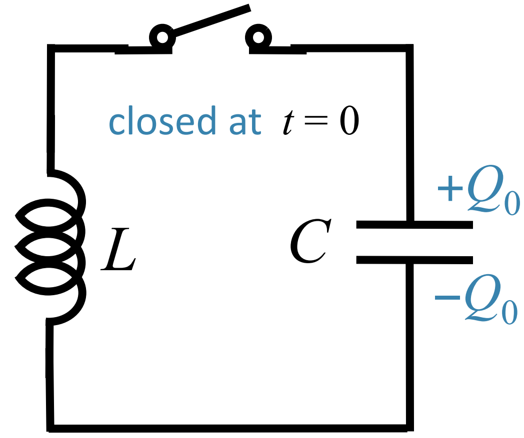
Energy proportional to $I^2 \Rightarrow C$ cannot be negative

Current is changing $\Rightarrow U_L$ is not constant

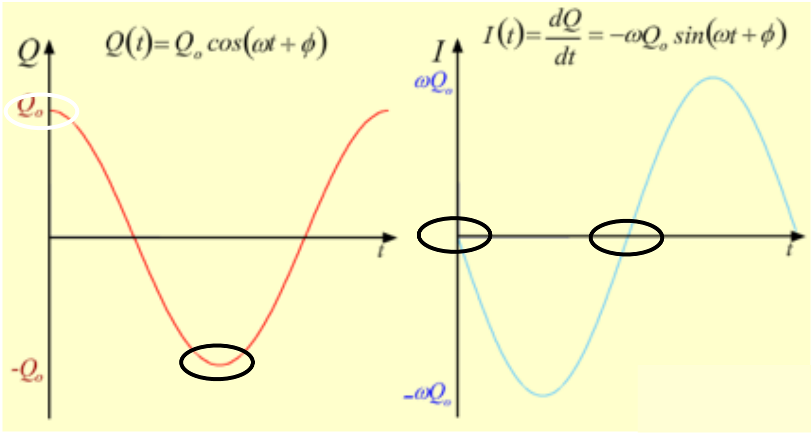
Initial current is zero

CheckPoint 12

When the energy stored in the capacitor reaches its maximum again for the first time after $t = 0$, how much charge is stored on the top plate of the capacitor?



A) $+Q_0$
B) $+Q_0/2$
C) 0
D) $-Q_0/2$
E) $-Q_0$



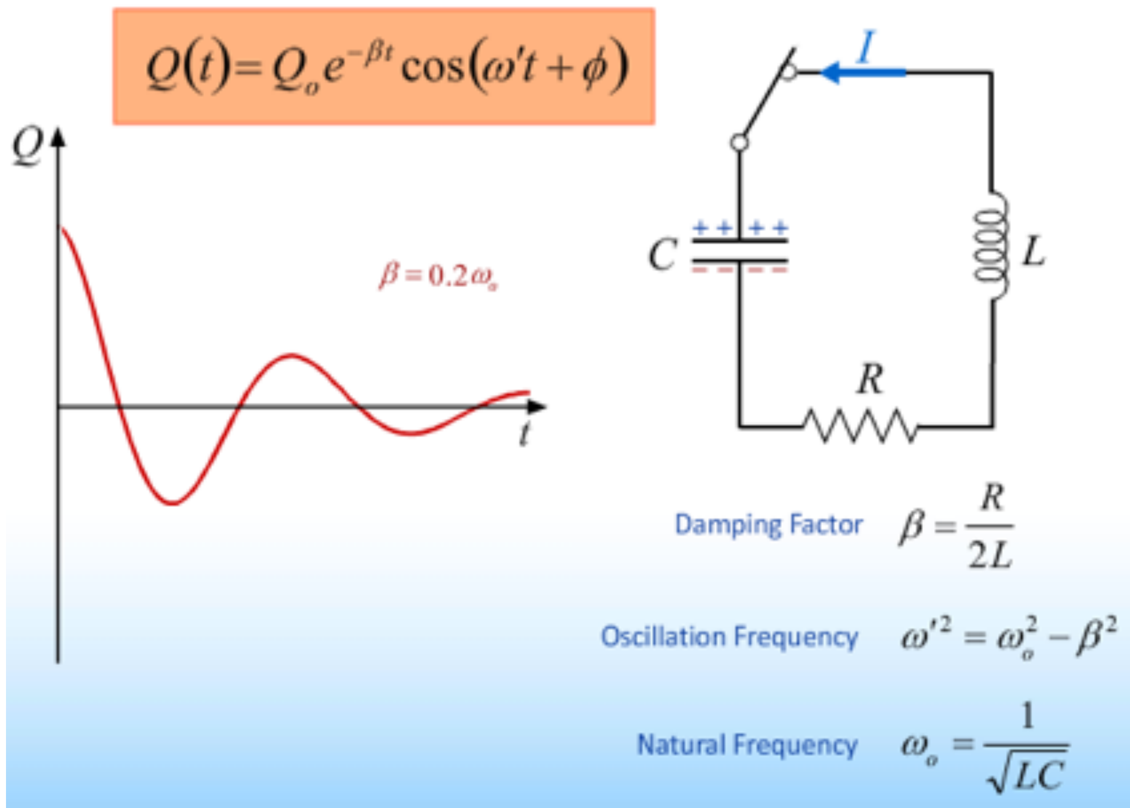
Q is maximum when current goes to zero

$$I = \frac{dQ}{dt}$$

Current goes to zero twice during one cycle

Add R: Damping

Just like LC circuit but the oscillations get smaller because of R



Concept makes sense… …but answer looks kind of complicated

Physics Truth #1:

Even though the answer sometimes looks complicated…

$$Q(t) = Q_o \cos(\omega t - \phi)$$

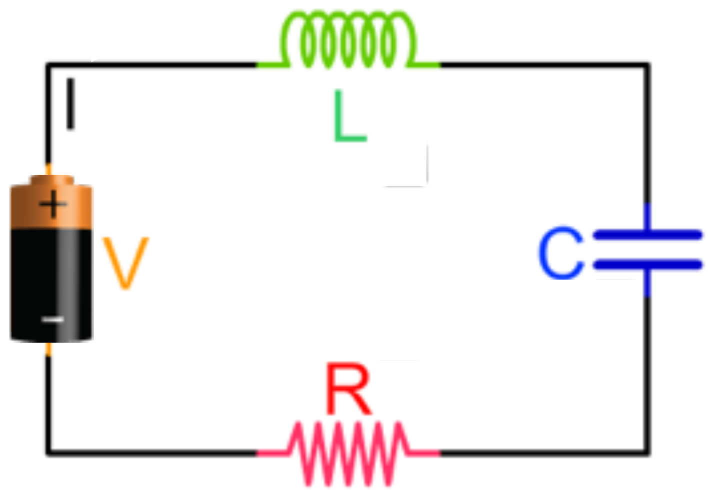
the physics under the hood is still very simple!

$$\frac{d^2Q}{dt^2} = -\omega^2 Q$$

The elements of a circuit are very simple:

$$V_L = L\frac{dI}{dt}$$

$$V = V_L + V_C + V_R$$



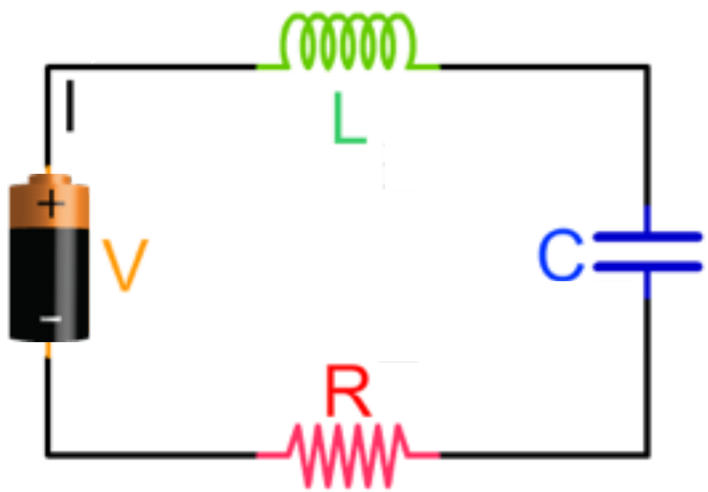
$$V_C = \frac{Q}{C}$$

$$I = \frac{dQ}{dt}$$

$$V_R = IR$$

This is all we need to know to solve for anything!

A Different Approach



Start with some initial V, I, Q, V_L

Now take a tiny time step dt (1 ms)

```
for (var t=0; t<tStepSec; t+=dt) {
    I += Vind_last*dt/L;
    Qcap += I*dt;
    Vcap = Qcap/C;
    Vres = I*R1;
    Vind_last = Vind;
    Vind = Va - Vres - Vcap;
}
```

$$dI = \frac{V_L}{L}dt$$

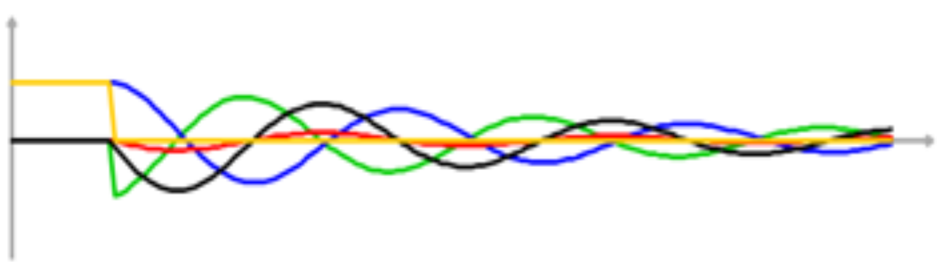
$$dQ = Idt$$

$$V_C = \frac{Q}{C}$$

$$V_R = IR$$

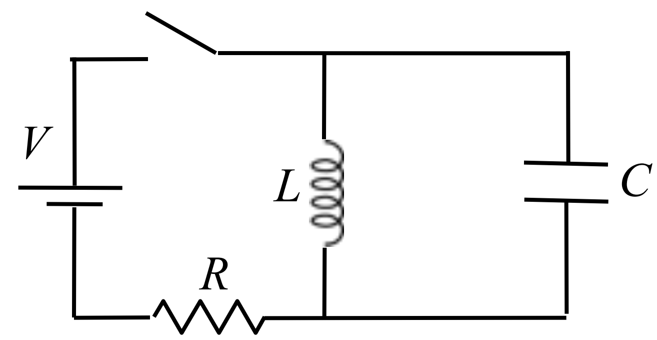
$$V_L = V - V_R - V_C$$

Repeat…



Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



What is Q_{MAX}, the maximum charge on the capacitor?

Conceptual Analysis

Once switch is opened, we have an LC circuit

Current will oscillate with natural frequency ω_0

Strategic Analysis

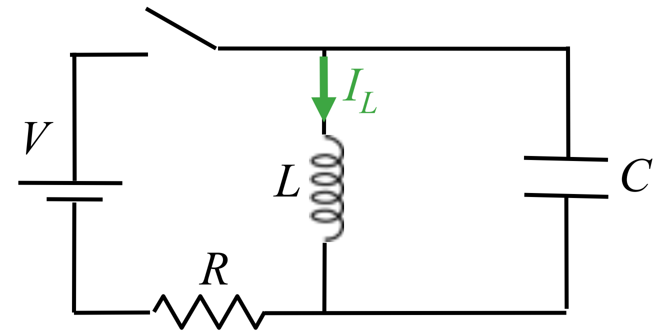
Determine initial current

Determine oscillation frequency ω_0

Find maximum charge on capacitor

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



What is I_L, the current in the inductor, immediately after the switch is opened? Take positive direction as shown.

A) $I_L < 0$ B) $I_L = 0$ C) $I_L > 0$

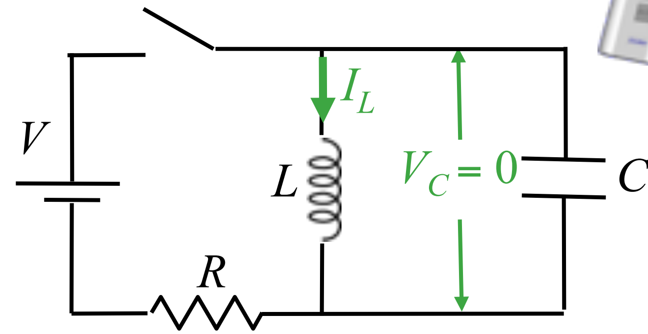
Current through inductor immediately after switch is opened
is the same as
the current through inductor immediately before switch is opened

before switch is opened:

all current goes through inductor in direction shown

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



$I_L(t = 0+) > 0$

The energy stored in the capacitor immediately after the switch is opened is zero.

A) TRUE B) FALSE

before switch is opened:

$dI_L/dt \sim 0 \Rightarrow V_L = 0$

BUT: $V_L = V_C$

since they are in parallel

→ $V_C = 0$

after switch is opened:

V_C cannot change abruptly

→ $V_C = 0$

→ $U_C = \frac{1}{2} C V_C^2 = 0$!

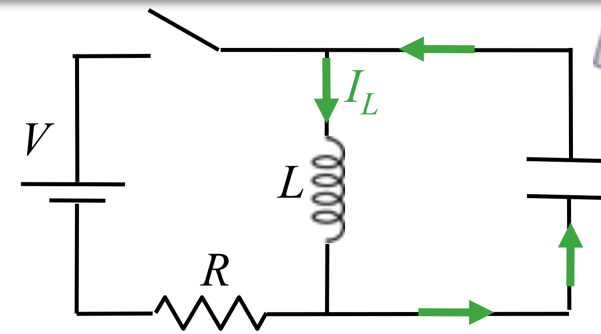
IMPORTANT: NOTE DIFFERENT CONSTRAINTS AFTER SWITCH OPENED

CURRENT through INDUCTOR cannot change abruptly

VOLTAGE across CAPACITOR cannot change abruptly

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



$I_L(t = 0+) > 0$ $V_C(t = 0+) = 0$

What is the direction of the current immediately after the switch is opened?

A) clockwise

B) counterclockwise

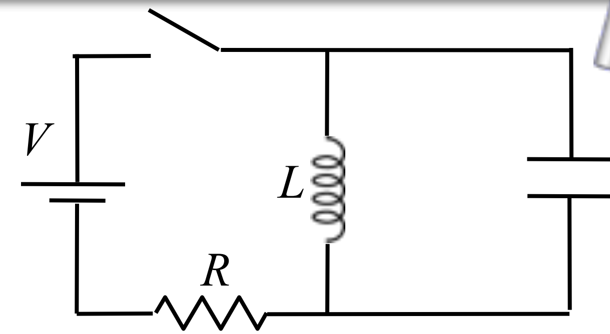
Current through inductor immediately after switch is opened
is the same as
the current through inductor immediately before switch is opened

Before switch is opened: Current moves down through L

After switch is opened: Current continues to move down through L

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



$I_L(t = 0+) > 0$ $V_C(t = 0+) = 0$

What is the magnitude of the current right after the switch is opened?

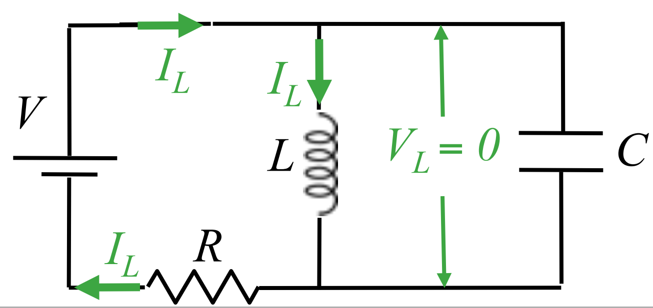
A) $I_o = V\sqrt{\frac{C}{L}}$ B) $I_o = \frac{V}{R^2}\sqrt{\frac{L}{C}}$ C) $I_o = \frac{V}{R}$ D) $I_o = \frac{V}{2R}$

Current through inductor immediately after switch is opened

is the same as

the current through inductor immediately before switch is opened

Before switch is opened:



$V_L = 0$

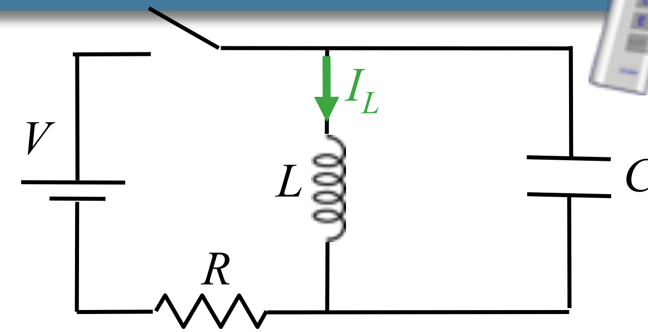
$\downarrow$

$V = I_L R$

Calculation

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.

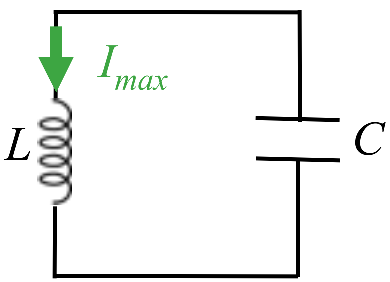
Hint: Energy is conserved



$I_L(t = 0+) = V/R$ $\quad V_C(t = 0+) = 0$

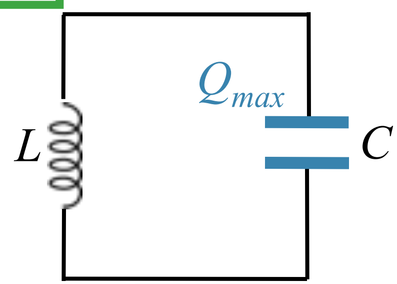
What is Q_{max}, the maximum charge on the capacitor during the oscillations?

A) $Q_{\max} = \frac{V}{R}\sqrt{LC}$ B) $Q_{\max} = \frac{1}{2}CV$ C) $Q_{\max} = CV$ D) $Q_{\max} = \frac{V}{R\sqrt{LC}}$



When I is max (and Q is 0)

$$U = \frac{1}{2}LI^2$$



When Q is max (and I is 0)

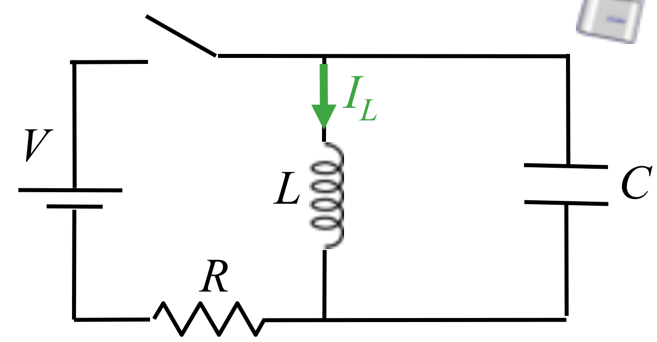
$$U = \frac{1}{2}\frac{Q_{\max}^2}{C}$$

$$\frac{1}{2}LI^2 = \frac{1}{2}\frac{Q_{\max}^2}{C}$$

$$Q_{\max} = I_{\max}\sqrt{LC} = \frac{V}{R}\sqrt{LC}$$

Follow-Up

The switch in the circuit shown has been closed for a long time. At $t = 0$, the switch is opened.



Is it possible for the maximum voltage on the capacitor to be greater than V ?

$I_{max} = V/R$ $\qquad Q_{max} = \frac{V}{R}\sqrt{LC}$

A) YES

B) NO

$$Q_{max} = \frac{V}{R}\sqrt{LC} \rightarrow V_{max} = \frac{V}{R}\sqrt{\frac{L}{C}} \rightarrow$$

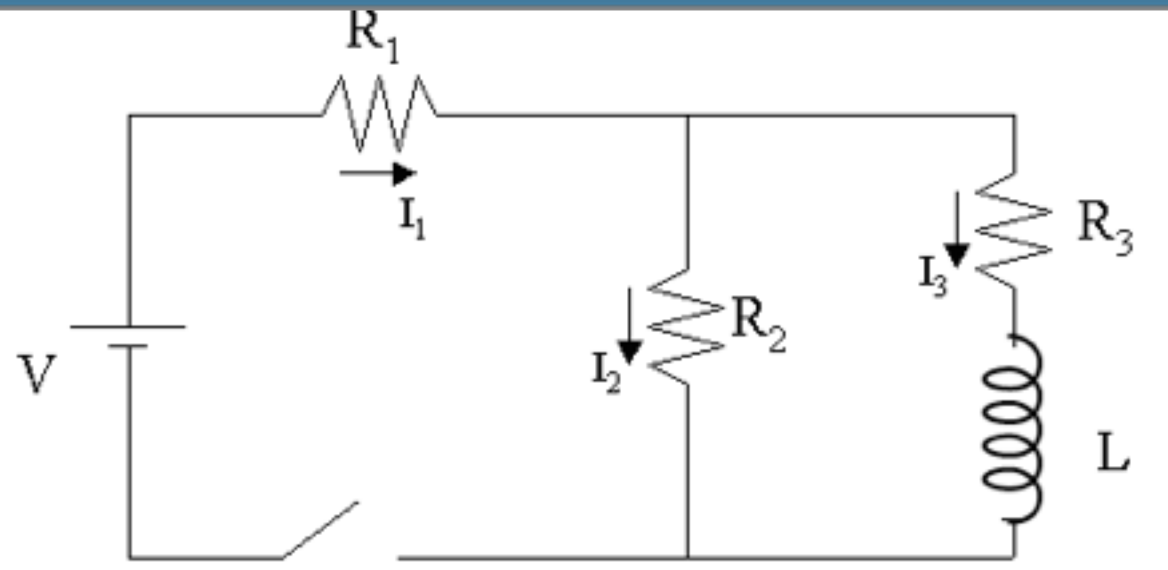
V_{max} can be greater than V IF: $\sqrt{\frac{L}{C}} > R$

We can rewrite this condition in terms of the resonant frequency:

$$\omega_0 L > R \quad \text{OR} \quad \frac{1}{\omega_0 C} > R$$

We will see these forms again when we study AC circuits!

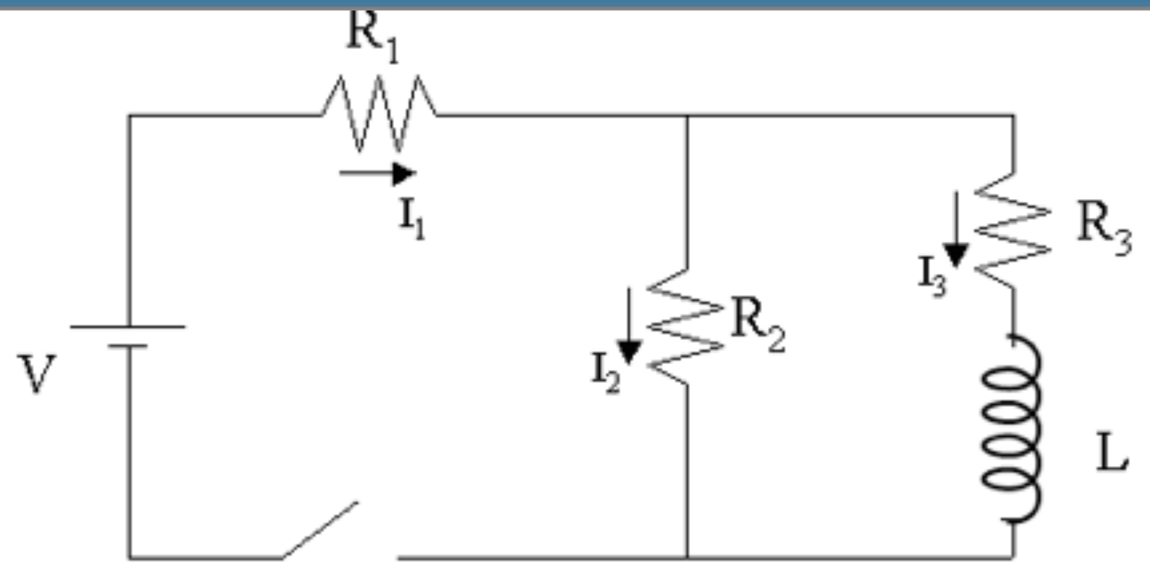
FlipItPhysics Problem



At what time, does the current through the inductor I_3 reach a value that is 63% of its maximum value?

Apply Kirchhoff's rules to (1) outer loop, (2) inner loop and (3) the junction.

FlipItPhysics Problem



(1) $V - I_1 R_1 - I_3 R_3 - L\frac{dI_3}{dt} = 0$

(2) $-I_3 R_3 - L\frac{dI_3}{dt} + I_2 R_2 = 0$

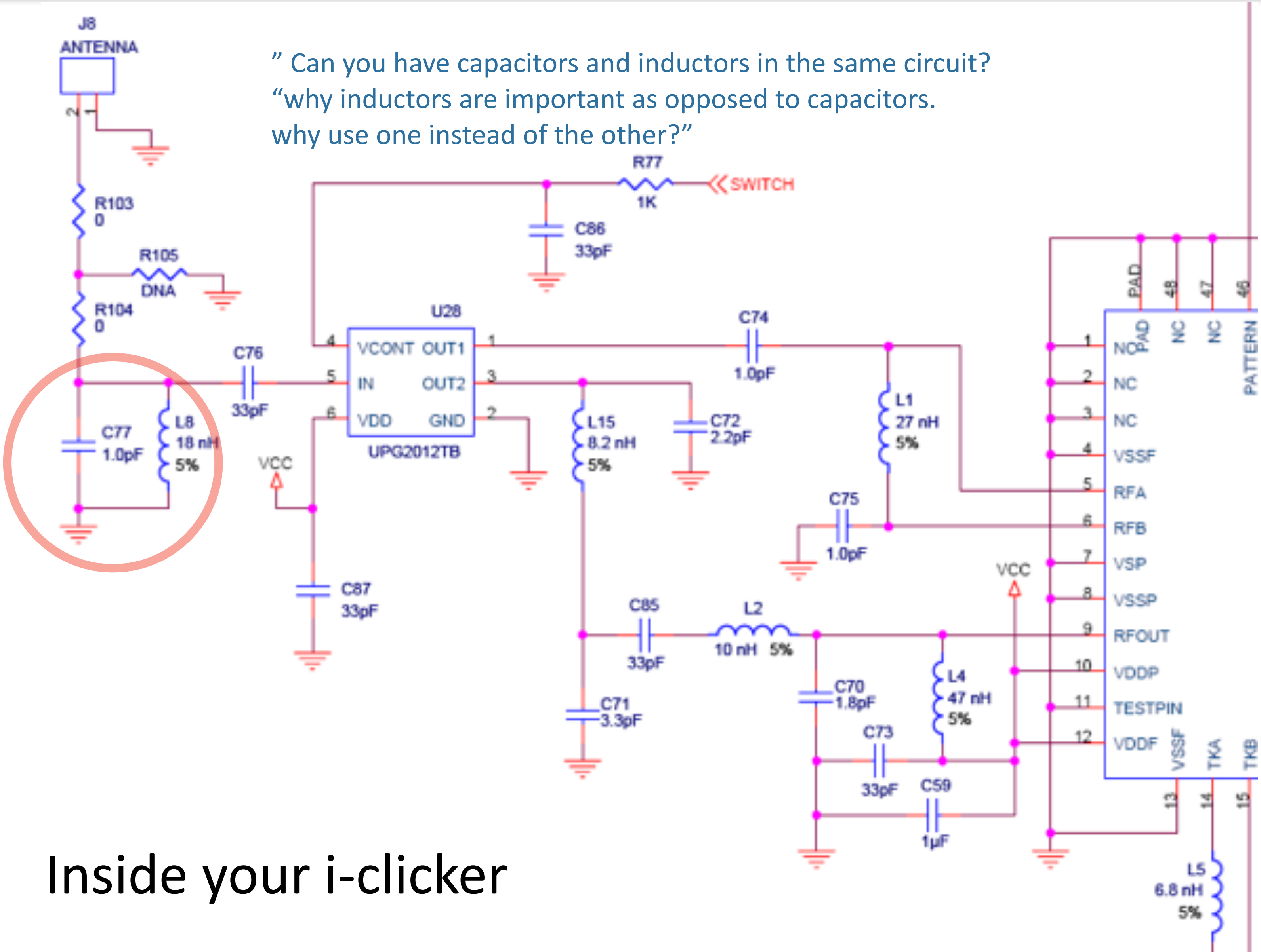
(3) $I_1 = I_2 + I_3$

Eliminate I_1 and I_2 and find the equation for I_3 of the form

$$V - AI_3 - B\frac{dI_3}{dt} = 0 \qquad \tau = \frac{B}{A}$$

What are Inductors and Capacitors Good For?

" Can you have capacitors and inductors in the same circuit?
"why inductors are important as opposed to capacitors.
why use one instead of the other?"



Inside your i-clicker