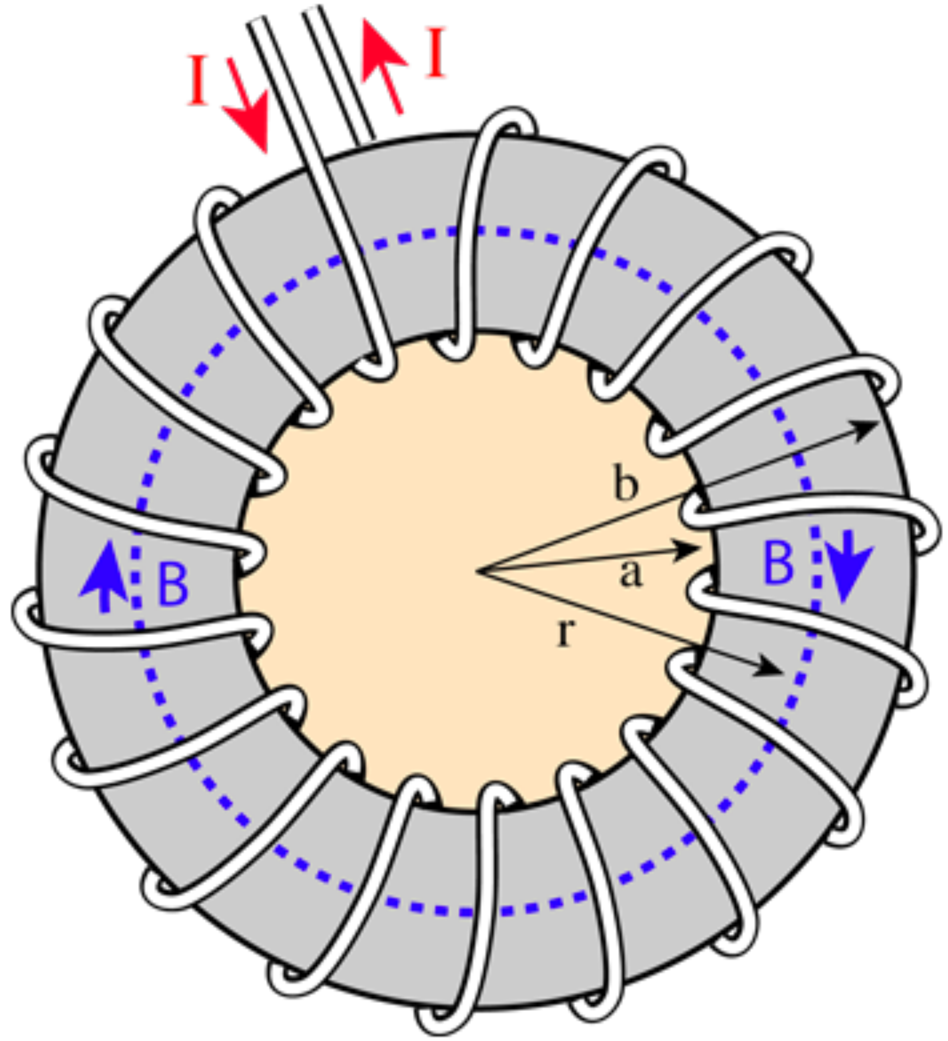
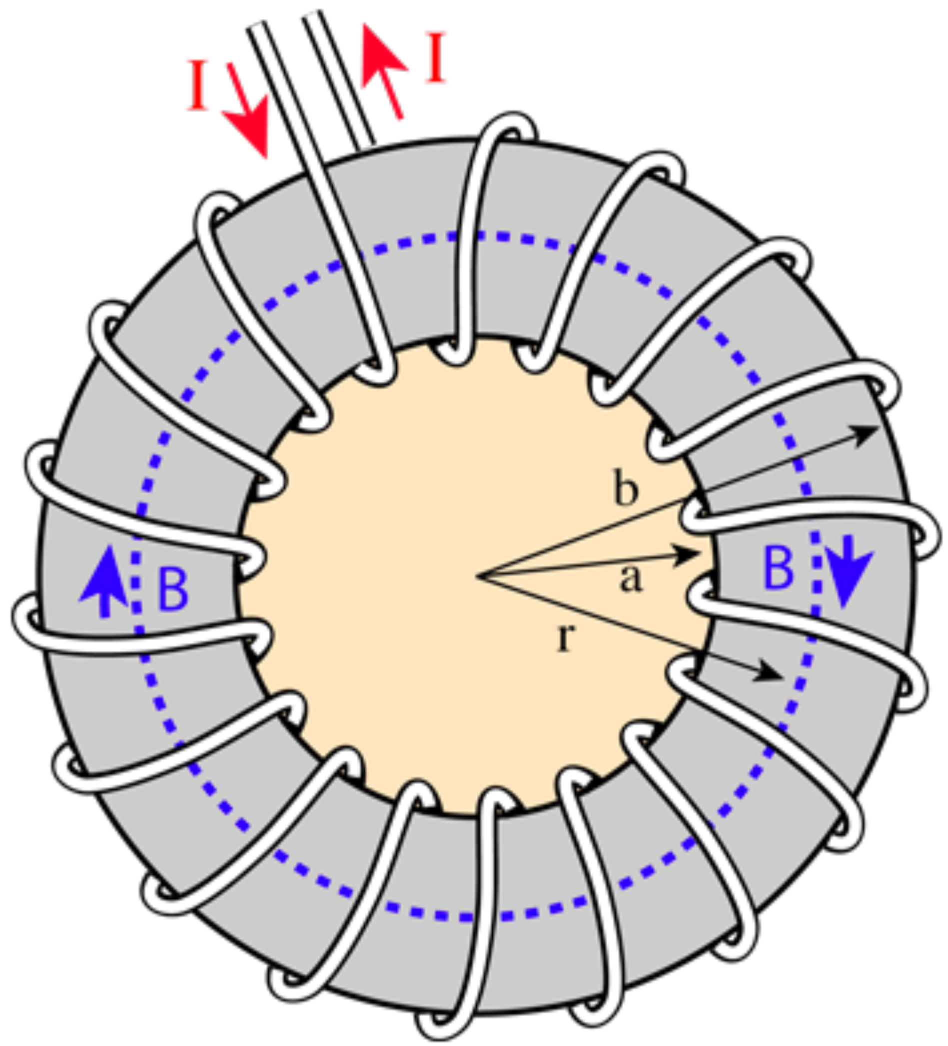
Toroidal Solenoid



$$\mu_0 I_{encl} = \oint \vec{B} \cdot d\vec{\ell}$$



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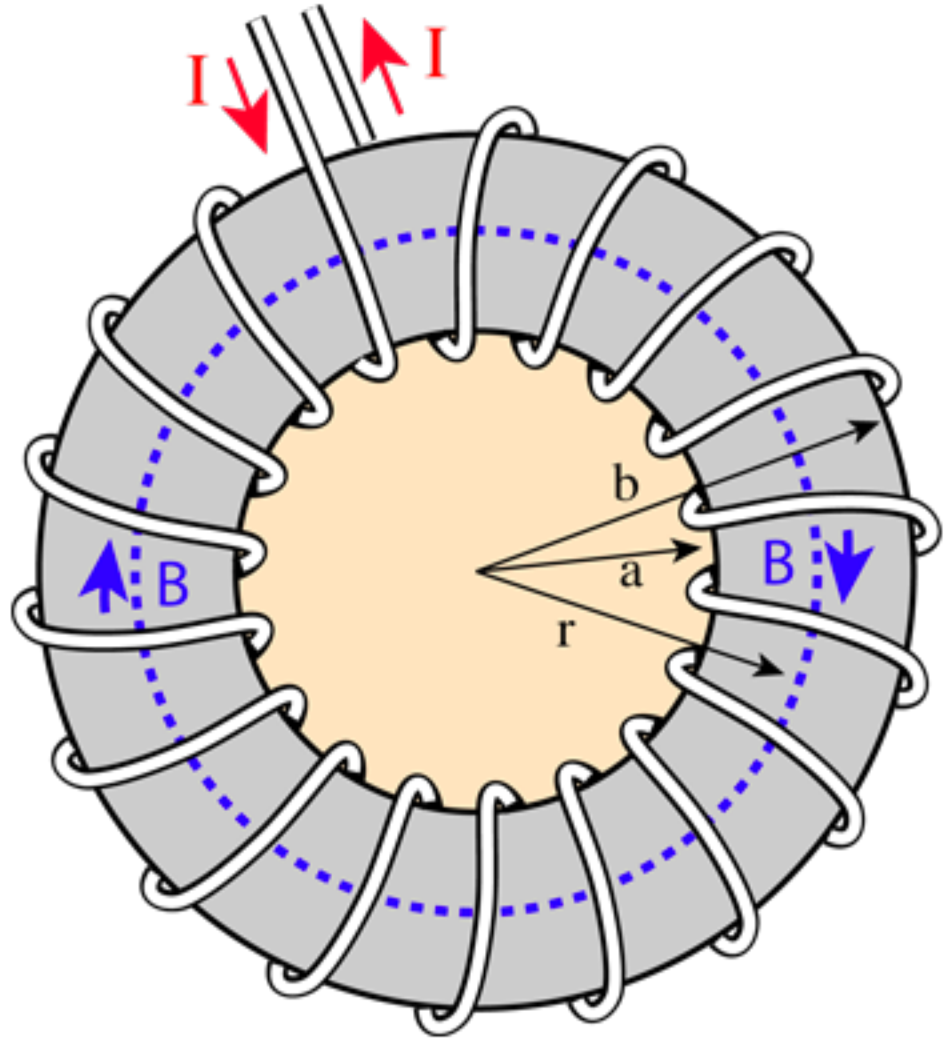
I_{encl} = NI = Number of loops x current in the wire



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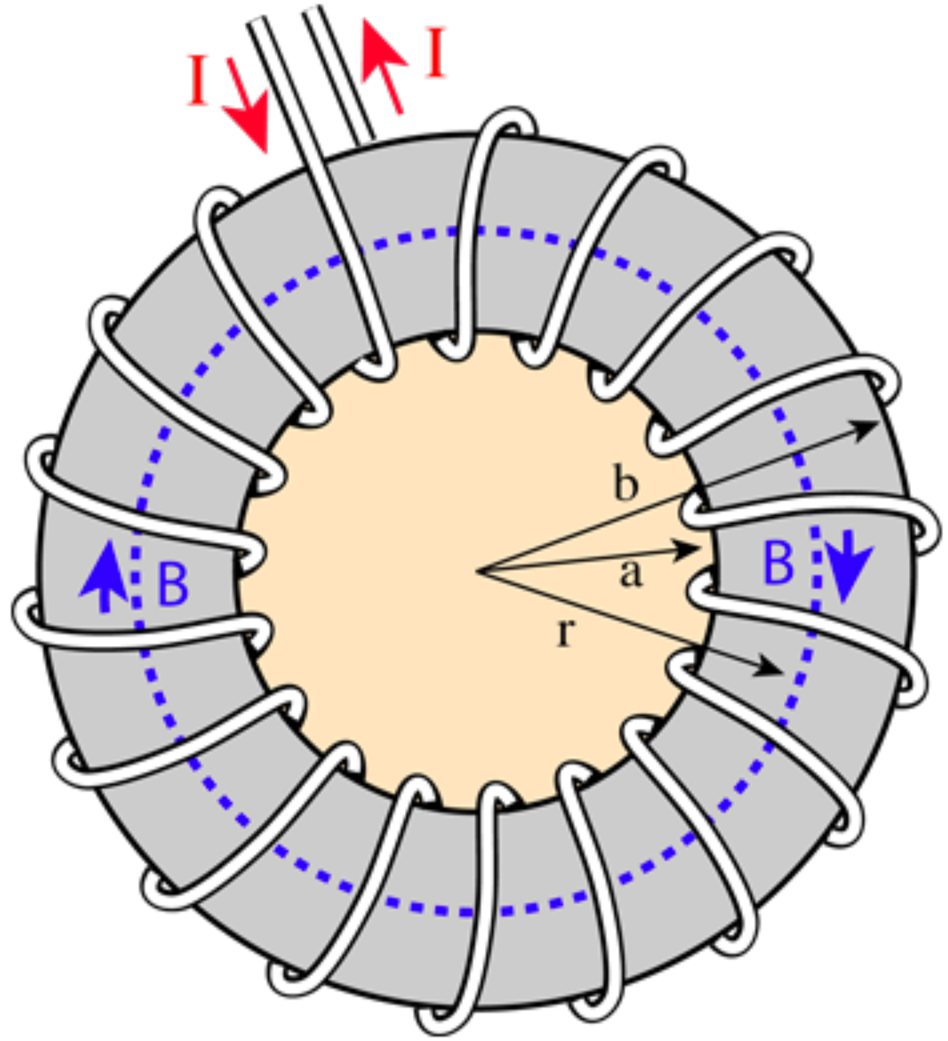
B is constant in magnitude and tangent to the dotted amperian loop.



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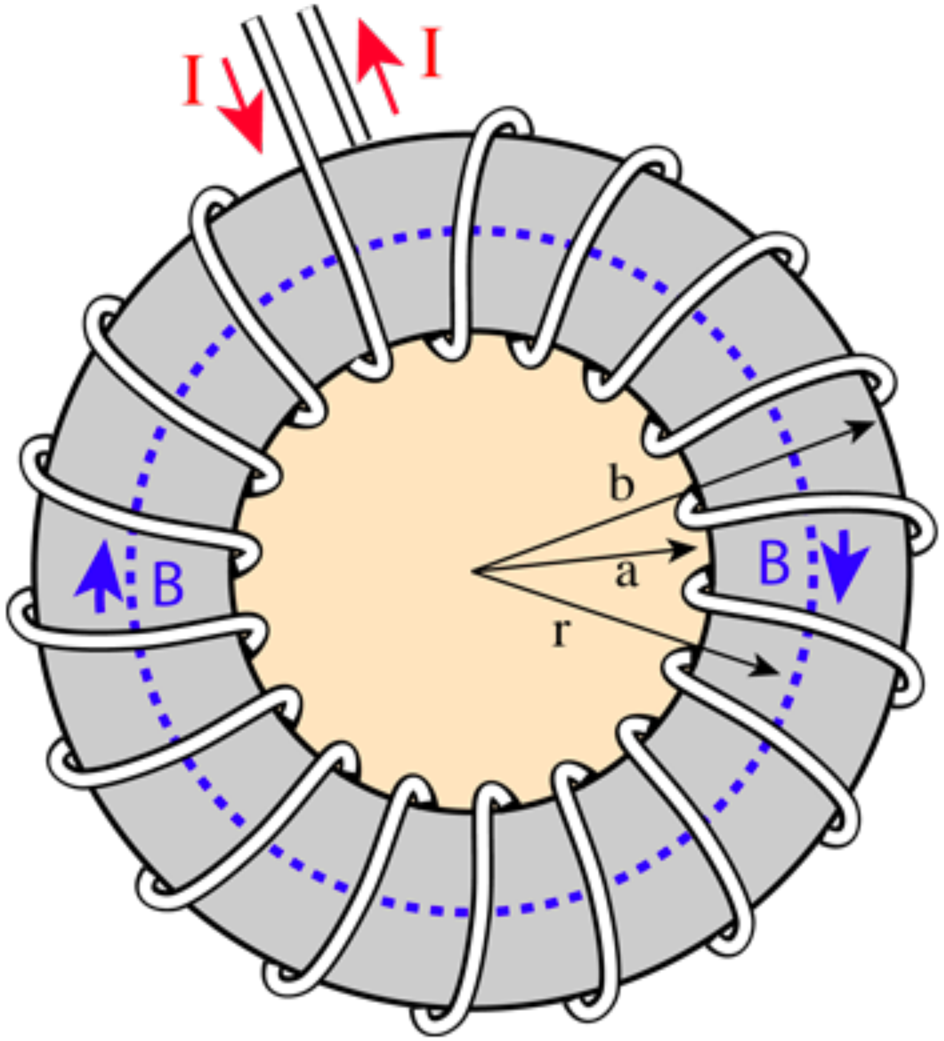
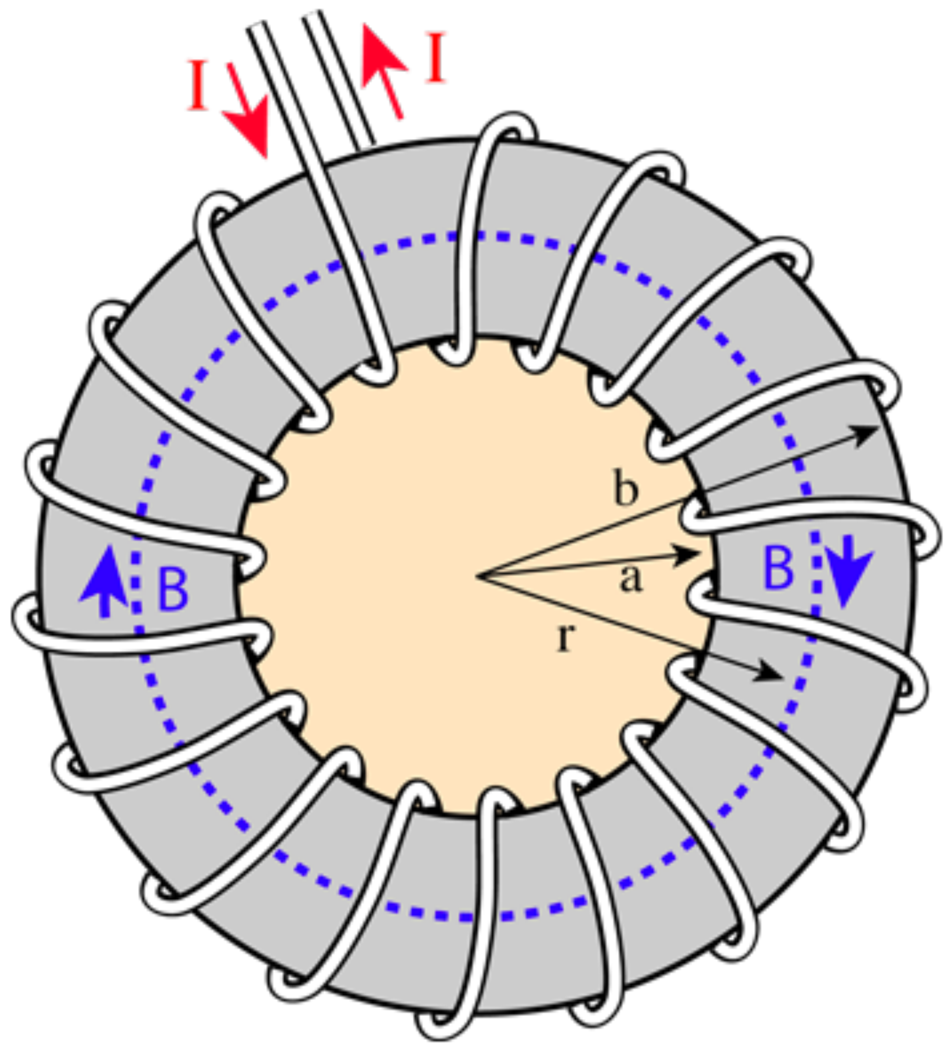
by symmetry



N loops

$$\mu_0 I_{encl} = \oint \vec{B} \cdot d\vec{\ell}$$

$$\mu_0 N I = B \oint d\ell = B(2\pi r)$$



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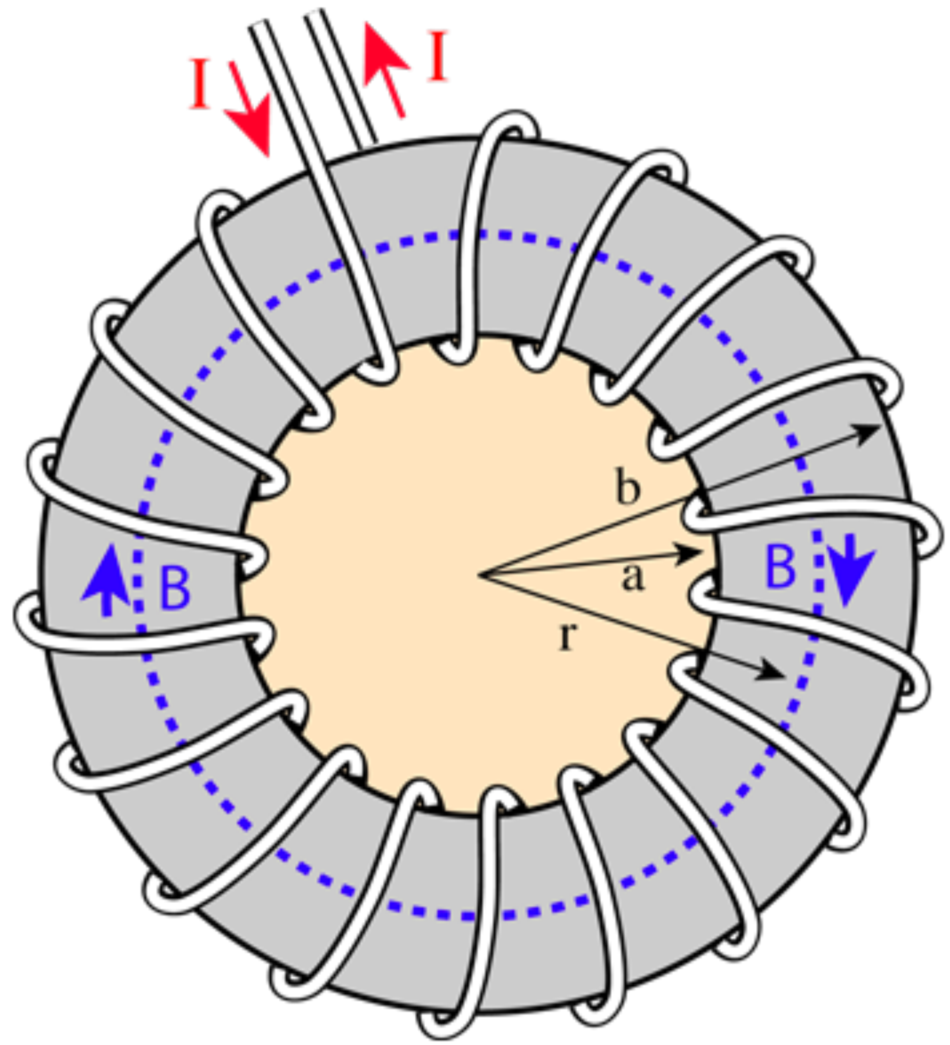

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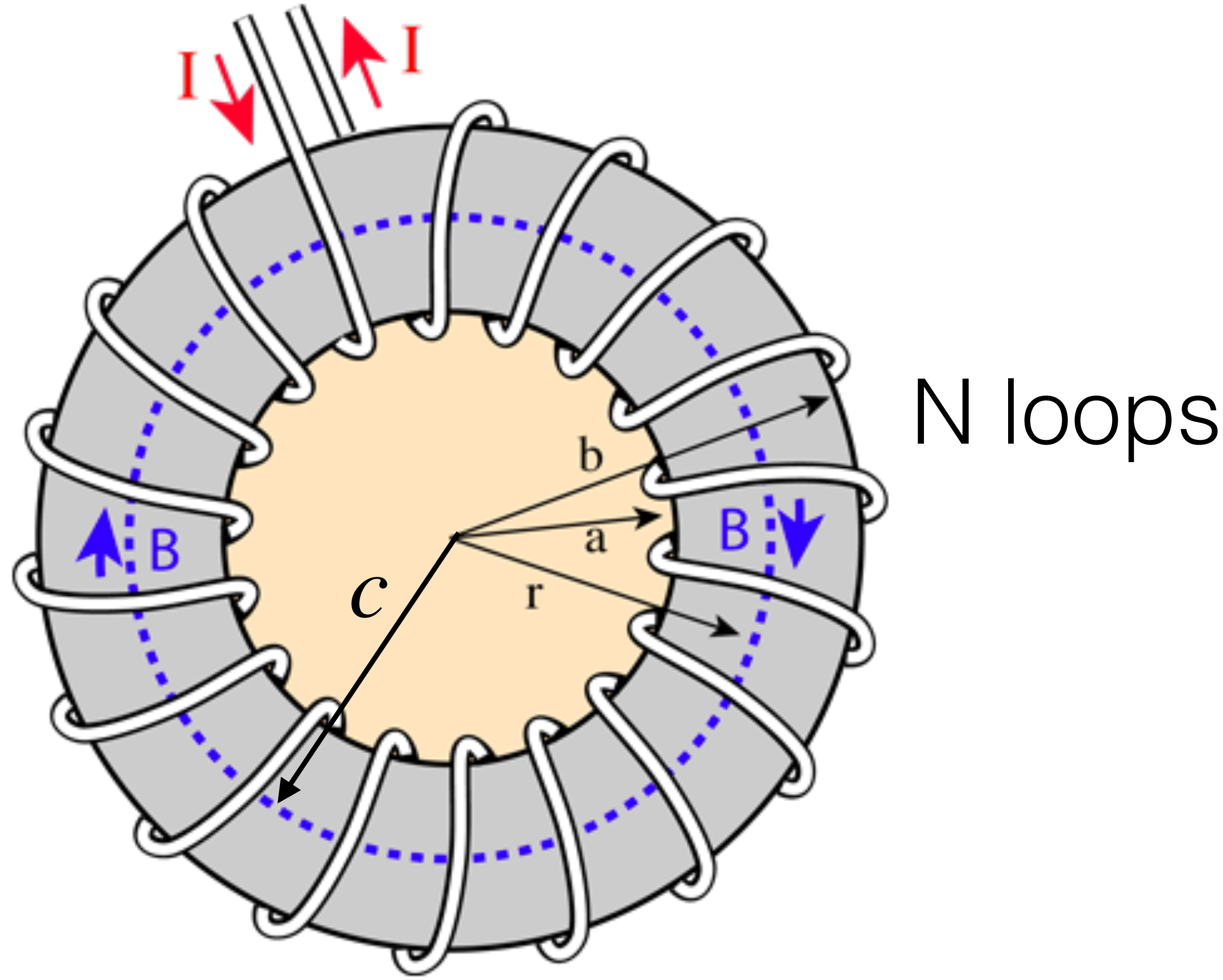

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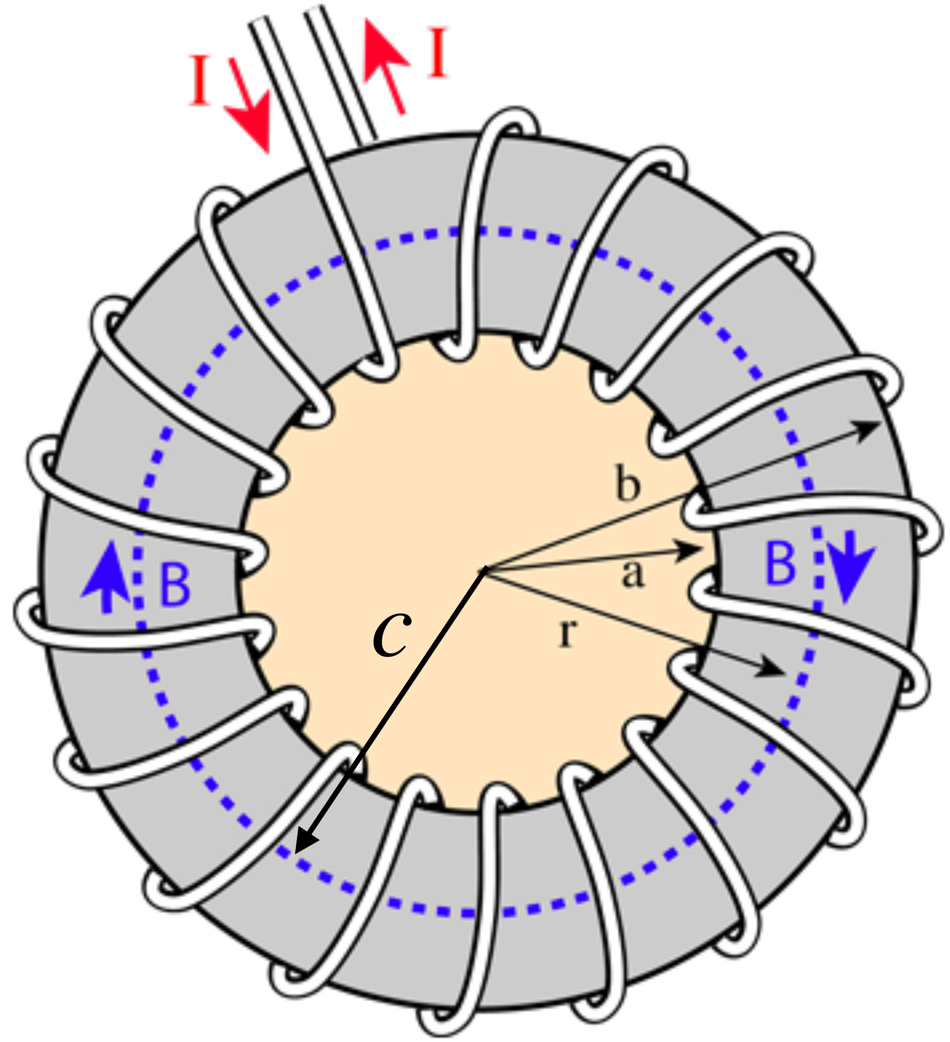

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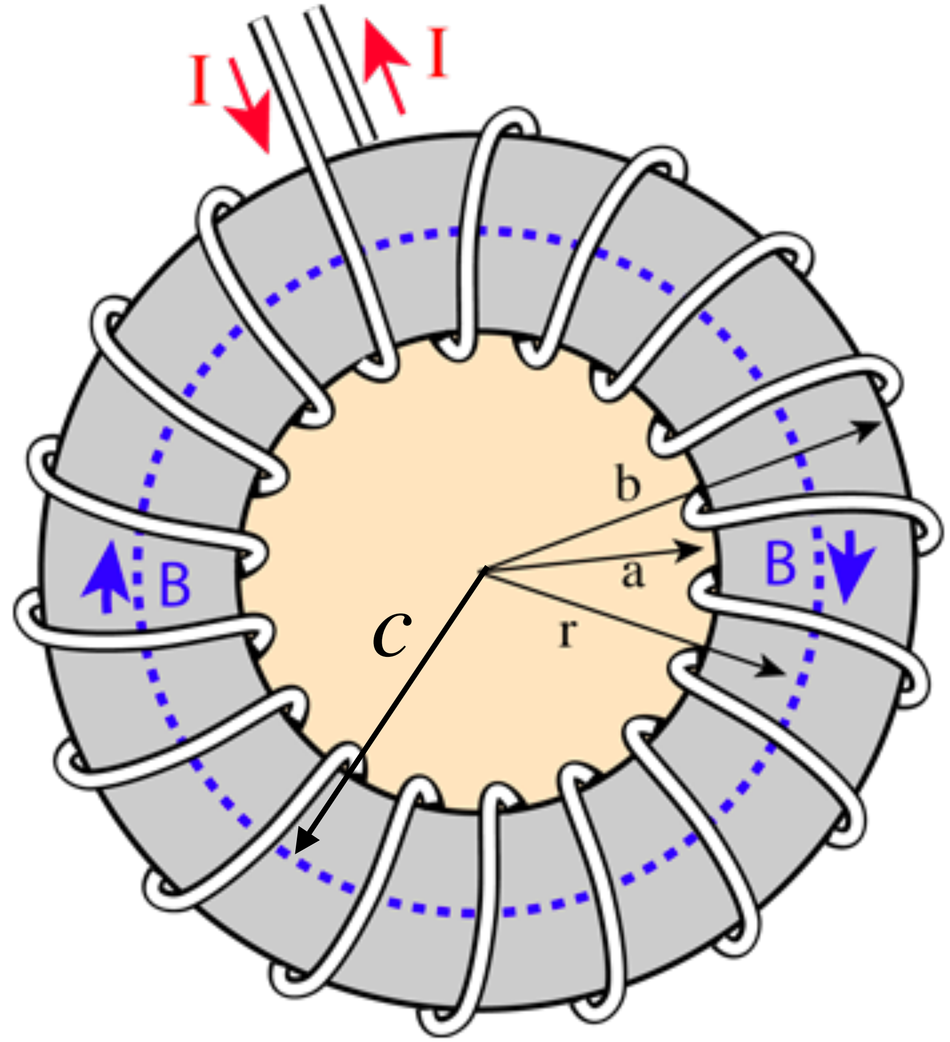
let c be the centre radius



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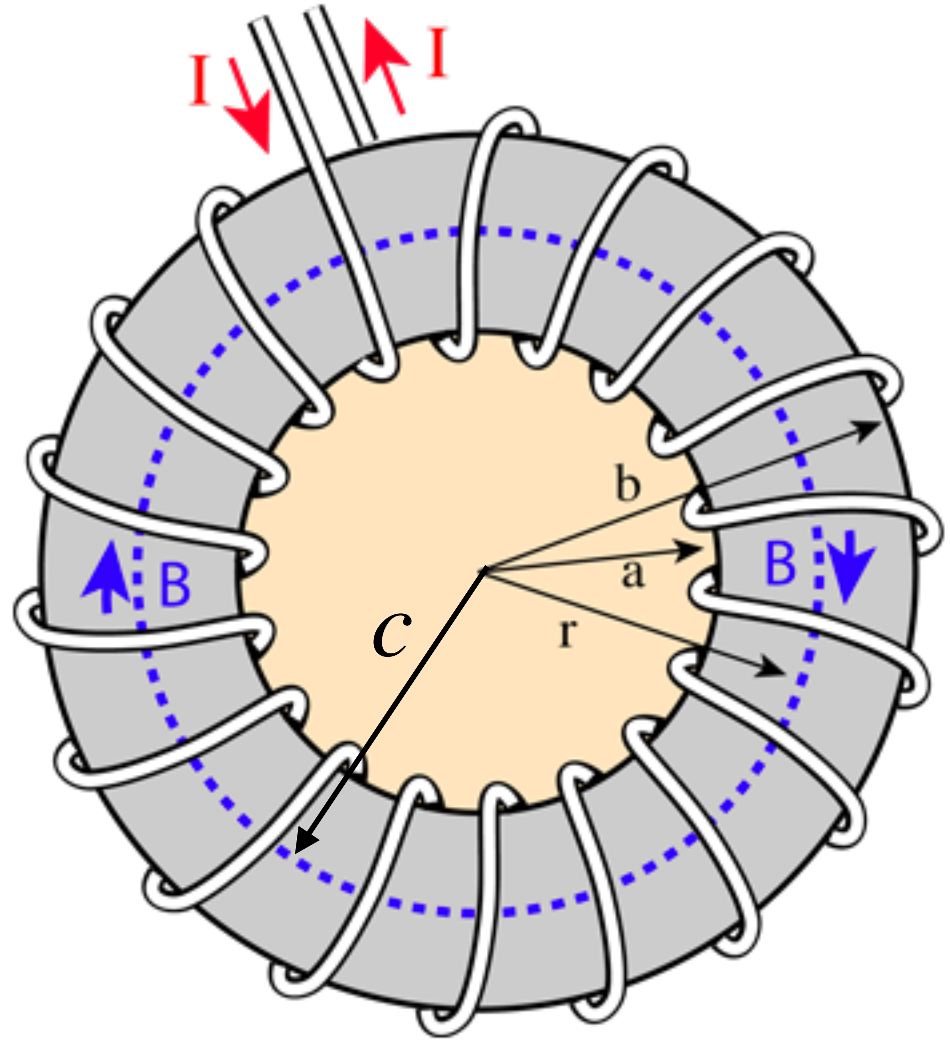
$$N = 2\pi c\, n$$



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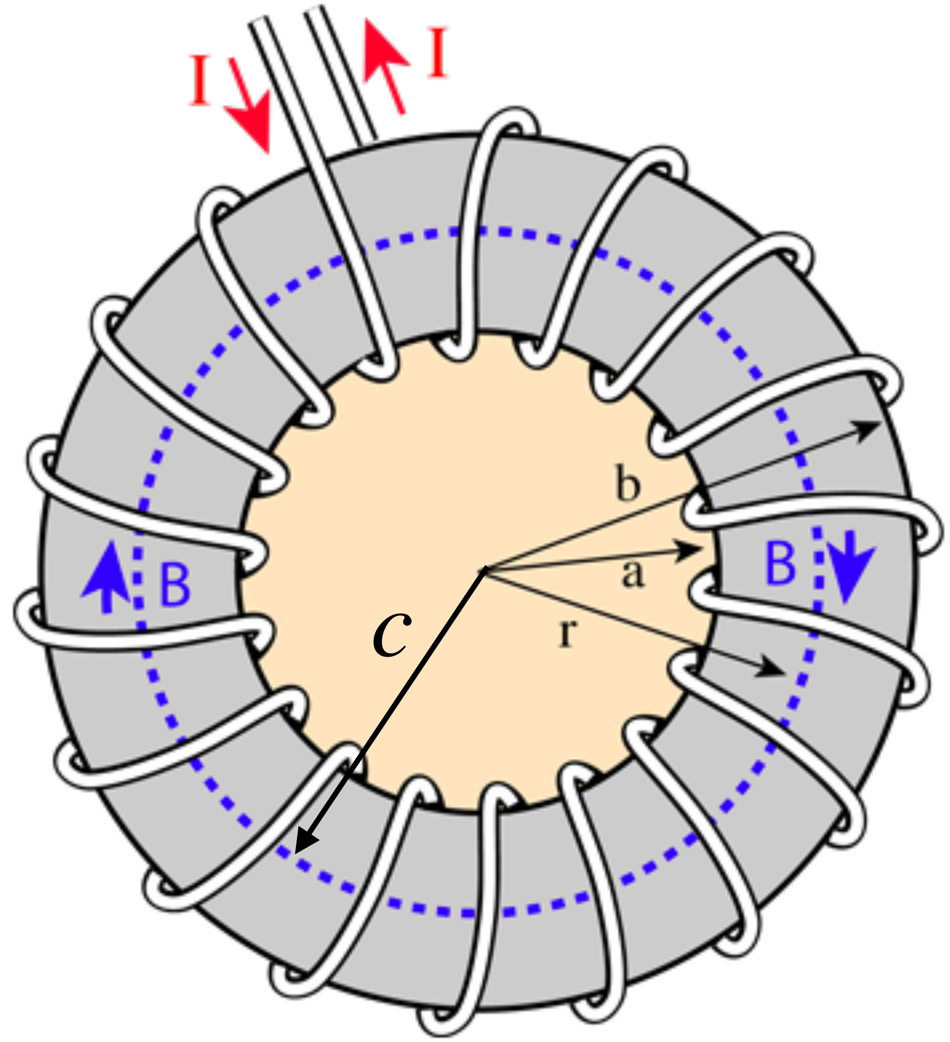

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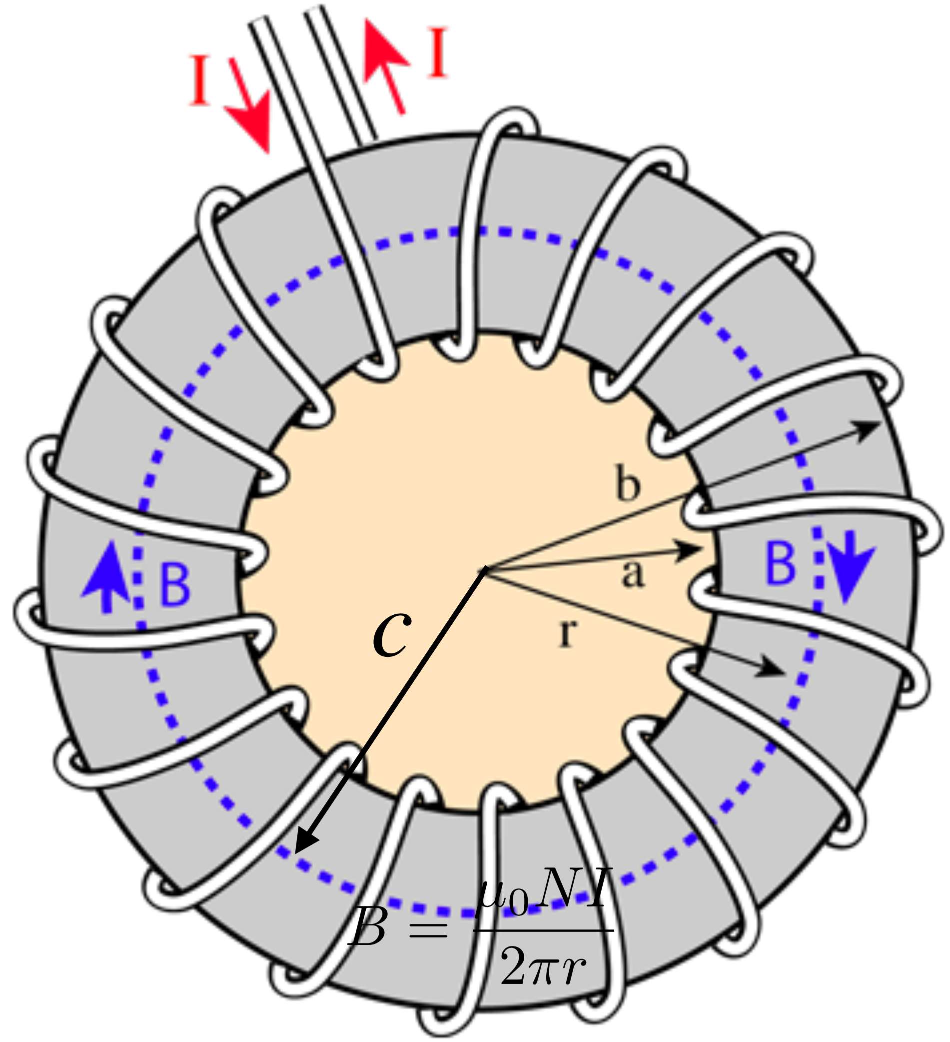
as the torus gets bigger $c\,/\,r \rightarrow 1$ then $B \rightarrow \mu_0 n I$



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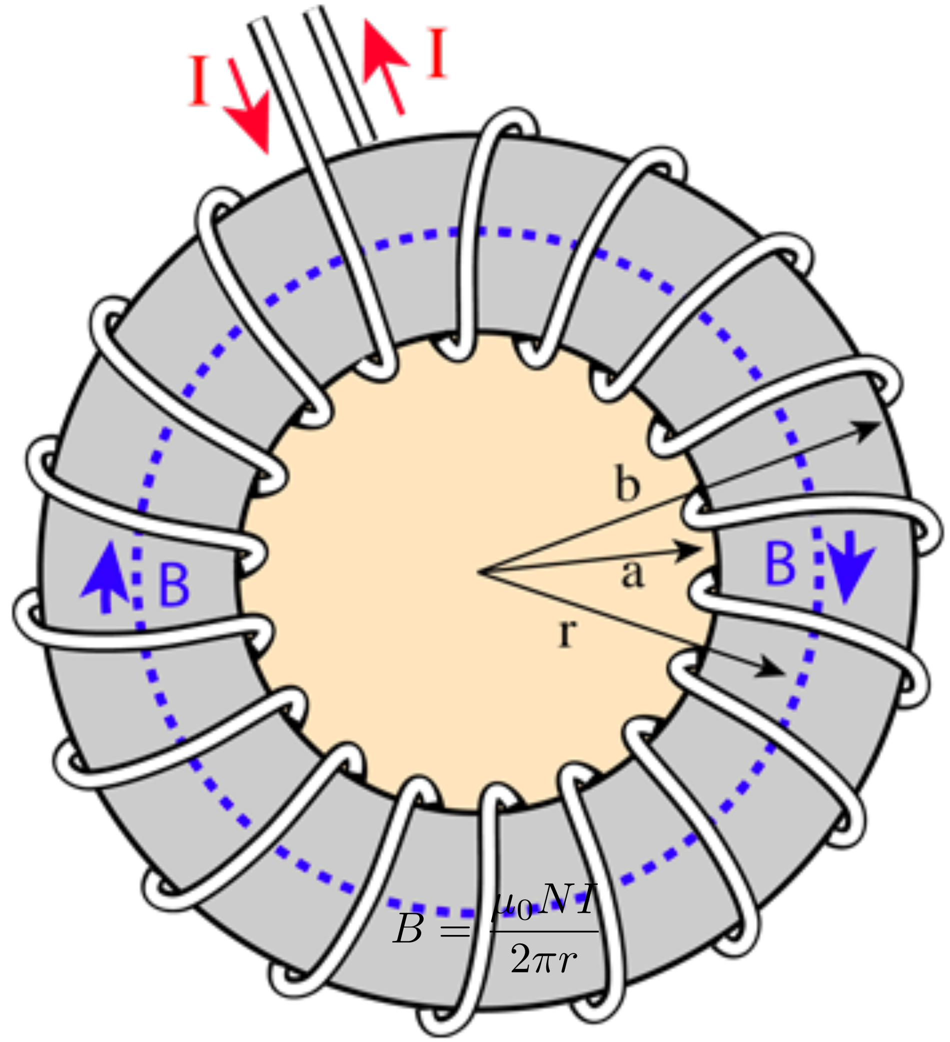

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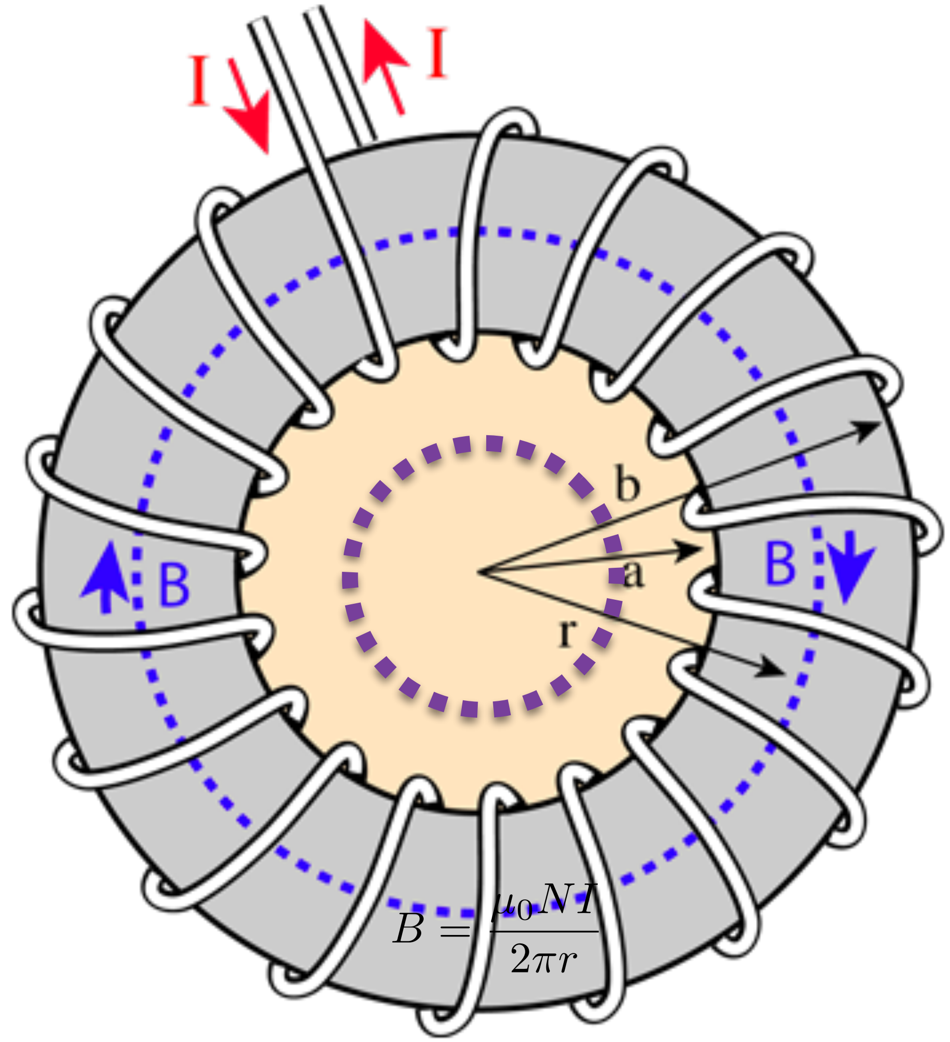

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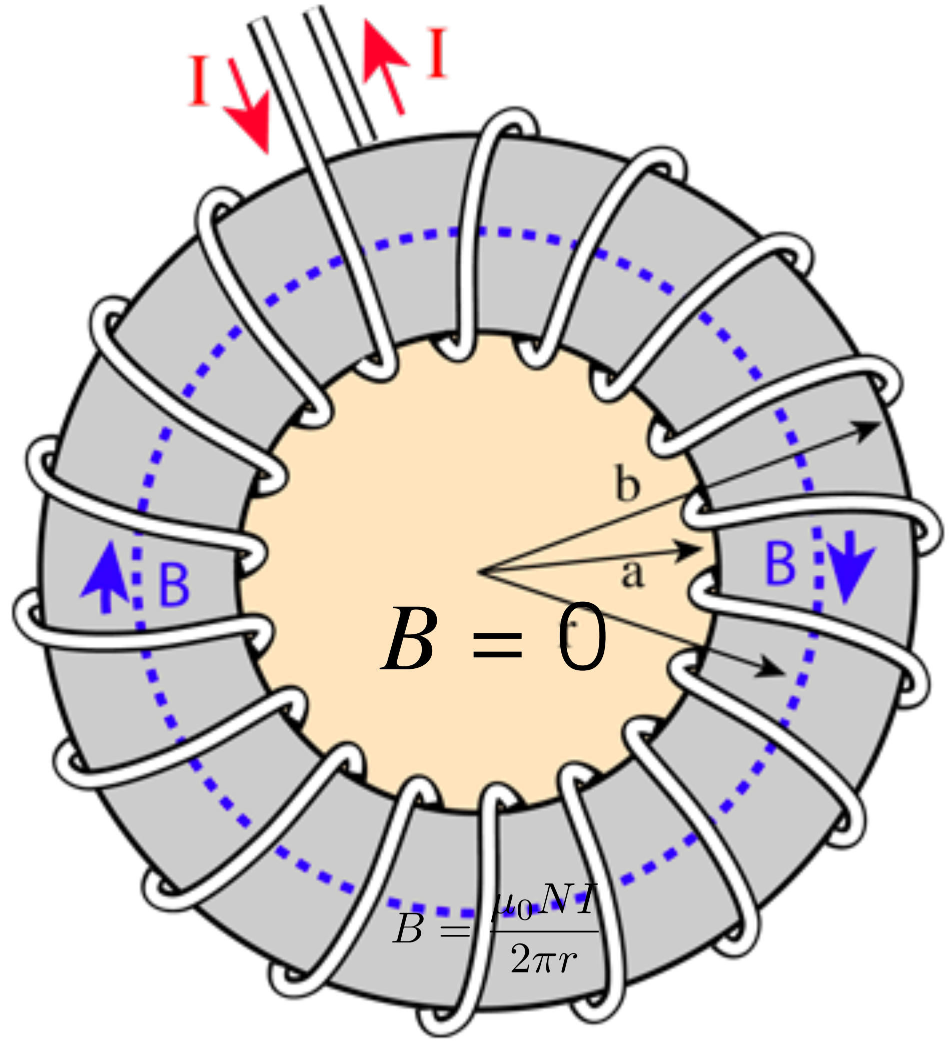
Amperian loop with radius < **a** encloses no current.

$$B = 0$$



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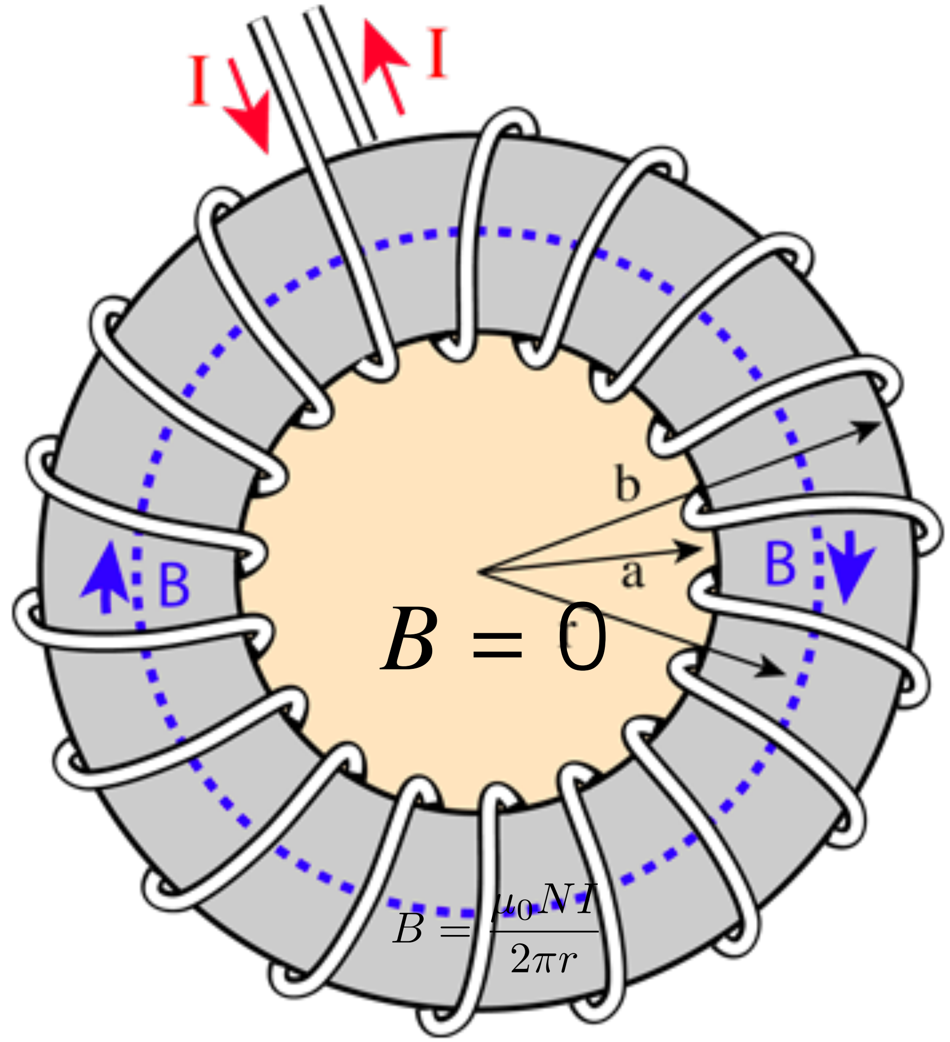

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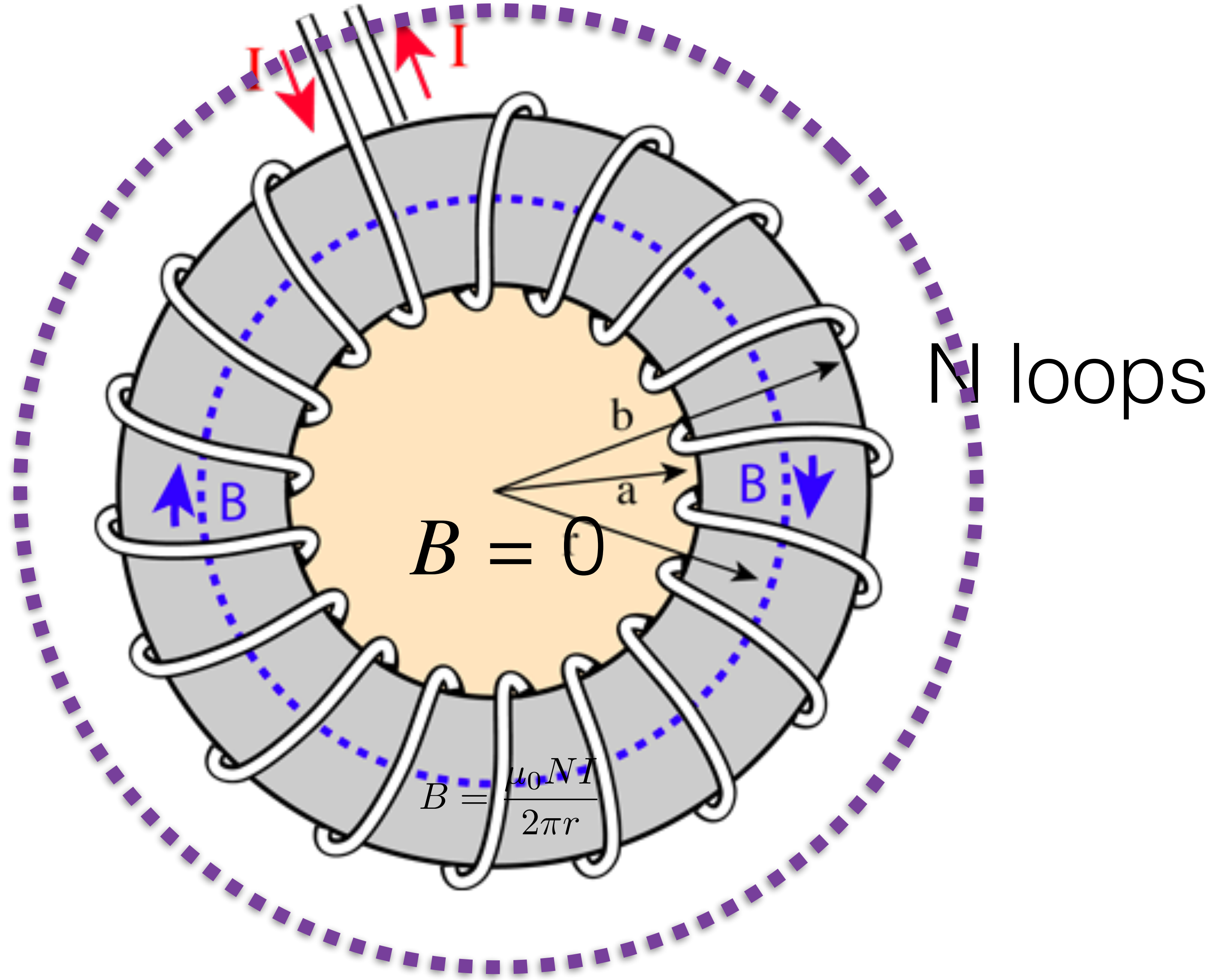
I
I
N loops
b
a
B
B
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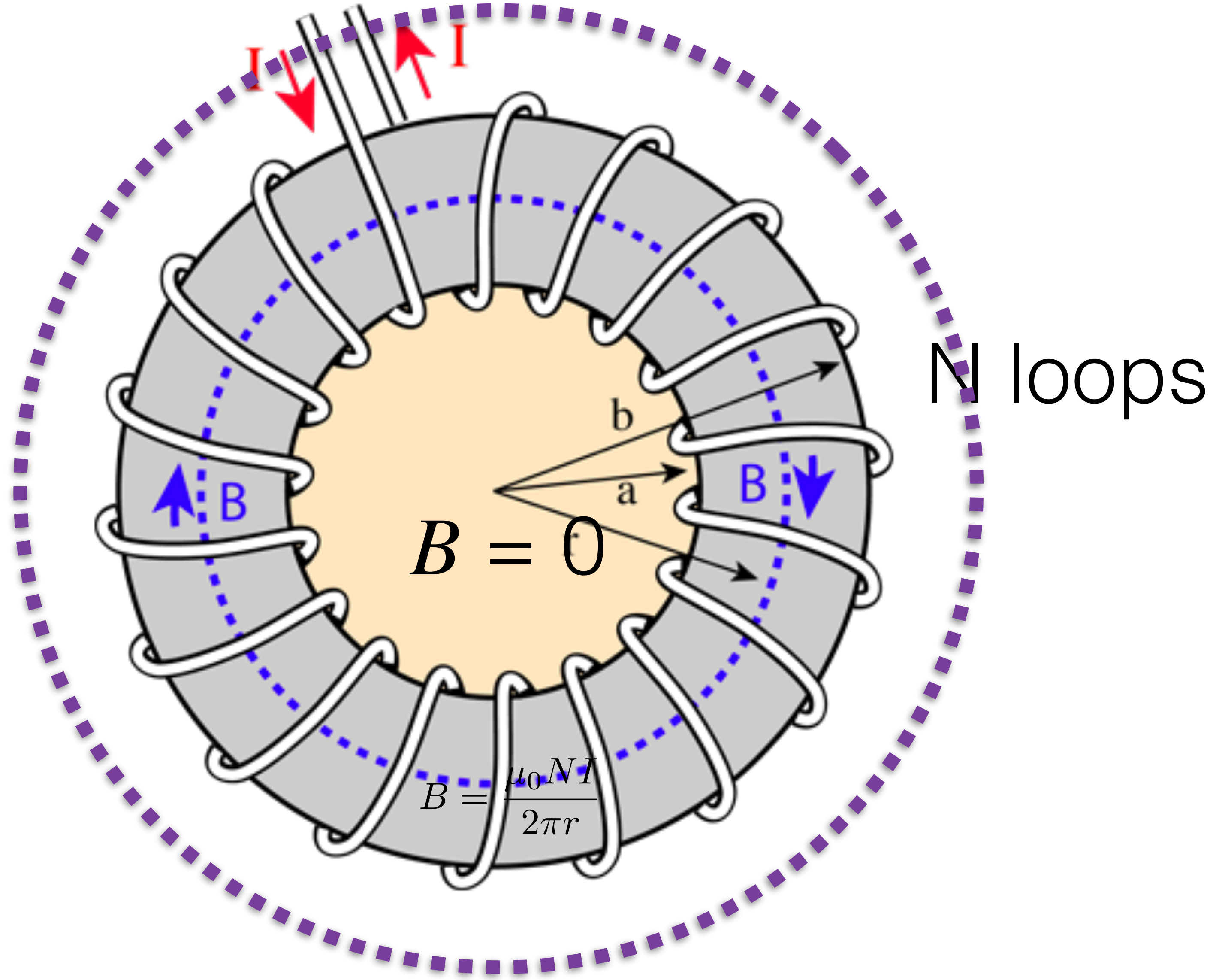
$B = \frac{\mu_0 NI}{2\pi r}$



Amperian loop with radius > **b** encloses no **net** current.

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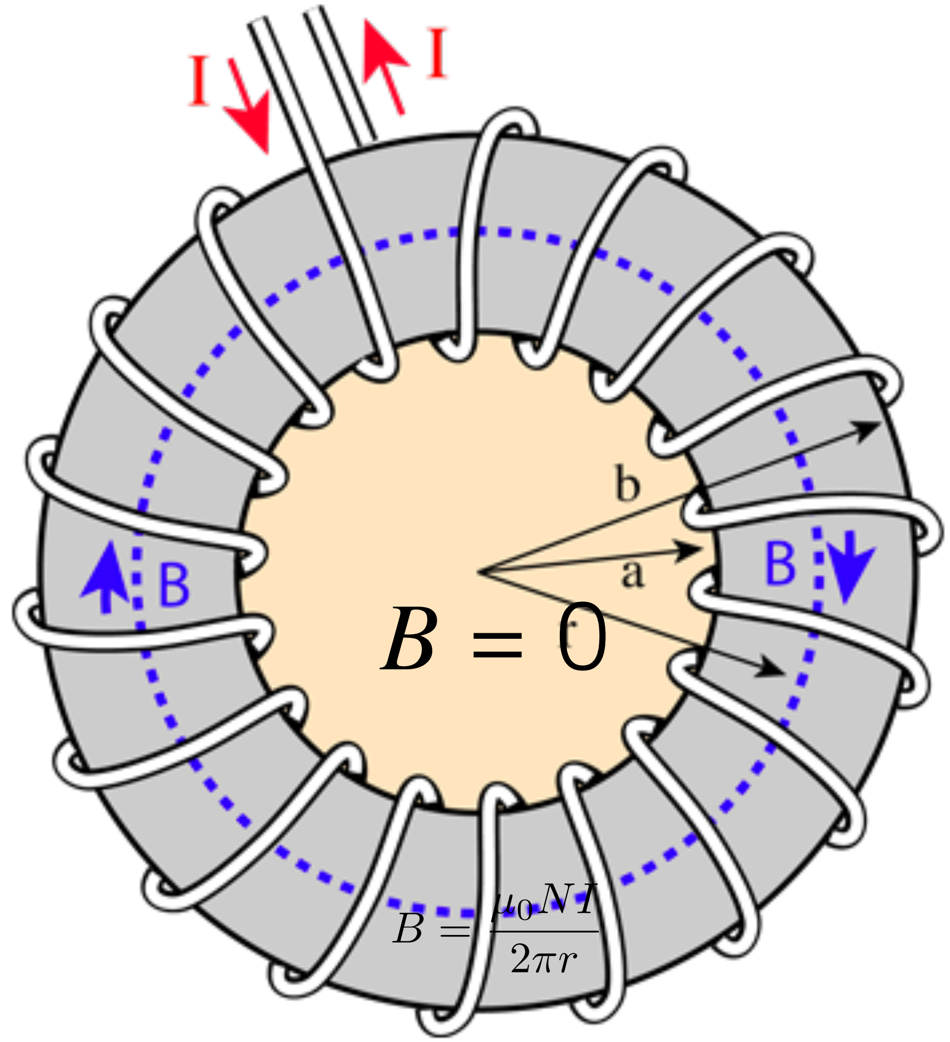

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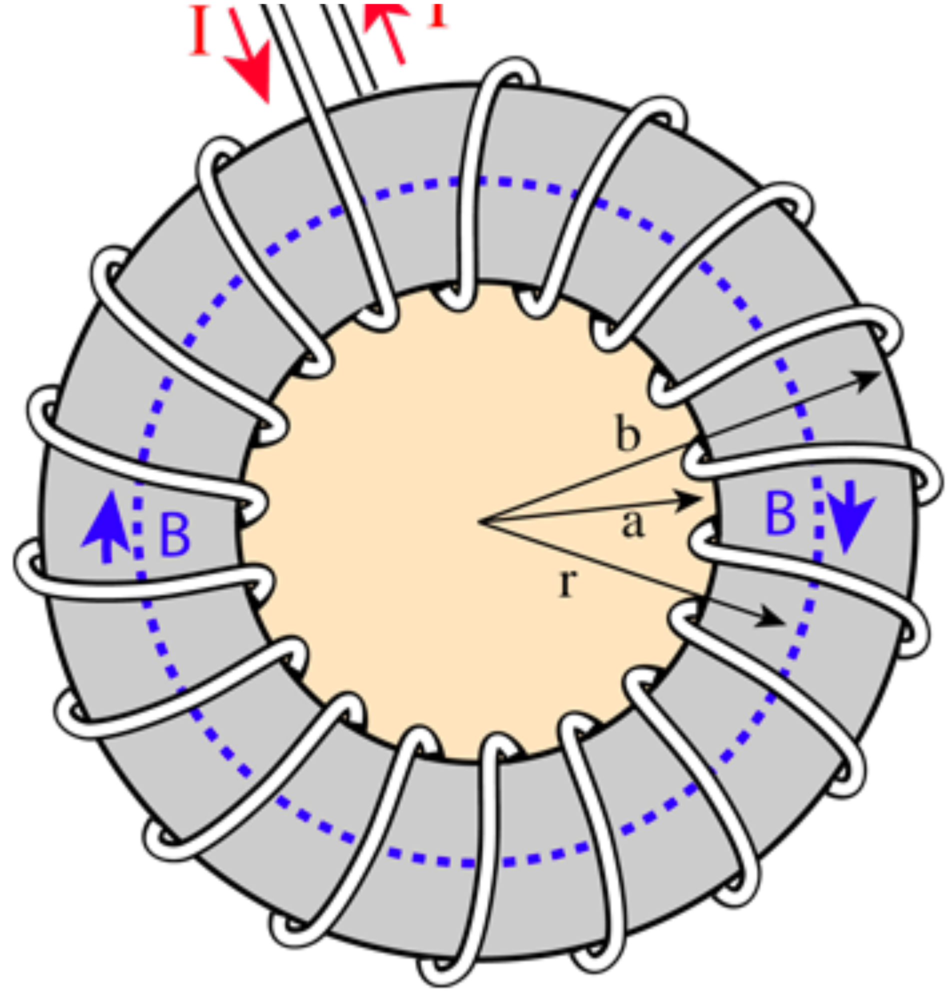
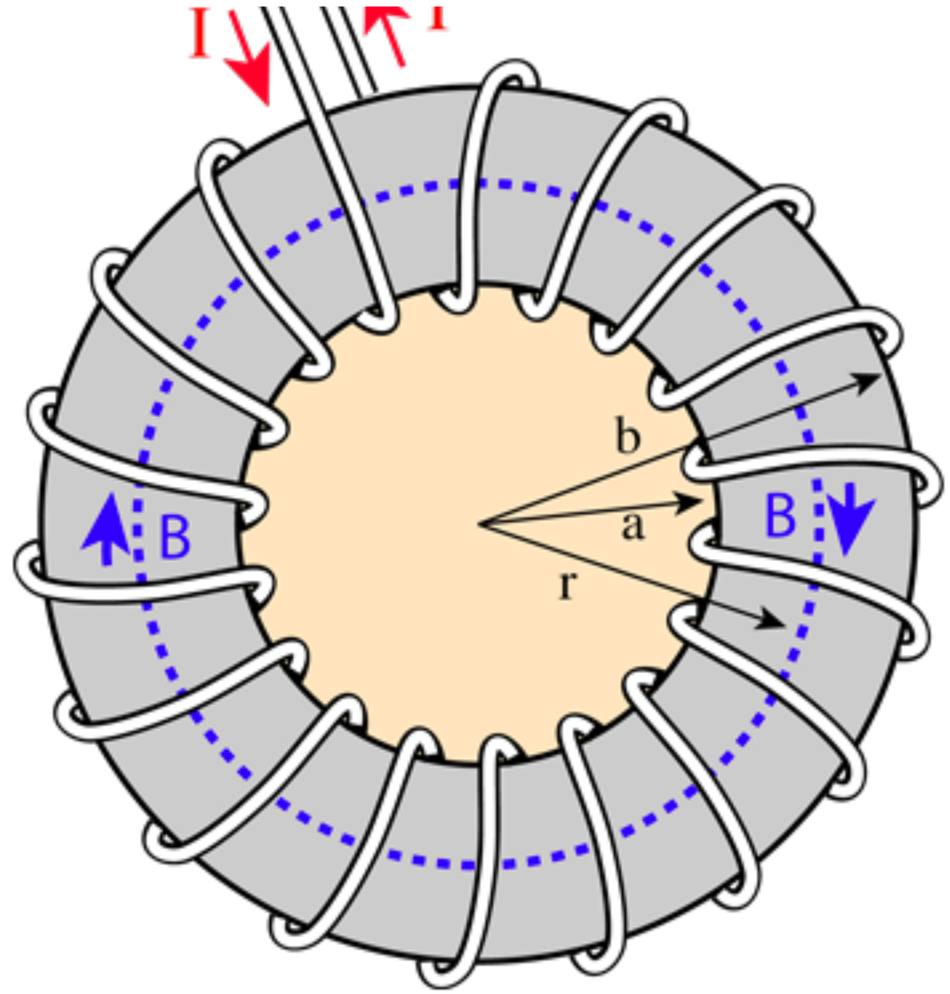
Numerical Example



N loops

Numerical Example

$r = 6$ cm
$a = 5$ cm
$b = 7$ cm
$I = 1$ A
$N = 100$



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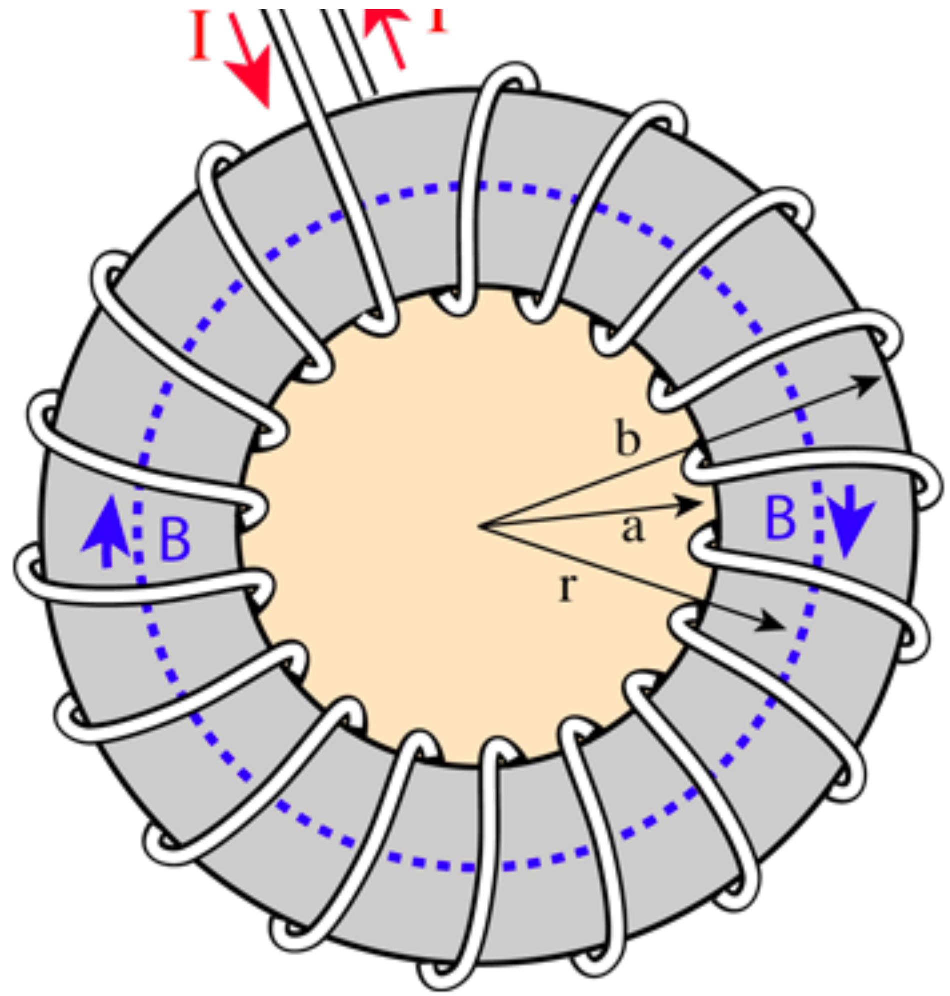

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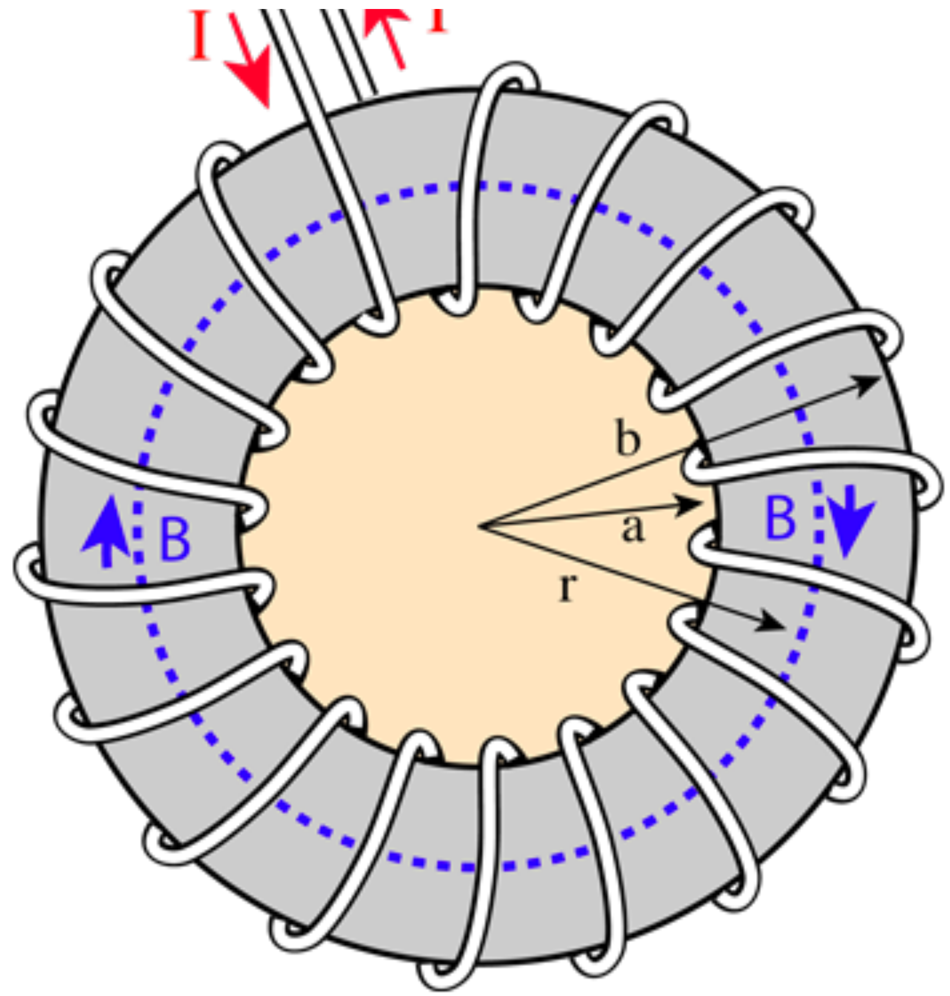

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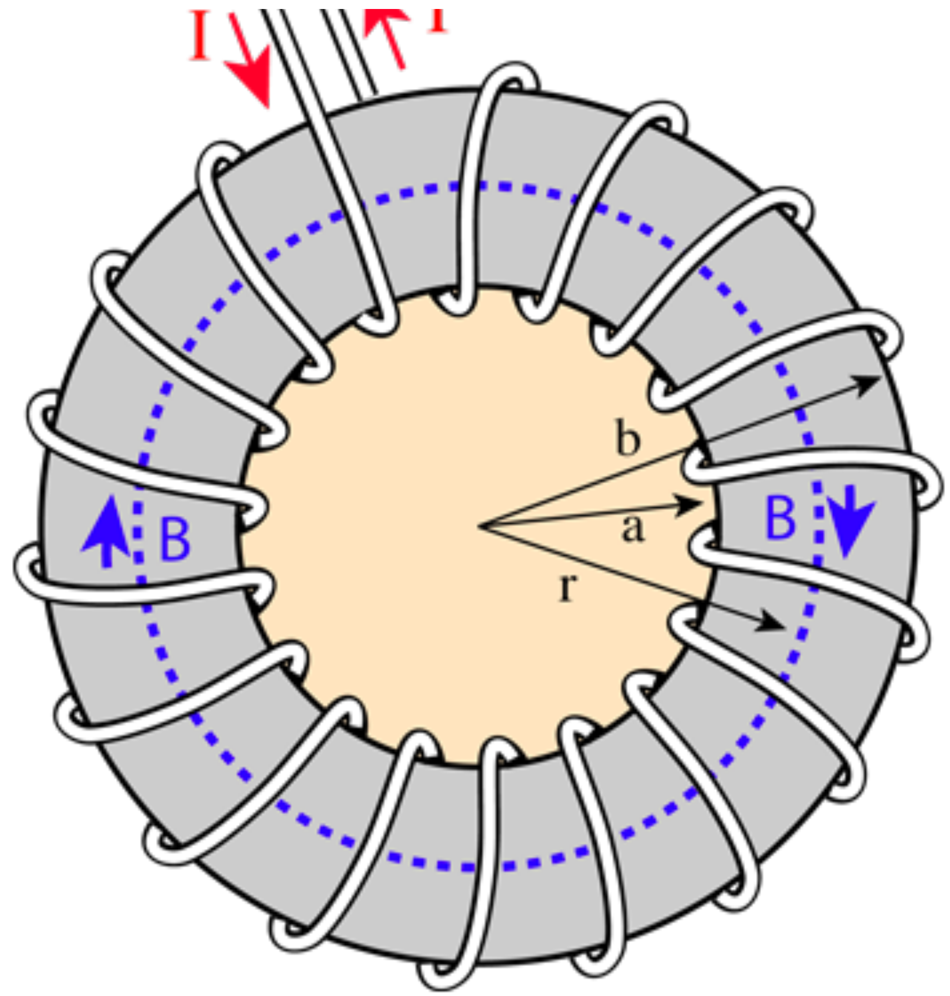

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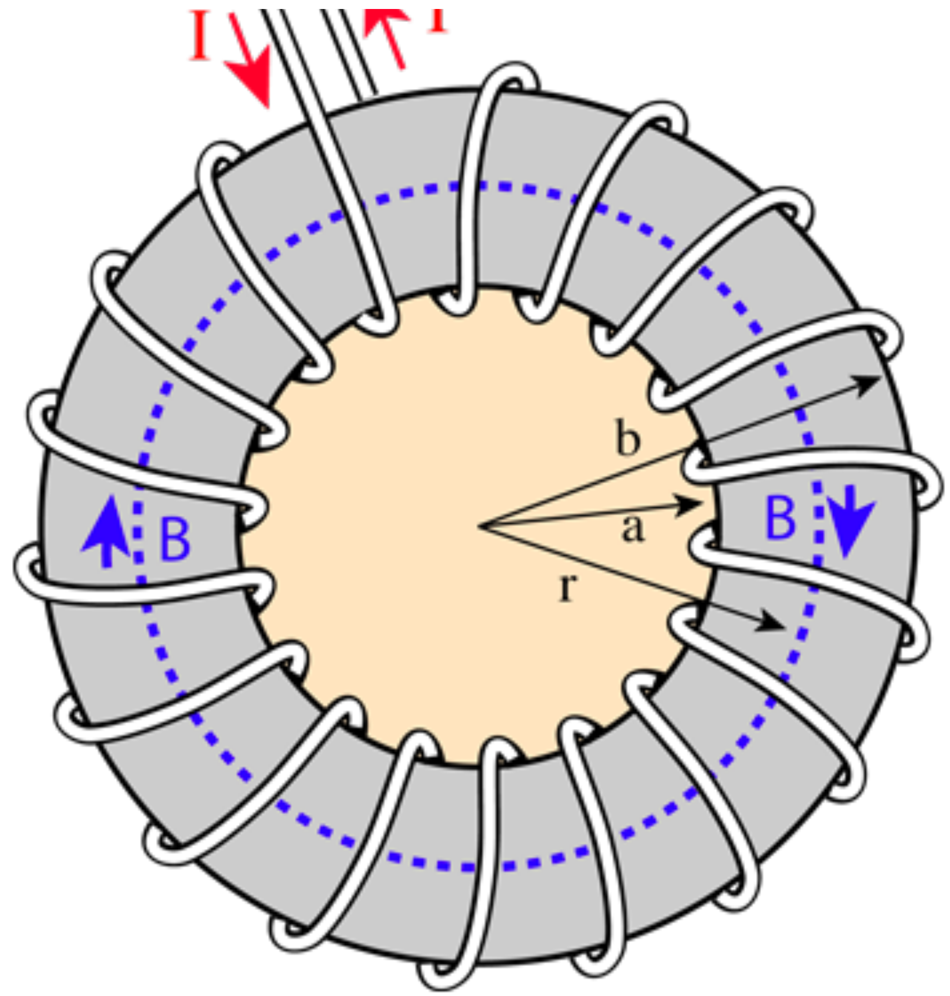

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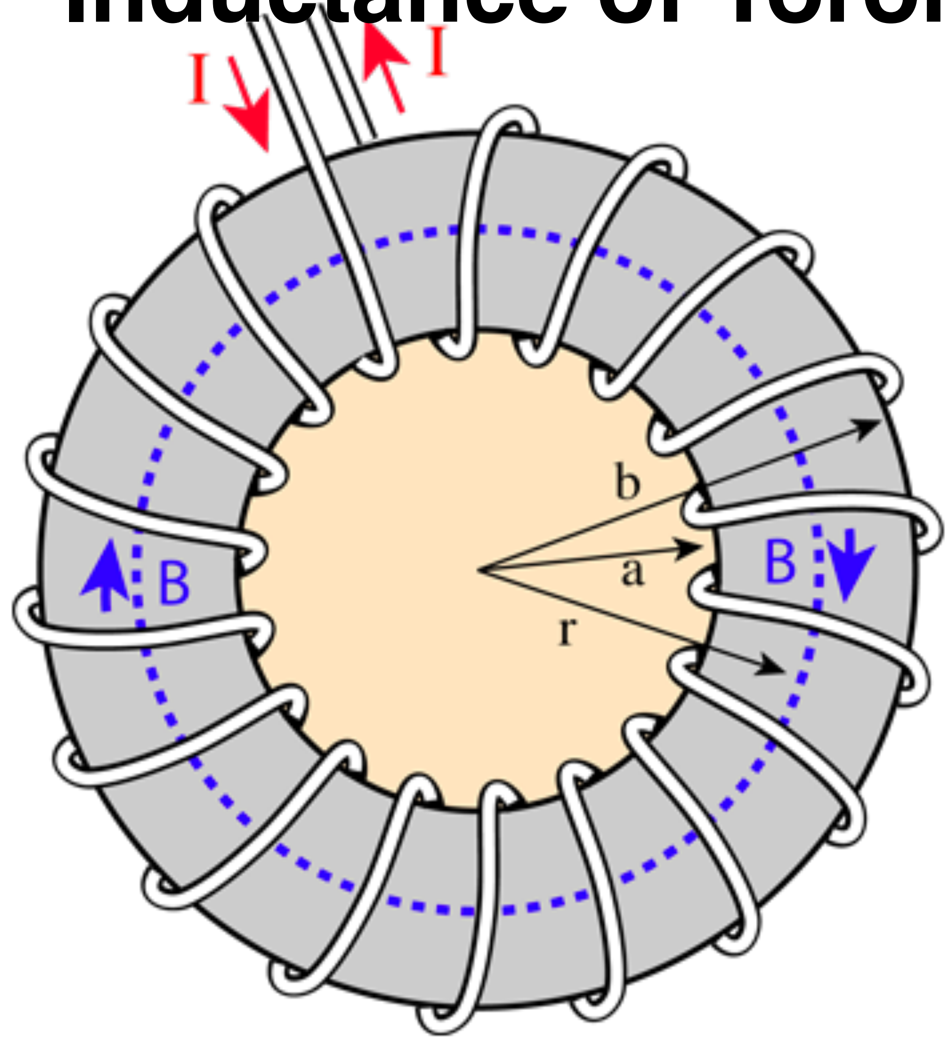
at radius **a:** B = 0.000400 T
at radius **b:** B = 0.000286 T

Inductance of Toroid



N loops

$$B = \frac{\mu_0 N I}{2\pi r}$$

Multiply by NA
and divide by I

$$L \approx B\frac{NA}{I} = \frac{\mu_0 N I}{2\pi r}\frac{NA}{I} = \frac{\mu_0 N^2 A}{2\pi r}$$

A = cross-sectional area

r = toroid radius to centerline

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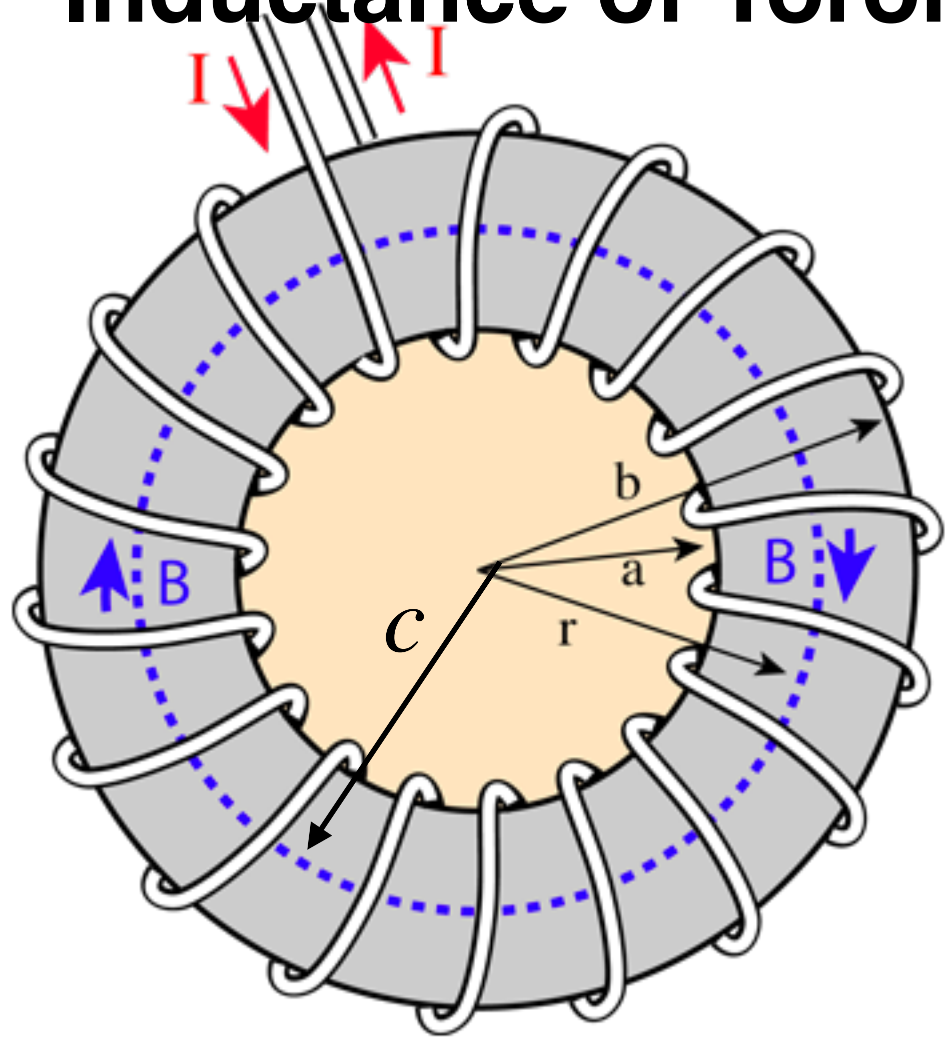

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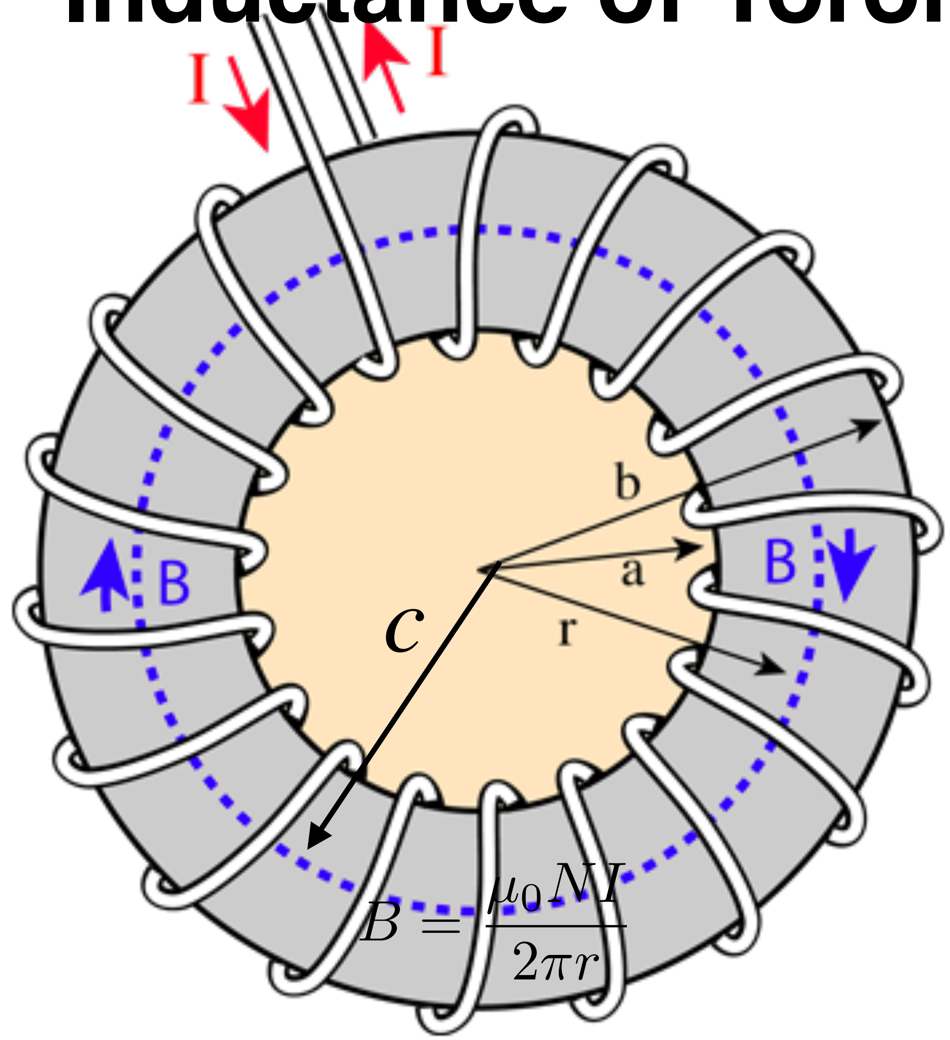

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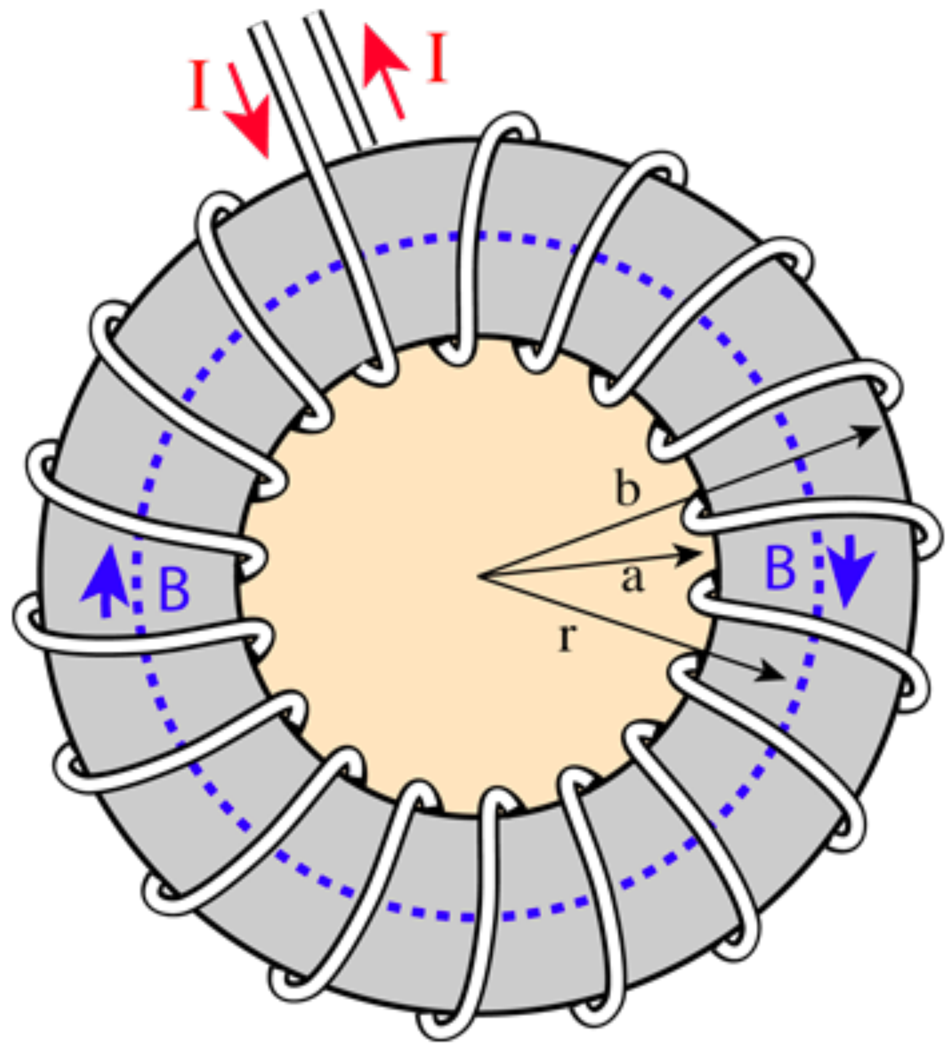
Air-core toroidal inductor with an inductance of 1 mH.



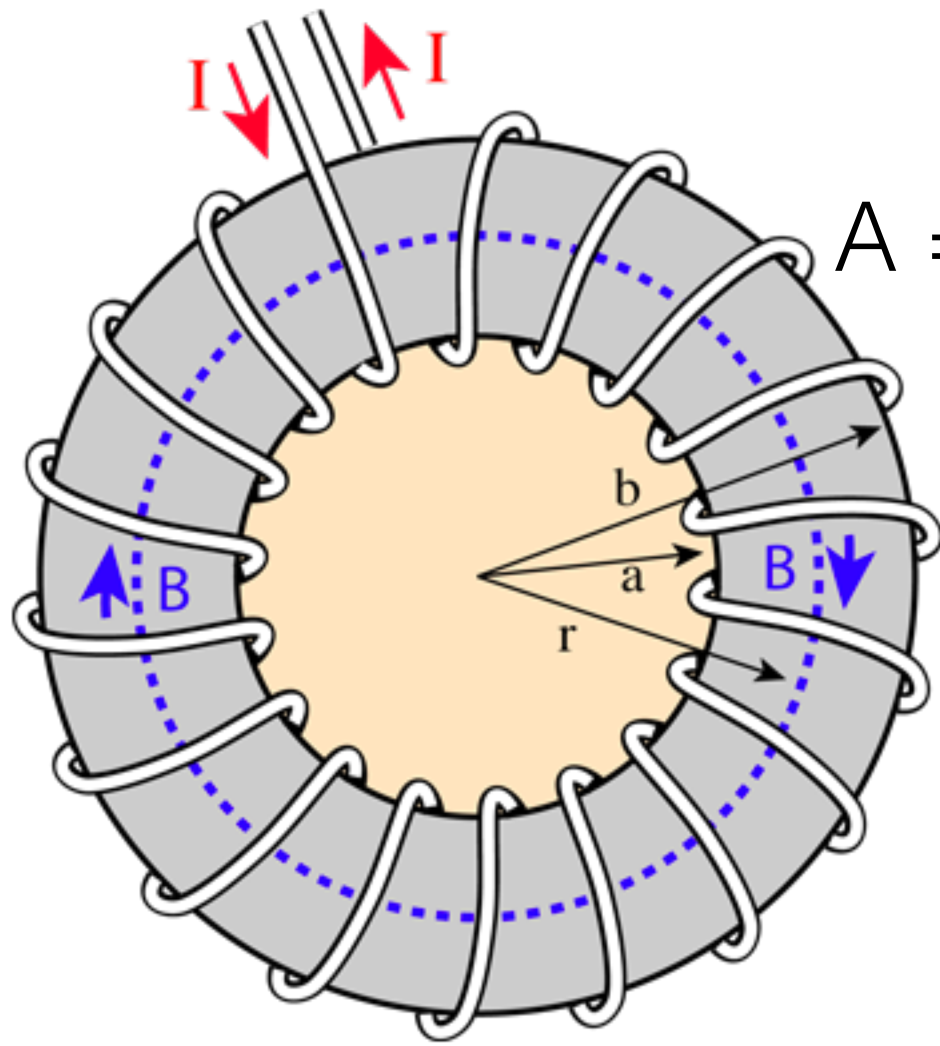
Design Problem:

Air-core toroidal inductor with an inductance of 1 mH.

$r = 6$ cm

$b - a = 2$ cm

$A = \pi \text{ cm}^2 = 0.000314 \text{ m}^2$



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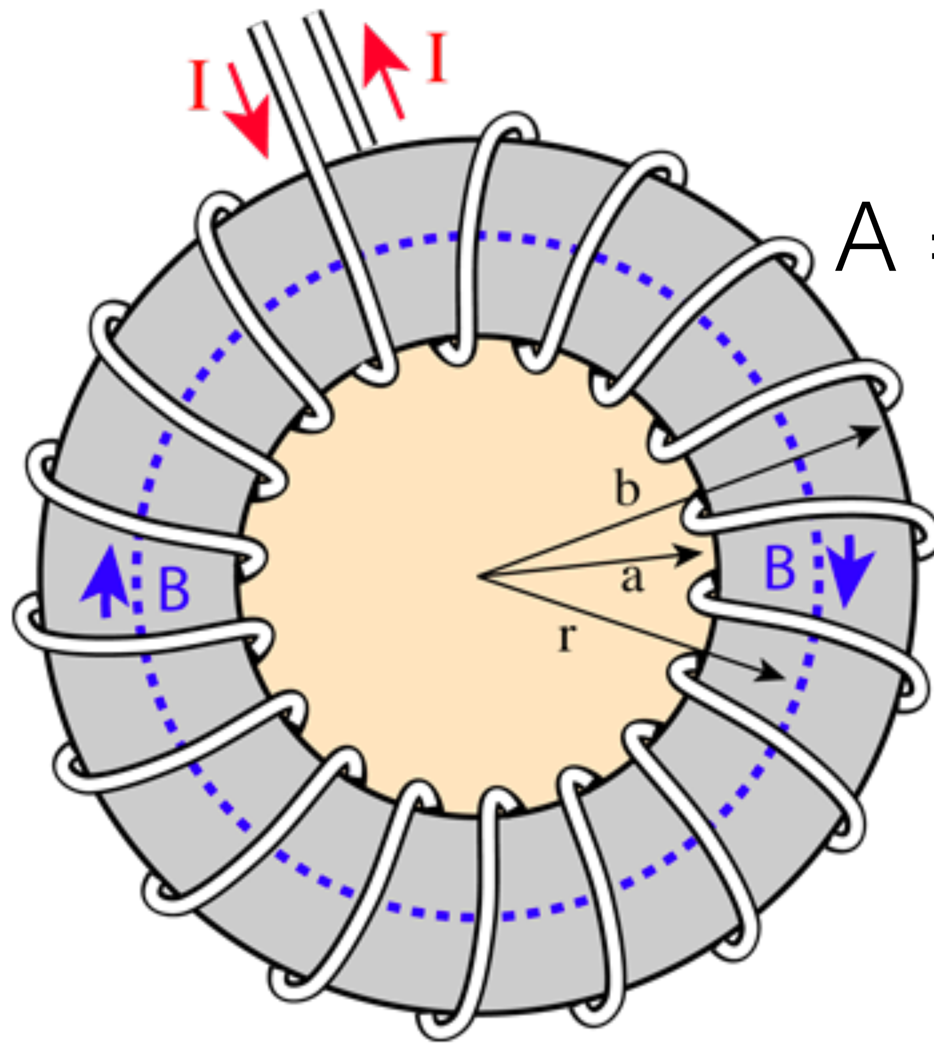

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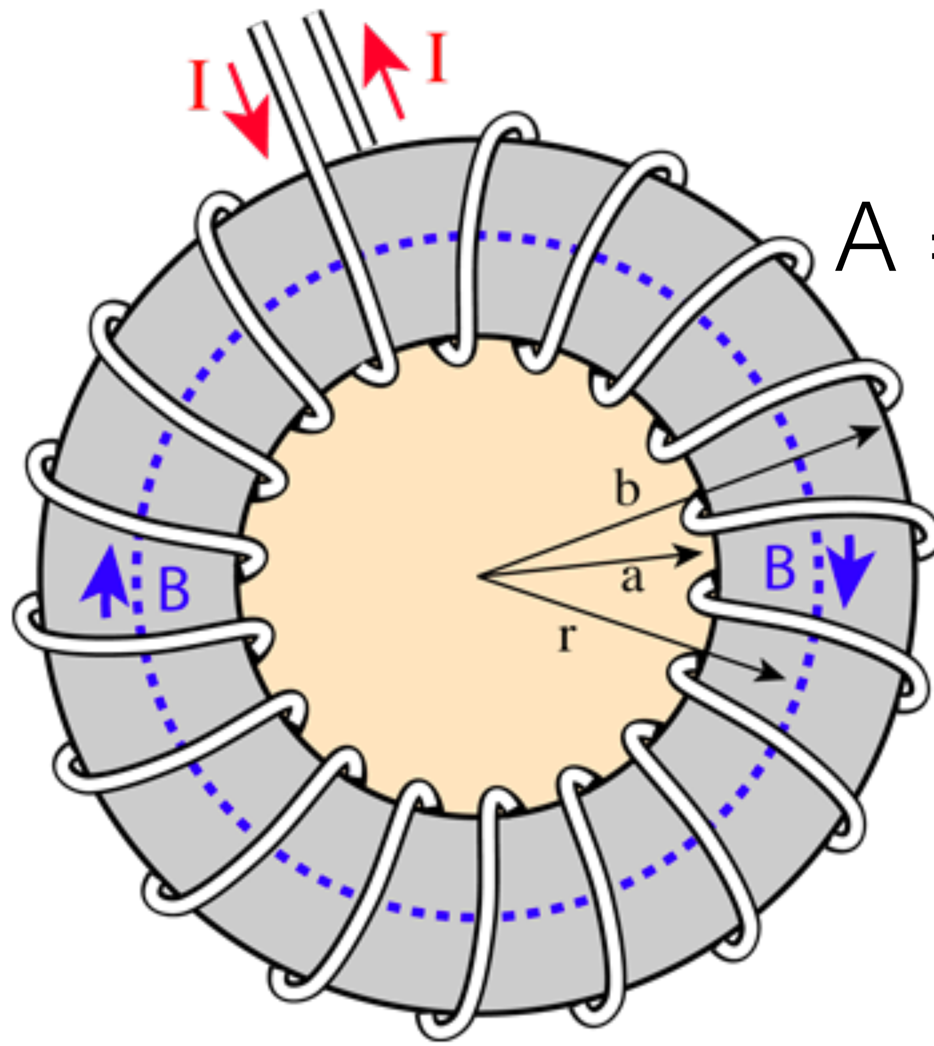

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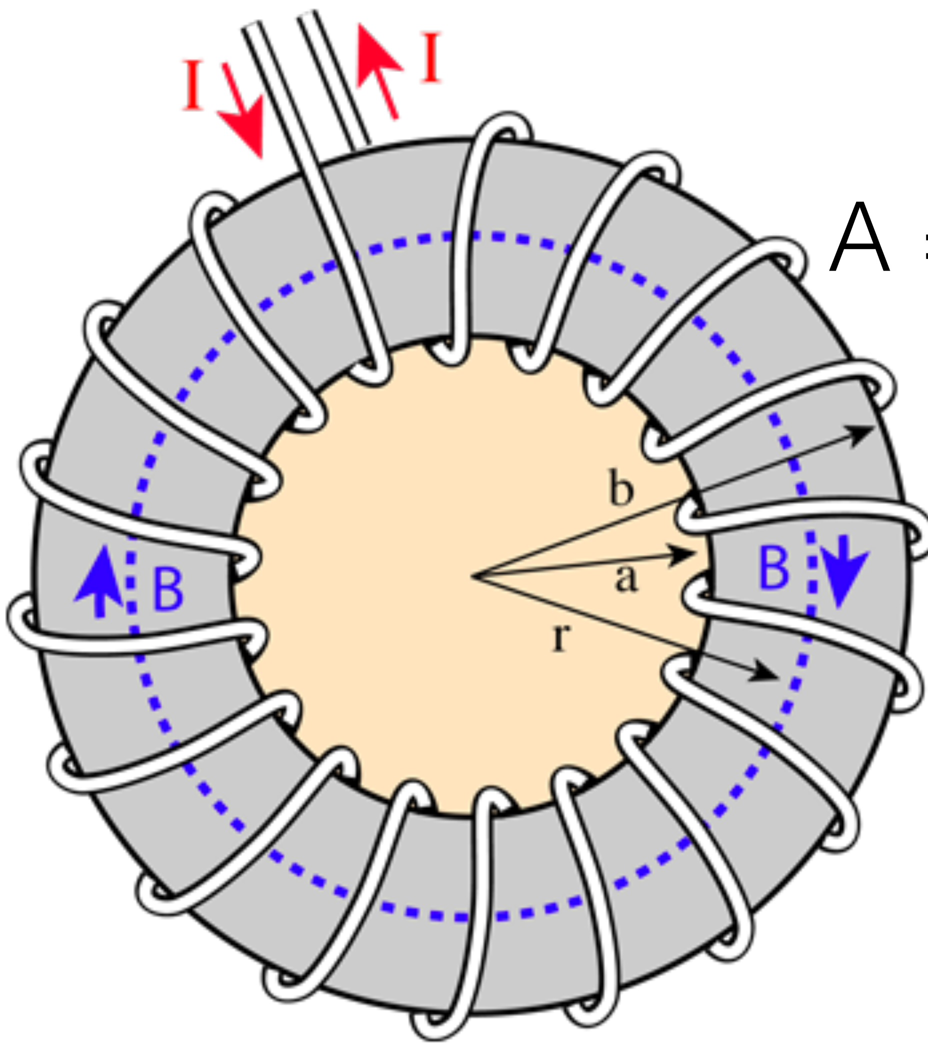

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=977 turns

Too many turns!

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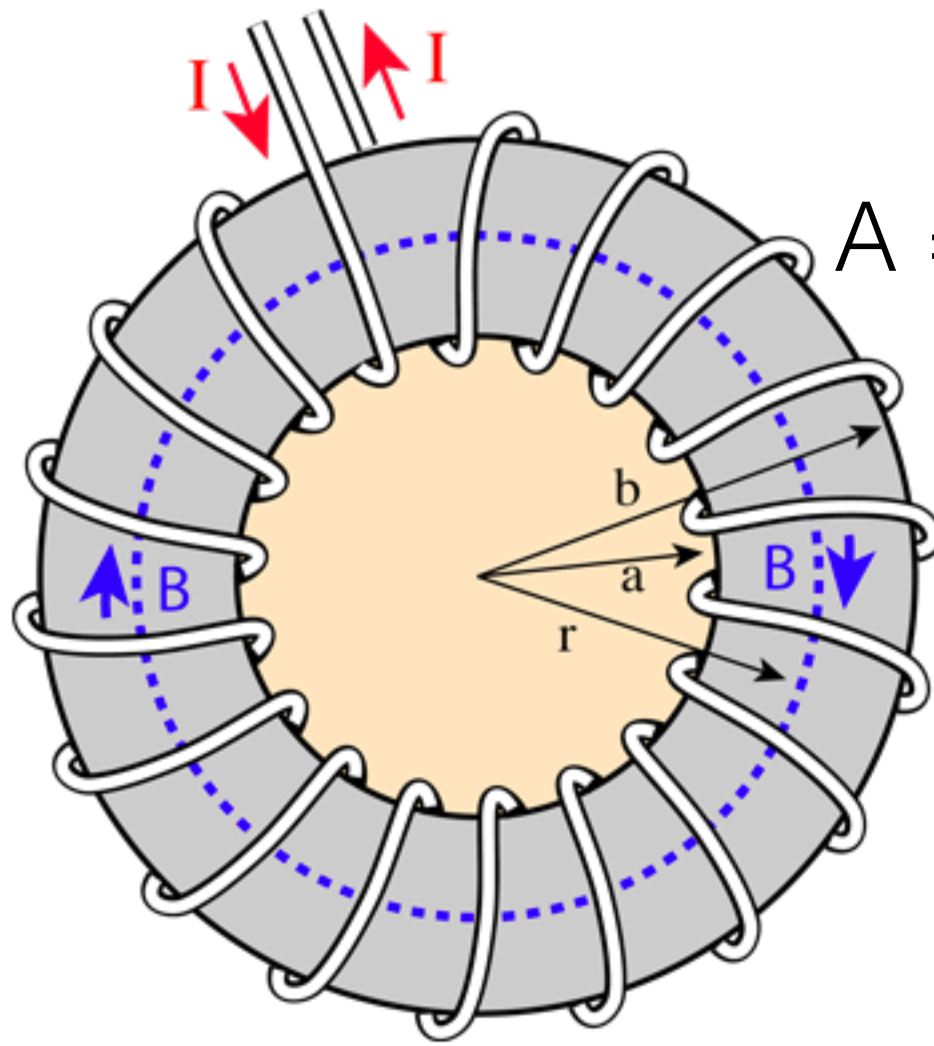

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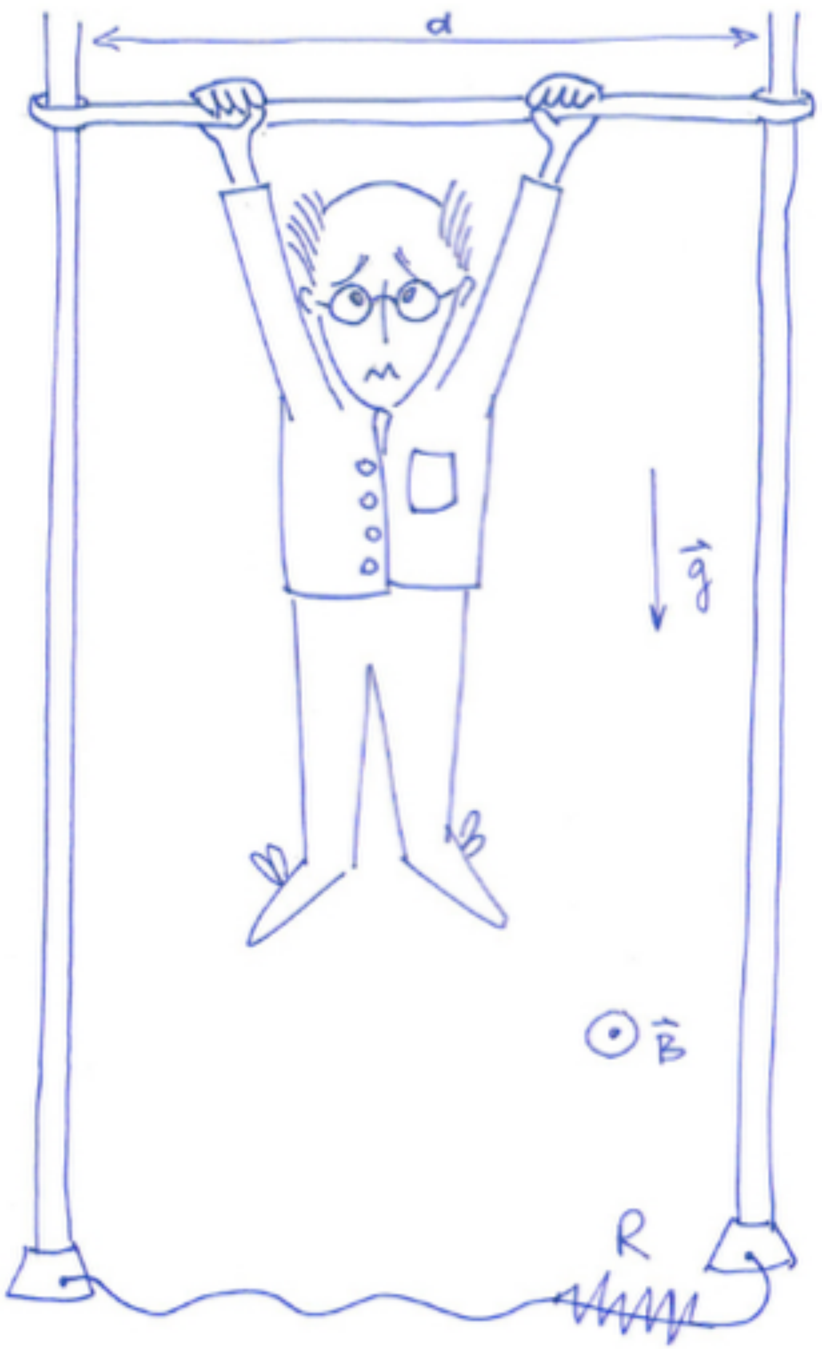
https://www.mag-inc.com/Products/Ferrite-Cores

Magnetic Parachute [edit]

A physics professor is demonstrating the concept of a “magnetic parachute” using the construction shown in the picture below. The poles and the bar are made of a conducting material that has a negligible resistance, and form a closed loop with a constant resistance *R*=100 Ohm. The distance between the poles is *d*=2m. The professor starts from a position that is sufficiently high to reach a constant terminal velocity. The room is filled with a uniform magnetic field $\vec{B}$ pointing out of the page, as shown. Suppose the professor weighs 80 kg. How strong should the magnetic field be in order for the terminal velocity to be within the safe landing range of $v_t < 8$m/s?



L. Pogosian, 2018