Title

EXPERIMENT A. Rubber Band Stretch SEPT. ANY YEAR

Date

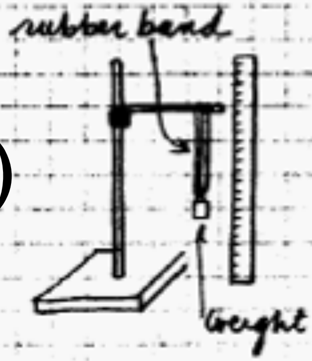
Lab Partner(s):

Lab Partner

Goal/Intro

This experiment is to see how a rubber band stretches under the influence of forces

Diagram(s)



YOUR OWN SKETCH WILL ALWAYS HELP DESCRIBE THE EXPERIMENT.

Procedure

I put different masses on the end of the rubber band and recorded the position of the top of the hook that holds the weight

Data

Room Temperature 26°C

Position of the top of rubber band 36.3 cm

INCLUDE ANY DATA YOU THINK MAY BE RELEVANT

mass (g)	Force (N)	Pos of Bottom (cm)	Extension (cm)	
(weights from set, no error) 0	0	44.0 ±.1	0	
10	.098	45.1 "	1.1±.2	
20	.196	45.8	1.8	
30	.294	46.8	2.8	
50	.490	49.6	5.6	second 20g weight is missing from set
60	.588	51.5	7.5	
70	.686	53.7	9.7	
80	.784	56.1	12.1	
100	.980	60.6	16.6	
80	.784	56.2	12.2	recheck

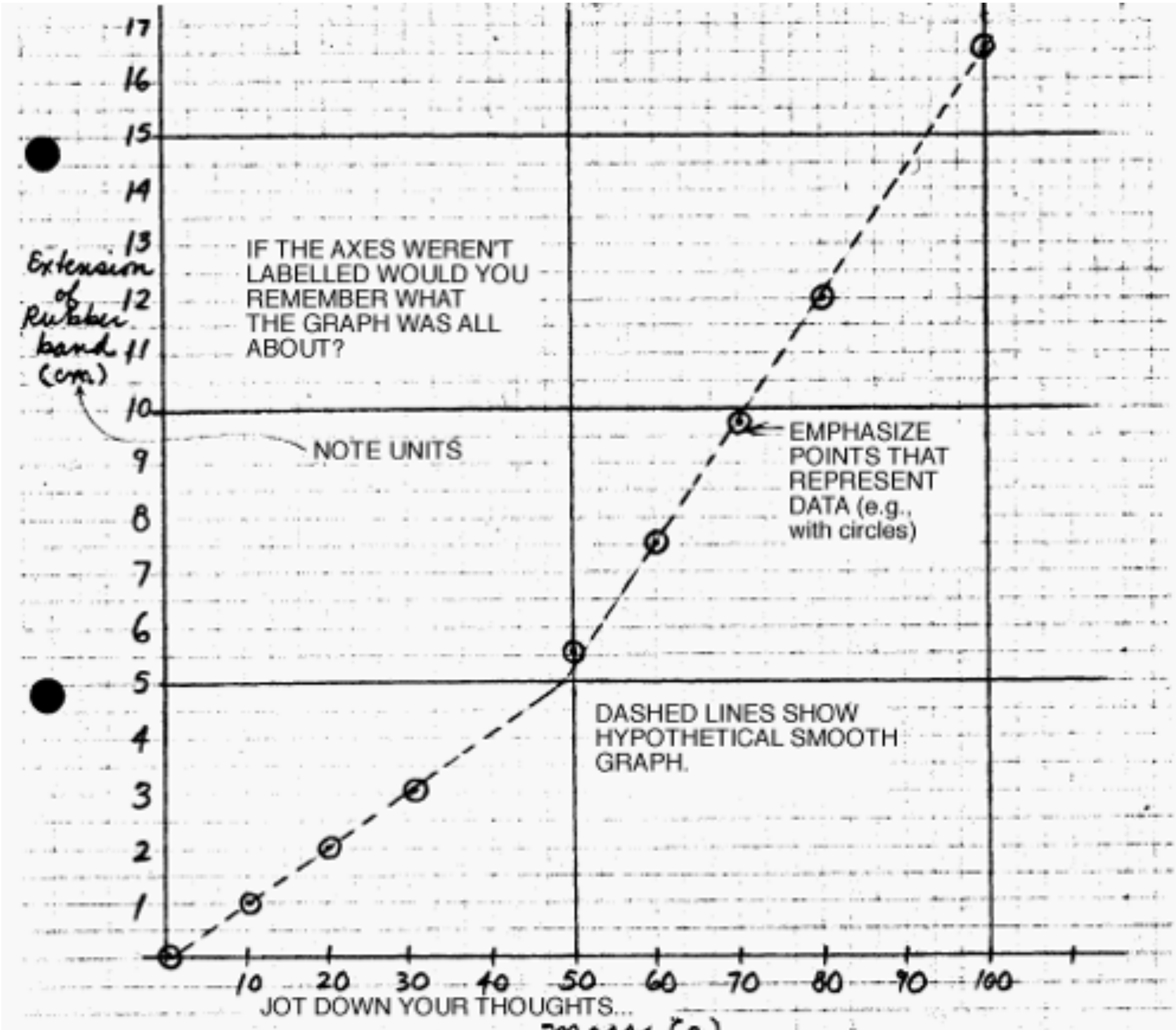
ALWAYS SHOW UNITS OF TABULATED QUANTITIES

ESTIMATE THE ERROR OF EVERY QUANTITY YOU MEASURE

INCLUDE COMMENTS ON YOUR DATA

KEEP DATA IN NEAT TABLES

Analysis,
Graph,
Calculations



JOT DOWN YOUR THOUGHTS...

INCLUDE POSSIBLE QUESTIONS— ESPECIALLY ONES YOU CAN'T ANSWER.

Discussion,
Conclusions

There are obviously two different straight lines. It would have been nice to see what it was at 40 gm, since that's just where the two lines cross.

The slopes of the two lines are the force constants $F=-kx$. For the first line $k=.105$ N/cm and for the second $k=.0438$ N/cm.

Grading

Notebooks:

8 marks	as follows
0:	No show
1:	Do experiment partially with incomplete results
2–3:	Do experiment with major error in method or analysis
4–5:	Do experiment satisfactorily but with minor faults
6:	Do experiment well
0—2	in addition to the above

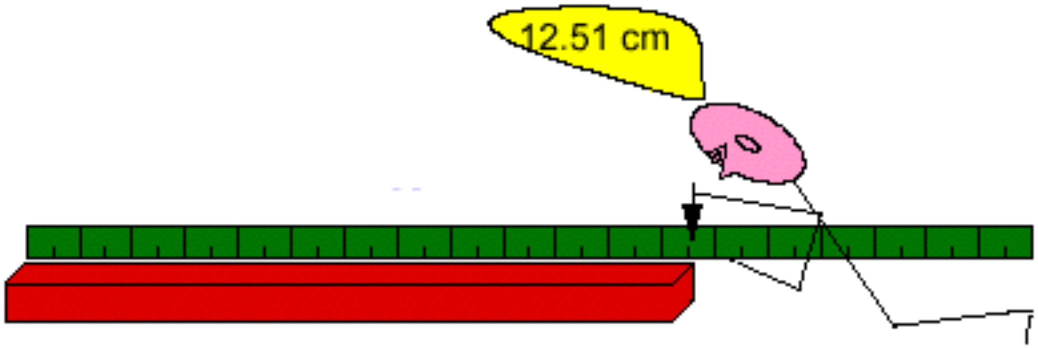
Error analysis:

Systematic vs. Random Errors

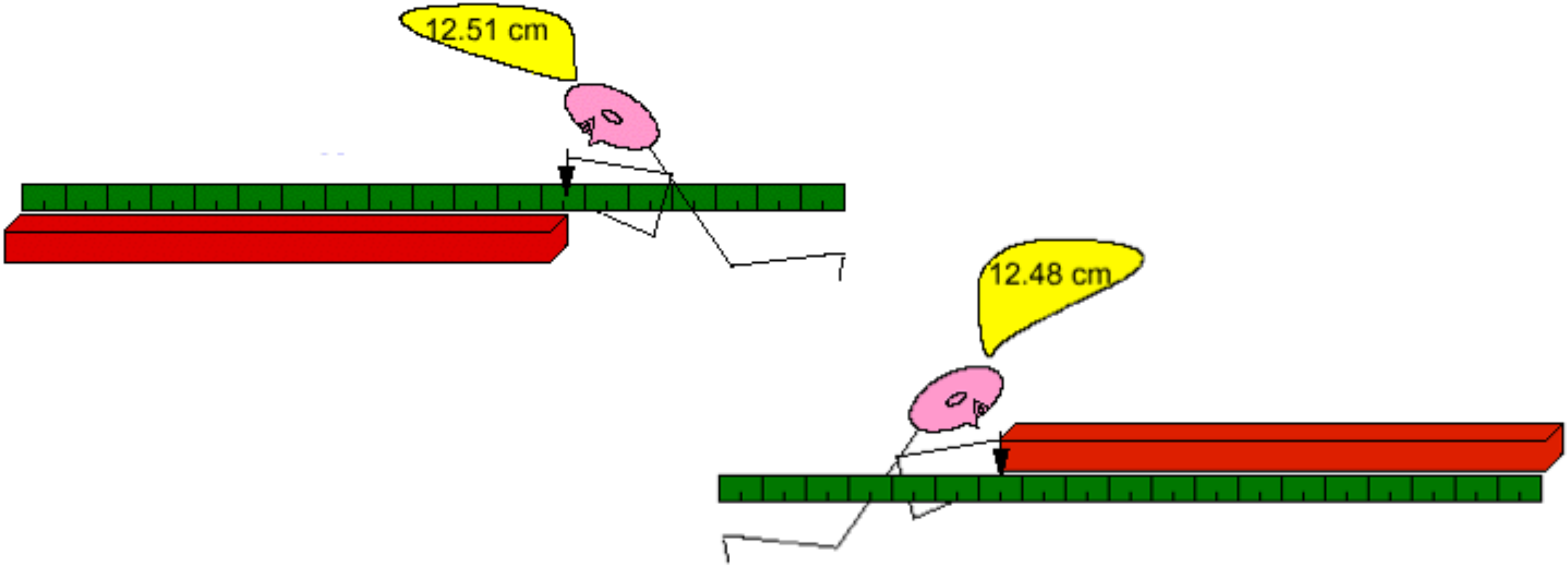
- **Systematic errors**:
  - Errors that are repeatable from measurement to measurement.
    - Stop clock limitation.
    - Meter stick offset
- **Random errors**:
  - Errors that give different result every time.
    - Reaction time etc.

Measuring

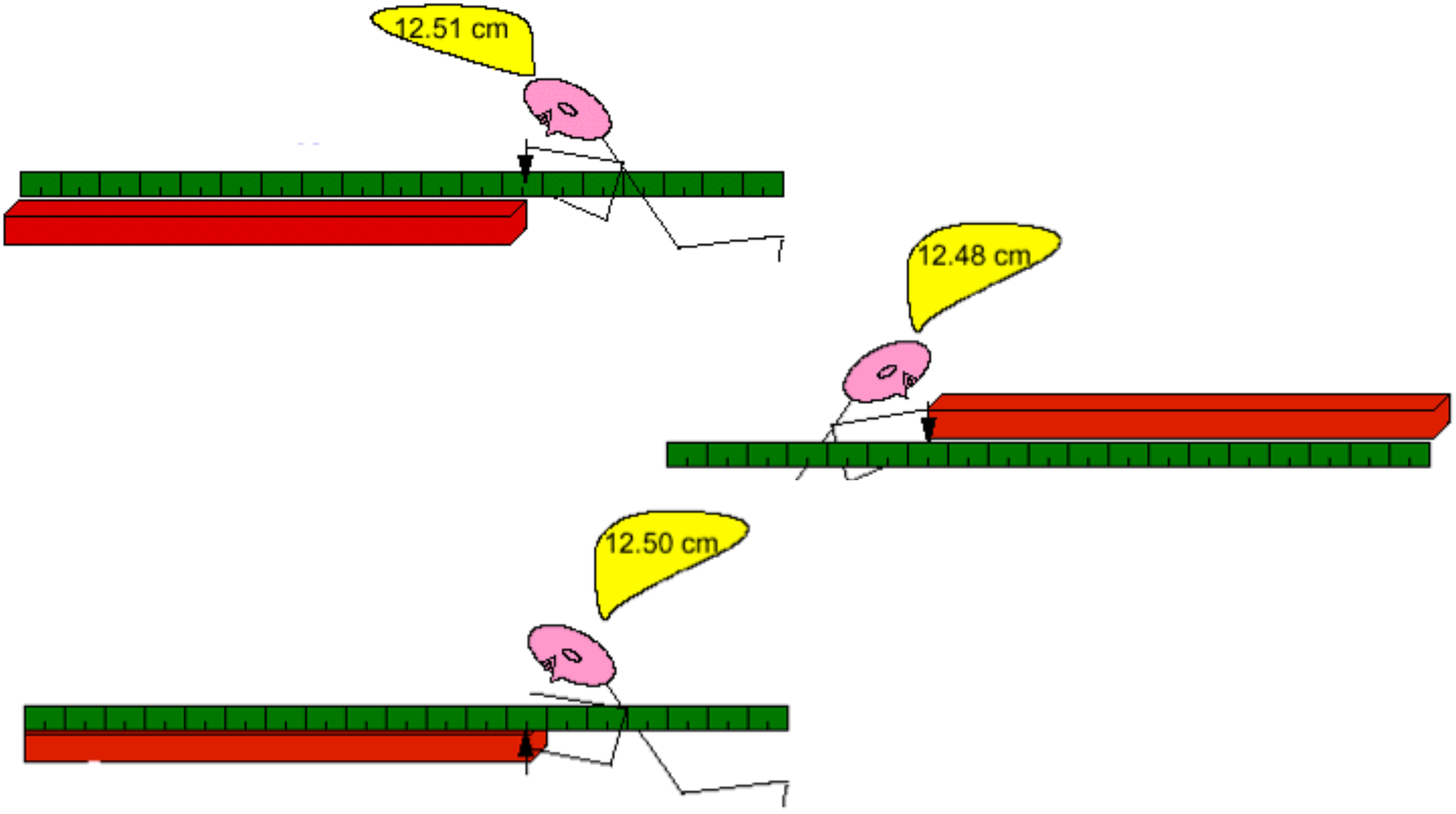
Measuring



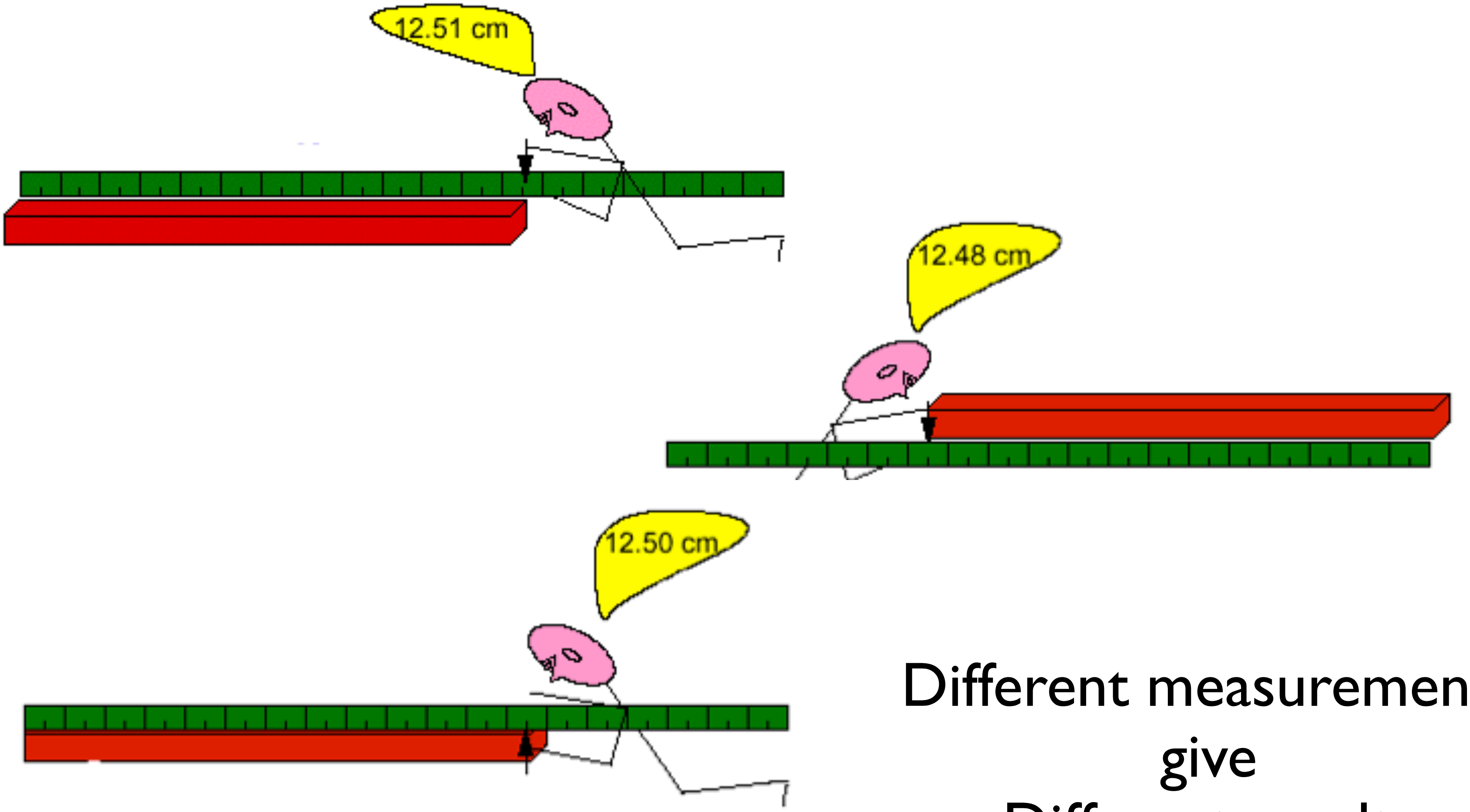
Measuring



Measuring



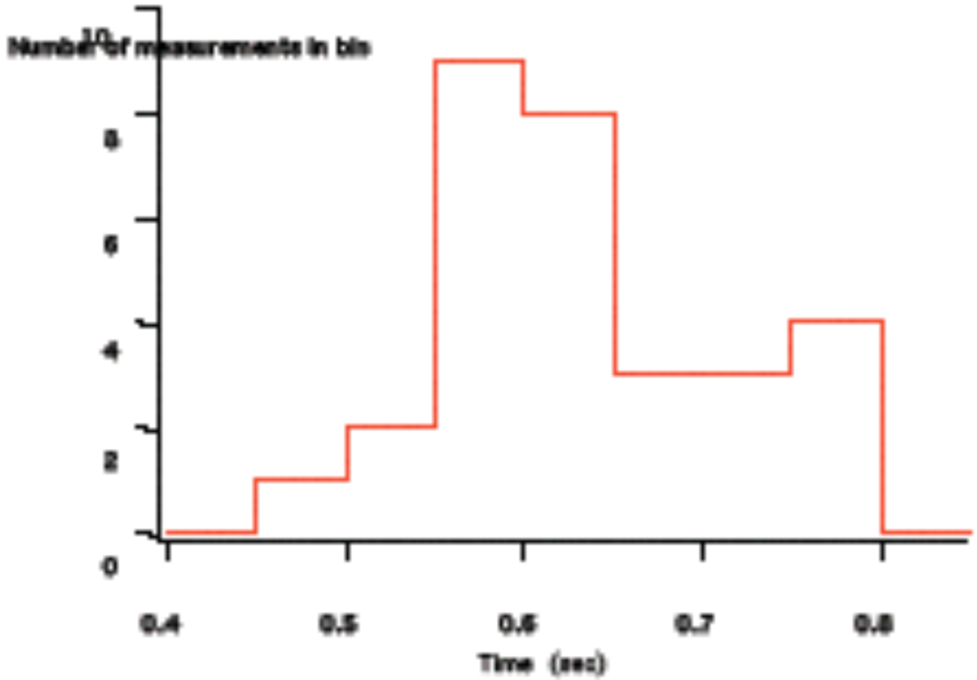
Measuring



Different measurements
give
Different results

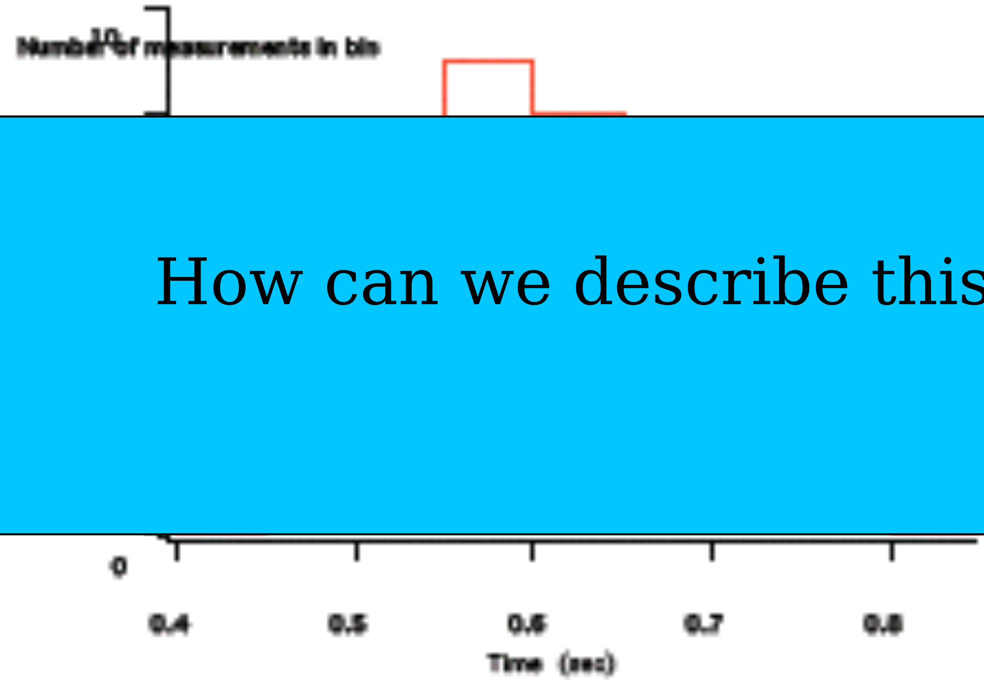
Histograms

- A graphical display of tabulated frequencies.



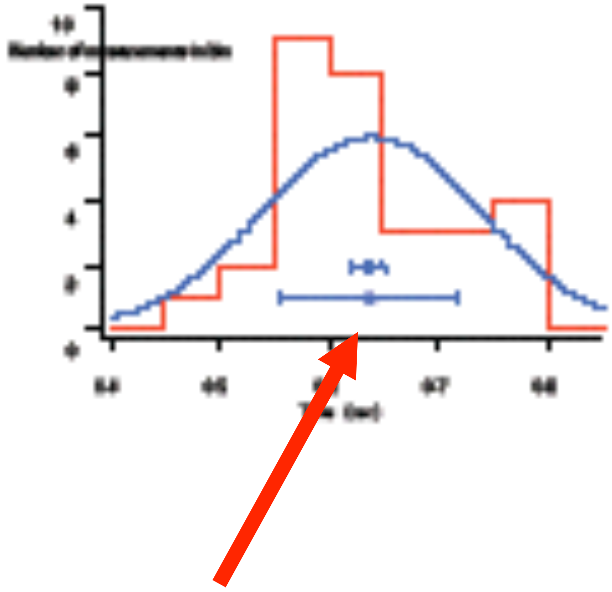
Histograms

- A graphical display of tabulated frequencies.



How can we describe this ?

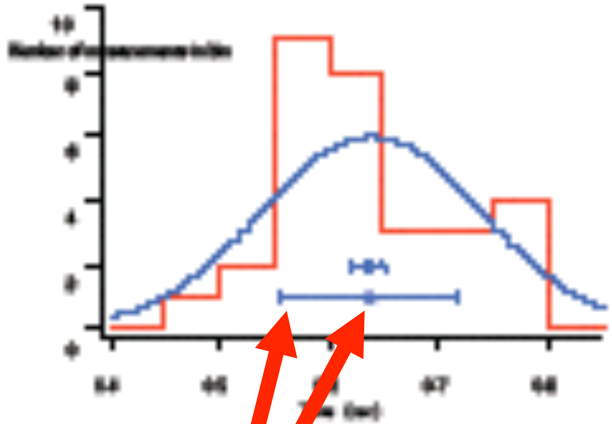
Bell Curve



Average Value:

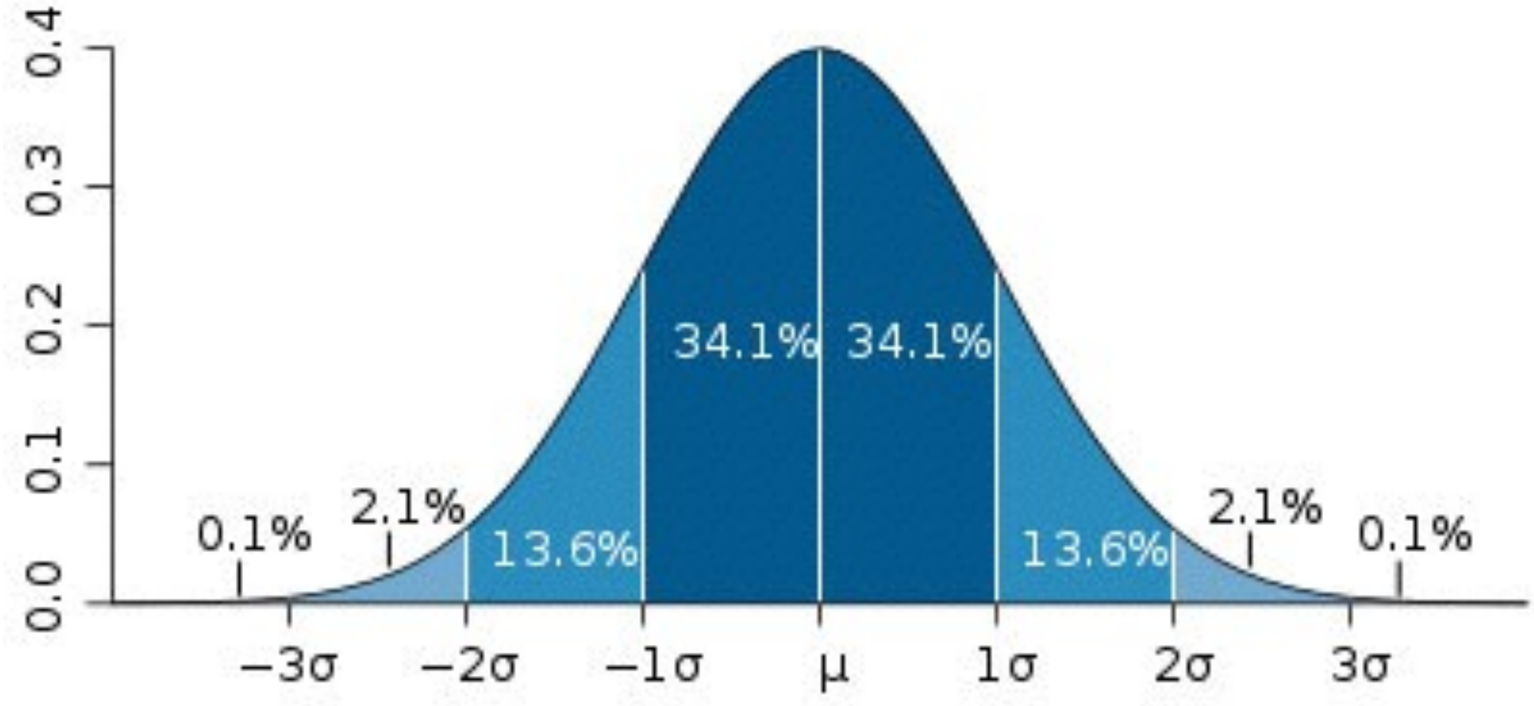
$$< t > = \frac{1}{N}\sum_{i=1}^{N} t_i$$

Bell Curve

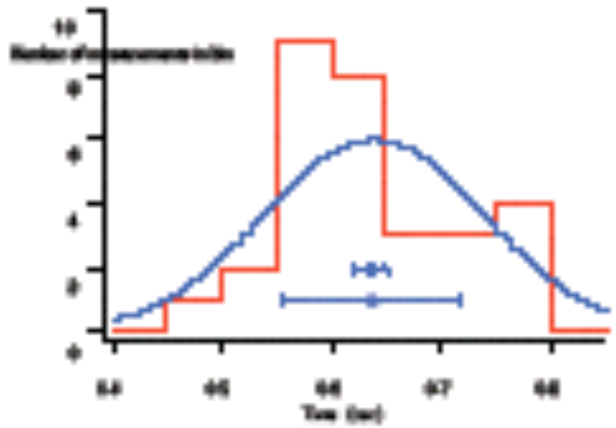


Standard Deviation:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(t_i - \langle t \rangle)^2}$$


0.4
0.3
0.2
0.1
0.0
34.1%
34.1%
13.6%
13.6%
2.1%
2.1%
0.1%
0.1%
−3σ
−2σ
−1σ
μ
1σ
2σ
3σ

Bell Curve



Standard Deviation of Mean:

$$\sigma_{avg} = \frac{\sigma}{\sqrt{N}}$$

Overall measurement values and uncertainties

One measurement

$$t_i = t_i + \sigma$$

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Average measurement

$$< t > = < t > + \sigma_{avg} = < t > + \frac{\sigma}{\sqrt{N}}$$

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One measurement

$$t_i = t_i + \sigma$$

Average measurement

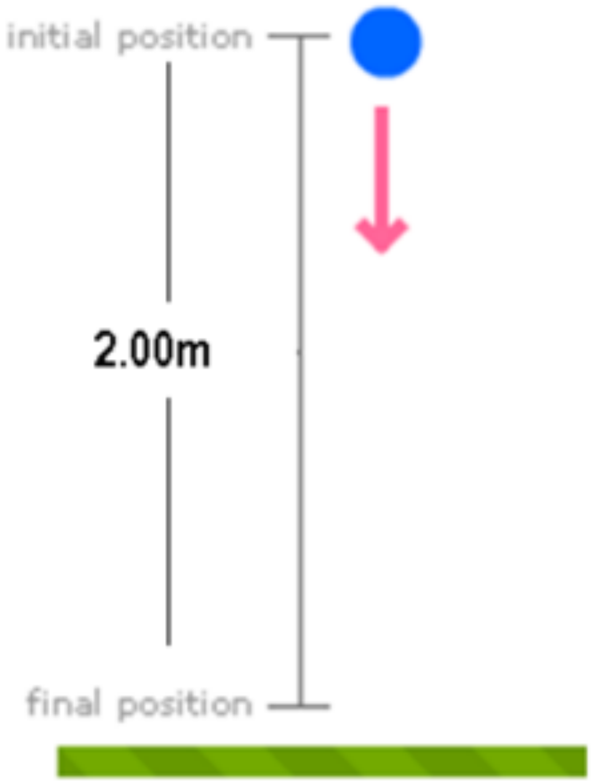
$$< t > = < t > + \sigma_{avg} = < t > + \frac{\sigma}{\sqrt{N}}$$

Therefore we want a large N (number of trials) to reduce the error.

Drop-a-Ball Experiment

Measure the time it takes for a ball at rest to drop by 2.000 ± 0.002 m.

Trial #	Time t_i (sec)	Deviation $d_i = t_i - \langle t \rangle$ (sec)	



Result: The time it takes for the ball to drop by 2.000 ± 0.002 m is

$$t = \langle t \rangle \pm \Delta t \text{ sec}$$

where typically $\Delta t = \sigma_{avg}$

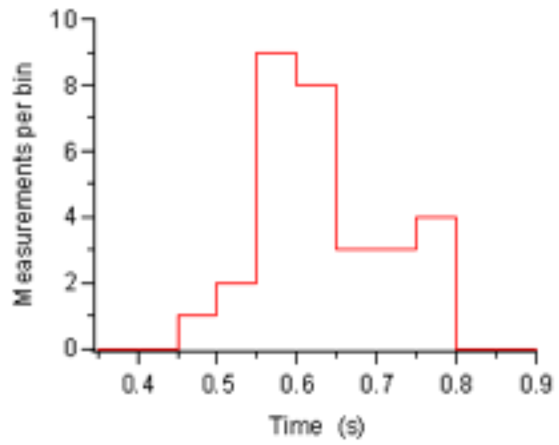
Statistical Analysis: Suppose we have many measurements, each with random errors.

Mean $$< t >= \frac{1}{N}\sum_{i=1}^{N} t_i$$

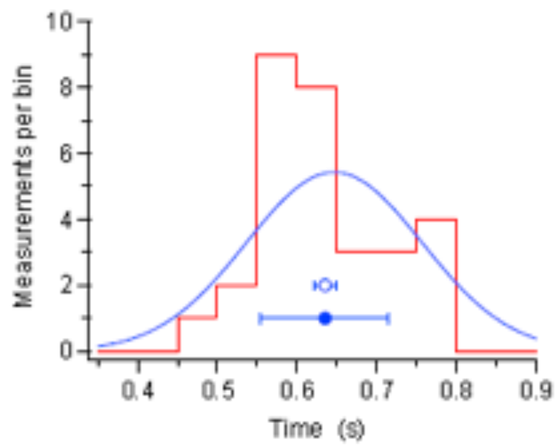
Standard Deviation $$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(t_i - < t >)^2}$$

Uncertainty in Mean $$\sigma_{avg} = \frac{\sigma}{\sqrt{N}}$$

(a)



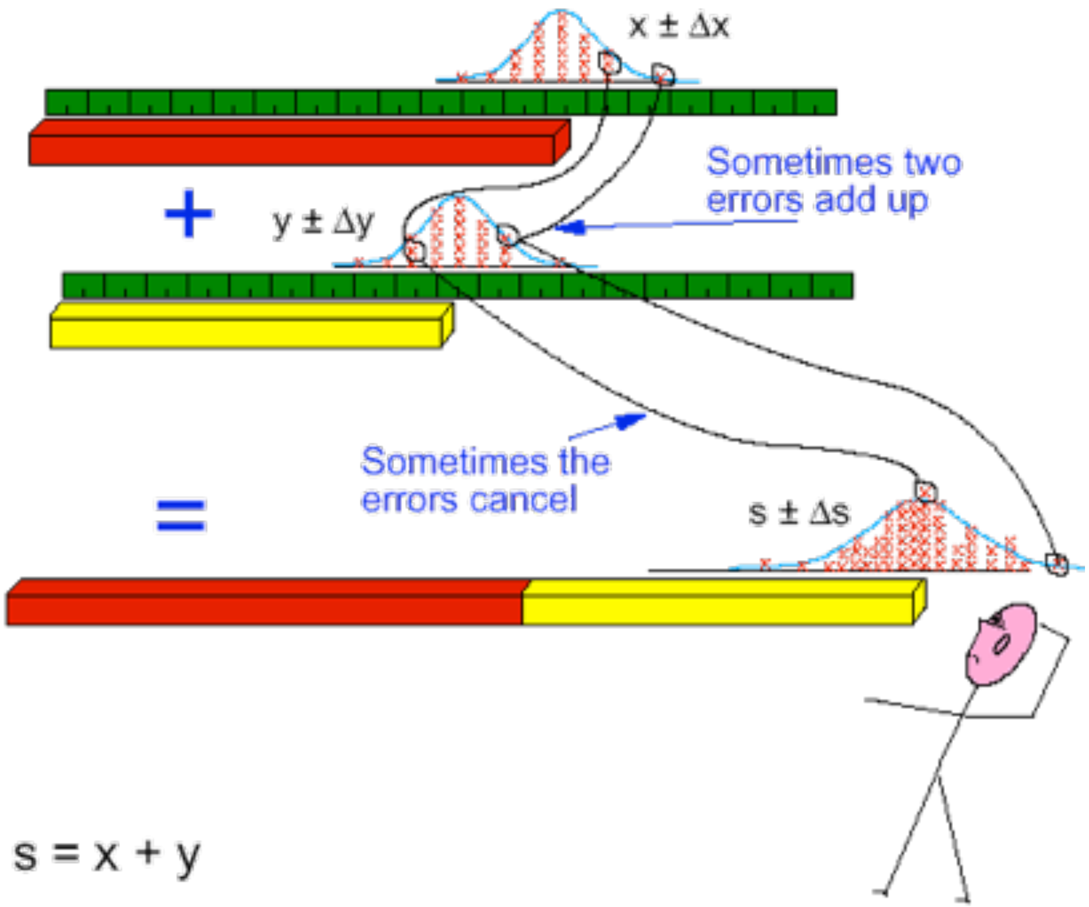
(b)



The mean and the standard deviation describe the distribution of data expected. These should not change as the number of measurements is increased.

Our estimate of the mean will improve, so we expect the standard deviation of the mean to get smaller as we do more measurements.

When you add two measurements
which have random errors

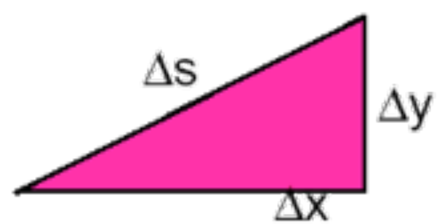


$s = x + y$

The error ranges of x and y are Δx and Δy

The error range of s is given by

$$\Delta s = \sqrt{\Delta x^2 + \Delta y^2}$$



Propagation of Errors

0.6.1 **Rule 1: A constant multiple**

If

$Y = k A$

where k is a constant, then

$\Delta Y = k \Delta A$

Example: $x = 1.4 \pm 0.1$ m, $2x = 2.8 \pm 0.2$m

Propagation of Errors

Rule 2: Addition and Subtraction (“provisional” rule)

$$\Delta Y = \Delta A + \Delta B$$

Propagation of Errors

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$$Y = A + B$$

$$Y = A - B$$

$$\Delta Y = \Delta A + \Delta B$$

Propagation of Errors

Rule 2: Addition and Subtraction ("provisional" rule)

$$Y = A + B$$

$$Y = A - B$$

$$\Rightarrow \quad \Delta Y = \Delta A + \Delta B$$

Propagation of Errors

Sum of two measurements

width $w = 0.24 \pm 0.03$ m

length $l = 0.89 \pm 0.04$ m

sum $s = w + l = 0.24 + 0.89 \text{ m} = 1.13 \text{ m}$

Propagation of Errors

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width $w = 0.24 \pm 0.03$ m

length $l = 0.89 \pm 0.04$ m

sum $s = w + l = 0.24 + 0.89$ m $= 1.13$ m

$s = 1.13 \pm 0.07$ m

Propagation of Errors

Rule 3: Multiplying or Dividing (“provisional” rule)

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Propagation of Errors

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$$Y = \frac{A}{B}$$

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$$Y = AB$$

$$Y = \frac{A}{B}$$

$$\Rightarrow \quad \frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

This means that you add "percentage" or "fractional" errors when quantities are multiplied or divided

Propagation of Errors

Area is the product of width and length

$A = wl = (0.24\ \text{m})\ (0.89\ \text{m}) = 0.2136\ \text{m}^2$

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$$\frac{\Delta w}{w} = \frac{0.03}{0.24} = 12.5\%$$

Propagation of Errors

Area is the product of width and length

$$A = wl = (0.24 \text{ m})\,(0.89 \text{ m}) = 0.2136 \text{ m}^2$$

$$\frac{\Delta w}{w} = \frac{0.03}{0.24} = 12.5\%$$

$$\frac{\Delta l}{l} = \frac{0.04}{0.89} = 4.5\%$$

Propagation of Errors

Area is the product of width and length

$$A = wl = (0.24 \text{ m})(0.89 \text{ m}) = 0.2136 \text{ m}^2$$

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$$\frac{\Delta l}{l} = \frac{0.04}{0.89} = 4.5\%$$

$$\Delta A = (0.17)(0.21360\text{m}^2) = 0.036 \text{ m}^2$$

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$$\Delta A = (0.17)(0.21360\text{m}^2) = 0.036\ \text{m}^2$$

so one should write: $A = 0.21 \pm 0.04\ \text{m}^2$

Propagation of Errors

Rule 4: Exponents or Powers

$$Y = A^x$$

$$\frac{\Delta Y}{Y} = x \frac{\Delta A}{A}$$

Propagation of Errors

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$$Y = A^x \quad \Rightarrow \quad \frac{\Delta Y}{Y} = x\frac{\Delta A}{A}$$

Propagation of Errors

Rule 4: Exponents or Powers

$$Y = A^x \quad \Rightarrow \quad \frac{\Delta Y}{Y} = x\frac{\Delta A}{A}$$

This means that you multiply the "percentage" or "fractional" error by the exponent.

Propagation of Errors

Rule 4: Exponents or Powers

Measure the radius of a circle at 5.1 ± 0.1 cm.

Area is $A = \pi r^2$

$$r = 5.1\text{ cm} \pm 0.1\text{ cm} \quad \text{that's } 2\% \text{ error}$$

Propagation of Errors

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so one should write: $A = 82 \pm 3 \text{ cm}^2$

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$$A = (\pi\, 5.1^2) = 81.7 \text{ cm}^2 \pm 4\%$$

so one should write: $A = 82 \pm 3 \text{ cm}^2$

What do you do with weird functions?

For example what is the possible error of

$$x = \cos(\theta)$$

when $\theta = 21° \pm 2°$?

$$x = \cos(21°) = 0.9335$$

$$\Delta\cos(\theta) = \frac{1}{2}\left|\cos(23°) - \cos(19°)\right|$$

$$= \frac{1}{2}\left|0.9205 - 0.9455\right| = \frac{0.025}{2}$$

$$= 0.0125$$

$$\boxed{x = 0.934 \pm 0.012}$$

Substitute for the maximum and minimum values:

$$\Delta\cos(\theta) = \frac{1}{2}\,|\cos(23°) - \cos(19°)|$$

$$= \frac{1}{2}\,|0.9205 - 0.9455| = \frac{0.025}{2}$$

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So write:

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