In this question, we will explore the effective inductance of series and parallel combinations of wire coils. In your calculations, you can assume that your coils are all very long compared to the radii of their circular turns … which allows you to approximate them as infinite solenoids.

(a) $\longleftarrow \quad l \quad \longrightarrow$ **(b), (c)**   **(d)**

L_1

L_2

L_1 L_2

n turns per meter
coil area *A*
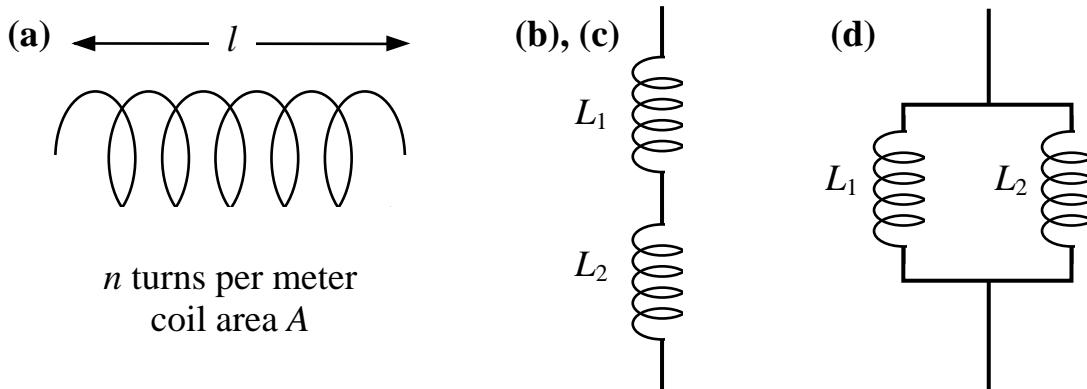
(a) Calculate the self-inductance *L* of the coil in figure (a).

The formula is *L* = Φ/*I* … just remember, the magnetic flux Φ we're talking about here is that caused by the coil's *own* field, not some external field as in earlier problems. (That's what we mean by *self*-inductance). So run some current *I* through the coil and see what flux it causes.

The field of the inductor is the field of a solenoid or B=$\mu_0 nI$.

This field pieces nL loops of area A and hence $\Phi = Bn\ell A$ or

$$\Phi = \mu_0 n^2 IA \rightarrow L = \frac{\Phi}{I} = \mu_0 n^2 A\ell$$

 (b) Work out the equivalent inductance for two inductors in series. Write down a relationship between the voltage and current through the *pair* of inductors, in terms of L_1 and L_2 for each coil. This relation should show that the pair of inductors is equivalent to a *single* coil with inductance L_s.

Let *V* be the voltage across the pair, and let V_1 and V_2 be the voltages across the individual inductors. Start with an equation relating these …

If the two coils are in series, one has the same I and dI/dt flowing through them.

The voltage drops are $\Delta V_1 = L_1 \dfrac{dI}{dt}$ and $\Delta V_2 = L_2 \dfrac{dI}{dt}$. The net voltage drop across

both coils is $\Delta V_{eq} = \Delta V_1 + \Delta V_2 = L_1 \dfrac{dI}{dt} + L_2 \dfrac{dI}{dt} = L_{equiv} \dfrac{dI}{dt} \rightarrow L_{equiv} = L_1 + L_2$

(d) What about two coils connected in *parallel*? Using the technique from part (b), determine the effective inductance L_p of this new combination.

If the coils are in parallel, the $\Delta V_{equiv} = \Delta V_1 = \Delta V_2$. The voltage drops are

related to the currents as $\Delta V_1 = \Delta V_{equiv} = L_1 \dfrac{dI_1}{dt}$ and $\Delta V_2 = \Delta V_{equiv} = L_2 \dfrac{dI_2}{dt}$ and $\Delta V_{equiv} = L_{equiv} \dfrac{dI}{dt}$

We now write the rate of change of currents:

$$\frac{\Delta V_{equiv}}{L_{equiv}} = \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} = \frac{\Delta V_{equiv}}{L_1} + \frac{\Delta V_{equiv}}{L_2} \rightarrow \frac{1}{L_{equiv}} = \frac{1}{L_1} + \frac{1}{L_2}$$

(e) Finally, here's a little design challenge, to help you develop some intuition about how inductors are constructed. Suppose you have a spool of thin wire of total length $D = 5$ m and radius $R = 0.2$ mm. You can make an inductor out of this wire by wrapping it into tight circular turns, thereby forming a spiral. How should you do the wrapping to achieve the <u>maximum</u> possible inductance? Given that you will make circular turns, there are only two parameters you can vary: the radius r of your turns, and the gap g between them (i.e. how tightly you pack your turns.) Let's try three designs and see which one gives the greatest self-inductance L.

   (i) <u>Design #1</u>: Use circular turns of radius $r = 1$ cm, and pack them as tightly as possible without causing any of the turns to overlap.

   (ii) <u>Design #2</u>: Double the turn radius r to 2 cm.

   (iii) <u>Design #3</u>: Go back to the 1 cm turn radius, but this time try packing the turns more loosely: leave a gap of 1 wire diameter between each turn.

   Hint- write L in terms of the winding radius r, the total wire length D, and the spacing

   between the wire centers.

We begin by writing L in a form that can be used for all three cases. We will write

all quantities in the expression $L = \mu_0 n^2 A \ell$ in terms of the winding radius r, the total length of wire D, and

the spacing between the centers of adjacent wires s. $\boxed{A = \pi r^2}$ Let N be the total number of number

of turns one can make with a length D of wire: $N = \dfrac{D}{2\pi r}$. Write ℓ in terms of spacing between

adjacent wires and total number of wires: $Ns = \ell \rightarrow \boxed{\dfrac{sD}{2\pi r} = \ell}$ and $\boxed{n = \dfrac{N}{\ell} = \dfrac{1}{s}}$

Substitute these forms for n , ℓ, and A to get $\boxed{L = \mu_0 n^2 A \ell = \mu_0 \left(\dfrac{1}{s}\right)^2 \left(\pi r^2\right)\left(\dfrac{sD}{2\pi r}\right) = \dfrac{\mu_0 r D}{2s}}$

Design #1 and #2 have $s = 2R$ since the wires are spaced apart by 1 wire diameter.

Hence $L = \dfrac{\mu_0 rD}{4R}$ $\boxed{\text{Thus } L_{\#2} = 2L_{\#1}}$ Since $r_{\#2} = 2\,cm$ and $r_{\#1} = 1\,cm$

Design #3 has $r_{\#3} = 1$ cm and $s_{\#3} = 4R = 2s_{\#1}$ Hence $L_{\#3} = \dfrac{1}{2}\,L_{\#1}$. Hence the order is $\boxed{L_{\#2} > L_{\#1} > L_{\#3}}$

$L_{\#1} = \dfrac{\mu_0 rD}{2R} = \dfrac{\left(4\pi \times 10^{-7}\right)(0.01)(5)}{2\left(0.1 \times 10^{-3}\right)} = 314\ \mu H\ ;\ L_{\#2} = 628\ \mu H\ ;\ L_{\#3} = 157\,\mu H$