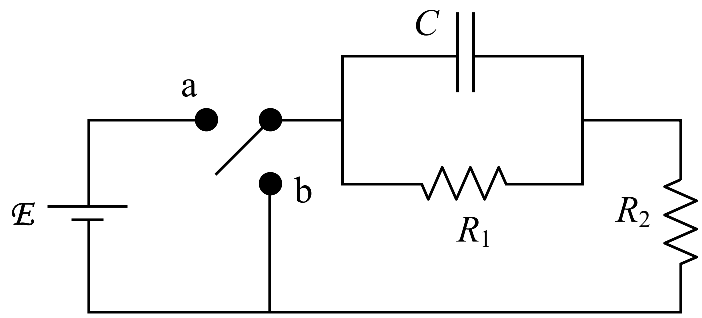
Discussion Question 7A
P212, Week 7
RC Circuits

The circuit shown initially has the capacitor uncharged, and the switch connected to neither terminal. At time $t = 0$, the switch is thrown to position a.



$\mathcal{E} = 12$ V
$C = 5$ μF
$R_1 = 3\ \Omega$
$R_2 = 6\ \Omega$

(a) At $t = 0+$, immediately after the switch is thrown to position a, what are the currents I_1 and I_2 across the two resistors?
What does the uncharged capacitor *look like* to the rest of the circuit at time 0? Does it offer *any* resistance to the flow of charge? (Why or why not?)

At t = 0, the capacitor acts like wire w/ no resistance to current flow. Hence the battery is effectively hooked to $R_2 = 6\Omega \Rightarrow I = \mathcal{E}/R_2 = 12V/6\Omega = 2$ A

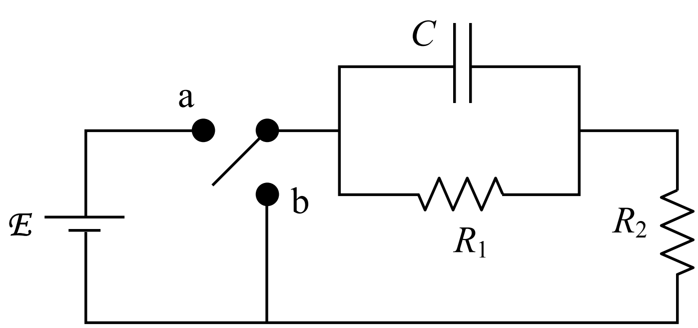
(b) After a very long time, what is the instantaneous power P dissipated in the circuit?
After a very long time, what will have happened to the capacitor? *Now* what will it look like to the rest of the circuit?

After a long time the capacitor acts like an open circuit and the battery current flows through $R_1 + R_2$. Hence I = $\mathcal{E}/(R_1 + R_2.)$ = 12/9 =4/3 A and the power is $P = \mathcal{E} I = (12V)\times(4/3A) = 16\ W$

(c) After a very long time, what is the Q charge on the capacitor?
To detemine Q, you need the voltage across the capacitor ...

From part (b), we know after a long time I=4/3 A. The voltage drop across the capacitor is the same as the voltage drop across $R_1 \rightarrow \Delta V_{cap} = IR_1 = 4/3\ A \times 3\Omega = 4V$.
$Q = C\Delta V_{cap} = (5\mu F)(4V) = 20\mu C$

Next, after a very long time T, the switch is thrown to position b.



(d) What is the time constant τ that describes the discharging of the capacitor?

We have a nice formula available for time constants: $\tau = RC$. But the R in the formula refers to the *total resistance through which the capacitor discharges*. Redrawing your circuit might help you to determine this R .

When the switch is thrown to b, the capacitor discharges through R_1 and R_2 in parallel.

$R_{equiv} = R_1 R_2 / (R_1 + R_2) = 3(6)/(3+6) = 2\Omega$; $\tau = CR_{equiv} = (5\mu F)(2\Omega) = 10\mu s$

(e) Write down an equation for the time dependence of the charge on the capacitor, for times $t > T$. Your answer for $Q(t)$ should depend only on the known quantities $\mathcal{E}$, R_1, R_2, C, and T.

You know the general form for the time dependence of a discharging capacitor. All you have to do is fix the constants in this expression to match the charge at $t = T$ and at $t = \infty$.

We know that Q will involve a exp(-t/τ) factor added to a possible constant. The boundary conditions are at $t = T^+, Q = 20\ \mu C$ and fades to zero at infinity.

The form that does this is $\boxed{Q = 20\mu C \exp\left(-\frac{t-T}{10\mu s}\right)}$

(f) What is the charge Q_{20} on the capacitor 20 μsec after time T?

$$Q = 20\mu C \exp\left(-\frac{t-T}{10\mu s}\right) = 20\mu C \exp\left(-\frac{20\mu s}{10\mu s}\right) = 20 e^{-2} = 2.71\ \mu C$$

(g) What is the current through R_2 20 μsec after time T ?

The voltage across R_2 is the $\Delta V_{cap} = Q_{20}/C = 2.71\mu C / 5\mu F = 0.542\ V$

$\Delta V_{cap} = I_2 R_2 \rightarrow \boxed{I_2 = (0.542\ V)/(6\Omega) = 0.0903\ A}$