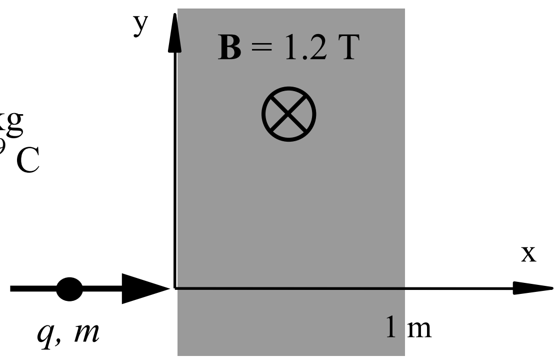
Discussion Question 7B
P212, Week 7
Lorentz Force on Moving Charges

The figure below depicts a region of space (shaded) where there is a constant magnetic field, B = 1.2 T, directed along the negative *z* axis (into the page). The magnetic field region extends infinitely far in the positive and negative *y* directions, but is constrained to the region between $x = 0$ and $x = 1$ m. A proton travels along the *x* axis toward this region, with initial speed v_0.

proton:
$m = 1.67 \times 10^{-27}$ kg
$q = e = 1.6 \times 10^{-19}$ C
$v_0 = 2 \times 10^8$ m/s



(a) What is the radius of curvature of the proton's path after it enters the magnetic field region?

To describe the particle's circular path, match the Lorentz force exerted by the **B** field to the centripetal force.

The Lorentz force provides the centripetal force to allow the proton to accelerate about a circle

Using $F = ma$ in a direction perpendicular to the proton velocity: $\vec{F} = q\vec{v} \times \vec{B} \rightarrow F_\perp = qvB$

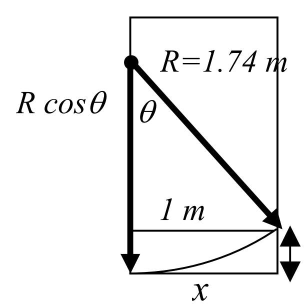
$$F_\perp = qvB = ma = m\frac{v^2}{r} \rightarrow r = \frac{mv}{qB} = \frac{(1.67e-27\text{ kg})(2e8\ m/s)}{(1.\text{e-}19\text{ C})(1.2\text{T})} = 1.74\text{ m}$$

(b) At what x and y positions will the proton leave the magnetic field region?

Sketch the proton's path on the figure, and use your knowledge of the radius of curvature.

From figure $x = 1\ m$ and $R \sin\theta = 1 \rightarrow \theta = \sin^{-1}(1/1.74) = 0.612$ rad.

$y = R - R\cos\theta = 1.74 - 1.74\cos(0.612\text{ rad}) = 0.316m$



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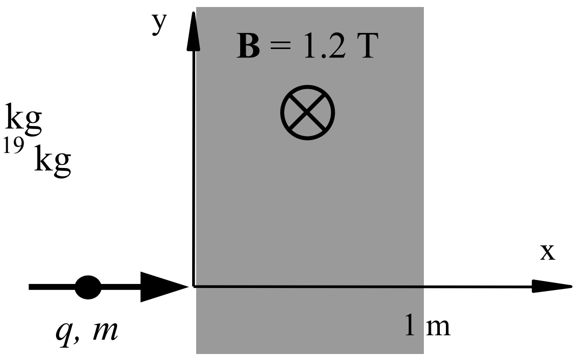
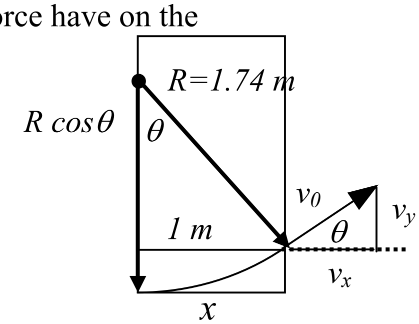


(c) What is the magnitude *v* of the proton's speed as it exits the magnetic field?
If the speed of the proton has changed, what has happened to its kinetic energy? Can the Lorentz force cause such an effect?

Because the Lorentz force is always $\perp$ to the proton path, it does no work $\Rightarrow$ no change in speed thus $v = v_0 = 2\times10^8$ m/s

(d) Suppose the proton had an initial component of velocity in the +*z* direction of $v_z = 1 \times 10^8$ m/s, and the same initial speed v_0 as before in the *xy* plane. What then would be its final velocity *v* as it exits the magnetic field? This time, express your answer as a vector with three components.
Analyze this situation by *components*. What effect does the Lorentz force have on the particle's motion in the *z* direction? What about in the *xy*-plane?



The Lorentz force is in the x-y plane hence no z acceleration. Thus v_z is unchanged $v_z = 1\times10^8$ m/s. From diagram you can
$v_x = v_0 \cos\theta = 1\times10^8 \cos(0.612) = 1.63\times10^8$ m/s
$v_y = v_0 \sin\theta = 1\times10^8 \sin(0.612) = 1.15 \times10^8$ m/s

(e) Finally, a constant electric field E is added in the shaded region. The effect of this field is that all charged particles launched with initial velocity v_0 in the *x* direction travel straight through the field region without being deflected at all! What is the magnitude and direction of the field E?

In order that the proton is undeflected, we need the electrical force to cancel the centripetal force due to the magnetic force. The magnetic force is initially along the $+\hat{y}$ axis. We thus want the electric force to be along the $-\hat{y}$ axis. To balance the forces we want:

$F_E = qE = F_B = qB_z v_0 \Rightarrow |E| = B_z v_0 = (1.2\text{ T})(2\times10^8\text{ m/s}) \rightarrow \boxed{\vec{E} = -2.4\times10^8\ \hat{y}\text{ N/C}}$