

### Chapter 1, Problem 24 (Grains of sand)

24. The metric prefixes (micro ( $\mu$ ), pico, nano, ...) are given for ready reference on the inside front cover of the textbook (see also Table 1–2) and Appendix E contains a variety of geometry formulas. The surface area  $A$  of each grain of sand of radius  $r = 50 \mu\text{m} = 50 \times 10^{-6} \text{ m}$  is given by  $A = 4\pi(50 \times 10^{-6} \text{ m})^2 = 3.14 \times 10^{-8} \text{ m}^2$ . A cube has six equal faces so the indicated surface area is  $6 \text{ m}^2$ . The number of spheres (the grains of sand)  $N$  which have a total surface area of  $6 \text{ m}^2$  is given by

$$N = \frac{6 \text{ m}^2}{3.14 \times 10^{-8} \text{ m}^2} = 1.91 \times 10^8.$$

We introduce the notion of density (which the students have probably seen in other courses):

$$\rho = \frac{m}{V}$$

so that the mass can be found from  $m = \rho V$ , where  $\rho = 2600 \text{ kg/m}^3$ . Thus, using  $V = 4\pi r^3/3$ , the mass of each grain is

$$m = \left( \frac{4\pi (50 \times 10^{-6} \text{ m})^3}{3} \right) \left( 2600 \frac{\text{kg}}{\text{m}^3} \right) = 1.36 \times 10^{-9} \text{ kg.}$$

Therefore, the total mass  $M$  is given by

$$M = Nm = (1.91 \times 10^8) (1.36 \times 10^{-9} \text{ kg}) = 0.260 \text{ kg.}$$