

Chapter 1, Problem 24 (Grains of sand)

24. The metric prefixes (micro (μ), pico, nano, ...) are given for ready reference on the inside front cover of the textbook (see also Table 1–2) and Appendix E contains a variety of geometry formulas. The surface area A of each grain of sand of radius $r = 50 \mu\text{m} = 50 \times 10^{-6} \text{ m}$ is given by $A = 4\pi(50 \times 10^{-6} \text{ m})^2 = 3.14 \times 10^{-8} \text{ m}^2$. A cube has six equal faces so the indicated surface area is 6 m^2 . The number of spheres (the grains of sand) N which have a total surface area of 6 m^2 is given by

$$N = \frac{6 \text{ m}^2}{3.14 \times 10^{-8} \text{ m}^2} = 1.91 \times 10^8.$$

We introduce the notion of density (which the students have probably seen in other courses):

$$\rho = \frac{m}{V}$$

so that the mass can be found from $m = \rho V$, where $\rho = 2600 \text{ kg/m}^3$. Thus, using $V = 4\pi r^3/3$, the mass of each grain is

$$m = \left(\frac{4\pi (50 \times 10^{-6} \text{ m})^3}{3} \right) \left(2600 \frac{\text{kg}}{\text{m}^3} \right) = 1.36 \times 10^{-9} \text{ kg}.$$

Therefore, the total mass M is given by

$$M = Nm = (1.91 \times 10^8) (1.36 \times 10^{-9} \text{ kg}) = 0.260 \text{ kg}.$$