Unit 11 HW Textbook problems

4. In each case, we use the fact that $W^{\text{grav}} = -\Delta U^{\text{grav}} = -mg\Delta y$.

(a) To point *A*, the work done by gravity is $W^{\text{grav}} = -mg\Delta y = 0 \text{ J}$.

(b) To point *B*, the work done by gravity is $W^{\text{grav}} = -mg\Delta y = -mg\left(-h/2\right) = mgh/2$.

(c) To point *C*, the work done by gravity is $W^{\text{grav}} = -mg\Delta y = -mg\left(-h\right) = mgh$.

(d) With $U_C^{\text{grav}} = 0 \text{ J.}$, we obtain $U_B^{\text{grav}} = mgh/2$.

(e) With $U_C^{\text{grav}} = 0 \text{ J.}$, we obtain $U_A^{\text{grav}} = mgh$.

(f) All the answers are proportional to the mass of the object. If the mass is doubled, all answers are doubled.

10. (a) The compression of the spring is d = 0.12 m. The work done by the force of gravity (acting on the block) is

$$W^{\text{grav}} = mgd = (0.25 \text{ kg})\left(9.8 \text{ m/s}^2\right)(0.12 \text{ m}) = 0.29 \text{ J}.$$

(b) The work done by the spring is

$$W^{\text{spring}} = -\tfrac{1}{2}kd^2 = -\tfrac{1}{2}(250\,\text{N/m})(0.12 \text{ m})^2 = -1.8 \text{ J}.$$

(c) The speed v_1 of the block just before it hits the spring is found from the work-kinetic energy theorem

$$\Delta K = 0 - \tfrac{1}{2}mv_1^2 = W^{\text{grav}} + W^{\text{spring}}.$$

Solving for the speed yields

$$v_1 = \sqrt{\frac{-2\left(W^{\text{grav}} + W^{\text{spring}}\right)}{m}} = \sqrt{\frac{-2\left(0.29 \text{ J} - 1.8 \text{ J}\right)}{0.25 \text{ kg}}} = 3.5 \text{ m/s}.$$

(d) If we instead had $v_1 = 6.94\,\text{m/s}$, we would have

$$\Delta K = 0 - \tfrac{1}{2}mv_1^2 = W^{\text{grav}} + W^{\text{spring}} = mgd - \tfrac{1}{2}kd^2.$$

Solving for *d* (and choosing the positive root) yields

$$\begin{aligned} d &= \frac{mg + \sqrt{m^2g^2 + mkv_1^2}}{k} \\ &= \frac{(0.25\text{ kg})(9.8\text{ m/s}^2) + \sqrt{(0.25\text{ kg})^2(9.8\text{ m/s}^2)^2 + (0.25\text{ kg})(250\text{ N/m})(6.94\,\text{m/s})^2}}{(250\text{ N/m})} \\ &= 0.23\text{ m}. \end{aligned}$$

15. The initial speed of the truck is $v_1 = 130\times10^3\text{ m}/3600\text{ s} = 36\text{ m/s}$. The ramp makes an angle $\theta = 15°$ with the horizontal.

(a) The change in the truck's potential energy as it climbs the ramp is $\Delta U^{\text{grav}} = mgL\sin\theta$. We conserve energy: $K_2 - K_1 = -\Delta U^{\text{grav}}$, which leads to $\tfrac{1}{2}m\left(v_2^2 - v_1^2\right) = -mgL\sin\theta$. We want the truck to come to a stop, so $v_2 = 0\text{ m/s}$. Therefore,

$$L = \frac{v_1^2}{2g\sin\theta} = \frac{(36\text{ m/s})^2}{2(9.8\text{ m/s}^2)(\sin 15°)} = 260\text{ m}.$$

(b) The required length does not depend on the mass of the truck. It remains the same if the mass is reduced.

(c) If the speed is decreased, *L* decreases since $L \propto v_1^2$.

46. The change in mechanical energy due to air drag is

$$\begin{aligned} \Delta E^{\text{mec}} &= \Delta K + \Delta U = \tfrac{1}{2}m\left(v_2^2 - v_1^2\right) + mg\Delta y \\ &= \tfrac{1}{2}(0.075\text{ kg})\left[(10.5\,\text{m/s})^2 - (12\,\text{m/s})^2\right] + (0.075\text{ kg})(9.8\,\text{m/s}^2)(1.0\text{ m}) \\ &= -0.53\text{ J}. \end{aligned}$$

54. The bundle of mass $m = 4.0\text{kg}$ moves up the $\theta = 30°$ incline from position x_1 to x_2 with an initial kinetic energy of $K_1 = 128$ J. The normal force acting on the bundle has a magnitude of $N = mg\cos\theta$, so the frictional force does work on the bundle equal to $W^{\text{friction}} = -f(x_2 - x_1) = -\mu^{\text{kin}} mg(x_2 - x_1)\cos\theta$. The gravitational potential energy of the bundle changes by $\Delta U^{\text{grav}} = mg(x_2 - x_1)\sin\theta$. The bundle reaches its maximum height x_2 when its kinetic energy is $K_2 = 0$ J. We conserve energy: $\Delta K + \Delta U = W^{\text{friction}}$. Therefore, $-K_1 + mg(x_2 - x_1)\sin\theta = -\mu^{\text{kin}} mg(x_2 - x_1)\cos\theta$. The bundle moves by a distance

$$x_2 - x_1 = \frac{K_1}{mg\left(\sin\theta + \mu^{\text{kin}}\cos\theta\right)}$$

$$= \frac{128\text{ J}}{(4.0\text{ kg})(9.8\,\text{m/s}^2)(\sin 30° + (0.30)\cos 30°)} = 4.3\text{ m.}$$

73. (a) Let $\vec{v}_2$ be the final velocity of the ball-gun system. Since the total momentum of the system is conserved $mv_1 = (m + M)v_2$. Therefore, $v_2 = mv_1/(m + M)$..

(b) The kinetic energy changes by

$$\Delta K = \tfrac{1}{2}(m + M)v_2^2 - \tfrac{1}{2}mv_1^2 = -\tfrac{1}{2}mv_1^2\left(\frac{M}{m + M}\right).$$

Since the initial kinetic energy is $K_1 = \tfrac{1}{2}mv_1^2$, the fraction of the initial kinetic energy that is stored in the spring is

$$\frac{-\Delta K}{K_1} = \frac{M}{m + M}.$$