Solutions to Unit 13 Text Book Hwk. Problems

16. If we write $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$, then (using Eq. 12-16) we find $\vec{r} \times \vec{F}$ is equal to

$$\left(yF_z - zF_y\right)\hat{i} + \left(zF_x - xF_z\right)\hat{j} + \left(xF_y - yF_x\right)\hat{k}.$$

(a) In the above expression, we set x = –2.0 m, y = 0 m, z = 4.0 m, F_x = 6.0 N, F_y = 0 N and F_z = 0 N. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = (24\,\mathrm{N\cdot m})\hat{\mathrm{j}}$.

(b) The values are just as in part (a) with the exception that now F_x = –6.0 N. We find $\vec{\tau} = \vec{r} \times \vec{F} = (-24\,\mathrm{N\cdot m})\,\hat{\mathrm{j}}$.

(c) In the above expression, we set x = –2.0 m, y = 0 m, z = 4.0 m, F_x = 0 N, F_y = 0 N and F_z = 6.0 N. We get $\vec{\tau} = \vec{r} \times \vec{F} = (12\,\mathrm{N\cdot m})\,\hat{\mathrm{j}}$.

(d) The values are just as in part (c) with the exception that now F_z = –6.0N. We find $\vec{\tau} = \vec{r} \times \vec{F} = (-12\,\mathrm{N\cdot m})\,\hat{\mathrm{j}}$.

21. If we write (for the general case) $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$, then (using Eq. 12-16) we find $\vec{r} \times \vec{v}$ is equal to

$$\left(yv_z - zv_y\right)\hat{i} + \left(zv_x - xv_z\right)\hat{j} + \left(xv_z - yv_x\right)\hat{k}$$

(a) The rotational momentum is given by the vector product $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r} = (3.0\text{ m})\hat{\mathrm{i}} + (8.0\text{ m})\hat{\mathrm{j}}$ is the position vector of the particle and m = 3.0 kg is its mass. The particle's rotational momentum about the origin is

$$\vec{\ell} = (3.0\text{ kg})\big((3.0\text{ m})(-6.0\,\mathrm{m/s}) - (8.0\text{ m})(5.0\,\mathrm{m/s})\big)\hat{\mathrm{k}} = \left(-1.7\times10^2\ \mathrm{kg\cdot m^2/s}\right)\hat{\mathrm{k}}\ .$$

(b) We write $\vec{F} = F_x\hat{\mathrm{i}} = (-7.0\mathrm{N})\hat{\mathrm{i}}$ and obtain

$$\vec{\tau} = \vec{r} \times \vec{F} = -yF_x\hat{\mathrm{k}} = -(8.0\text{ m})(-7.0\text{ N})\hat{\mathrm{k}} = (56\text{ N}\cdot\text{m})\hat{\mathrm{k}}.$$

(c) According to Eq. 12-24, $\vec{\tau} = d\vec{\ell}/dt$, so the rate of change of the rotational momentum is 56 $\mathrm{kg\cdot m^2/s^2}$, in the positive z direction.

28. If we write $\vec{r}' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}}$ and $\vec{v} = v_x\,\hat{\mathrm{i}} + v_y\,\hat{\mathrm{j}}$, then $\vec{r}' \times \vec{v} = \left(x'v_y - y'v_x\right)\hat{\mathrm{k}}$.

(a) Here, $\vec{r}' = \vec{r} = (3.0 \text{ m})\hat{\text{i}} - (4.0 \text{ m})\hat{\text{j}}$. Thus, with $\vec{v} = (30 \text{ m/s})\hat{\text{i}} + (60 \text{ m/s})\hat{\text{j}}$ and $m = 2.0$ kg, we obtain the rotational momentum relative to the origin:

$$\vec{\ell} = m(\vec{r} \times \vec{v}) = (2.0 \text{ kg})((3.0 \text{ m})(60 \text{ m/s}) - (-4.0 \text{ m})(30 \text{ m/s}))\hat{\text{k}} = (600 \text{ kg} \cdot \text{m}^2/\text{s})\hat{\text{k}}.$$

(b) Now, with $\vec{r}_0 = (-2.0 \text{ m})\hat{\text{i}} - (2.0 \text{ m})\hat{\text{j}}$, $\vec{r}' = \vec{r} - \vec{r}_o = (5.0 \text{ m})\hat{\text{i}} - (2.0 \text{ m})\hat{\text{j}}$. The rotational momentum relative to the point $\vec{r}_0$ is

$$\vec{\ell}' = m(\vec{r}' \times \vec{v}) = (2.0 \text{ kg})((5.0 \text{ m})(60 \text{ m/s}) - (-2.0 \text{ m})(30 \text{ m/s}))\hat{\text{k}} = (720 \text{ kg} \cdot \text{m}^2/\text{s})\hat{\text{k}}.$$

33. (a) The total rotational inertia is $I = m(3d)^2 + m(2d)^2 + m(d)^2 = 14md^2$

(b) The rotational momentum of the middle particle (particle B) is given by $L_B = I_B\omega = m(2d)^2\,\omega = 4md^2\omega.$

40. (a) Let the first disk be disk A and the second disk B. Rotational momentum is conserved. The rotational speed after coupling is therefore

$$\begin{aligned}|\omega_2| &= \frac{I_A|\omega_{A1}| + I_B|\omega_{B\,1}|}{I_A + I_B}\\ &= \frac{(3.3 \text{ kg}\cdot\text{m}^2)(450 \text{ rev/min}) + (6.6 \text{ kg}\cdot\text{m}^2)(900 \text{ rev/min})}{3.3 \text{ kg}\cdot\text{m}^2 + 6.6 \text{ kg}\cdot\text{m}^2}\\ &= 750 \text{ rev/min}.\end{aligned}$$

(b) In this case, we obtain

$$|\omega_2| = \left|\frac{I_A|\omega_{A1}| - I_B|\omega_{B\,1}|}{I_A + I_B}\right| = 450 \text{ rev/min}.$$

The direction of $\vec{\omega}_2$ in this case is the same as the rotational velocity of disk B.

54. Let m and v be the mass and initial speed of the ball and R the radius of the merry-go-round. The initial rotational momentum is

$$L_1 = mvR\sin\theta,$$

where the ball approaches the "lever arm" at an angle of $\theta = 127°$. With I as the rotational inertia of the merry-go-round and m' as the mass of the ball, the rotational momentum after the ball is caught can be expressed as

$$L_2 = \left[\left(m + m'\right)R^2 + I\right]\omega_2,$$

where ω_2 is the rotational velocity of the merry-go-round after the ball is caught. Conservation of rotational momentum leads to $mvR\sin\theta = \left[\left(m + m'\right)R^2 + I\right]\omega_2$, so the rotational speed of the merry-go-round is

$$\left|\omega_2\right| = \frac{mvR\sin\theta}{\left(m + m'\right)R^2 + I} = \frac{\left(1.0\text{ kg}\right)\left(12\text{ m/s}\right)\left(2.0\text{ m}\right)\left(\sin 127°\right)}{\left(30\text{ kg} + 1.0\text{ kg}\right)\left(2.0\text{ m}\right)^2 + 150\text{ kg}\cdot\text{m}^2} = 0.070\text{ rad/s}.$$