Solutions to Unit 14 Text Book Hwk. Problems

1. (a) During simple harmonic motion, the velocity is momentarily zero at the endpoints of its travel, when $x(t) = \pm X$. From endpoint to the next constitutes a half period of the motion, so the period is $T = 0.50\,\text{s}$.

(b) From Eq. 16-2, $f = 1/T = 1/0.50\ \text{s} = 2.0\ \text{Hz}$.

(c) At one of the endpoints, $x(t) = X$. At the other, $x(t) = -X$. The distance between these two points is given as 36 cm, so the amplitude is $X = 18$ cm.

3. (a) The motion repeats every 0.500 s so the period must be $T = 0.500$ s.

(b) From Eq. 16-2, $f = 1/T = 1/0.500\ \text{s} = 2.00\ \text{Hz}$.

(c) From Eq. 16-6, $\omega = 2\pi f = 2\pi\,(2.00\ \text{Hz}) = 12.6\,\text{rad/s}$.

(d) From Eq. 16-11, the spring constant is

$$k = m\omega^2 = (0.500\text{kg})(12.6\,\text{rad/s})^2 = 79.0\,\text{kg/s}^2.$$

(e) The velocity amplitude is related to the amplitude $X = 35.0\ \text{cm}$ by

$$V = \omega X = (12.6\,\text{rad/s})(0.350\text{m}) = 4.40\,\text{m/s}.$$

(f) The force is at its maximum magnitude when $x(t) = \pm X$, so
$F^{\max} = kX = (79.0\,\text{kg/s}^2)(0.350\text{m}) = 27.6\ \text{N}$.

4. From Eq. 16-16, simple harmonic motion with an amplitude of $X = 2.20\ \text{cm}$ and a frequency of $f = 6.60\ \text{Hz}$ has a maximum acceleration of

$$A = (2\pi f)^2\, X = (2\pi\,(6.60\ \text{Hz}))^2\,(0.0220\ \text{m}) = 37.8\ \text{m/s}^2.$$

8. (a) The maximum force is related to the maximum acceleration by $F^{\max} = mA$, and Eq. 16-16 relates the maximum acceleration to the amplitude X. The maximum force is

$$F^{\max} = mA = m\omega^2 X = \frac{(2\pi)^2\, mX}{T^2} = \frac{(2\pi)^2\,(0.12\text{kg})(0.085\text{m})}{(0.20\ \text{s})^2} = 10\ \text{N}.$$

(b) Hooke's law leads to $k = F^{\max}/X = 10\text{ N}/0.085\text{ m} = 120\,\text{N/m}$.

30. The weight of the three cars has a component acting down the $\theta = 30°$ incline, causing the cable to stretch by a distance $\Delta x = 15$ cm. The spring constant of the cable is therefore given by

$$k = \frac{F}{\Delta x} = \frac{3mg\sin\theta}{\Delta x} = \frac{3(10000\text{kg})(9.8\,\text{m/s}^2)(\sin 30°)}{0.15\text{m}} = 9.8\times10^5\ \text{N/m}.$$

(a) With only two cars remaining attached to the cable, the frequency of oscillations is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{9.8\times10^5\ \text{N/m}}{2(10000\,\text{kg})}} = 1.1\ \text{Hz}.$$

(b) The amplitude of the oscillations is just the difference between the initial equilibrium position with three cars and the new equilibrium position with two cars. Therefore,

$$X = \frac{3mg\sin\theta}{k} - \frac{2mg\sin\theta}{k} = \frac{mg\sin\theta}{k} = \frac{(10000\text{kg})(9.8\,\text{m/s}^2)(\sin 30°)}{9.8\times10^5\ \text{N/m}} = 5.0\text{ cm}.$$

34. (a) Eq. 16-24 gives

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{17\text{m}}{9.8\,\text{m/s}^2}} = 8.3\text{ s}.$$

(b) The period is independent of the ball's mass (for a simple pendulum).

52. (a) The bullet collides inelastically with the block. We conserve momentum in the collision to obtain the speed of the block immediately after the collision:

$$V = \frac{mv}{m+M}.$$

(b) The speed of the system reaches zero at the maximum compression of the spring, so conservation of mechanical energy leads to $\frac{1}{2}(m+M)V^2 = \frac{1}{2}kX^2$. Using the result from part (a), this leads to a maximum displacement of

$$X = \frac{mv}{\sqrt{k(m+M)}}.$$