Print this page

3. Hints for Problems

7

You need to know the time taken by the dive. Use what you know about free fall to compute the time, then divide the rotational displacement ($2.5\ rev$) by the duration of the dive.

[Ans: $1.75\ rev/s$ ($11\ rad/s$)]

13

Take the time at the start of the interval to be $t_1 = 0$. Then the time at the end of the interval is $t_2 = 4.0\ s$, and the angle of rotation is $\theta_2 = \omega_1 t_2 + \frac{1}{2}\alpha t_2^2$. Solve for ω_1. Now let t_2 be the time when the wheel is at rest and then use $\omega_2 = \omega_1 + \alpha t_2$, with $\omega_2 = 0$, to calculate t_2.

[Ans: $8.0\ s$]

17

(a) Use the equations for rotation with constant rotational acceleration: $\theta_2 = \theta_1 + \omega_1(t_2 - t_1) + \frac{1}{2}\alpha(t_2 - t_1)^2$ and $\omega_2 = \omega_1 + \alpha(t_2 - t_1)$., where θ_1 is the rotational position and ω_1 is the rotational velocity at time t_1, θ_2 is the rotational position and ω_2 is the rotational velocity at time t_2, and α is the rotational acceleration. Use the second equation to eliminate α from the first and obtain $\theta_2 - \theta_1 = \frac{1}{2}(\omega_2 + \omega_1)(t_2 - t_1)$. Solve for $t_2 - t_1$. When you evaluate the result, convert $40\ rev$ to radians.

(b) Solve $\omega_2 = \omega_1 + \alpha(t_2 - t_1)$ for α.

(c) Solve $\theta_2 - \theta_1 = \omega_1(t_2 - t_1) + \frac{1}{2}\alpha(t_2 - t_1)^2$ for $t_2 - t_1$.

[Ans: (a) $3.4 \times 10^2\ s$; (b) $-4.5 \times 10^{-3}\ rad/s^2$; (c) $98\ s$]

25

(a) In the time light takes to go from the wheel to the mirror and back again, the wheel turns through a rotational displacement of $\Delta\theta = 2\pi/500 = 1.26 \times 10^{-2}\ rad$. That time is $\Delta t = 2L/c$, where c is the speed of light. Divide the rotational displacement by the duration of the time interval.

(b) If r is the radius of the wheel, the translational speed of a point on its rim is $v = \omega r$, where ω is the rotational speed.

[Ans: (a) $3.8 \times 10^3\ rad/s$; (b) $1.9 \times 10^2\ m/s$]

27

(a) Earth makes one rotation per day. Divide the rotational displacement in radians ($2\pi\ rad$) by the duration of a day in seconds.

(b) Use $v = |\omega| r$, where $|\omega|$ is the rotational speed and r is the radius of the orbit. A point on Earth at a latitude of $40°$ goes around a circle of radius $r = R\cos 40°$, where R is the radius of Earth ($6.37 \times 10^6\ m$).

(c) At the equator and all other points on Earth the value of ω is the same.

(d) The radius of the orbit is now R.

[Ans: (a) $7.3 \times 10^{-5}\,rad/s$; (b) $3.6 \times 10^{2}\,m/s$; (c) $7.3 \times 10^{-5}\,rad/s$; (d) $4.6 \times 10^{2}\,m/s$]

33

The kinetic energy is given by $K = \frac{1}{2}I|\omega|^2$, where I is the rotational inertia and $|\omega|$ is the rotational speed. Solve for I. Covert the given rotational speed to radians per second.

[Ans: $12.3\ kg \cdot m^2$]

37

(a) The rotational inertia of a rod of length d and mass M, rotating about an axis through its center and perpendicular to it, is $\frac{1}{12}Md^2$ (see Table 11-2). Use the parallel-axis theorem to find the rotational inertias of the rods in this problem. For one rod the rotation axis is $d/2$ from its center and for the other it is $3d/2$ from its center. The rotational inertia of either of the particles is the product of its mass and the square of its distance from the rotation axis. Sum the rotational inertias to obtain the total rotational inertia.

(b) The rotational kinetic energy is given by $K = \frac{1}{2}I|\omega|^2$, where I is the total rotational inertia.

[Ans: (a) $0.023\ kg \cdot m^2$; (b) $11\ mJ$]

41

Use the parallel-axis theorem. According to Table 11-2, the rotational inertia of a uniform slab about an axis through the center and perpendicular to the large faces is given by $I_{com} = (M/12)(a^2 + b^2)$. A parallel axis through a corner is a distance $h = \sqrt{(a/2)^2 + (b/2)^2}$ from the center.

[Ans: $4.7 \times 10^{-4}\,kg \cdot m^2$]

47

Take the positive direction to be out of the page. Then a torque that tends to cause a counterclockwise rotation from rest is positive and a torque that tends to cause a clockwise rotation from rest is negative. The magnitude of either of the torques is given by $rF\sin\theta$, where r is the distance from O to the point of application of the force, F is the magnitude of the force, and θ is the angle between the force and the displacement of the application point from O, when they are drawn with their tails at the same point.

[Ans: (a) $r_AF_A\sin\theta_A - r_BF_B\sin\theta_B$; (b) $-3.85\ N \cdot m$]

49

(a) Assume the rotational acceleration is constant and that the launch occurs at time $t_1 = 0$. Use the kinematic equation $\omega_2 = \omega_1 + \alpha(t_2 - t_1)$, where ω_1 is the initial rotational velocity (at time t_1), ω_2 is the final rotational velocity (at time t_2), and α is the rotational acceleration. Solve for α.

(b) If I is the rotational inertia of the diver, then according to Newton's second law for rotation the magnitude of the torque acting on her is $\tau = I\alpha$.

[Ans: (a) $28.2\ rad/s^2$; (b) $3.38 \times 10^2\ N \cdot m$]

55

(a) Use constant translational acceleration kinematics. If down is taken to be the positive y direction, the origin is placed at the starting point of the heavier block, and $\vec{a}$ is the acceleration of the heavier block, then the coordinate of that block is given by $y_2 = \frac{1}{2}a_y(t_2 - t_1)^2$. Solve for a. The lighter block has an acceleration of the same magnitude but upward.

(b) Newton's second law for the heavier block is $Mg - T_h = Ma_y$, where T_h is the tension force of the cord on that block. Evaluate this expression.

(c) Newton's second law for the lighter block is $mg - T_l = -ma_y$, where T_l is the tension force on that block. Evaluate this expression.

(d) Since the cord does not slip on the pulley, the tangential acceleration of a point on the rim of the pulley must be the same as the acceleration of the blocks, so the magnitude of the rotational acceleration of the pulley is given by $|\alpha| = |a_y|/R$, where R is the radius of the pulley.

(e) The net torque acting on the pulley is $\tau = (T_h - T_l)R$. Equate this to $I\alpha$ and solve for I.

[Ans: (a) $6.00 \times 10^{-2}\,m/s^2$; (b) $4.87\,N$; (c) $4.54\,N$; (d) $1.20\,rad/s^2$; (e) $1.38 \times 10^{-2}\,kg \cdot m^2$]

61

(a) The rotational kinetic energy is $K = \frac{1}{2}I|\omega|^2$, where I is the rotational inertia of the rod and $|\omega|$ is its rotational speed in radians per second at the lowest point. Use Table 11-2 of the text and the parallel axis theorem to calculate the rotational inertia.

(b) If the center of mass rises a distance h, the potential energy increases by Mgh and the kinetic energy decreases to zero. The conservation of energy equation is $\frac{1}{2}I\omega^2 = Mgh$. Solve for h.

[Ans: (a) $0.63\,J$; (b) $0.15\,m$]

63

Let ℓ be the length of the stick. Since its center of mass is $\ell/2$ from either end, the potential energy changes by $-mg\ell/2$, where m is its mass. The initial kinetic energy is zero and the final kinetic energy is $\frac{1}{2}I|\omega|^2$, where I is its rotational inertia of the stick for rotation about an axis through one end and $|\omega|$ is its rotational speed just before it hits the floor. Conservation of energy yields $\frac{1}{2}mg\ell = \frac{1}{2}I|\omega|^2$. Solve for $|\omega|$. The free end of the stick is a distance ℓ from the rotation axis, so its speed as it hits the floor is $v = |\omega|\ell$. Use Table 11-2 and the parallel-axis theorem to calculate the rotational inertia.

[Ans: $5.42\,m/s$]