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3. Hints for Problems

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(a) Since the tires do not skid on the road the speed v of the automobile and the rotational speed $|\omega|$ of the tires are related by $v = |\omega|R$, where R is the radius of a tire.

(b) Use $|\omega_2|^2 - |\omega_1|^2 = 2\alpha\,\Delta\theta$, where $|\omega_1|$ is the initial rotational speed, $|\omega_2|$ is the final rotational speed, α is the rotational acceleration, and $\Delta\theta$ is the rotational displacement. Solve for α. Be sure to use consistent angular measure.

(c) The distance d the car travels is related to the rotational displacement of the wheels by $d = |\Delta\theta|\,R$.

[Ans: (a) $59.3\ rad/s$; (b) $9.31\ rad/s^2$; (c) $70.7\ m$]

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(a) An expression for the acceleration is derived in the text and appears as Eq. 12-6:

$$a_{\text{com } y} = -\frac{g}{1 + I_{\text{com}}/MR_0^2},$$

where M is the mass of the yo-yo, I_{com} is its rotational inertia about the center of mass, and R_0 is the radius of its axle.

(b) Solve the kinematic equation $Y_{\text{com}} = \frac{1}{2}a_{\text{com } y}(\Delta t)^2$ for Δt.

(c) As it reaches the end of the string its translational speed is $v_{\text{com}} = a_{\text{com}}\,\Delta t$.

(d) The translational kinetic energy is $K = \frac{1}{2}mv_{\text{com } y}^2$.

(e) The rotational speed is given by $|\omega| = v_{\text{com } y}/R_0$ and the rotational kinetic energy is $K = \frac{1}{2}I_{\text{com}}|\omega|^2$.

(f) The rotational speed is $|\omega| = v_{\text{com } y}/R_0$.

[Ans: (a) $13\ cm/s^2$; (b) $4.4\ s$; (c) $55\ cm/s$; (d) $1.8 \times 10^{-2}\ J$; (e) $1.4\ J$; (f) $27\ rev/s$]

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(a) Recall that the vector product of two vectors is perpendicular to both of the vectors and that the scalar product of two vectors is proportional to the cosine of the angle between the vectors.

(b) The magnitude of $\vec{b} \times \vec{a}$ is $ab\sin\phi$. Since $\vec{b} \times \vec{a}$ is perpendicular to $\vec{a}$, the magnitude of $\vec{a} \times (\vec{b} \times \vec{a})$ is $a|\vec{b} \times \vec{a}|$.

[Ans: (b) $a^2 b \sin\phi$]

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(a) Let $\vec{F} = F_x\,\hat{i} + F_y\,\hat{j}$ and $\vec{r} = x\,\hat{i} + y\,\hat{j}$. Then

$$\vec{\tau} = \vec{r} \times \vec{F} = (x\,\hat{i} + y\,\hat{j}) \times (F_x\,\hat{i} + F_y\,\hat{j}) = (xF_y - yF_x)\,\hat{k}\,.$$

The last result can be obtained by multiplying out the quantities in parentheses and using $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{i} = -\hat{k}$, $\hat{i} \times \hat{i} = 0$, and $\hat{j} \times \hat{j} = 0$.

(b) Use the definition of the vector product: $|\vec{r} \times \vec{F}| = rF \sin\phi$, where ϕ is the angle between $\vec{r}$ and $\vec{F}$ when they are drawn with their tails at the same point. Also use $r = \sqrt{x^2 + y^2}$ and $F = \sqrt{F_x^2 + F_y^2}$.

[Ans: (a) $(50\ N \cdot m)\ \hat{k}$; (b) $90°$]

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(a) The rotational momentum is given by the vector product $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector of the particle and $\vec{v}$ is its velocity. Since the position and velocity vectors are in the xy plane we may write $\vec{r} = x\,\hat{i} + y\,\hat{j}$ and $\vec{v} = v_x\,\hat{i} + v_y\,\hat{j}$. Thus $\vec{r} \times \vec{v} = (xv_y - yv_x)\,\hat{k}$.

(b) The torque is given by $\vec{\tau} = \vec{r} \times \vec{F}$. Since the force has only an x component we may write $\vec{F} = F_x\,\hat{i}$ and find that $\vec{\tau} = -yF_x\,\hat{k}$. (c) According to Newton's second law, $\vec{\tau} = d\vec{\ell}/dt$.

[Ans: (a) $(-1.7 \times 10^2\ kg \cdot m^2/s)\ \hat{k}$; (b) $(56\ N \cdot m)\ \hat{k}$; (c) $(56\ kg \cdot m^2/s^2)\ \hat{k}$]

25

The rotational momentum of a particle is given by $\vec{\ell} = m\vec{r} \times \vec{v}$, where m is its mass, $\vec{r}$ is its relative to the origin, and $\vec{v}$ is its velocity. The magnitude is $\ell = mrv \sin\phi$, where ϕ is the angle between $\vec{r}$ and $\vec{v}$. For each of the particles in this problem $\vec{r}$ and $\vec{v}$ are perpendicular to each other, so $\ell = mrv$. To find the direction use the right hand rule: Move $\vec{r}$ so its tail is at the tail of $\vec{v}$, then curl the fingers of your right hand so they tend to rotate $\vec{r}$ toward $\vec{v}$. Your thumb points in the direction of the vector product. Find the rotational momentum of each particle and vectorially sum the rotational momenta.

[Ans: (a) $9.8\ kg \cdot m^2/s$; (b) positive z direction]

27

(a) Use $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector of the object, $\vec{v}$ is its velocity vector, and m is its mass. Write $\vec{r} = x\,\hat{i} + z\,\hat{k}$ and $\vec{v} = v_x\,\hat{i} + v_y\,\hat{j}$. Then $\vec{\ell} = -m(zv_y\,\hat{i} + zv_x\,\hat{j} + xv_y\,\hat{k})$.

(b) Use $\vec{\tau} = \vec{r} \times \vec{F}$, with $\vec{F} = F\,\hat{j}$. Then $\vec{\tau} = -zF_y\,\hat{i} + xF_y\,\hat{k}$.

[Ans: (a) $-(10\ kg \cdot m^2/s)\,\hat{i} + (6.0\ kg \cdot m^2/s)\hat{j} + (20\ kg \cdot m^2/s)\,\hat{k}$; (b) $(8.0\ N \cdot m)\,\hat{i} + (8.0\ N \cdot m)\,\hat{k}$]

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(a) Since $\vec{\tau} = d\vec{L}/dt$ the average torque acting during any interval is given by $<\vec{\tau}>= (\vec{L}_2 - \vec{L}_1)/\Delta t$, where $\vec{L}_1$ is the initial rotational momentum, $\vec{L}_2$ is the final rotational momentum, and Δt is the time interval.

(b) The angle turned is $\Delta\theta = \omega_1 \Delta t + \frac{1}{2}\alpha(\Delta t)^2$. If the rotational acceleration α is uniform, then so is the torque and $\alpha = \tau/I$. Furthermore, $\omega_1 = L_1/I$.

(c) The work done on the wheel is $W = \tau\,\Delta\theta$.

(d) The average power is the work done by the flywheel (the negative of the work W done on the flywheel) divided by the time interval: $< P >= -W/\Delta t$.

[Ans: (a) $1.47\ N \cdot m$; (b) $20.3\ rad$; (c) $-29.8\ J$; (d) $19.9\ W$]

33

(a) A particle contributes mr^2 to the rotational inertia. Here r is the distance from the origin O to the particle. The strings do not contribute since they are massless. Sum the contributions of the individual particles.

(b) The rotational momentum of the middle particle is given by $L_{\text{middle}} = I_{\text{middle}}\omega$, where $I_{\text{middle}} = 4md^2$ is its rotational inertia.

(c) The magnitude of the total rotational momentum is $I\omega$.

[Ans: (a) $14md^2$; (b) $4md^2\omega$; (c) $14md^2\omega$]

37

(a) No external torques act on the system consisting of the man, bricks, and platform, so the total rotational momentum of that system is conserved. Let I_1 be the initial rotational inertia of the system and let I_2 be the final rotational inertia. If $|\omega_1|$ is the initial rotational speed and $|\omega_2|$ is the final rotational speed, then $I_1|\omega_1| = I_2|\omega_2|$. Solve for $|\omega_2|$.

(b) The initial kinetic energy is $K_1 = \frac{1}{2}I_1|\omega_1|^2$, the final kinetic energy is $K_2 = \frac{1}{2}I_2|\omega_2|^2$. Calculate $K_2 - K_1$.

[Ans: (a) $3.6\ rev/s$; (b) 3.0; (c) The man did work in decreasing the rotational inertia by pulling the bricks closer to his body. This energy came from the man's store of internal energy.]

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No external torques act on the system consisting of the train and wheel, so the rotational momentum of that system is conserved. The total rotational momentum of the system is initially zero and remains zero. Let I $(= MR^2)$ be the rotational inertia of the wheel. The magnitude of its final rotational momentum is $L_w = I\omega = MR^2\omega$, where M is the mass of the wheel. The speed of the track is ωR and the speed of the train is $\omega R - v$. The magnitude of the rotational momentum of the train is $L_t = m(\omega R - v)R$, where m is its mass. Substitute into the conservation of rotational momentum equation and solve for ω.

[Ans: $\dfrac{mv}{(M+m)R}$]

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The total rotational momentum of the record-putty system is conserved as the putty sticks to the record. The magnitude of the rotational momentum of the system before the putty sticks is $I|\omega_1|$, where I is the rotational inertia of the record and $|\omega_1|$ is its rotational speed. After the putty sticks the magnitude of the total rotational momentum is $(I + mR^2)|\omega_2|$, where m is the mass of the putty, R is the radius of the record, and $|\omega_2|$ is the new rotational speed. Equate the two expressions for the rotational momentum and solve for $|\omega_2|$.

[Ans: $3.4\ rad/s$]

53

(a) The rotational momentum of the two-disk system is conserved as the small disk slides. Let I_A be the rotational inertia of the large disk, I_{B1} be the rotational inertia of the small disk in its initial location, and I_{B2} be the rotational inertia of the small disk in its final location, all about the axis through the center of the large disk. If $|\omega_1|$ is the initial rotational speed of the disks and $|\omega_2|$ is their final rotational speed, then conservation of rotational momentum yields $(I_A + I_{B1})|\omega_1| = (I_A + I_{B2})|\omega_2|$. The center of the small disk moved a distance of $2r$. The parallel axis theorem tells us that $I_{B2} = I_{B1} + m(2r)^2$. According to Table 11-2, the rotational inertia of a uniform disk with mass M and radius R is $MR^2/2$. You can now solve for $|\omega_2|$.

(b) The old kinetic energy is $K_1 = \frac{1}{2}(I_A + I_B)\omega_1^2$ and the new kinetic energy is $K_2 = \frac{1}{2}(I_A + I'_B)\omega_2^2$.

[Ans: (a) $18\ rad/s$; (b) 0.92]

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Use conservation of mechanical energy to show that the speed of the particle just before it hits the rod is $v = \sqrt{2gh}$. Use conservation of angular momentum to show that the angular speed of the rod and particle just after the collision is $|\omega| = mvd/(I + md^2)$, where I is the rotational inertia of the rod about an end. To compute I see Table 11-2 and use the parallel-axis theorem. Use conservation of mechanical energy to find an expression for θ. The initial kinetic energy is the rotational kinetic energy of the rod and particle.

[Ans: $\cos^{-1}\left[1 - \frac{6m^2h}{d(2m+M)(3m+M)}\right]$]