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3. Hints for Problems

1

The points of zero velocity are the end points of the motion, so it takes half a period to travel between them and the distance between them is twice the amplitude. The frequency is the reciprocal of the period.

[Ans: (a) $0.50\ s$; (b) $2.0\ Hz$; (c) $18\ cm$]

5

The magnitude of the maximum acceleration is given by $A = \omega^2 X$, where ω is the angular frequency and X is the amplitude. The angular frequency for which the magnitude of the maximum acceleration is g is given by $\omega = \sqrt{g/X}$ and the corresponding frequency is given by $f = \omega/2\pi$.

[Ans: $5.0 \times 10^2\ Hz$]

7

The angular frequency ω is given by $\omega = 2\pi f = 2\pi/T$, where f is the frequency and T is the period. The relationship $f = 1/T$ was used to obtain the last form. The maximum speed V and maximum displacement X are related by $V = \omega X$.

[Ans: (a) $6.28 \times 10^5\ rad/s$; (b) $1.59 \times 10^{-3}\ m$]

9

The amplitude is half the range of the displacement. The maximum speed V is related to the amplitude X by $V = \omega X$, where ω is the angular frequency. Furthermore, the angular frequency is $\omega = 2\pi f$, where f is the frequency. The maximum acceleration is $A = \omega^2 X = (2\pi f)^2 X$.

[Ans: (a) $1.0\ mm$; (b) $0.75\ m/s$; (c) $570\ m/s^2$]

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(a) The displacement x and acceleration a at any instant of time are related by $x = -\omega^2 a$, where ω is the angular frequency. The frequency is $f = \omega/2\pi$.

(b) The angular frequency, mass m, and spring constant k are related by $\omega^2 = k/m$.

(c) If X is the amplitude then we may take the displacement to be $x = X \cos(\omega t)$. The velocity is $v = -\omega X \sin(\omega t)$. Use the trigonometric identity $\sin^2(\omega t) + \cos^2(\omega t) = 1$ to find an expression for X.

[Ans: (a) $5.58\ Hz$; (b) $0.325\ kg$; (c) $0.400\ m$]

29

(a) First consider a single spring with spring constant k and unstretched length L. One end is attached to a wall and the other is attached to an object. If the spring is elongated by Δx the magnitude of the force it exerts on the object is $F = k\,\Delta x$. Now consider it to be two springs, with spring constants k_1 and k_2, arranged so spring 1 is attached to the object. If spring 1 is elongated by Δx_1 then the magnitude of the force exerted on the object is $F = k_1\,\Delta x_1$. This must be the same as the force of the single spring, so $k\,\Delta x = k_1\,\Delta x_1$. We must determine the relationship between Δx and Δx_1.

The springs are uniform so the elongation of any portion of the spring is proportional to its unstretched length. This means spring 1 is elongated by $\Delta x_1 = CL_1$ and spring 2 is elongated by $\Delta x_2 = CL_2$, where C is a constant of proportionality. The total elongation is $\Delta x = \Delta x_1 + \Delta x_2 = C(L_1 + L_2) = CL_1(n+1)/n$, where $L_2 = L_1/n$ was used to obtain the last form. Solve for C and substitute into $\Delta x_1 = CL_1$.

(b) Now suppose the object is placed at the other end of the composite spring, so spring 2 exerts a force on it, and carry out a similar analysis.

(c) and (d) The frequency when spring 1 alone is attached to an block of mass m is given by $f_1 = (1/2\pi)\sqrt{k_1/m}$. Substitute for k_1 and recognize that $f = (1/2\pi)\sqrt{k/m}$ is the frequency with the entire spring in place. Carry out a similar calculation for the situation when spring 2 alone is attached to the block.

[Ans: (a) $k(n+1)/n$; (b) $k(n+1)$; (c) $\sqrt{\frac{n+1}{n}}f$; (c) $\sqrt{n+1}f$]

33

The period of a simple pendulum is given by $T = 2\pi\sqrt{L/g}$, where L is its length. Solve for L.

[Ans: $0.99\ m$]

35

Since the pivot point is a distance d from the center of mass the period is given by $T = 2\pi\sqrt{I/mgd}$, where m is the mass of pendulum and I is its rotational inertia about the pivot point. Use the parallel axis theorem to find I. According to Table 11-2 the rotational inertia of a uniform stick about an axis through its center is $I_{com} = (1/12)mL^2$, where L is the length of the stick. The rotational inertia about an axis that is a distance d from the center is $I = I_{com} + md^2$. Substitute for I in the expression for T, then solve for d.

[Ans: $5.6\ cm$]

39

(a) A uniform disk pivoted at its center has a rotational inertia of $\frac{1}{2}MR^2$, where M is its mass and R is its radius. See Table 11-2. The disk of this problem rotates about a point that is displaced from its center by $R + L$, where L is the length of the rod. Use the parallel-axis theorem to find the rotational inertia about the pivot point.

(b) Put the origin at the pivot. The center of mass of the disk is $\ell_d = L + R$ away and the center of mass of the rod is $\ell_r = L/2$ away, on the same line. The distance from the pivot point to the center of mass of the disk-rod system is $d = (M\ell_d + m\ell_r)/(M+m)$.

(c) The period of oscillation is given by $T = 2\pi\sqrt{I/(M+m)gd}$.

[Ans: (a) $0.205\ kg \cdot m^2$; (b) $0.477\ m$; (c) $1.50\ s$]

43

(a) The frequency for small amplitude oscillations is $f = (1/2\pi)\sqrt{g/L}$, where L is the length of the pendulum.

(b) and (c) The forces acing on the pendulum are the tension force $\vec{T}$ of the rod and the force of gravity $\vec{F}_{grav}$. The magnitude of $\vec{F}_{grav}$ is mg, where m is the mass. Newton's second law yields $\vec{T} + m\vec{g} = m\vec{a}$, where $\vec{a}$ is the acceleration of the pendulum. Let $\vec{a} = \vec{a}_e + \vec{a}'$, where $\vec{a}_e$ is the acceleration of the elevator and $\vec{a}'$ is the acceleration of the pendulum relative to the elevator. Newton's second law can then be written $m(\vec{g} - \vec{a}_e) + \vec{T} = m\vec{a}'$. Relative to the elevator the motion is exactly the same as it would be in an inertial frame where the gravitational acceleration is

$\vec{g} - \vec{a}_e$.

[Ans: (a) $0.35\ Hz$; (b) $0.39\ Hz$; (c) 0]

51

(a) Use $f = (1/2\pi)\sqrt{k/m}$, where k is the spring constant and m is the mass of the object.

(b) The potential energy is given by $U = \frac{1}{2}kx^2$.

(c) The kinetic energy is given by $K = \frac{1}{2}mv^2$, where v is the speed of the object.

(d) The mechanical energy, which is the sum of the kinetic and potential energies, is $\frac{1}{2}kX^2$, where X is the amplitude of the oscillation.

[Ans: (a) $2.25\ Hz$; (b) $1125\ J$; (c) $250\ J$; (d) $86.6\ cm$]

53

(a) and (b) The mechanical energy is given by $E^{\text{mec}} = \frac{1}{2}kX^2$, where k is the spring constant and X is the amplitude. When $x = \frac{1}{2}X$ the potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{8}kX^2$ and the kinetic energy is $E^{\text{mec}} - U$. Calculate the ratios U/E^{mec} and K/E^{mec}.

(c) Since $E^{\text{mec}} = \frac{1}{2}kX^2$ and $U = \frac{1}{2}kx^2$, $U/E^{\text{mec}} = x^2/X^2$. Solve x^2/X^2.

[Ans: (a) $1/4$; (b) $3/4$; (c) $X/\sqrt{2}$]

57

Let $A = Xe^{-bt/2m}$. You want to evaluate $A/X = e^{-bt/2m}$ for $t = 20T$, where T is the period. Values for b ($70\ g/s$) and m ($250\ g$) are given in Touchstone Example 16-4, and T is found in that example to be $0.34\ s$.

[Ans: 0.385]