

## 2. Overview

### 10-2 Work and Path Dependence

- **Potential energy** is an energy associated with the configuration of two or more interacting objects; that is, it depends primarily on the positions of the objects. For example, the potential energy associated with the gravitational interaction between Earth and another object depends on the separation of Earth and the object. The greater the separation the greater the gravitational potential energy.
- A potential energy is associated with a force only if it is conservative. As a system changes from one configuration to some other configuration the work done by a conservative force is the same regardless of the intermediate configurations.
- Gravitational and spring forces are conservative. The work done by gravity, for example, depends only on the initial and final separations of Earth and the object, not on the paths they may take.
- The work done by a conservative force acting on a particle as the particle goes around a closed path is zero.
- If a frictional force acts on a object the work it does depends on the length of the path, so frictional forces cannot be conservative.

### 10-3 Potential Energy as "Stored Work"

- The change  $\Delta U$  in the potential energy of a system when its configuration changes is the negative of the work done by internal forces:  $\Delta U = -W_{\text{sys}}^{\text{int}}$
- This means there is a potential energy value associated with each configuration of the system and we can compute the work done by internal conservative forces by calculating the negative of the difference in the potential energy values for the beginning and ending configurations.
- Potential energy may be thought of as being stored in the system. For example, the gravitational potential energy of a ball and Earth is stored in the Earth-ball system.
- The work done by a frictional force, on the other hand, depends on the path, so we cannot associate potential energy values with the various configurations.
- Potential energy is a scalar. If two or more conservative forces are present, the total potential energy is simply the algebraic sum of the individual potential energies.
- The SI unit for potential energy is the joule.
- The change in the gravitational potential energy of a system consisting of Earth and an object of mass  $m$ , near its surface, as the object goes from height  $y_1$  to height  $y_2$  is given by
 
$$\Delta U = +mg(y_2 - y_1)$$

where the  $y$  axis is taken to be positive in the upward direction. Gravitational potential energy is often ascribed to the object alone, but it is actually a property of the Earth-object system.

- Gravitational potential energy depends only on the separation of the object and Earth, even if the object moves horizontally as well as vertically.
- If the object-Earth separation is increased the work done by gravity is negative and the potential energy change is positive. If the separation is decreased, the work done by gravity is positive and the potential energy change is negative.
- Remember that the work done by the force of an ideal spring on an object attached to one end, as the object goes from  $x_1$  to  $x_2$ , is given by  $W_{\text{spring}} = -\frac{1}{2}k(x_2^2 - x_1^2)$ , where  $k$  is the spring constant. During this motion the potential energy of the spring-object system changes by
 
$$\Delta U_{\text{spring}} = +\frac{1}{2}k(x_2^2 - x_1^2),$$

The greater the elongation or compression of the spring the greater the change in potential energy.

- An external agent can increase the spring potential energy by compressing or extending the spring. When a spring is elongated from its equilibrium length, the work done by the spring is negative and the change in the spring potential energy is positive. When the spring is compressed from its equilibrium length, the work done by the spring is again negative and the change in the spring potential energy is again positive. This stored potential

energy is converted to kinetic energy when an elongated or compressed spring is released.

- For a two-body system we use

$$\Delta U = -W_{\text{int}} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r}$$

to compute the change in the potential energy as body A moves from  $\vec{r}_1$  to  $\vec{r}_2$  relative to body B. Here  $\vec{F}$  is the force of body B on body A and is assumed to be conservative.

- Only *changes* in the potential energy are physically meaningful. Usually the potential energy is chosen to have some value for a particular reference configuration of the system. Then, the work done by forces of the system as the system goes from the reference configuration to another configuration is computed. If  $U_{\text{ref}}$  represents the potential energy in the reference configuration and  $U$  represents the potential energy in the other configuration, then  $U = U_{\text{ref}} - W$ , where  $W$  is the work.  $U_{\text{ref}}$  is often taken to be zero.

## 10-4 Mechanical Energy Conservation

- The **mechanical energy** of a system is the sum of its potential and kinetic energies:  $E^{\text{mec}} = K + U$ . The mechanical energy of a system is **conserved** (remains constant) if a potential energy is associated with each of the forces between objects of the system and no external agents do work on the system. This follows directly from the work-kinetic energy theorem and the definition of potential energy.
- If a ball is thrown upward, the force of gravity slows it down; the kinetic energy of the Earth-ball system decreases but the gravitational potential energy system increases. Similarly, as the ball falls the kinetic energy of the system increases and the gravitational potential energy of the system decreases. If the gravitational force is the only force acting, the sum of the potential and kinetic energies remains constant during the flight of the ball; the mechanical energy of the Earth-ball system is conserved. The equation  $\frac{1}{2}mv^2 + mgy = \text{constant}$  can be used to solve for one of the quantities that appear in it.
- If a block on a horizontal frictionless surface is attached to an ideal spring and the block is pulled aside and released, it oscillates back and forth, repeatedly coming back to its original position. As the block moves toward the equilibrium point ( $x = 0$ ) it speeds up and the spring becomes less extended or compressed; as it moves away from the equilibrium point it slows down and the spring becomes more extended or compressed. In the first case, the block gains kinetic energy and the spring loses potential energy; in the second case, the block loses kinetic energy and the spring gains potential energy. The sum of the potential energy stored in the spring and the kinetic energy of the block remains constant throughout; the mechanical energy of the spring-block system is conserved.  $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$  can be used to solve for one of the quantities that appear in it.
- If frictional forces act, the mechanical energy of a system decreases. Mechanical energy is said to be **dissipated**.

## 10-5 Reading a Potential Energy Curve

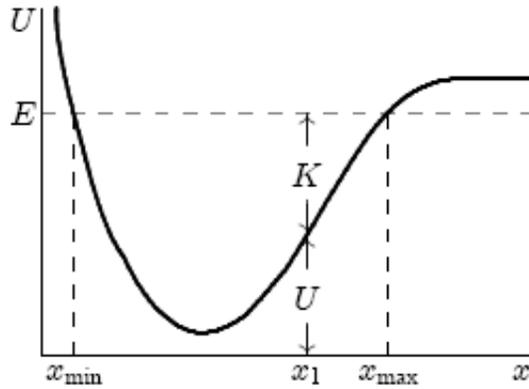
- If the potential energy function is known, the force can be computed by evaluating its derivatives with respect to coordinates. If  $U(x)$  is the potential energy for an object moving on the  $x$  axis and subjected to an internal conservative force  $F_{\text{int}}$ , then

$$F_{\text{int}}(x) = -\frac{dU(x)}{dx}$$

On a graph of the potential energy as a function of  $x$  the force is the negative of the slope of the line that is tangent to the curve at the coordinate of the particle. A positive slope indicates a force in the negative  $x$  direction and a negative slope indicates a force in the positive  $x$  direction.

- When dealing with potential energy curves, assume that the agent exerting the force is stationary, so only the object whose motion is being considered has kinetic energy.
- A potential energy curve is shown to the right for an object that moves along the  $x$  axis with constant mechanical energy  $E$ , represented by a horizontal dotted line. The potential and kinetic energies when the object is at  $x_1$  are indicated by arrows. The object cannot move to the left of  $x_{\text{min}}$  or to the right of  $x_{\text{max}}$  because the potential energy is greater than the mechanical energy in those regions. The kinetic energy and the velocity are zero at  $x_{\text{min}}$  and  $x_{\text{max}}$ . When the object is at  $x_{\text{min}}$ , the force acting on it is positive (the slope of the curve is negative) and the object is pushed to the right. When it is at  $x_{\text{max}}$ , the force on it is negative and the object is pushed to

the left.  $x_{\min}$  and  $x_{\max}$  are called the **turning points** of the motion.



- Suppose the object starts at  $x_1$  and travels in the negative  $x$  direction. Its speed increases until it gets to the coordinate corresponding to the minimum in the potential energy curve; then it slows. When it reaches  $x_{\min}$ , its velocity is zero but a positive force acts on it and it starts moving in the positive  $x$  direction. Its speed increases, then decreases until it gets to  $x_{\max}$ , where its velocity is zero. It then starts moving back again and the motion is repeated. If the energy is decreased, it oscillates over a narrower range of coordinates.
- **Equilibrium points** are places where the force vanishes. The slope of the potential energy curve is zero at these points and the potential energy is either a maximum or a minimum. When an object is released from rest near a point where the curve has a minimum (a point of **stable equilibrium**), its subsequent motion is toward the equilibrium point. When an object is released from rest near a point where the curve has a maximum (a point of **unstable equilibrium**), its subsequent motion is away from the equilibrium point. When an object is released from rest in a region where the curve is horizontal (a region of **neutral equilibrium**), it remains at rest.

## 10-6 Nonconservative Forces and Energy

- If frictional forces act on objects of a system, the thermal energy  $E_{\text{thermal}}$  of the objects must be included in the total energy of the system and the energy equation becomes  $\Delta E_{\text{mec}} + \Delta E_{\text{thermal}} = 0$  if no external agents do work on the system.  $\Delta E_{\text{mec}}$  is the change in the mechanical energy of the system, and  $\Delta E_{\text{thermal}}$  is the change in the thermal energy of the system.
- If two objects in the system slide a distance  $d$  over each other and exert a constant frictional force of magnitude  $f$  on each other  $\Delta E_{\text{thermal}} = fd$ . The thermal energy is shared by the two objects.

## 10-7 Conservation of Energy

- The total energy of a system is the sum of its mechanical energy and its energy due to nonconservative forces, such as friction. If external agents do work on the objects of a system, the total energy of the system is not conserved. In fact, the energy equation becomes

$$W_{\text{ext}} = \Delta E_{\text{mec}} + \Delta E_{\text{noncons}},$$

where  $W_{\text{ext}}$  is the net work done *on* objects of the system by outside agents and  $\Delta E_{\text{noncons}}$  is the change in energy associated with internal nonconservative forces, such as frictional forces. An external force doing work on objects in a system transfers energy to or from the system. For example, the work done by a person carrying a crate up some stairs increases the mechanical energy of the crate-Earth system. If the speed of the crate does not change, the increase in energy appears as potential energy; the crate is further away from Earth than previously. If the crate's speed increases, both the potential and kinetic energies increase.

- If  $W_{\text{ext}} = 0$  then the total energy of the system is conserved.

## 10-8 One-Dimensional Energy and Momentum Conservation

- Be careful to distinguish between conservation of momentum and conservation of energy. The momentum of a system is conserved if the net external force on the system is zero. Total energy is conserved if the net external work on the system is zero and mechanical energy is conserved if, in addition, the internal forces are all conservative.
- If the total kinetic energy of a system of two colliding objects is conserved then the collision is said to be **elastic**.

Otherwise it is said to be **inelastic**.

## 10-9 One-Dimensional Elastic Collisions

- Suppose object A, with mass  $m_A$  moves with velocity  $\vec{v}_{A1}$  along the  $x$  axis and collides elastically with object B, which has mass  $m_B$  and is initially at rest. After the collision object A moves along the  $x$  axis with velocity  $\vec{v}_{A2}$  and object B moves along the  $x$  axis with velocity  $\vec{v}_{B2}$ . The equation that expresses conservation of momentum during the collision is

$$m_A v_{Ax}(t_1) = m_A v_{Ax}(t_2) + m_B v_{Bx}(t_2)$$

and the equation that expresses conservation of kinetic energy during the collision is

$$\frac{1}{2} m_A v_{Ax}^2(t_1) = \frac{1}{2} m_A v_{Ax}^2(t_2) + \frac{1}{2} m_B v_{Bx}^2(t_2),$$

where time  $t_1$  is before the collision and time  $t_2$  is after the collision. The solution to these two equations is

$$v_{Ax}(t_2) = \frac{m_A - m_B}{m_A + m_B} v_{Ax}(t_1) \quad \text{and} \quad v_{Bx}(t_2) = \frac{2m_A}{m_A + m_B} v_{Ax}(t_1).$$

- Some special cases:
  - If the two masses are the same, then  $v_{Ax}(t_2) = 0$  and  $v_{Bx}(t_2) = v_{Ax}(t_1)$ . The objects have interchanged their velocities.
  - If the target object, object B, is very massive compared to the incident object, object A, then  $v_{Ax}(t_2) \approx -v_{Ax}(t_1)$  and  $v_{Bx}(t_2) \approx (2m_A/m_B)v_{Ax}(t_1)$ . The incident object bounces off the target with only a slight loss of speed and the target moves slowly away from the collision.
  - If object A is very massive compared to object B, then  $v_{Ax}(t_2) \approx v_{Ax}(t_1)$  and  $v_{Bx}(t_2) \approx 2v_{Ax}(t_1)$ . The incident object continues with only a slight loss in speed and the target moves away with a speed that is twice the original speed of the incident object.
- The velocity of the center of mass of the two-object system is not changed by the collision. If the target object is initially at rest, the velocity of the center of mass is given by  $\vec{v}_{\text{com}} = m_A \vec{v}_{Ax}(t_1) / (m_A + m_B)$ .
- If both the target and incident object are initially moving, then

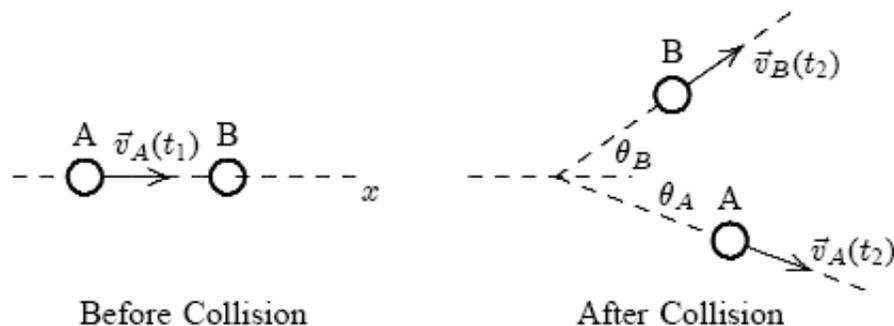
$$v_{Ax}(t_2) = \frac{m_A - m_B}{m_A + m_B} v_{Ax}(t_1) + \frac{2m_B}{m_A + m_B} v_{Bx}(t_1)$$

and

$$v_{Bx}(t_2) = \frac{2m_A}{m_A + m_B} v_{Ax}(t_1) + \frac{m_B - m_A}{m_A + m_B} v_{Bx}(t_1).$$

## 9-10 Two-Dimensional Energy and Momentum Conservation

- A two-dimensional elastic collision is diagramed below.



Object B is initially at rest and object A impinges on it with speed  $v_A(t_1)$  along the  $x$  axis. Object A leaves the collision with speed  $v_A(t_2)$  along a line that is below the  $x$  axis and makes the angle  $\theta_A$  with that axis. Object B leaves the collision with speed  $v_B(t_2)$  along a line that is above the  $x$  axis and makes the angle  $\theta_B$  with that axis.

Take the  $y$  axis to be upward in the diagram. Conservation of total momentum  $\vec{p}_{\text{total}}$  leads to the equations  $m_A v_A(t_1) = m_B v_A(t_2) \cos \theta_A + m_B v_B(t_2) \cos \theta_B$  (conservation of the  $x$  component) and  $0 = -m_A v_A(t_2) \sin \theta_A + m_B v_B(t_2) \sin \theta_B$  (conservation of the  $y$  component). Since the collision is elastic  $\frac{1}{2} m_A v_A^2(t_1) = \frac{1}{2} m_A v_A^2(t_2) + \frac{1}{2} m_B v_B^2(t_2)$ . These equations can be solved for three unknowns.

- Note that the masses and the initial velocity of object A alone do not determine the outcome of a two-dimensional elastic collision. The magnitude and direction of the impulses acting on the object during the collision also play important roles. Often these are not known, so the outcome cannot be predicted. Experimenters usually measure  $\theta_A$  or  $\theta_B$ , then use the equations to calculate other quantities.