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2. Overview

11-1 Translation and Rotation

- Distinguish between translational and rotational motion. For an object in pure translation all points on it have the same velocity at any selected time. For an object in pure rotation each point on an object follows a circular path and the velocities of different points are different (in either magnitude or direction). Some motions are combinations of translation and rotation.

11-2 The Rotational Variables

- If a rigid body undergoes pure rotation about a fixed axis, then the path followed by each point on the body is a circle, centered on the **axis of rotation**.
- The **rotational position** of the body is described by giving the angle between a **reference line**, fixed to the body, and a non-rotating coordinate axis. If the body rotates around the z axis, reference line that rotates in the xy plane might be chosen and the x axis might be chosen as the fixed axis.
- If the body rotates from θ_1 to θ_2, then its **rotational displacement** is $\Delta\theta = \theta_2 - \theta_1$.
- If the body rotates through more than one revolution, θ continues to increase beyond 2π rad. An angle of 2π rad is not equivalent to 0.
- The sign of the displacement is determined by a right-hand rule. Suppose the rotation axis coincides with the z axis. Curl the fingers of your right hand around the axis in the direction of rotation; if your thumb points in the positive z direction the displacement is positive, if it points in the negative z direction the displacement is negative. If you sight along the axis from the positive side of the origin counterclockwise rotation is positive and clockwise rotation is negative.
- If the body undergoes an rotational displacement $\Delta\theta$ in time Δt, its **average rotational velocity** during the interval is

  $$< \omega >= \frac{\Delta\theta}{\Delta t} \, .$$

  Its **instantaneous rotational velocity** at any time is given by the derivative

  $$\omega = \frac{d\theta}{dt} \, .$$

  These are actually vector components along the rotation axis.

- Rotational speed is the magnitude of the rotational velocity and denoted by $|\omega|$ or by $|\vec{\omega}|$.
- If the rotational velocity of the body changes with time, then the body has a non-vanishing **rotational acceleration**. If the rotational velocity changes from ω_1 to ω_2 in the time interval Δt, then the average rotational acceleration in the interval is

  $$< \alpha >= \frac{\Delta\omega}{\Delta t} \, ,$$

  where $\Delta\omega = \omega_2 - \omega_1$. The **instantaneous rotational acceleration** at any time t is given by the derivative

  $$\alpha = \frac{d\omega}{dt} \, .$$

  These are also a vector components along the rotation axis. Instantaneous rotational acceleration is usually called simply rotational acceleration.

- Common units of rotational velocity are deg/s, rad/s, rev/s, and rev/min. Corresponding units of rotational acceleration are deg/s^2, rad/s^2, rev/s^2, and rev/min^2.
- A positive rotational acceleration does not necessarily mean the rotational speed is increasing and a negative rotational acceleration does not necessarily mean the rotational speed is decreasing. The rotational speed is decreasing if ω and α have opposite signs and is increasing if they have the same sign, no matter what the signs are.
- The values of $\Delta\theta$, ω, and α are the same for every point in a rigid body. The rotational positions of different points may be different, of course, but when one point rotates through any angle, all points rotate through the same angle. All points rotate through the same angle in the same time and their rotational velocities change at the same rate.

11-3 Rotation with Constant Angular Acceleration

- If a body is rotating around a fixed axis with constant rotational acceleration α, with ω_1 the rotational velocity at time t_1 and ω_2 the rotational velocity at time t_2, then

  $$\omega_2 = \omega_1 + \alpha(t_2 - t_1) \qquad \text{and} \qquad \theta_2 = \theta_1 + \omega_1(t_2 - t_1) + \frac{1}{2}\alpha(t_2 - t_1)^2 .$$

  These are the basic equations for rotation with constant rotational acceleration.

  The reference line can usually be placed so $\theta_1 = 0$. These two equations can be solved simultaneously to find values for two of the symbols that appear in them. Another useful equation is obtained when of the basic equations is used to eliminate $t_2 - t_1$ from the other. It is

  $$\omega_2^2 - \omega_1^2 = 2\alpha(\theta_2 - \theta_1) .$$
- Use a consistent set of units: θ in degrees, ω in $degrees/s$, and α in $degrees/s^2$ or θ in radians, ω in $radians/s$, and α in $radians/s^2$ or θ in revolutions, ω in $revolutions/s$, and α in $revolutions/s^2$. Do not mix units.

11-4 Relating the Translational and Rotational Variables

- Each point in a rotating body has a translational velocity and acceleration and these are related to the rotational variables. Consider a point in the body a perpendicular distance r from the axis of rotation. If the body turns through the angle $|\theta|$ (in radians), the point moves a distance $s = |\theta|r$ along its circular path. The speed of the point is $v = ds/dt = (d|\theta|/dt)r = |\omega|r$. The units of ω MUST be rad/s for these relationships to be valid.
- If the rotational velocity is constant the period of the rotation is given by $T = 2\pi/|\omega|$, where again radian measure must be used.
- If a rotating object is speeding up or slowing down each point in it has an acceleration component along the line that is tangent at the point to the circular path of the point. The magnitude of the tangential component of the acceleration is $|a_t| = dv/dt = (d|\omega|/dt)r = |\alpha|r$, where radian measure MUST be used for α.
- Because the point is moving in a circular path its acceleration also has a radial component. Its magnitude is $|a_r| = v^2/r$. In terms of the rotational speed, $|a_r| = \omega^2 r$.
- In terms of the components a_r and a_t, the magnitude of the total acceleration is given by $a = \sqrt{a_r^2 + a_t^2}$.
- Notice that s, v, a_t, and a_r are proportional to r. Compared to a point on the rim of a rotating wheel, for example, a point halfway out travels half the distance, has half the speed, has half the tangential acceleration, and has half the radial acceleration.

11-5 Kinetic Energy of Rotation

- Suppose a rigid body, rotating about a fixed axis, is made up of N particles. The total kinetic energy is given by $K = \sum \frac{1}{2} m_i v_i^2$, where m_i is the mass of particle i, v_i is its speed, and the sum is over all particles in the body. Substitute $v_i = \omega r_i$, where r_i is the distance of particle i from the axis, and obtain $K = \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$. ω MUST be in rad/s.

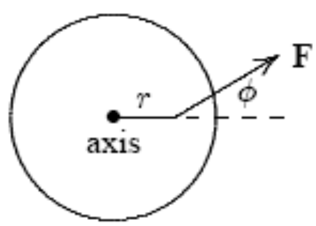
- The sum $\sum m_i r_i^2$ is called the **rotational inertia** of the body and is denoted by I. Thus the kinetic energy of a rotating body is $K = \frac{1}{2} I \omega^2$.

11-6 Calculating the Rotational Inertia

- Rotational inertia is a property of a body that depends on the distribution of mass in the body and on the axis of rotation. A particle far from the axis of rotation contributes more to the rotational inertia than a particle of the same mass closer to the axis.
- The sum $\sum m_i r_i^2$ may be difficult to evaluate for a body with more than a few particles. Some bodies, however, can be approximated by a continuous distribution of mass and techniques of the integral calculus can be used to find the rotational inertia. The body is divided into a large number of small regions, each with mass dm. The contribution of each region to the rotational inertia is computed as if the region were a particle; then the results are summed: $I = \int r^2\, dm$. Table 11-2 of the text gives the rotational inertias of several bodies.
- The defining equation for the rotational inertia is a sum over all particles in the body. If we like, we can consider the body to be composed of two or more parts and calculate the rotational inertia of each part about the same axis, then add the results to obtain the rotational inertia of the complete body.
- The defining equation for rotational inertia is used to prove the parallel-axis theorem. Consider two identical bodies, one rotating about an axis through the center of mass and the other rotating about an axis that is parallel to the first axis but is a distance h from the center of mass. The rotational inertia I for the second body is related to the rotational inertia I_{com} for the first by $I = I_{\text{com}} + Mh^2$, where M is the total mass of the body.
- The parallel-axis theorem tells us that of all the places we can position the axis of rotation, the one that leads to the smallest rotational inertia is the one through the center of mass and that the rotational inertia increases as the axis moves away from the center of mass (remaining parallel to its original orientation, of course).

11-7 Torque

- A **torque** is associated with any force that is applied to an object and is not along the line from the axis of rotation to the point of application. If the force is perpendicular to the rotation axis, then the torque is given by $\tau = rF_t$, where r is the distance from the axis of rotation to the point of application and F_t is the component of the force perpendicular to the line from the axis to the application point. In the diagram on the right, the torque associated with **F** is $\tau = rF \sin \phi$. Notice that the magnitude of a torque depends on the distance from the axis to the point of application. A force produces a greater torque if it is applied far from the axis than if it applied closer.



- For rotation about a fixed axis, a torque is taken to be positive if it tends to turn the object counterclockwise and negative if it tends to turn the body clockwise. This convention is consistent with the one introduced earlier for the signs of the rotational velocity and acceleration. Later you will learn that torque is a vector. The quantity used here is actually its component along the rotation axis.
- If several torques act on an object the net torque is the sum of the individual torques, taking their signs into account.

11-8 Newton's Second Law for Rotation

- This law relates the net torque τ^{net} acting on a rigid body to the rotational acceleration α of the body:

  $$\tau^{\text{net}} = I\alpha\,,$$

  where I is the rotational inertia of the body about the axis of rotation. It is as important to the study of rotational motion about a fixed axis as $\vec{F}^{\text{net}} = m\vec{a}$ is to the study of translational motion. It can be derived directly from Newton's second law.

- If the torque is in newton-meters, then the rotational acceleration must be in radians per second squared.
- Be careful that you use the net torque in this equation. You must identify and sum all the torques that are acting and you must be careful about signs. You must also be careful to use radian measure for the rotational acceleration α.

11-9 Work and Rotational Kinetic Energy

- Consider a particle traveling counterclockwise in a circular orbit, subjected to a force with tangential component F_t. As it moves through an infinitesimal arc length ds the magnitude of the work done by the force is $dW^{\text{rot}} = F_t\, ds$. Since $ds = r\, d\theta$, where $d\theta$ is the infinitesimal rotational displacement, and $\tau = F_t r$, the expression for the work becomes $dW^{\text{rot}} = \tau\, d\theta$. As the particle travels from θ_1 to θ_2 the work done by the torque is given by the integral
$$W^{\text{rot}} = \int_{\theta_1}^{\theta_2} \tau\, d\theta\,.$$
- The work done by a torque may be positive or negative. If the torque and rotational displacement are in the same direction, both clockwise or both counterclockwise, then the work is positive; if they are in opposite directions, one clockwise and one counterclockwise, then the work is negative. If more than one torque acts, sum the works done by the individual torques to find the total work.
- In terms of the torque τ and rotational velocity ω, the power supplied by the torque is given by $P = dW^{\text{rot}}/dt = \tau\omega$.
- A work-kinetic energy theorem is valid for rotational motion: the net work done by all torques acting on a body equals the change in its rotational kinetic energy: $W^{\text{net-rot}} = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$.