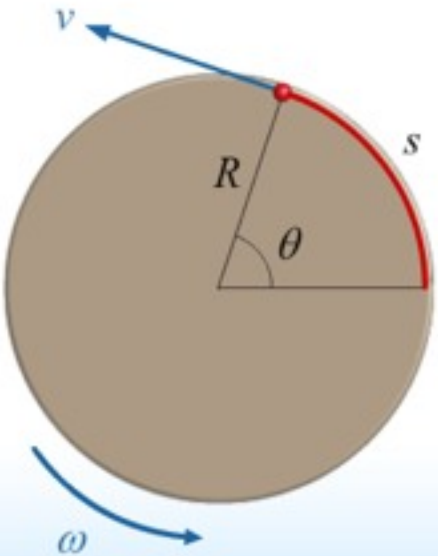
Classical Mechanics
Lecture 14

Today's Concepts:

a) Rotational Motion

b) Moment of Inertia

Summary of Rotations



Arc Length

$s = R\theta$

where θ is measured in radians

Tangential Speed

$v = R\omega$

Tangential Acceleration

$a = R\alpha$

For constant angular acceleration

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_o + \alpha t$$

$$\omega^2 - \omega_o^2 = 2\alpha(\theta - \theta_o)$$

Rotational Kinetic Energy

$$K_{system} = \frac{1}{2} I \omega^2$$

Angular velocity ω is measured in radians/sec

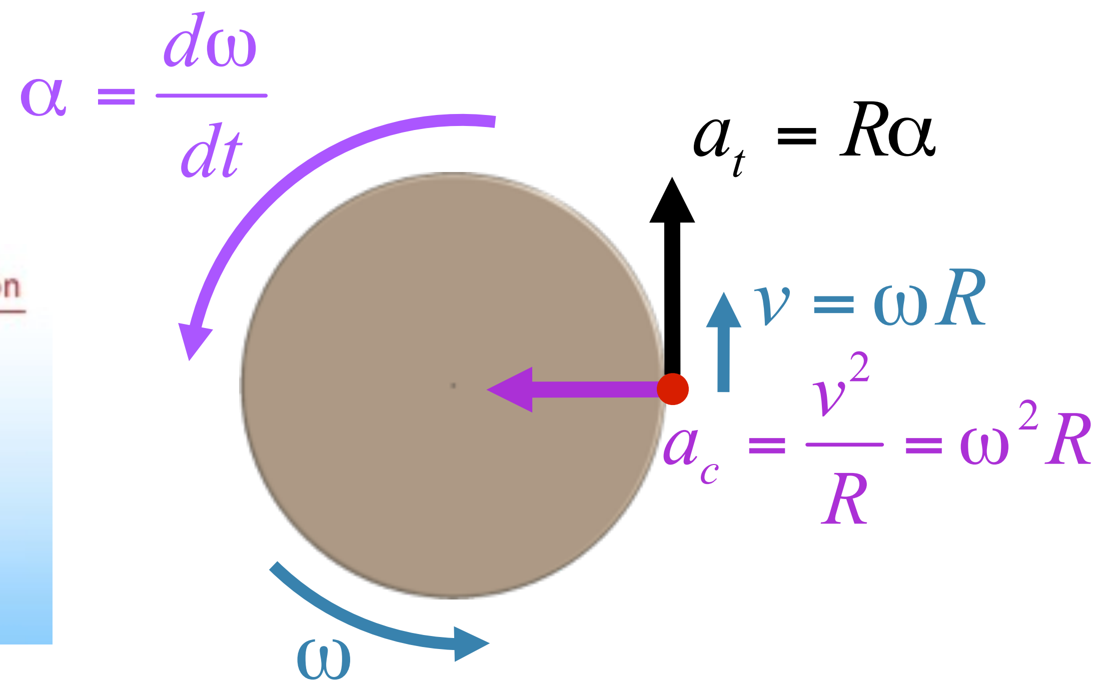
Frequency f is measured in revolutions/sec

1 revolution = 2π radians

Period $T = 1/f$ →

$$\omega = \frac{2\pi}{T}$$

Another Summary



For constant angular acceleration

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_o + \alpha t$$

$$\omega^2 - \omega_o^2 = 2\alpha(\theta - \theta_o)$$

Constant α does not mean constant ω

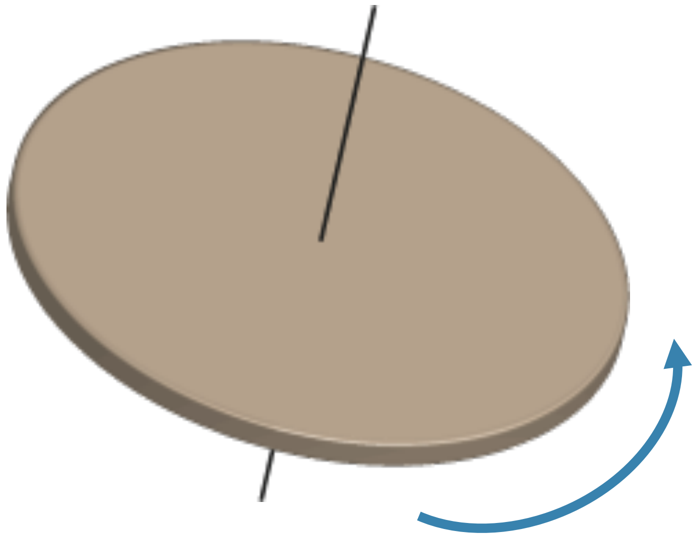
A disk spins at 2 revolutions/sec.

What is its period?

A) $T = 2$ sec

B) $T = 2\pi$ sec

C) $T = \frac{1}{2}$ sec



Clicker Question

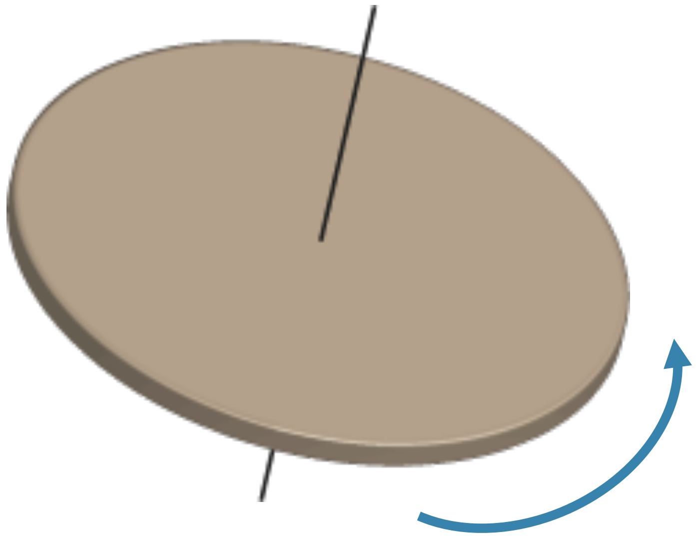
A disk spins at 2 revolutions/sec.

What is its angular velocity?

A) $\omega = 2\pi$ rad/sec

B) $\omega = \frac{\pi}{2}$ rad/sec

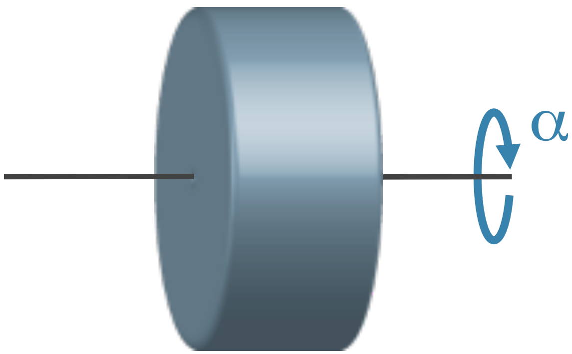
C) $\omega = 4\pi$ rad/sec



A wheel which is initially at rest starts to turn with a constant angular acceleration. After 4 seconds it has made 4 complete revolutions.

How many revolutions has it made after 8 seconds?

A) 8 B) 12 C) 16



Only about half got this right so let's try again...

CheckPoint Response

After 4 seconds it has made 4 complete revolutions.

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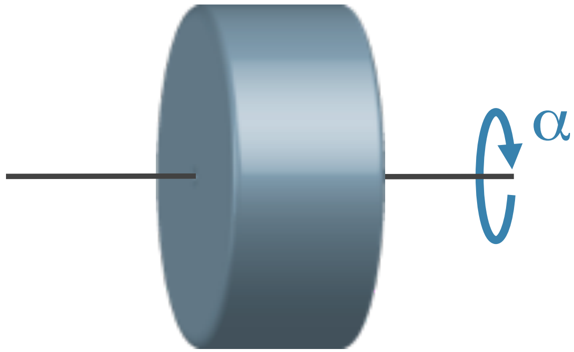
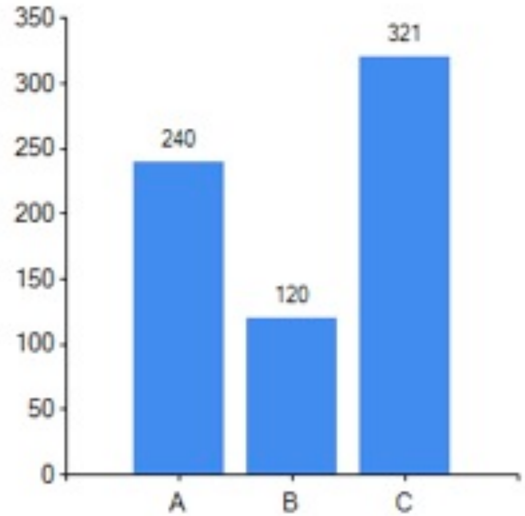
A) 8 B) 12 C) 16





A) Since it made 4 revolutions in 4 seconds, its angular velocity is 1 revolution per second. Therefore, in 8 seconds, it will have made 8 revolutions.

B) it makes 4 in the first 4 seconds and then 4 + 4 in the second 4 seconds.
$4 + 4 + 4 = 12$

C) The number of revolutions is proportional to time squared.

Calculating Moment of Inertia

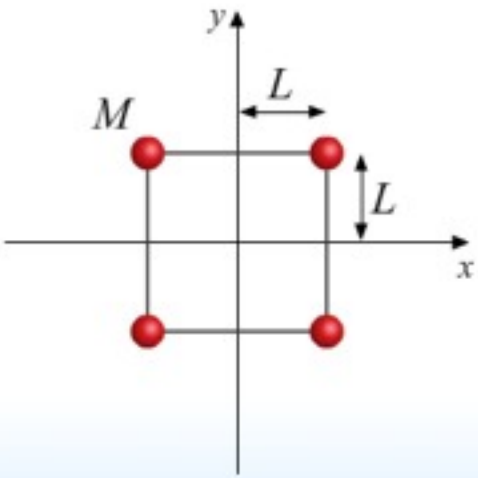
Moment of Inertia

For Discrete Distributions

$$I \equiv \sum m_i r_i^2$$

For Continuous Distributions

$$I = \int r^2 dm$$



Moment of Inertia

$$I \equiv \sum m_i r_i^2$$

$$I_y = I_x = 4ML^2$$

$$I_z = 8ML^2$$

Depends on rotation axis

CheckPoint

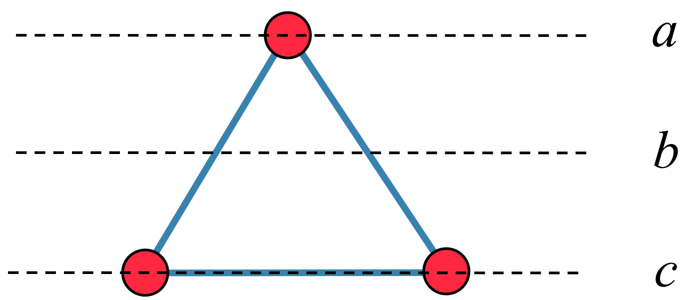
A triangular shape is made from identical balls and identical rigid, massless rods as shown. The moment of inertia about the a, b, and c axes is I_a, I_b, and I_c respectively.

Which of the following orderings is correct?

A) $I_a > I_b > I_c$

B) $I_a > I_c > I_b$

C) $I_b > I_a > I_c$



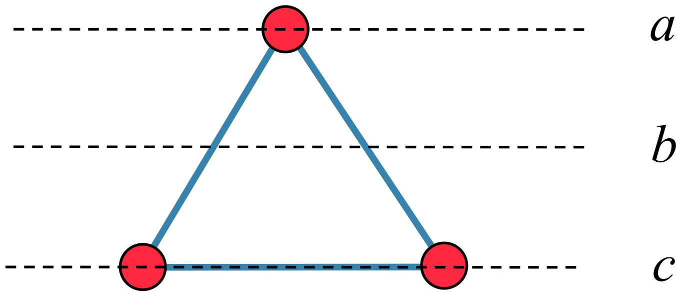
Only about half got this right so let's try again...

CheckPoint Response

A) $I_a > I_b > I_c$

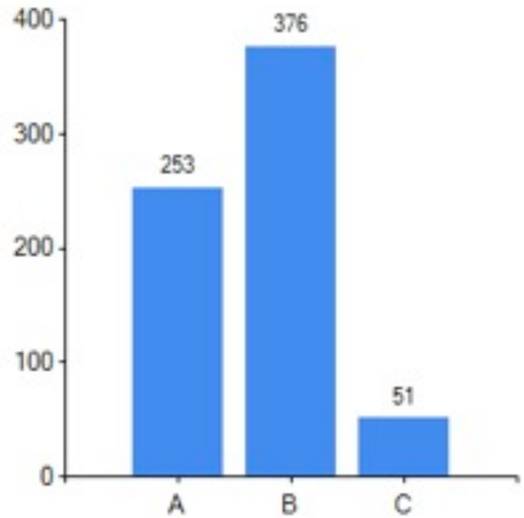
B) $I_a > I_c > I_b$

C) $I_b > I_a > I_c$

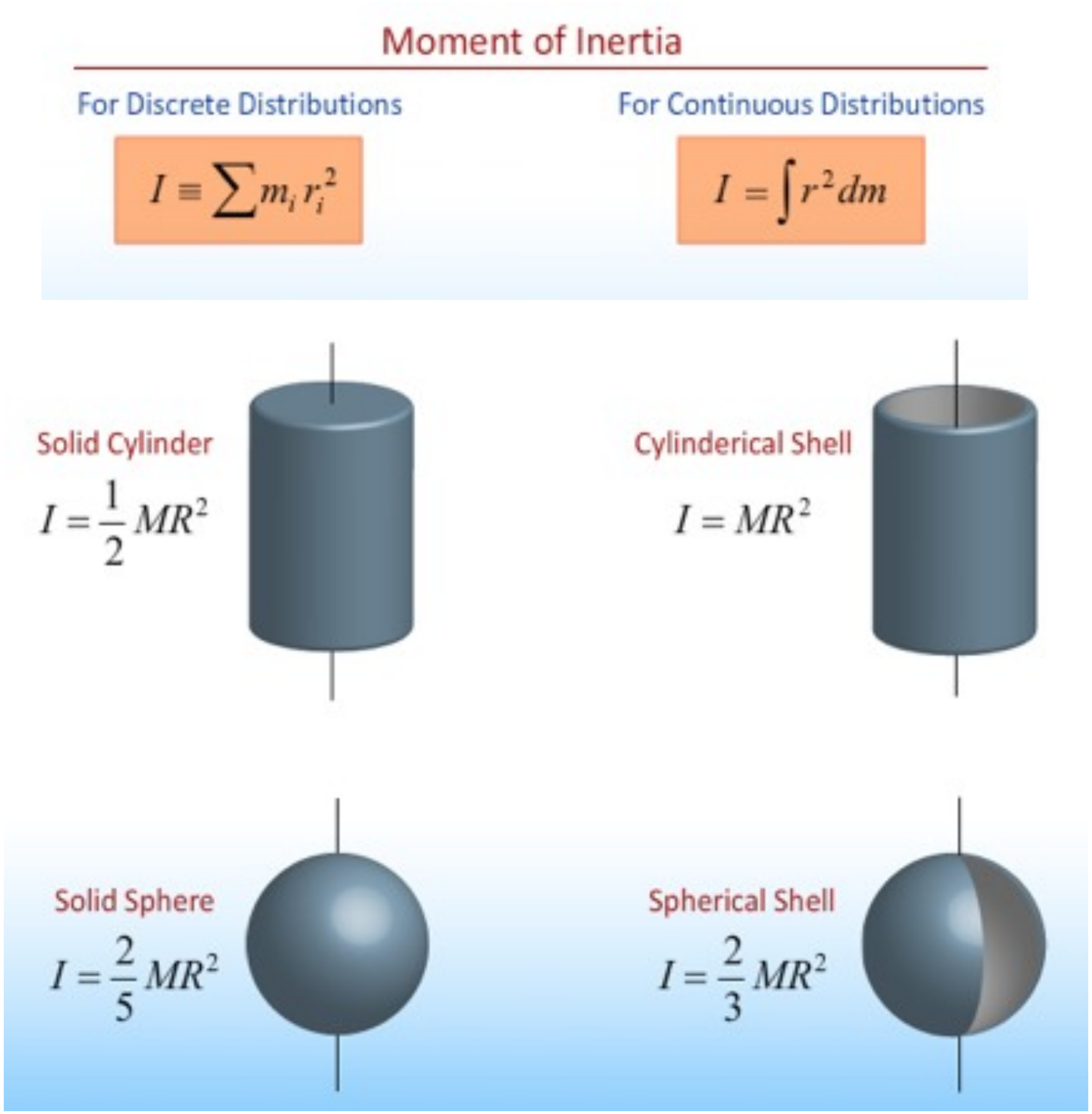


A) $I_a = 8mr^2 \quad I_b = 3mr^2 \quad I_c = 2mr^2$

B) $I_a = 8mr^2 \quad I_b = 3mr^2 \quad I_c = 4mr^2$



Calculation Moment of Inertia



Bigger when the mass is further out

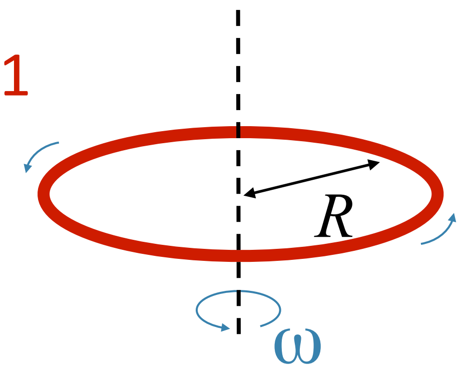
Clicker Question

In both cases shown below a hula hoop with mass M and radius R is spun with the same angular velocity about a vertical axis through its center. In Case 1 the plane of the hoop is parallel to the floor and in Case 2 it is perpendicular.

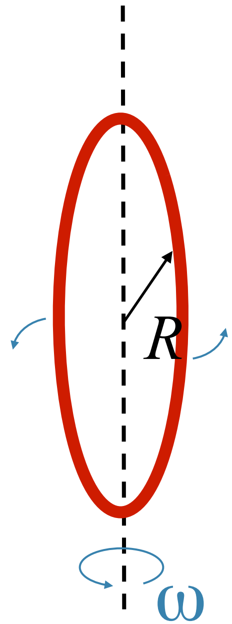
In which case does the spinning hoop have the most kinetic energy?

A) Case 1 B) Case 2 C) Same

Case 1



Case 2



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Clicker Question

In which case does the spinning hoop have the most kinetic energy?

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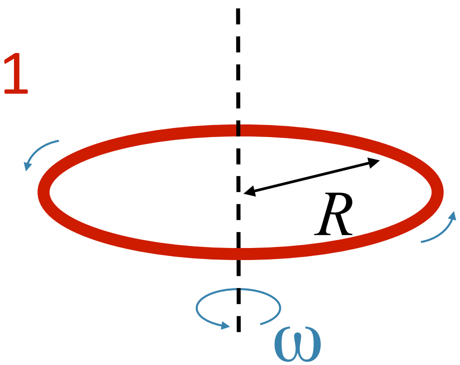
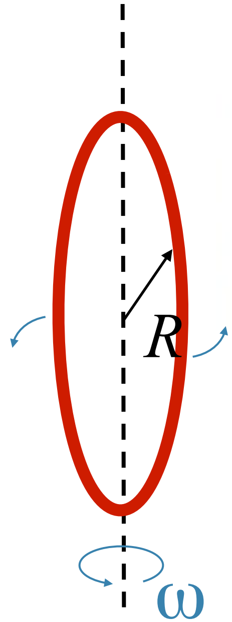
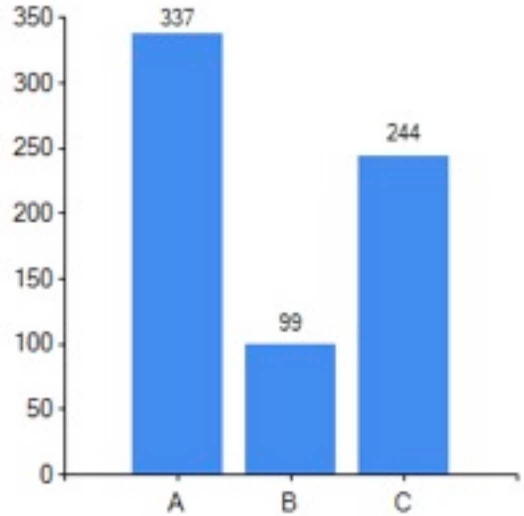
Case 1



Case 2

$$K = \frac{1}{2} I \omega^2$$





A) In case one, more mass is located away from its axis, so it has larger moment of inertia. Therefore it has more kinetic energy.

C) The radius, angular velocities and masses are the same so the inertia is the same which means the kinetic energy is the same for both.

Clicker Question

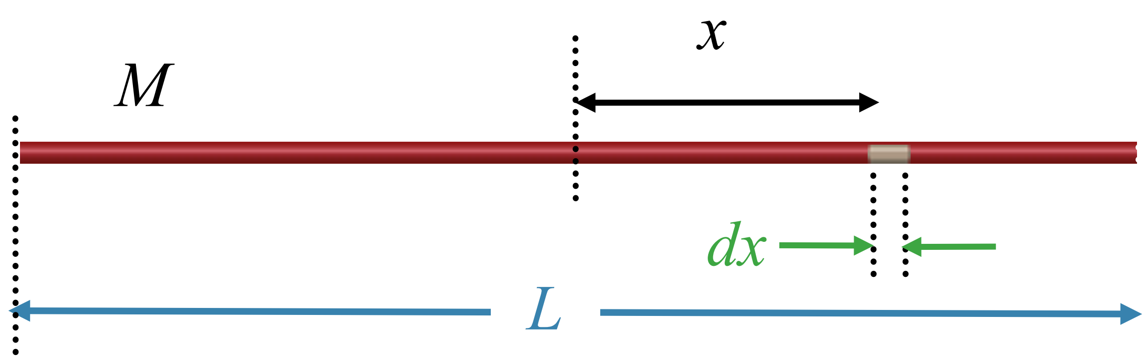
A mass M is uniformly distributed over the length L of a thin rod. The mass inside a short element dx is given by:

A) $M\,dx$

B) $\dfrac{dx}{M}$

C) $\dfrac{M}{L}dx$

D) $\dfrac{L}{M}dx$



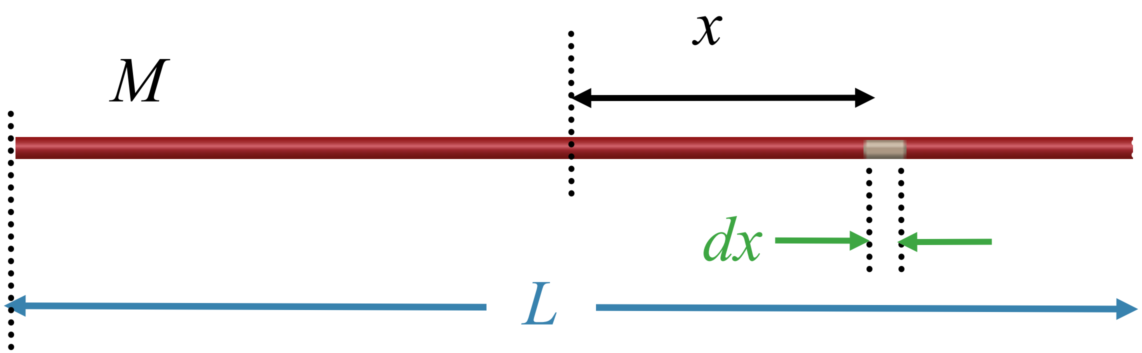
Clicker Question

A mass M is uniformly distributed over the length L of a thin rod. The contribution to the rod's moment of inertia provided by element dx is given by:

A) $x^2 \frac{M}{L} dx$

B) $\frac{1}{x^2} \frac{M}{L} dx$

C) $\frac{M}{L} dx^2$



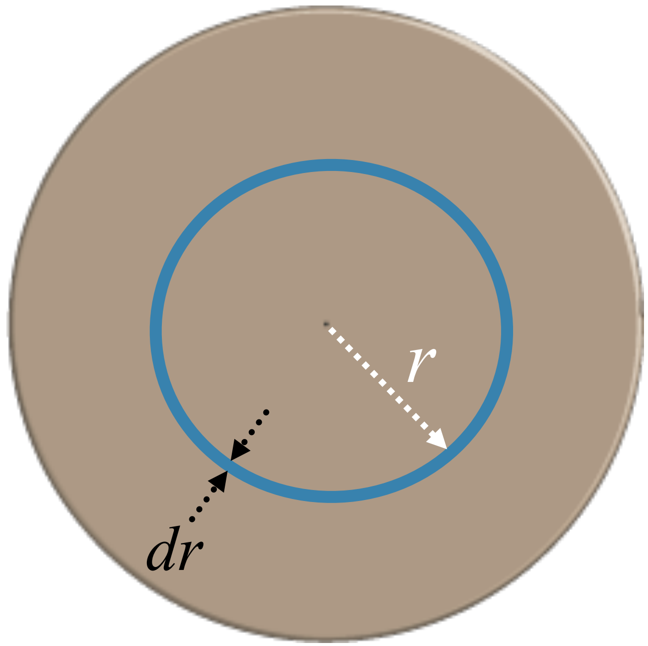
Clicker Question

A disk has a radius R. The area of a thin ring inside the disk with radius r and thickness dr is:

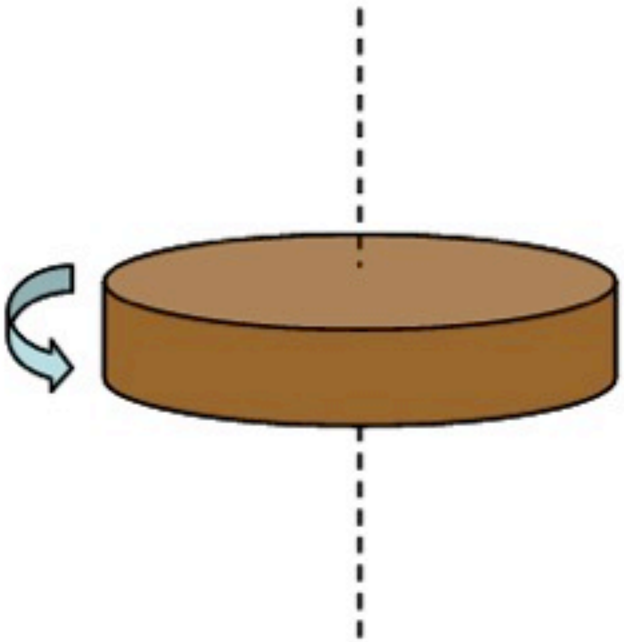
A) $\pi r^2 dr$

B) $2\pi r dr$

C) $4\pi r^3 dr$



Spinning Disk



(i) $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

(ii) $\omega = \omega_0 + \alpha t$

(iii) $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

A disk with mass m = 11.2 kg and radius R = 0.34 m begins at rest and accelerates uniformly for t = 17.4 s, to a final angular speed of ω = 29 rad/s.

1) What is the angular acceleration of the disk?

rad/s^2 Submit

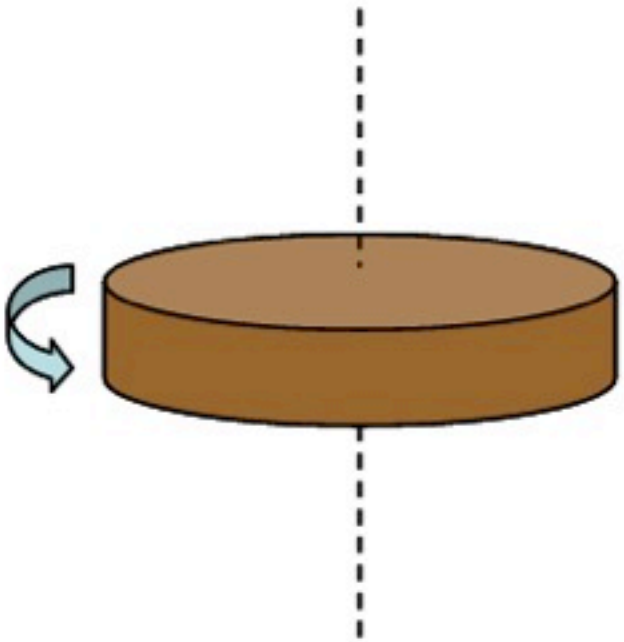
Using (ii) $\alpha = \frac{\omega - \omega_0}{t}$

2) What is the angular displacement over the 17.4 s?

rad Submit

Using (i) $\theta = \frac{1}{2}\alpha t^2$

Spinning Disk



(iv) $$I_{DISK} = \frac{1}{2}MR^2$$

(v) $$K = \frac{1}{2}I\omega^2$$

A disk with mass m = 11.2 kg and radius R = 0.34 m begins at rest and accelerates uniformly for t = 17.4 s, to a final angular speed of ω = 29 rad/s.

3) What is the moment of inertia of the disk?

kg-m^2 Submit

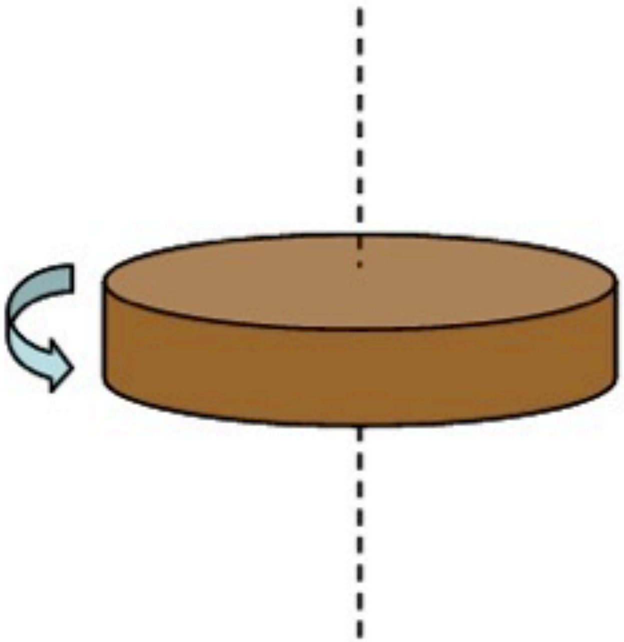
Use (iv)

4) What is the change in rotational energy of the disk?

J Submit

Use (v)

Spinning Disk



(vi) $d = R\theta$

(vii) $v = R\omega$

(viii) $a_T = R\alpha$

(ix) $a_c = \frac{v^2}{R} = \omega^2 R$

A disk with mass m = 11.2 kg and radius R = 0.34 m begins at rest and accelerates uniformly for t = 17.4 s, to a final angular speed of ω = 29 rad/s.

5) What is the tangential component of the acceleration of a point on the rim of the disk when the disk has accelerated to half its final angular speed?

[ ] m/s^2 Submit

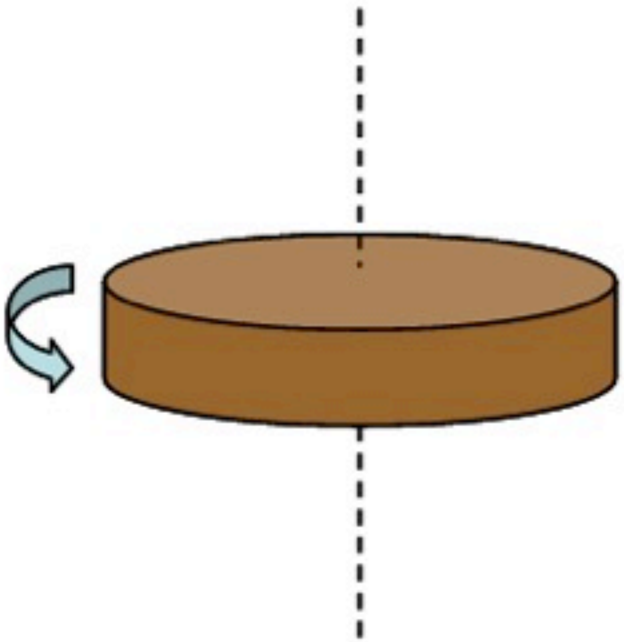
Use (viii)

6) What is the radial component of the acceleration of a point on the rim of the disk when the disk has accelerated to half its final angular speed?

[ ] m/s^2 Submit

Use (ix)

Spinning Disk



(vi) $d = R\theta$

(vii) $v = R\omega$

(viii) $a_T = R\alpha$

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A disk with mass m = 11.2 kg and radius R = 0.34 m begins at rest and accelerates uniformly for t = 17.4 s, to a final angular speed of ω = 29 rad/s.

7) What is the final speed of a point on the disk half-way between the center of the disk and the rim?

[ ] m/s Submit

Use (vii)

8) What is the total distance a point **on the rim** of the disk travels during the 17.4 seconds?

[ ] m Submit

Use (vi)