UNIT 9 HOMEWORK AFTER SESSION ONE

(You'll note that some of these problems are greyed out. These are not assigned this semester but we leave them visible so you can see what you are missing.)

➡ **SmartPhysics:** *View the Prelecture 12 "Elastic Collisions" before the class of next session and answer the checkpoint questions. (The online Homework is due the next day.)*

- *Work Supplemental Problems SP9-1 shown below*

SP9-1) [20 pts] *How does the centre of mass of a system of two equal masses move?* You are to graph the motion of the position average for moving carts of equal mass and interpret the graph. (In the special case of equal masses the position average and the *centre of mass*, which we will define more carefully in the next session, are the same thing.) Re-analyse the two same-mass cart collision movie you made in this unit in order to obtain a graph of the x-value of the position average, <x>, as a function of time. The position average can be obtained by using previously taken or new data on the positions of the carts in metres (i.e. you must perform a horizontal calibration). Once these data are transferred to a spreadsheet you can set up a column for calculating the average position of the two cart system. By rearranging columns on the spreadsheet you can graph <x> in metres as a function of time in seconds.

(a) Do a fit to find an equation that describes your data. Display the equation for <x> vs. t on the graph. Upload a copy of your plot to WebCT assignments.

(b) Is the velocity of cart 1 a constant (i.e. the same) before and after the collision occurs or does it change as a result of the collision? How about cart 2? Examine your graph. Is the velocity of the average position (i.e. the centre of mass) a constant before and after the collision? During the collision?

(c) Based on the equation you determined, what is the magnitude of velocity of the centre of mass (average position) of the carts? How does the total mass ($M = m+m = 2m$) times the velocity compare with the average total momentum of this two-cart system?

SP 9-2) [4 pts] **(a)** Consider the "before" picture in Figure 9-6. According to the Law of Momentum Conservation what quantity is constant before, during, and after the collision if no outside forces are present?

(b) Use the definition of instantaneous velocity and the fact that a constant can be pulled out of a derivative to show that if $\vec{p} = \vec{p}_1 + \vec{p}_2$ and if the objects only move along a line in one dimension then,

$$p = m_1 \frac{dx_1}{dt} + m_2 \frac{dx_2}{dt}$$

(c) Noting that any constant can be pulled in or out of a derivative, show that

$$p = \frac{d(m_1 x_1 + m_2 x_2)}{dt}$$

(d) Now by defining the centre of mass of the two particle system as

$$x_{cm} = \frac{(m_1 x_1 + m_2 x_2)}{M} \qquad \text{where} \quad M = m_1 + m_2$$

show that

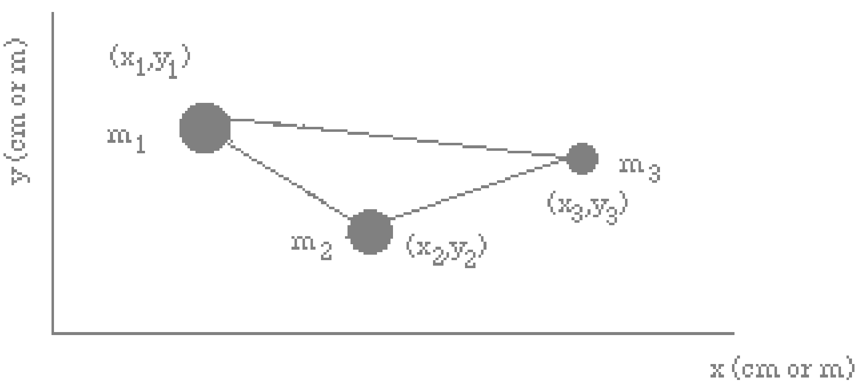
$$p = \frac{d(m_1 x_1 + m_2 x_2)}{dt} = M v_{cm}$$

SP 9-3) [4 pts] Most objects or systems of particles extend in all three dimensions rather than lying along a line. The definition of centre of mass can be readily extended to two and three dimensions. For example, the defining equations for the two-dimensional case can be given as follows:

$$\vec{r}_{cm} = X_{cm}\hat{i} + Y_{cm}\hat{j}$$

where $$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

and $$Y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$



Suppose the three masses located in the *xy* plane have the following coordinates: a 3.0-kg mass has coordinates given by (3.0, -2.0) m; a 4.0-kg mass has coordinates (-2.0, 4.0) m; a 1.0-kg mass has coordinates (2.0, 2.0) m. Find the coordinates of the centre of mass.

UNIT 9 HOMEWORK AFTER SESSION TWO

➡ **SmartPhysics:** *View the Prelecture 13 "Collisions, Impulse and Reference Frames" before the class of next session and answer the checkpoint questions. (The online Homework is due later.)*

• *Complete Unit 9 entries in the Activity Guide* You probably didn't have time to finish in class.

• *Work Supplemental Problems 9-4, 9-5, 9-6 and 9-7.*

SP9-4) [2 pts] The mass of the Sun is 329,390 Earth masses and the mean distance from the centre of the Sun to the centre of the Earth is 1.496×10^8 km. Treating the Earth and Sun as particles, with each mass concentrated at the respective geometric centre, how far from the centre of the Sun is the C.M. of the Earth-Sun system? Compare this distance with the mean radius of the Sun (6.96×10^5 km).

SP9-5) [20 pts] *Momentum components* Since momentum is a vector, the Law of Conservation of Momentum in two dimensions requires that if the vector conservation equation is broken into components then the conservation law must also hold for each of the vector components. Thus, if we consider the interaction of three or more objects, and if

$$\Sigma \vec{p} = \vec{p}_{1,i} + \vec{p}_{2,i} + \vec{p}_{3,i} + ... = \vec{p}_{1,f} + \vec{p}_{2,f} + \vec{p}_{3,f} + \ldots = \text{a constant}$$

then

$$\Sigma p_x = p_{1x,i} + p_{2x,i} + p_{3x,i} + ... = p_{1x,f} + p_{2x,f} + p_{3x,f} + \ldots = \text{a constant}$$

and

$$\Sigma p_y = p_{1y,i} + p_{2y,i} + p_{3y,i} + ... = p_{1y,f} + p_{2y,f} + p_{3y,f} + \ldots = \text{a constant}$$

If a coordinate system is chosen and a given momentum vector makes an angle θ with respect to the designated x-axis then the momentum vector can be broken into components in the usual way:

$$\vec{p} = p_x\hat{i} + p_y\hat{j} = p\cos\theta\hat{i} + p\sin\theta\hat{j}$$

Let's consider the fairly complex interaction of three air pucks that you examined in Activity 9-8.

(a) Determine the *x* and *y*-components of the momentum for each of the three pucks before the collision. Use these to find *x* and *y*-components of the total initial momentum of the system.
(b) Determine the *x* and *y*-components of the momentum for each of the three pucks after the collision. Use these to find *x* and *y*-components of the total final momentum of the system.
(c) Does momentum appear to be conserved (within reasonable experimental uncertainties) in this collision? Justify your answer.

SP9-6) [10 pts] *Momentum as a vector*
(a) Use the results from SP9-5 (a) to determine the magnitude and angular direction of the total initial momentum of the system.

(b) Use the results from SP9-5 (b) to determine the magnitude and angular direction of the total final momentum of the system.
(c) According to these results, does momentum appear to be conserved (within reasonable experimental uncertainties) in this collision? Justify your answer.

SP9-7) [20 pts] *What does an observer moving with the centre of mass of a system with two equal masses see?* We know that in the laboratory coordinate system (where the observer is at rest in the laboratory) the velocities of the two carts are different from each other most of the time. What velocities does a moving observer see? You can re-analyse one more time the two same-mass cart collision movie in order to obtain a graph of the velocity of cart 1 and the velocity of cart 2 as measured by our observer who is moving with the centre of mass of the system. Imagine you are moving so that you are always just opposite the point that lies in the centre between the two carts as they move. You already *know* you will be moving with a constant velocity of magnitude V which you determined in part (c) in SP9-1. What do you see as the carts interact? This point of view can be simulated by choosing a moving origin which lies halfway between the carts in each frame of the cart movie. (No calculations are needed, but you may want to hold a ruler up to the computer monitor screen to help you determine the halfway points.) The software will then calculate how far each of the carts is from this moving origin on a frame-by-frame basis. You should transfer your data to a spreadsheet and calculate the velocities of the carts as a function of time.

(a) Create a correctly labelled overlay graph of the positions of the two carts (x_1 vs. t(s) and x_2 vs. t(s)) relative to the centre-of-mass location. Upload a copy of your plot to WebCT assignments.
(b) Calculate best values for the velocities of the carts before and after the collision by determining the slope of the position vs. time graph for cart 1 before collision and the slope of the position vs. time for cart 1 after the collision. Do the same for cart 2.
(c) In the centre-of-mass coordinate system, how does the magnitude of the cart 1 velocity compare to the magnitude of the cart 2 velocity before and after the collision?
(d) What is the total momentum of the two cart system when it is viewed by the observer in the centre-of-mass coordinate system? Does this view of the collision bear any relationship to the collisions you observed in Activity 9-2?
(e) Examine your graph (no need for calculations). Does the moving observer still believe momentum is conserved in the process? Explain.

(Practice only Do not hand in.) Ball and Spring Gun In Fig. 10-56, a ball of mass m is shot with speed v1 into the barrel of a spring gun of mass M initially at rest on a frictionless surface. The ball sticks in the barrel at the point of maximum compression of the spring. Assume that the increase in thermal energy due to friction between the ball and the barrel is negligible. (a) What is the speed of the spring gun after the ball stops in the barrel? (b) What fraction of the initial kinetic energy of the ball is stored in the spring?

-