

Classical Mechanics

Lecture 10

Today's Concept:

Center of Mass

- Finding it
- Using it

Your Comments

Center of mass just got a little more difficult. What is r relative to? The origin? Do we define this origin? Also will we actually have to do 3D center of mass problems? If so that would make me sad, very sad.

Just because the prelecture guy tells me integration is easy and does it in a happy voice doesn't make it so. If we could go over how the CM velocity and CM acceleration formulas are found from the position that would be awesome

I want to do more number examples, especially with finding the center of mass of objects with integrals. We do too much variable stuff, I need to see some numbers.

Need real donuts in lecture, not just conceptual ones.//can we get more doughnuts in class?

why doesn't the moon crash into the earth, causing widespread panic and destruction?

i found the prelecture hard to follow...maybe its the headache from all this h/w..

Please explain the two men pulling a pole on ice question. I get it, but not quite.... I need a better explanation.

Center of Mass

Center of Mass

(for Discrete Distributions)

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \sum_{i=1}^N m_i \vec{r}_i$$

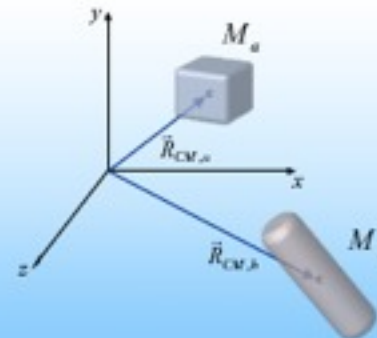
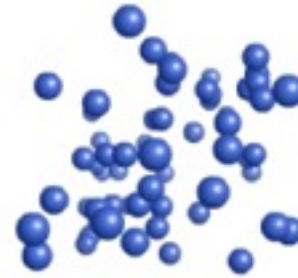
THIS IS IT!

(for Continuous Distributions)

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \int \vec{r} dm$$

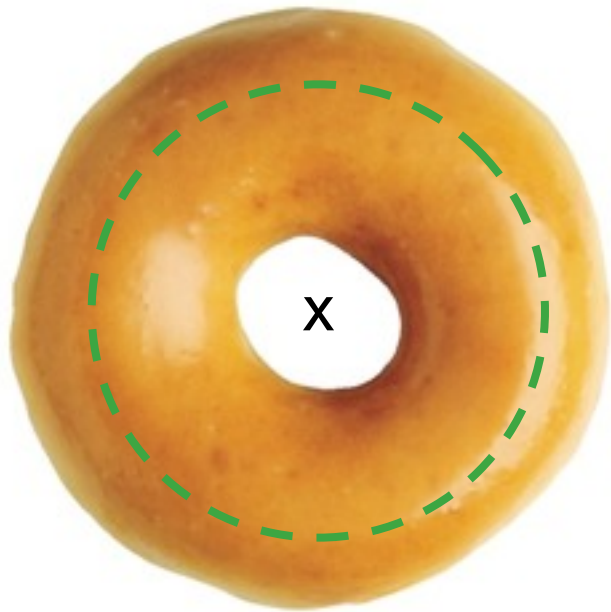
(for System of Solid Objects)

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \sum_{i=1}^N M_i \vec{R}_{CM,i}$$



CheckPoint

A yummy glazed doughnut is shown below. Where is the center of mass of this fantastic culinary delight?

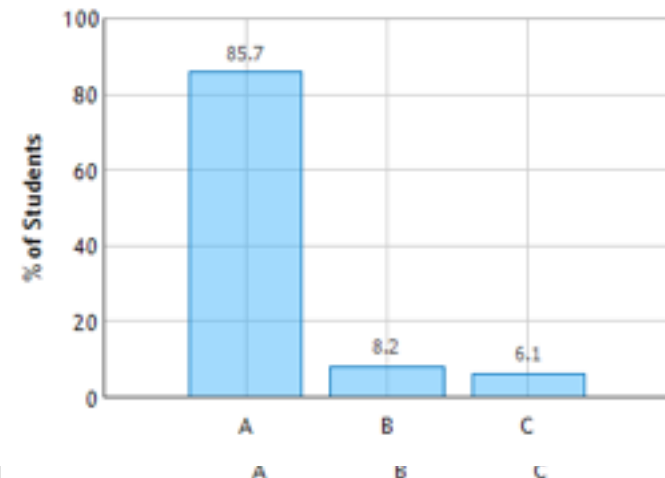


- A) In the center of the hole.
- B) Anywhere along the blue dashed line going through the solid part of the dough.
- C) The center of mass is not defined in cases where there is missing mass.

The yummy doughnut is round and has equal weight distribution all over the doughnut, therefore, the center of mass must be in the middle



Doughnut: Question 1 (N = 49)



Wrong Reasons

A yummy glazed doughnut is shown below. Where is the center of mass of this fantastic culinary delight?

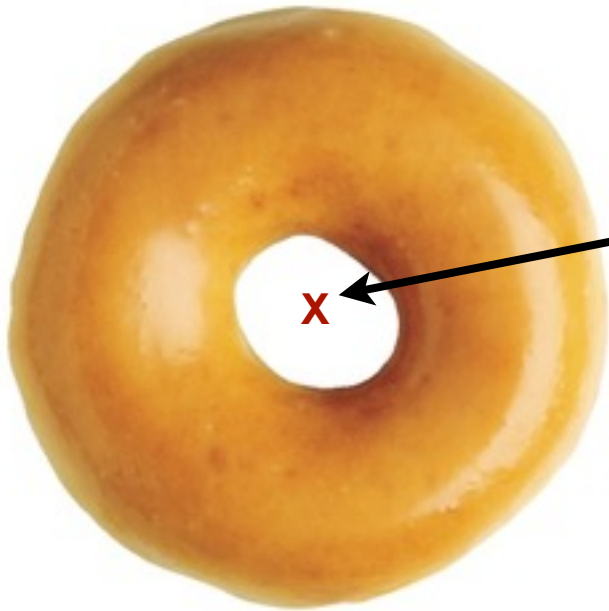


- A) In the center of the hole.
- B) Anywhere along the blue dashed line going through the solid part of the dough.
- C) The center of mass is not defined in cases where there is missing mass.

(B) It can't be in the centre of the doughnut because there is no mass so it has to be somewhere in the donut

(C) The center of mass is not defined where the deliciousness does not exist.

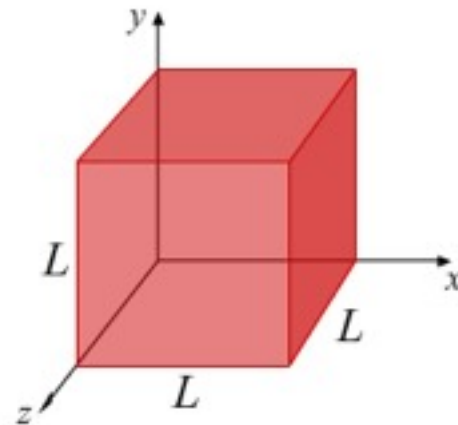
Health Considerations



The centre of mass is the healthiest part!

Cube Example

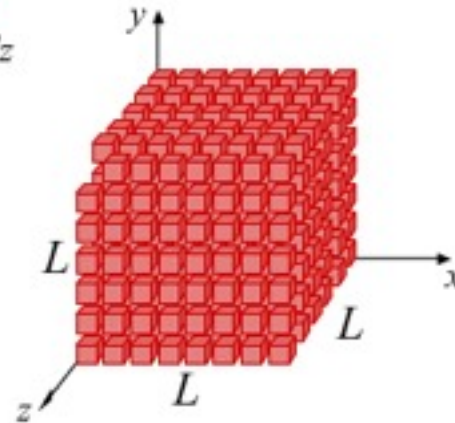
$$\vec{R}_{CM} = \frac{1}{M_{Total}} \int \vec{r} \, dm$$



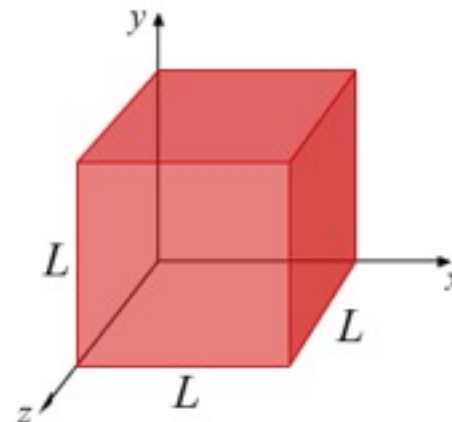
Cube Example: Using Volume Mass Density

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \int \vec{r} dm$$

mass $dm = \rho dxdydz$
volume $dxdydz$



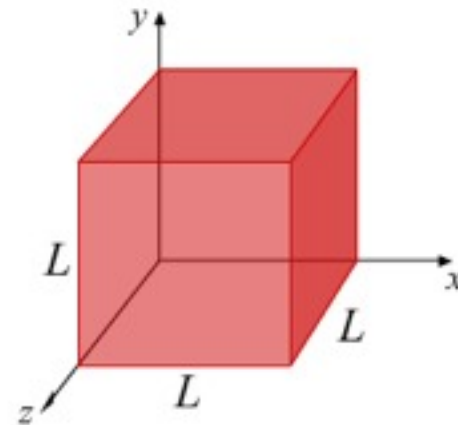
$$\vec{R}_{CM} = \frac{1}{M_{Total}} \iiint \vec{r} \rho dxdydz$$



Cube Example: CM Calculation

Center of Mass
(for Continuous Distributions)

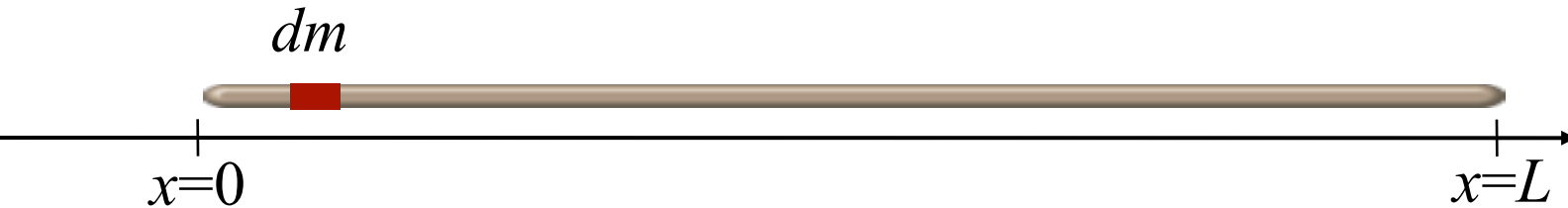
$$\begin{aligned} X_{CM} &= \frac{1}{M_{Total}} \rho \int_0^L \int_0^L \int_0^L x \, dx \, dy \, dz \\ &= \frac{1}{M_{Total}} \rho \int_0^L x \, dx \int_0^L dy \int_0^L dz \\ &= \frac{1}{M_{Total}} \rho \left(\frac{1}{2} x^2 \Big|_0^L \right) \left(y \Big|_0^L \right) \left(z \Big|_0^L \right) \\ &= \frac{1}{M_{Total}} \rho \left(\frac{1}{2} L^2 \right) (L)(L) \\ &= \frac{1}{M_{Total}} \left(\frac{L}{2} \right) (\rho L^3) \end{aligned}$$



Example

A uniform stick of length L and mass M .

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \int \vec{r} dm$$



$$dm = M \frac{dx}{L} = \frac{M}{L} dx$$

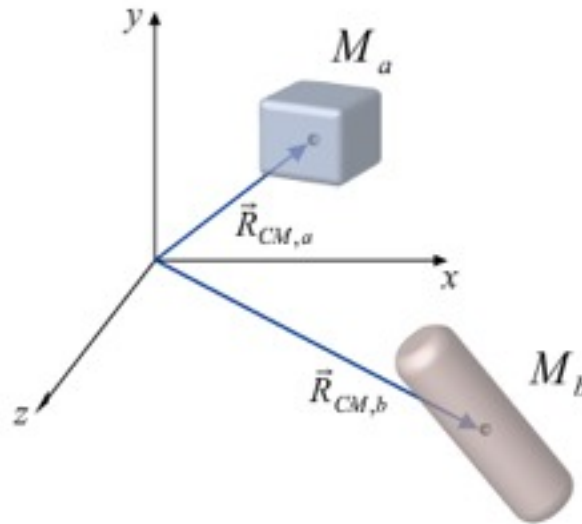
$$x_{cm} = \frac{1}{M} \int x dm = \frac{1}{M} \int_0^L x \frac{M}{L} dx = \frac{1}{L} \int_0^L x dx$$

$$= \frac{1}{L} \left(\frac{1}{2} x^2 \right)_0^L = \frac{1}{L} \frac{1}{2} (L^2 - 0^2) = \frac{L}{2}$$

Center of Mass for System of Objects

Center of Mass
(for System of Solid Objects)

$$\vec{R}_{CM} = \frac{1}{M_{Total}} \sum_{i=1}^N M_i \vec{R}_{CM,i}$$



Clicker Question



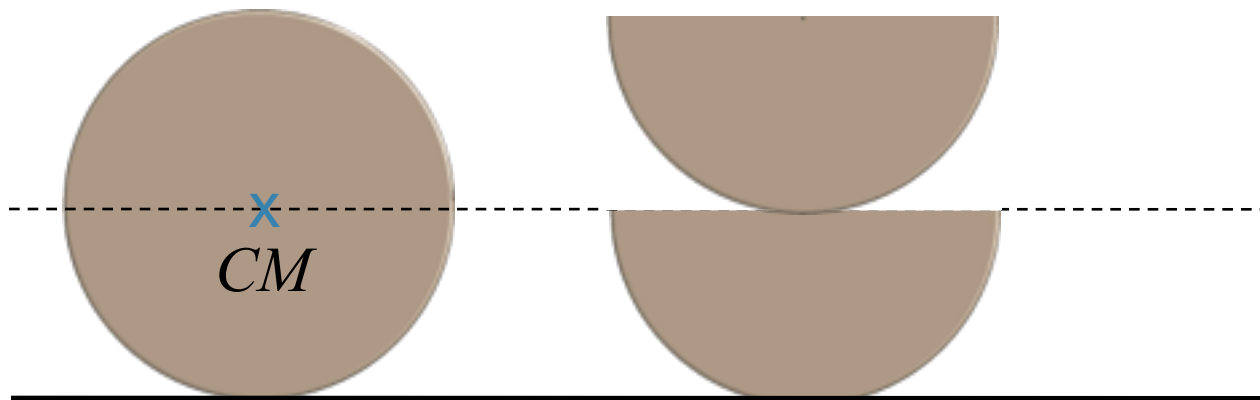
The disk shown in **Case 1** clearly has its **CM** at the center. Suppose the disk is cut in half and the pieces arranged as shown in **Case 2**

In which case is the center of mass highest?

A) Case 1

B) Case 2

C) same



Case 1

Case 2

Clicker Question



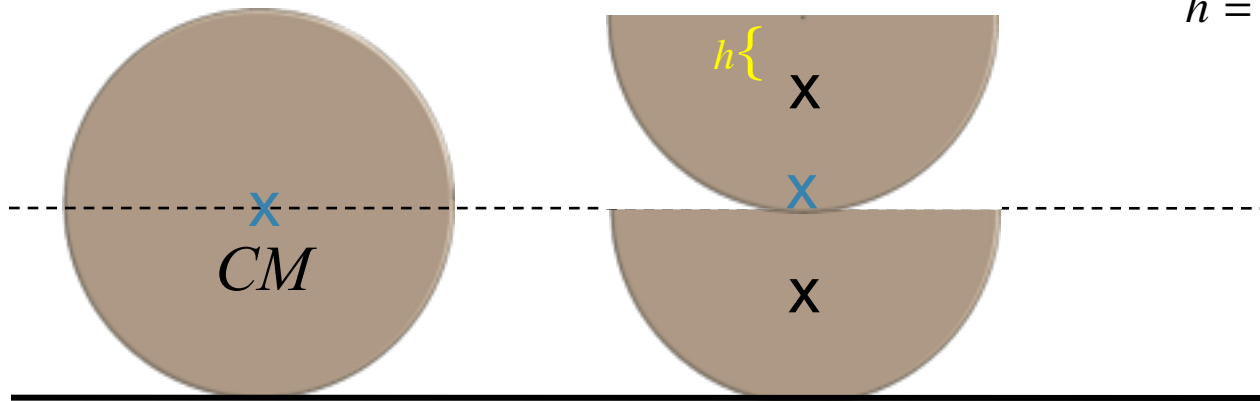
The disk shown in Case 1 clearly has its *CM* at the center. Suppose the disk is cut in half and the pieces arranged as shown in Case 2

In which case is the center of mass highest?

A) Case 1

B) Case 2

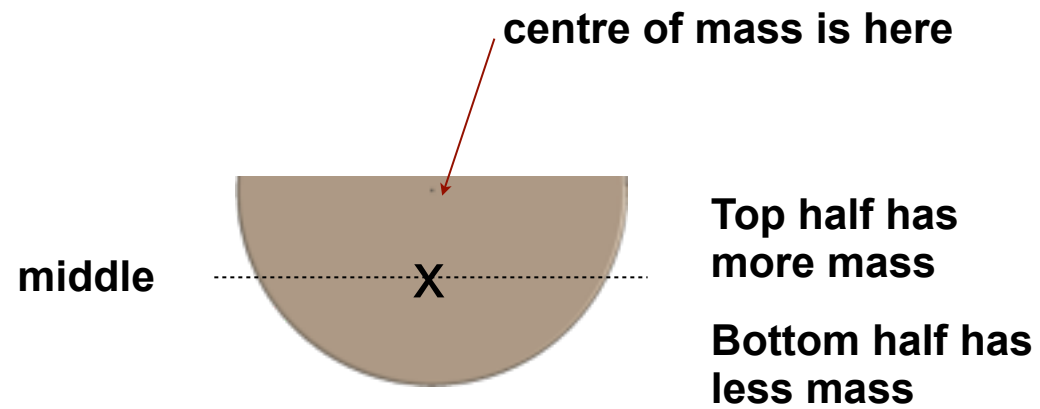
C) same



$$h = \frac{4r}{3\pi} < \frac{r}{2}$$

Case 1

Case 2



Kinematic Quantities

Displacement of Center of Mass

$$\vec{R}_{CM} = \frac{\sum_{i=1}^N m_i \vec{r}_i}{M_{Total}}$$

Velocity of Center of Mass

$$\vec{V}_{CM} = \frac{\sum_{i=1}^N m_i \vec{v}_i}{M_{Total}}$$

Acceleration of Center of Mass

$$\vec{A}_{CM} = \frac{\sum_{i=1}^N m_i \vec{a}_i}{M_{Total}}$$

$$\vec{A}_{CM} = \frac{\vec{F}_{Net, External}}{M_{Total}}$$

If the total force is zero, the *CM* won't accelerate

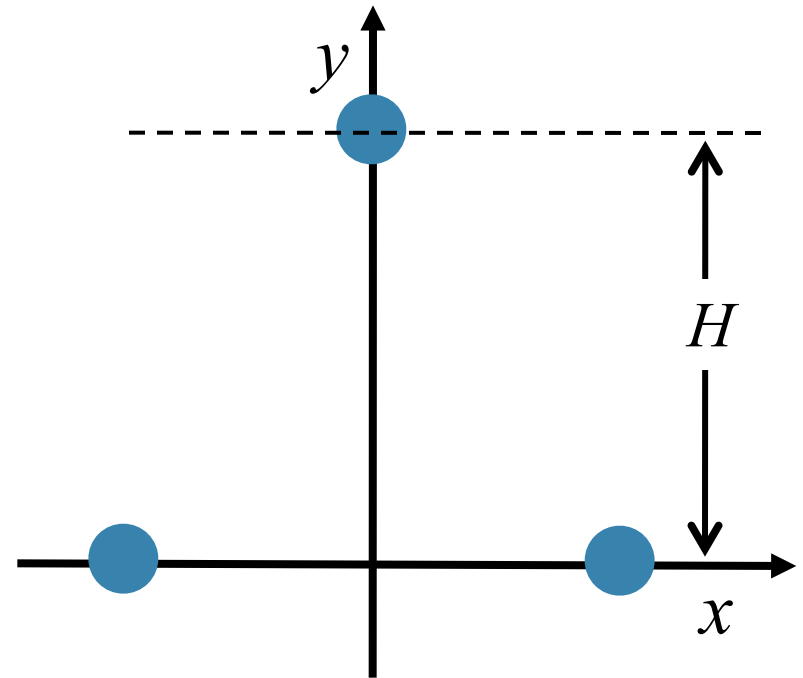
Clicker Question



Three tiny equal masses are placed at the corners of an **equilateral triangle**. When the masses are released, they attract and quickly move to a single point. **What are the coordinates of that point?**

X_{CM} Y_{CM}

- A) 0 0
- B) 0 $H/2$
- C) 0 $H/3$
- D) $H/4$ $H/4$
- E) $H/4$ 0

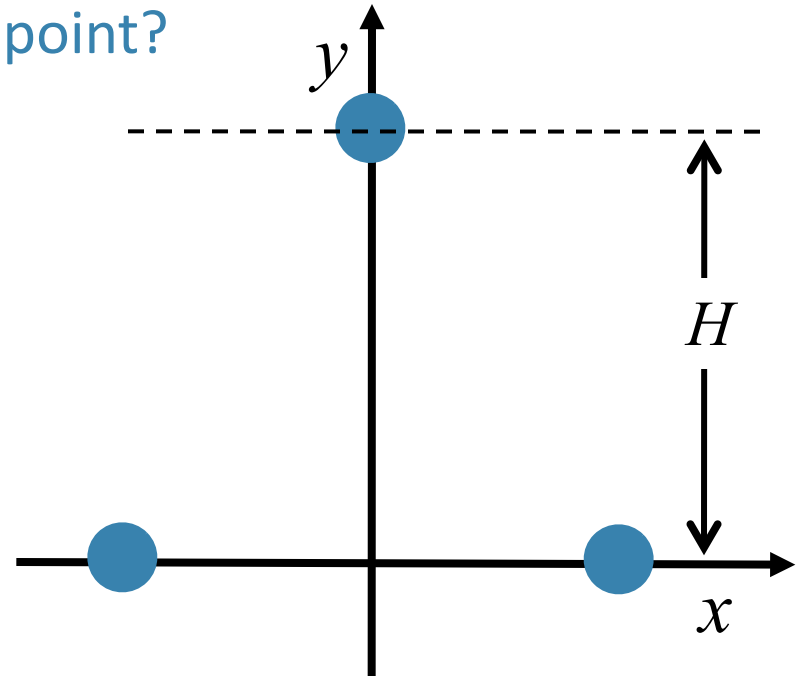


Three tiny equal-mass magnets are placed on a horizontal frictionless surface at the corners of an **equilateral triangle**. When the magnets are released, they attract and quickly slide to a single point. **What are the coordinates of that point?**

$$\begin{array}{cc} X_{CM} & Y_{CM} \\ \text{c) } 0 & H/3 \end{array}$$

$$X_{CM} = 0 \text{ (symmetry)}$$

$$Y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{m \times 0 + m \times 0 + m \times H}{3m} = \frac{H}{3}$$



Homework Problem

System of Particles

Four particles are in a 2-D plane with masses, x- and y- positions, and x- and y- velocities as given in the table below:

	m	x	y	v _x	v _y
1	7.8 kg	-2.8 m	-4.7 m	3.2 m/s	-4.2 m/s
2	7.8 kg	-3.7 m	3.7 m	-5.2 m/s	5.2 m/s
3	7.8 kg	4.7 m	-5.7 m	-6.2 m/s	2.2 m/s
4	7.8 kg	5.7 m	2.7 m	4.2 m/s	-3.2 m/s

1) What is the x position of the center of mass? m

2) What is the y position of the center of mass? m

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4}{m_1 + m_2 + m_3 + m_4} = 0.975 \text{ m}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4}{m_1 + m_2 + m_3 + m_4} = -1 \text{ m}$$

Homework Problem

System of Particles

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4	7.8 kg	5.7 m	2.7 m	4.2 m/s	-3.2 m/s

3) What is the speed of the center of mass? m/s

$$\left. \begin{aligned} v_{cm,x} &= \frac{m_1 v_{1,x} + m_2 v_{2,x} + m_3 v_{3,x} + m_4 v_{4,x}}{m_1 + m_2 + m_3 + m_4} \\ v_{cm,y} &= \frac{m_1 v_{1,y} + m_2 v_{2,y} + m_3 v_{3,y} + m_4 v_{4,y}}{m_1 + m_2 + m_3 + m_4} \end{aligned} \right\} v_{cm} = \sqrt{v_{cm,x}^2 + v_{cm,y}^2}$$



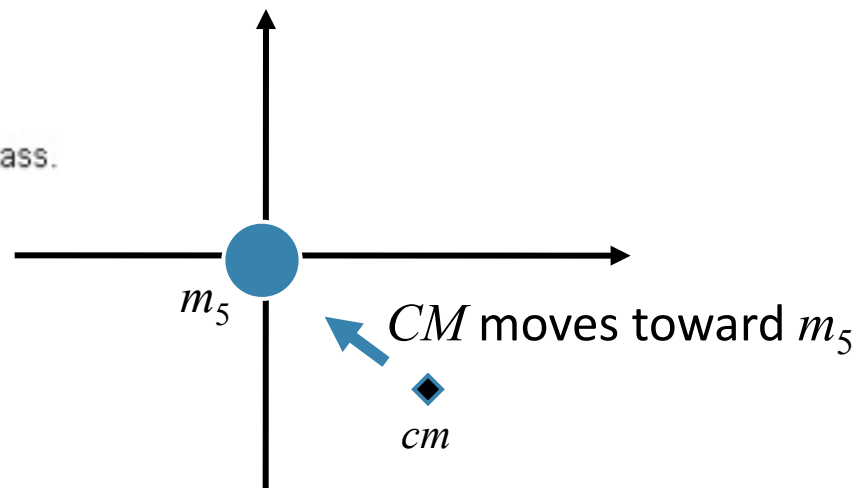
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4	7.8 kg	5.7 m	2.7 m	4.2 m/s	-3.2 m/s

4)  When a fifth mass is placed at the origin, what happens to the horizontal (x) location of the center of mass?

- A ☐ It moves to the right.
- B ☐ It moves to the left.
- C ☐ It does not move.
- D ☐ It can not be determined unless you know the mass.



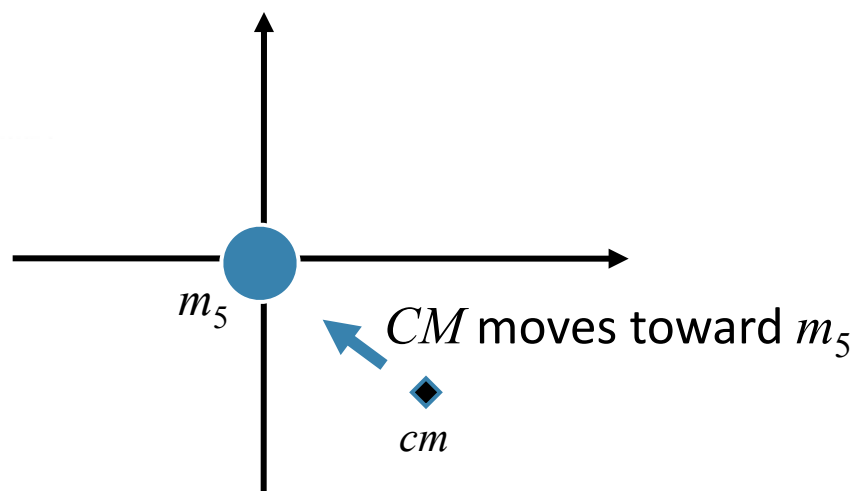
System of Particles

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3	7.8 kg	4.7 m	-5.7 m	-6.2 m/s	2.2 m/s
4	7.8 kg	5.7 m	2.7 m	4.2 m/s	-3.2 m/s

5)  When a fifth mass is placed at the center of mass, what happens to the vertical (y) location of the center of mass?

- A ☐ It moves up.
- B ☐ It moves down.
- C ☐ It does not move.



Clicker Question

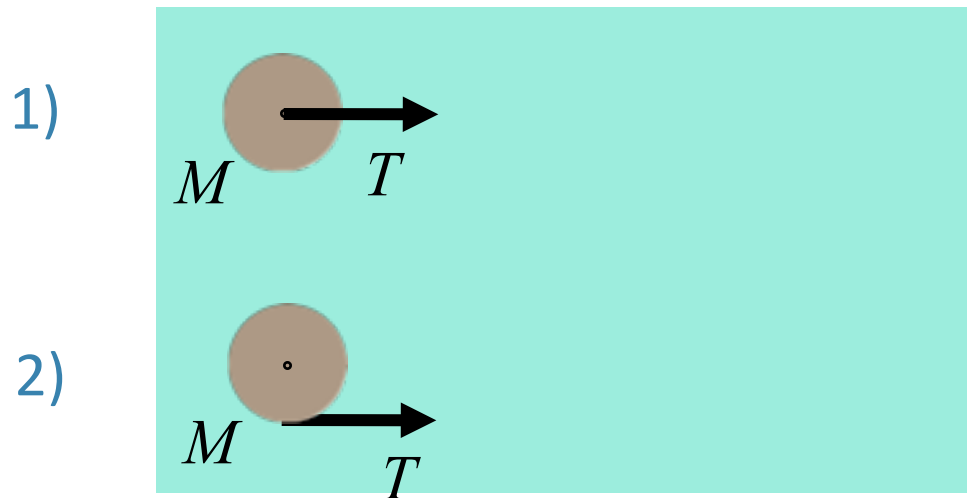


Two pucks of equal mass, on a frictionless table, are being pulled at different points with equal forces. Which one gets to the end of the table first?

A) Puck 1

B) Puck 2

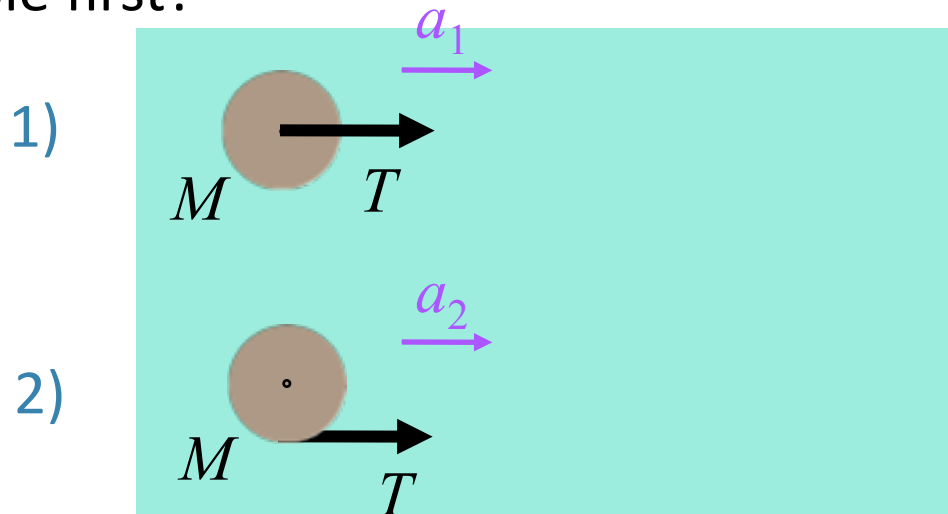
C) Same



Clicker Question



Two pucks of equal mass, on a frictionless table, are being pulled at different points with equal forces. Which one gets to the end of the table first?



C) Same

$$\vec{a}_{CM} = \frac{\vec{F}_{Net, External}}{M_{Total}} \quad \text{SAME}$$

CheckPoint

Two objects, one having twice the mass of the other, are initially at rest. Two forces, one twice as big as the other, act on the objects in opposite directions as shown.



The diagram shows two brown circular objects. The left object is labeled $2M$ and has a thick black arrow pointing left labeled $2F$. The right object is labeled M and has a thick black arrow pointing right labeled F .

$$\vec{a}_{CM} = \frac{\vec{F}_{Net, External}}{M_{Total}}$$

Which of the following statements about the acceleration of the center of mass of the system is true:

- A) $a_{CM} = F/M$ to the right
- B) $a_{CM} = F/(3M)$ to the right
- C) $a_{CM} = 0$
- D) $a_{CM} = F/(3M)$ to the left
- E) $a_{CM} = F/M$ to the left

CheckPoint



$$\vec{a}_{CM} = \frac{\vec{F}_{Net, External}}{M_{Total}}$$

Which of the following statements about the acceleration of the center of mass of the system is true:

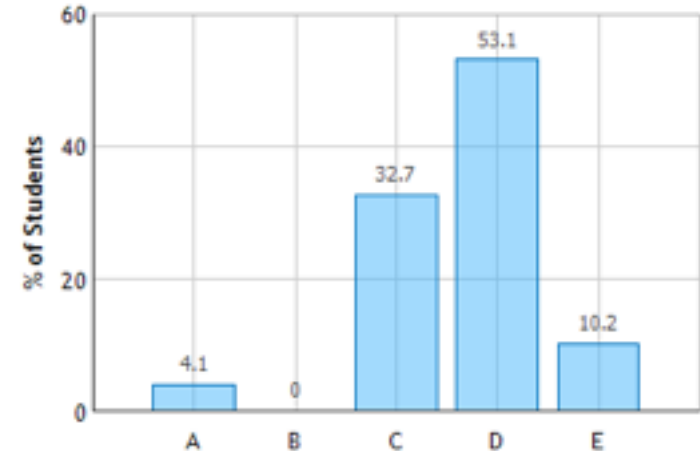
- C) $a_{CM} = 0$
- D) $a_{CM} = F/(3M)$ to the left
- E) $a_{CM} = F/M$ to the left

C) $F=Ma$. The $2M$ and the $2F$ would cancel each other out, making the bigger object accelerate at the same rate as the smaller ball. Therefore, the acceleration would be zero.

D) Acceleration is equal to the net force divided by the total mass. The net force is F to the left and the total mass is $3M$

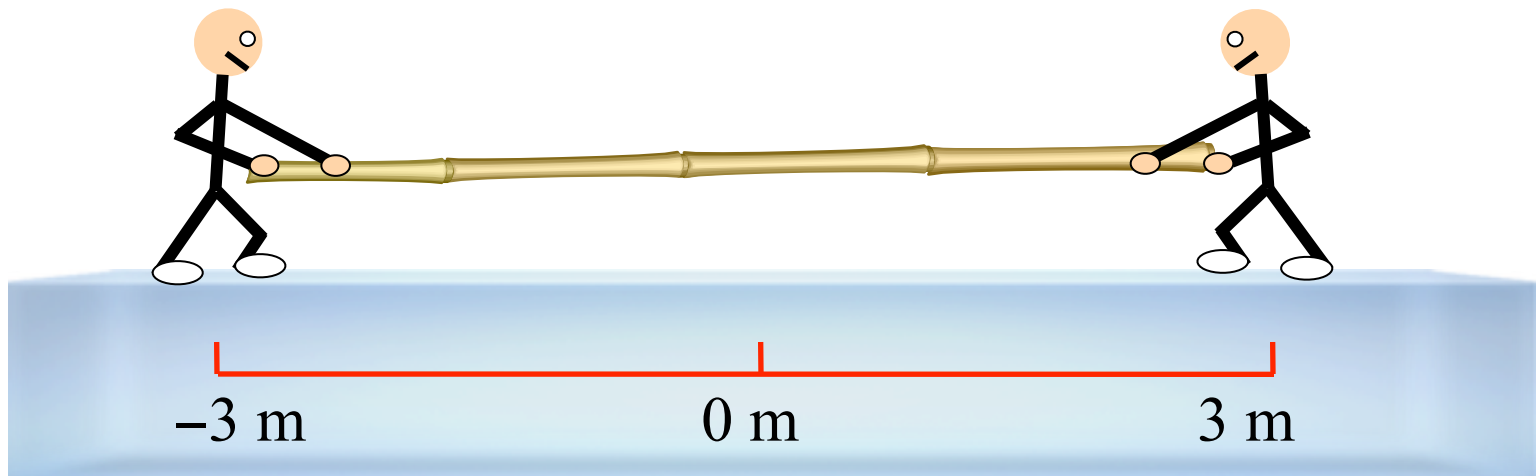
E) $A_{CM} = F \text{ total} / M \text{ total} = 3F/3M = F/M$

System of Two Masses: Question 1 (N = 49)



CheckPoint

Two guys who weigh the same are holding onto a **massless** pole while standing on horizontal **frictionless** ice. If the guy on the left starts to pull on the pole, where do they meet?



A) -3 m

B) 0 m

C) 3 m

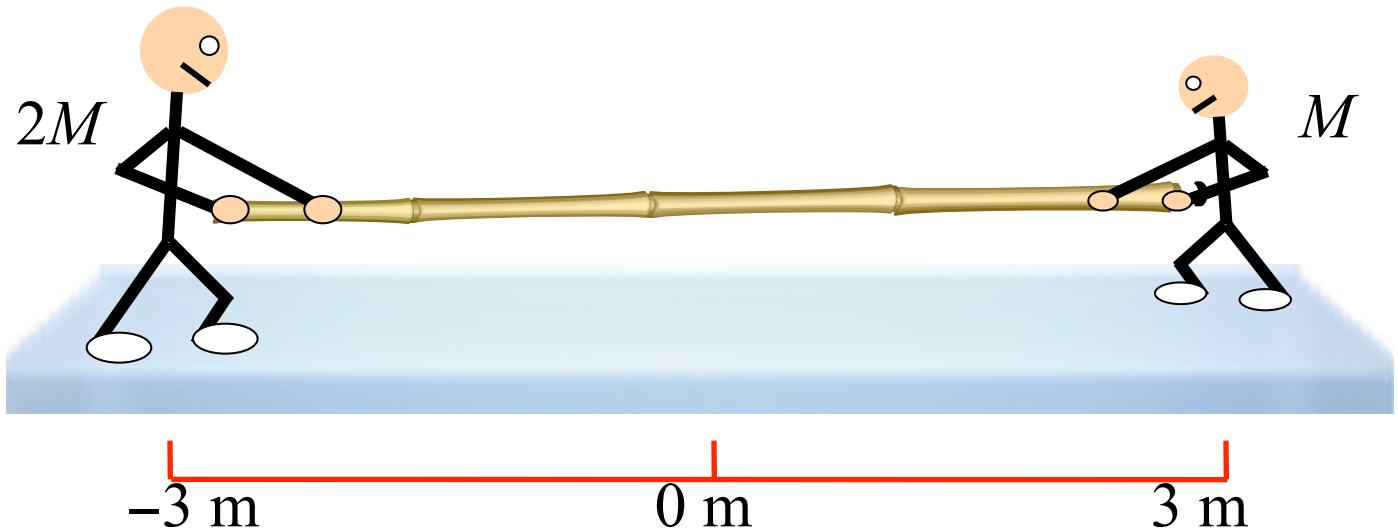
Two Men Pulling: Question 1 (N = 49)



Clicker Question

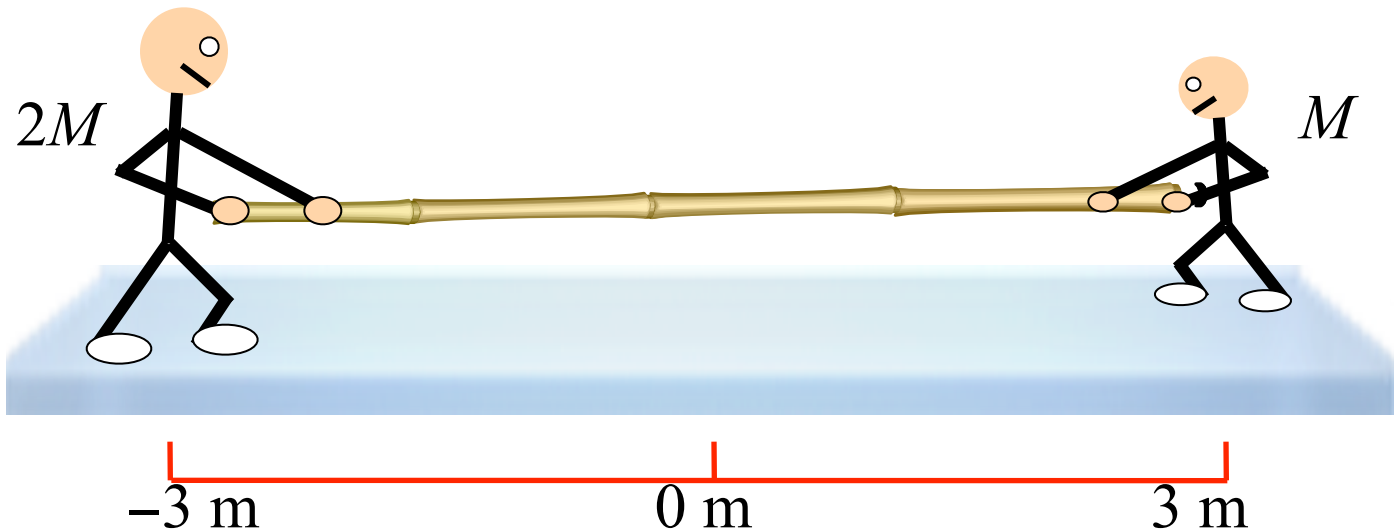


A large skinny guy with mass $2M$ and a smaller guy with mass M are holding onto a **massless** pole while standing on **frictionless** ice, as shown below. If the little guy pulls himself toward the big guy, where would they meet?



- A) -3 m
- B) -1 m
- C) 0 m
- D) 1 m
- E) 3 m

A large skinny guy with mass $2M$ and a smaller guy with mass M are holding onto a **massless** pole while standing on **frictionless** ice, as shown below. If the little guy pulls himself toward the big guy, where would they meet?



A) -3 m

B) -1 m

C) 0 m

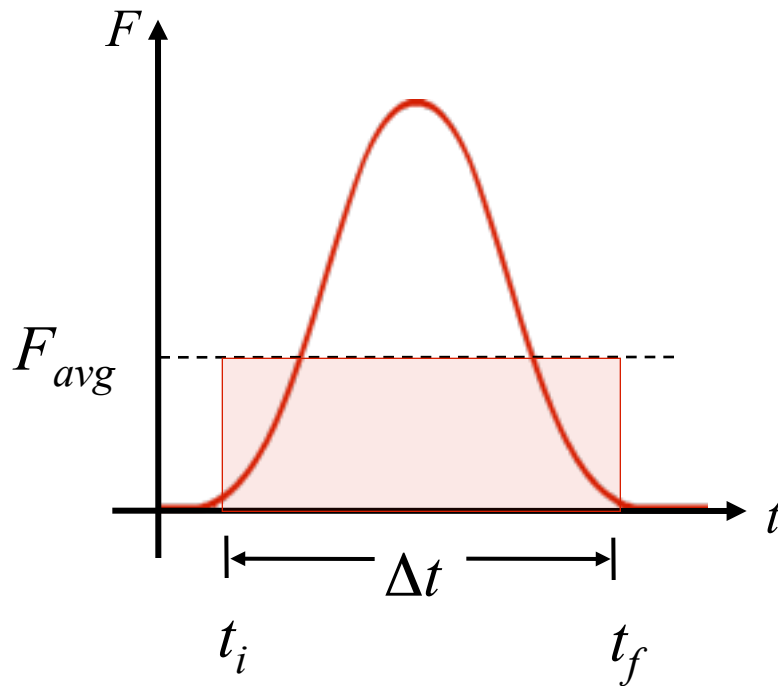
D) 1 m

E) 3 m

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{2M(-3m) + M(3m)}{3M} = -1\text{ m}$$

Preview: Forces during Collisions

$$\vec{F}_{tot} = m\vec{a} \quad \vec{F}_{tot} = \frac{d\vec{P}}{dt} \quad \vec{F}_{tot} dt = d\vec{P} \quad \underbrace{\int_{t_1}^{t_2} \vec{F}_{tot} dt}_{\vec{F}_{avg} \Delta t} = \underbrace{\vec{P}(t_2) - \vec{P}(t_1)}_{\Delta \vec{P}}$$



$$\Delta \vec{P} = \vec{F}_{avg} \Delta t$$

$$\int_{t_1}^{t_2} \vec{F}_{tot} dt \equiv \vec{F}_{avg} \Delta t$$

Impulse