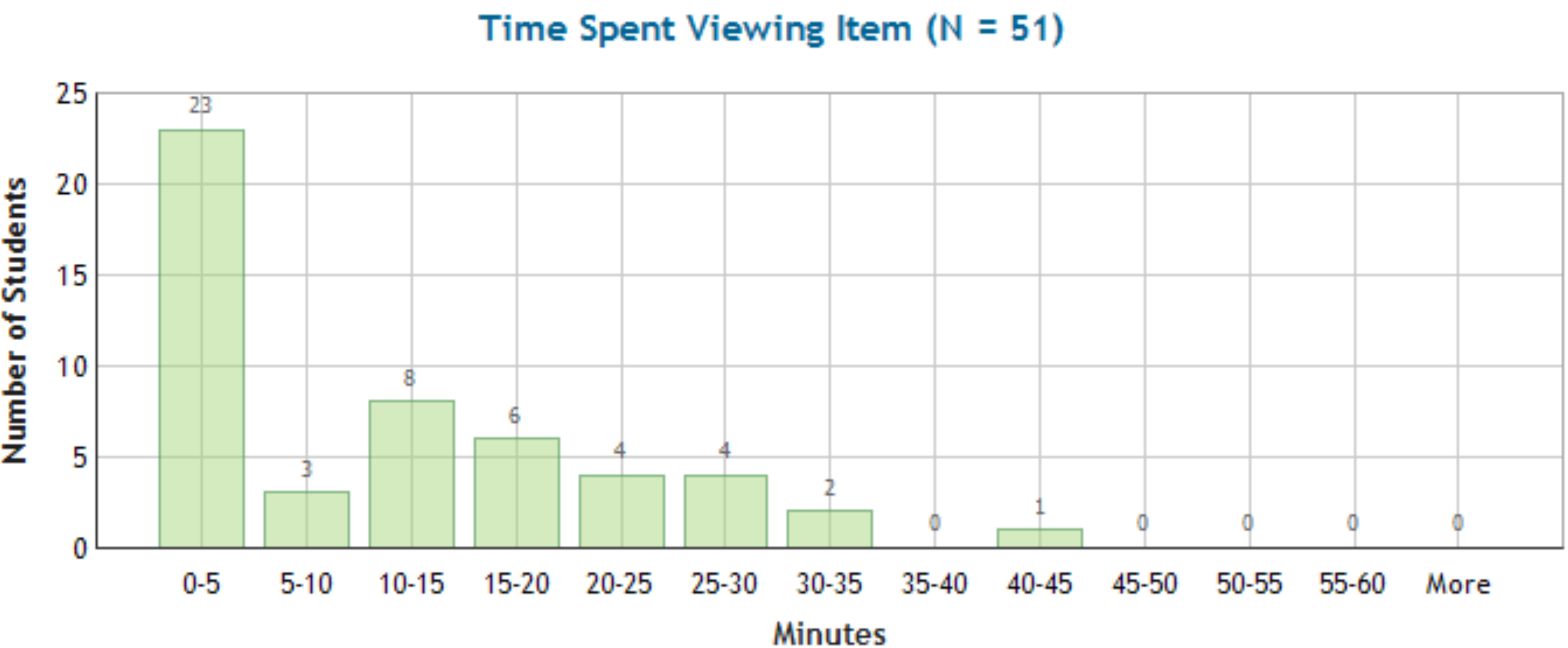
Classical Mechanics
Lecture 22

Today's Concept:

Simple Harmonic Motion: *Motion of a Pendulum*

I haven't been "watching" the pre-lectures. I am sorry.



Your Comments

I love playing with pendulums and different effects that can appear within systems of pendulums.

What does phi represent again? Also, what is the difference between angular velocity and angular frequency? They look and sound to be the same.

Who's the genius who decided omega should have two meanings? Did they run out of Greek letters? Why don't they fly over there and get some more? It would probably help boost their economy at this point.

I am finding it difficult to understand how the moment of inertia and the radius are both being used in the equation. Isn't the moment of inertia dependent on the radius?

Is the period proportional to Rcm then?

talking about harmonic motions , Lets all dance "GANGNOM style " , its a perfect practical example !

Why my roommate keeps on ramming Grape Fantas into my mini fridge is way more confusing than anything that we've covered this year

Can we do a potluck on Friday?

Conceptual Problems workbook

Don’t Panic!

http://www.youtube.com/watch?v=7rOMGIbY-9s

http://www.youtube.com/watch?v=Iexp-OwOs0M

I want to know why the answer to life is 42!

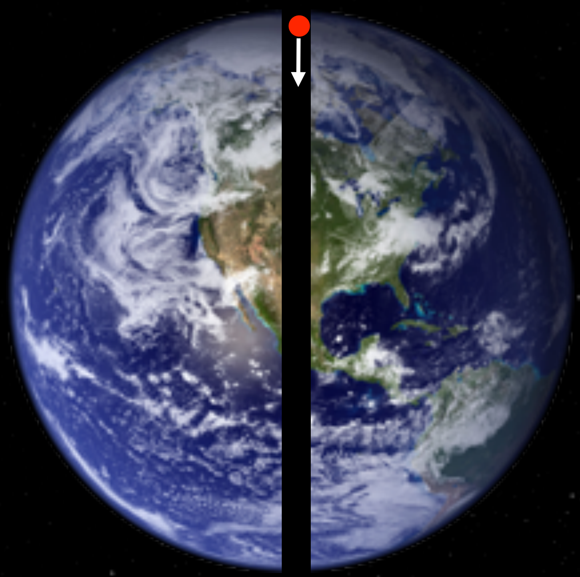
Drill a hole through the earth and jump in – what happens?

Just for fun – you don't need to know this.

I want to know why the answer to life is 42!

Drill a hole through the earth and jump in – what happens?

- You will oscillate like a mass on a spring with a period of 84 minutes.
- It takes 42 minutes to come out the other side!



$k = mg/R_E$

I want to know why the answer to life is 42!

Drill a hole through the earth and jump in – what happens?

You will oscillate like a mass on a spring with a period of 84 minutes.
It takes 42 minutes to come out the other side!

The hole doesn't even have to go through the middle – you get the same answer anyway as long as there is no friction.

I want to know why the answer to life is 42!

This is also the same period of an object orbiting the earth right at ground level.

Just for fun – you don't need to know this.

Panic!

"Is there such a thing as Rotational Harmonic Motion? There better not be..."

Yes there is.

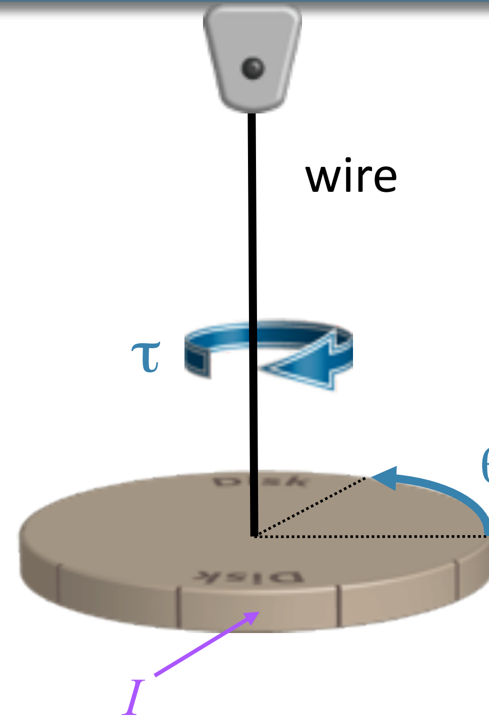
Are you ready?

Torsion Pendulum

$$\tau = I\alpha$$

$$-\kappa\theta \qquad I\frac{d^2\theta}{dt^2} \quad \rightarrow \quad \frac{d^2\theta}{dt^2} = -\omega^2\theta$$

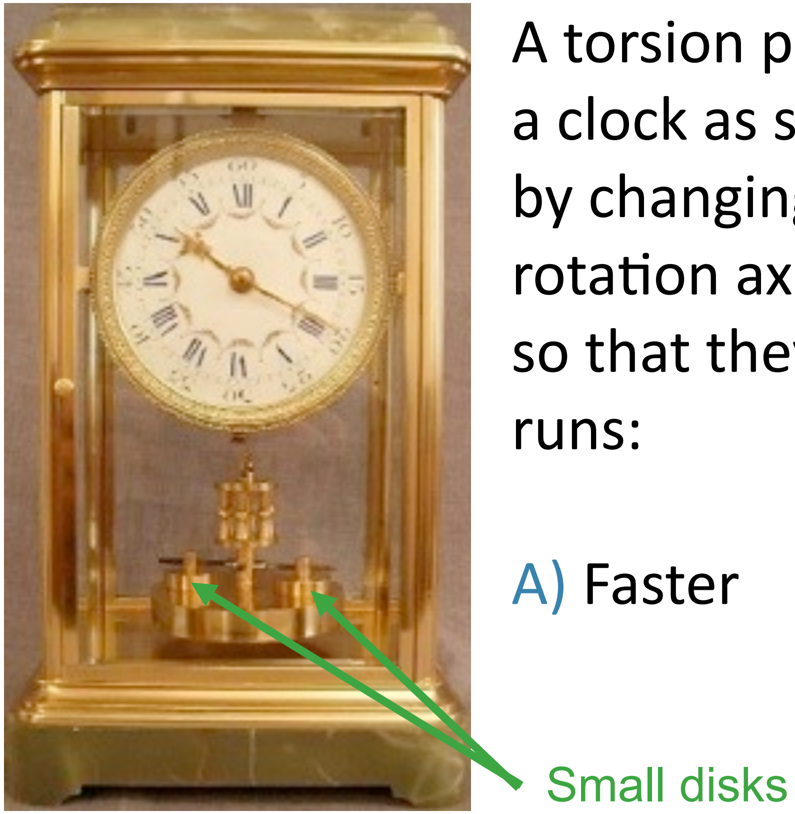
$$\omega = \sqrt{\frac{\kappa}{I}}$$



$$\theta(t) = \theta_{max}\cos(\omega t + \phi)$$

Q: In the prelecture the equation for restoring torque is given as $\tau=-\kappa\theta$ in clockwise direction..so if the restoring torque is in counter clockwise directions then would τ be positive?

CheckPoint



A torsion pendulum is used as the timing element in a clock as shown. The speed of the clock is adjusted by changing the distance of two small disks from the rotation axis of the pendulum. If we adjust the disks so that they are closer to the rotation axis, the clock runs:

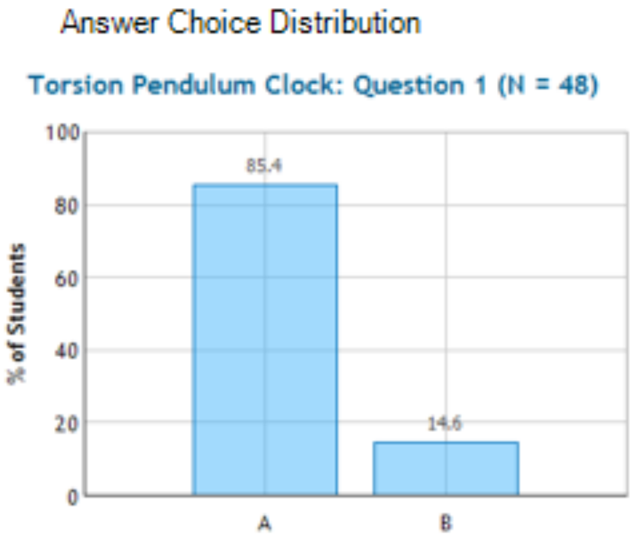
A) Faster   B) Slower

CheckPoint

If we adjust the disks so that they are closer to the rotation axis, the clock runs

A) Faster B) Slower

$$\omega = \sqrt{\frac{\kappa}{I}}$$



A) The moment of inertia decreases, so the angular frequency increases, which makes the period shorter and thus the clock faster.

B) T = 2pi * sqrt(I/MgRcm). If Rcm decreases, T will increase, making the clock run slower.

Grading

Unit 14 and 15 Activity Guides will not be graded

Please turn in:

- Unit 14 Written Homework on Dec 1
- The Mini-labbook on your SHM or Karate Project, Dec 9

Triumf Lectures

This weekend, Sat. Nov. 29

- 10 am Earthquakes
- 11 am Earthquake engineering

snacks

Pendulum

$$\tau = I\alpha$$

$$\tau \rightarrow -MgX_{CM}$$

For small θ

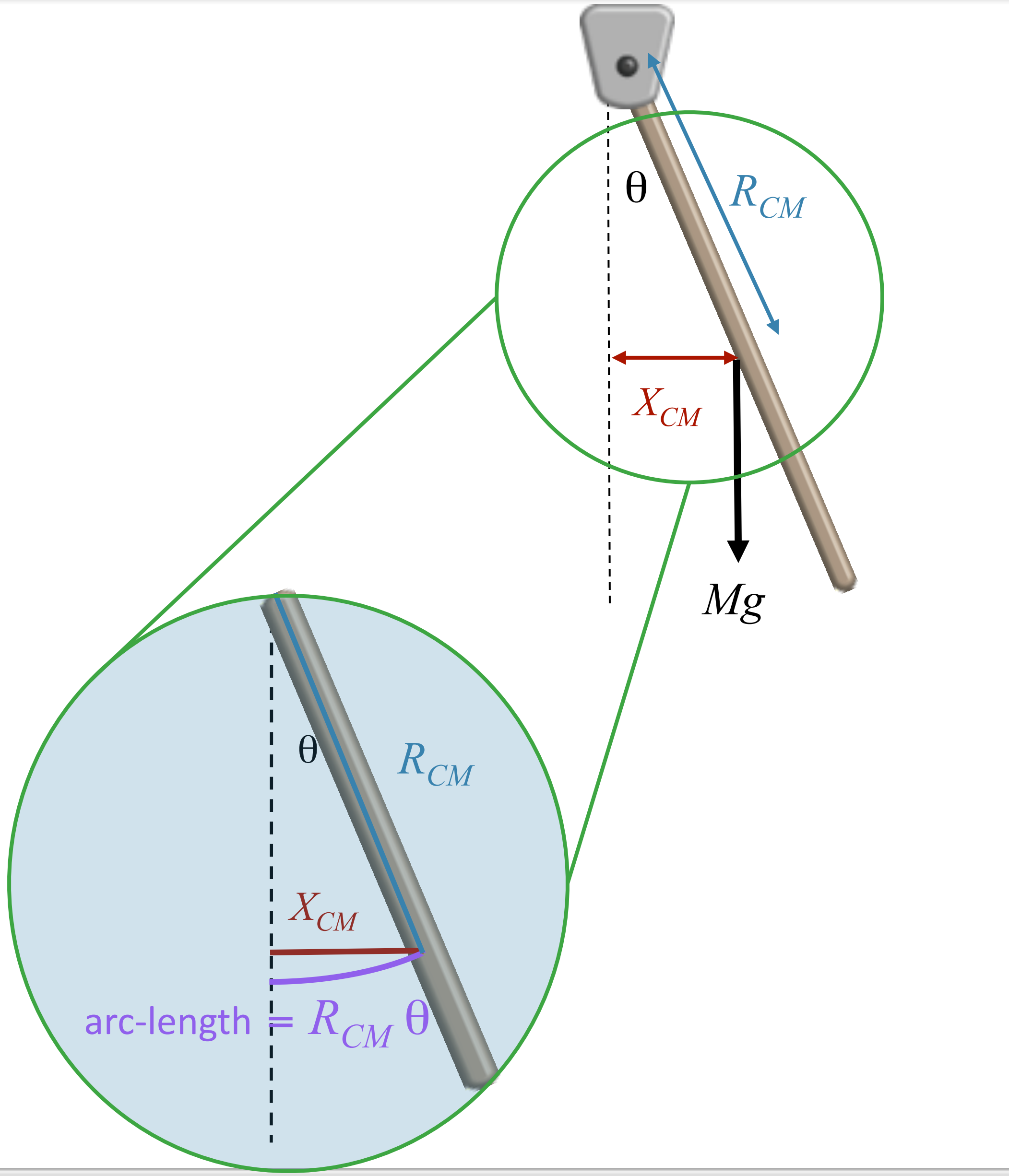
$$-MgX_{CM} \rightarrow -MgR_{CM}\theta$$

$$I\alpha \rightarrow I\frac{d^2\theta}{dt^2}$$

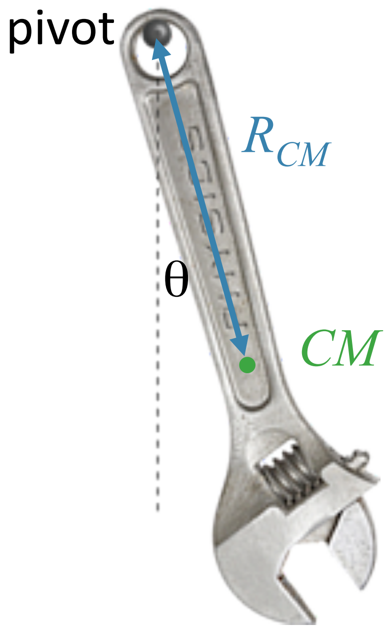
$$\frac{d^2\theta}{dt^2} = -\frac{MgR_{CM}}{I}\theta$$

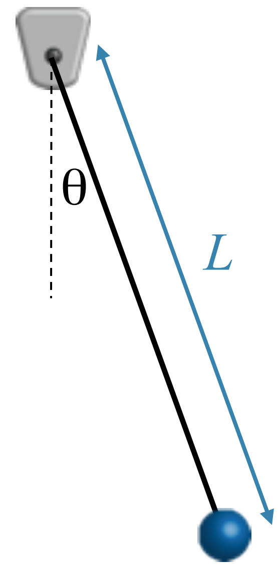
$$\rightarrow \frac{d^2\theta}{dt^2} = -\omega^2\theta$$

$$\omega = \sqrt{\frac{MgR_{CM}}{I}}$$



The Simple Pendulum





The simple case

$$\omega = \sqrt{\frac{MgR_{CM}}{I}}$$

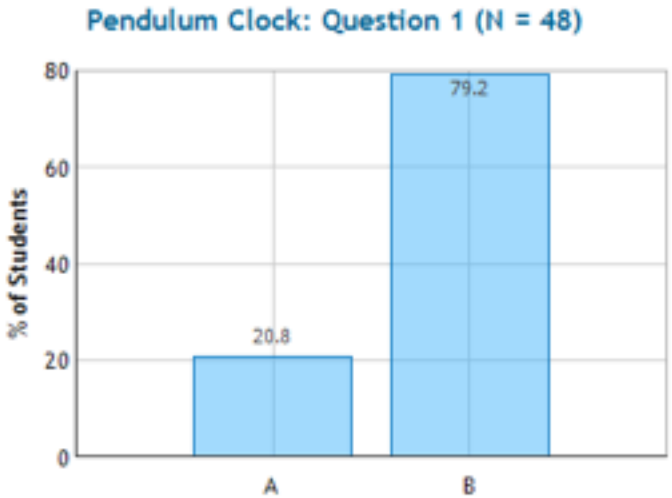
$$\omega = \sqrt{\frac{MgL}{ML^2}} = \sqrt{\frac{g}{L}}$$

The general case

CheckPoint

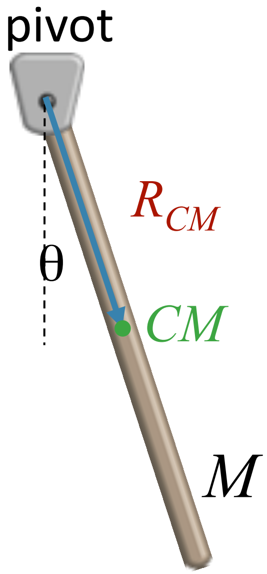
If the clock is running too fast, the weight needs to be moved A) Up B) Down

$$\omega = \sqrt{\frac{g}{L}}$$



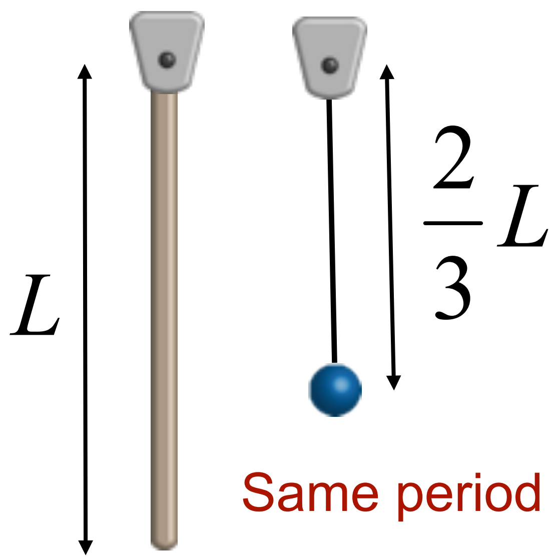
If the clock is running too fast then we want to reduce it's period, T, and to do that we need to increase omega, the frequency it moves with and to do that we need the position of the center of mass to be further from the pivot, which is achieved by moving the weight down.

The Stick Pendulum



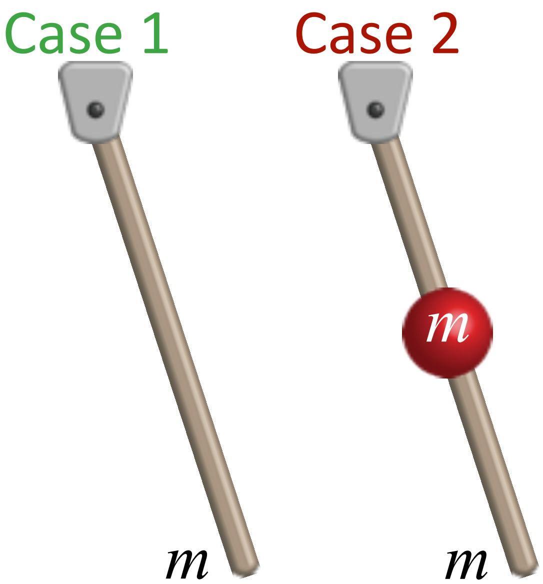
$$\omega = \sqrt{\frac{MgR_{CM}}{I}} \qquad R_{CM} \rightarrow \frac{L}{2} \qquad I \rightarrow \frac{1}{3}ML^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{\frac{2}{3}L}}$$



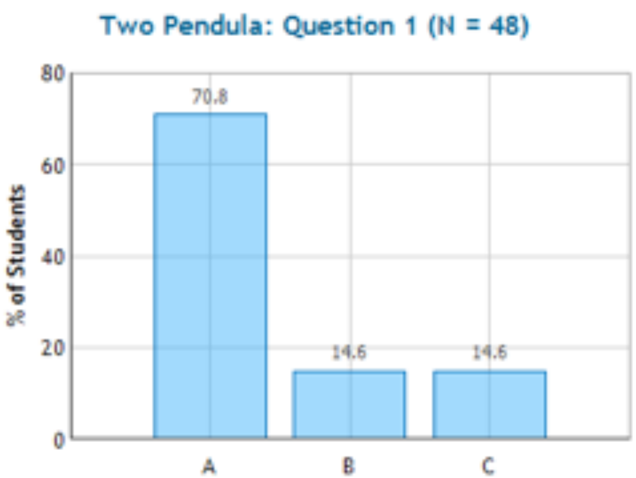
Same period

CheckPoint



In Case 1 a stick of mass m and length L is pivoted at one end and used as a pendulum. In Case 2 a point particle of mass m is attached to the center of the same stick. In which case is the period of the pendulum the longest?

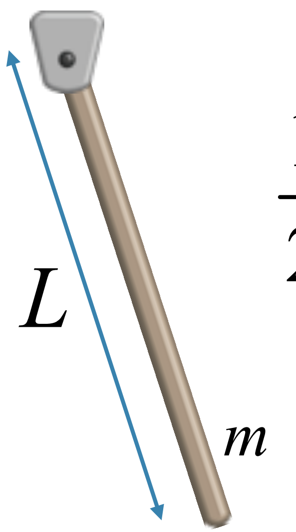
A) Case 1 B) Case 2 C) Same



C is not the right answer.

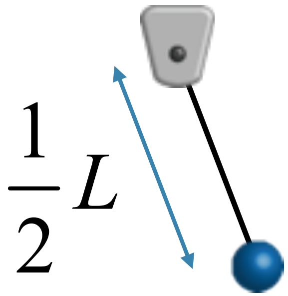
Lets work through it

Case 1



$$\omega = \sqrt{\frac{g}{\frac{2}{3}L}}$$

Case 2



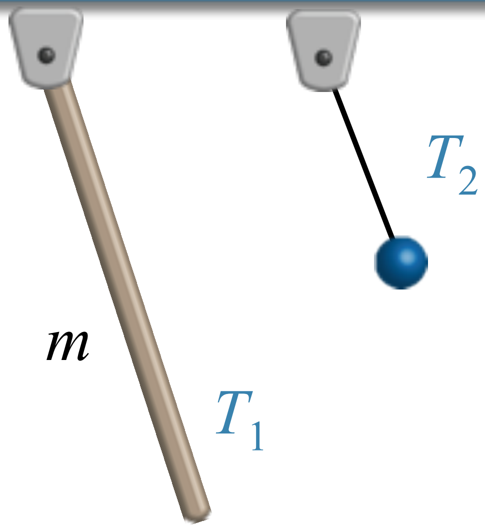
$$\omega = \sqrt{\frac{g}{\frac{1}{2}L}}$$

In Case 1 a stick of mass m and length L is pivoted at one end and used as a pendulum. In Case 2 a point particle of mass m is attached to a string of length $L/2$?

In which case is the period of the pendulum longest?

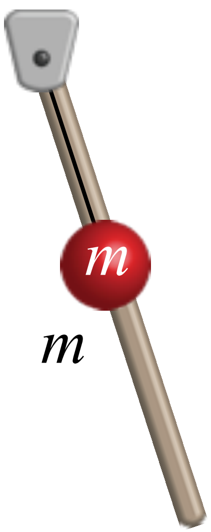
$\boxed{\text{A) Case 1}}$ B) Case 2 C) Same

$$T = 2\pi\sqrt{\frac{\frac{2}{3}L}{g}} \qquad T = 2\pi\sqrt{\frac{\frac{1}{2}L}{g}}$$



Suppose you start with 2 different pendula, one having period T_1 and the other having period T_2.

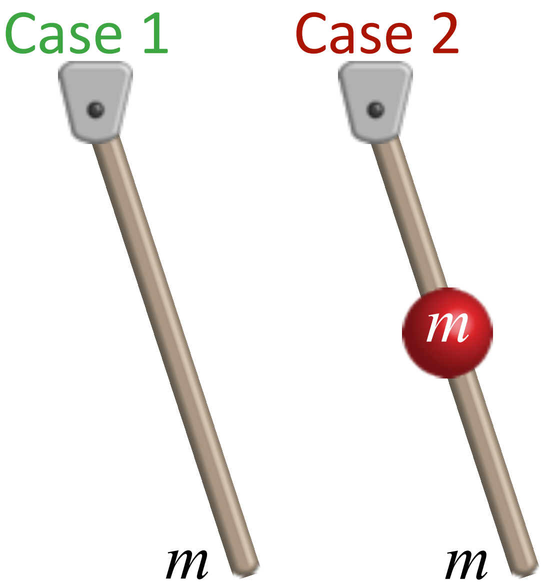
$T_1 > T_2$



Now suppose you make a new pendulum by hanging the first two from the same pivot and gluing them together.

What is the period of the new pendulum?

A) T_1 B) T_2 $\boxed{\text{C) In between}}$

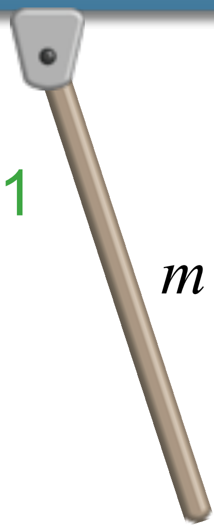


In Case 1 a stick of mass m and length L is pivoted at one end and used as a pendulum. In Case 2 a point particle of mass m is attached to the center of the same stick. In which case is the period of the pendulum the longest?

A) Case 1 B) Case 2 C) Same

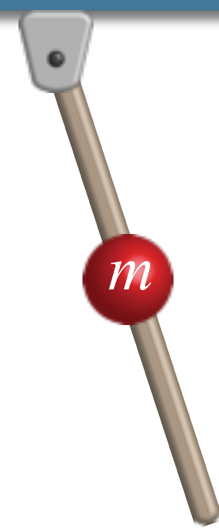
Now lets work through it in detail

Case 1



m

Case 2



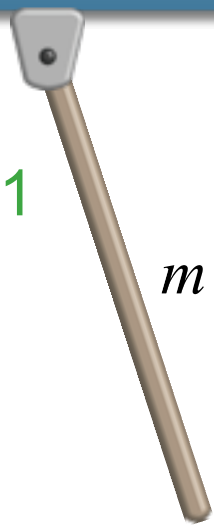
m

Lets compare $\omega = \sqrt{\dfrac{MgR_{CM}}{I}}$ for each case.

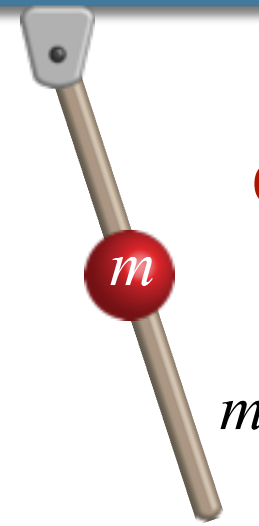
$$mg\frac{L}{2}$$

$$2mg\frac{L}{2}$$

Case 1



Case 2



Lets compare $\omega = \sqrt{\dfrac{MgR_{CM}}{I}}$ for each case.

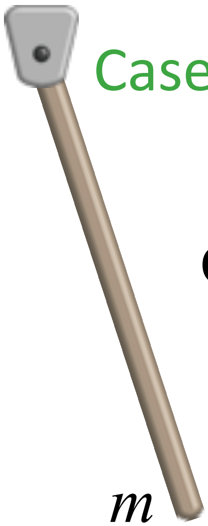
$$\frac{1}{3}mL^2$$

$$\frac{1}{3}mL^2 + mL^2 = \frac{4}{3}mL^2 \quad \text{(A)}$$

$$\frac{1}{3}mL^2 + \frac{1}{2}mL^2 = \frac{5}{6}mL^2 \quad \text{(B)}$$

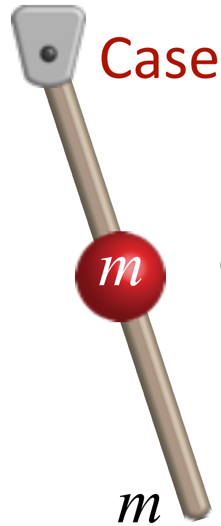
$$\frac{1}{3}mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{7}{12}mL^2 \quad \text{(C)}$$

So we can work out $\omega = \sqrt{\dfrac{MgR_{CM}}{I}}$



Case 1

$$\omega = \sqrt{\frac{g}{\frac{2}{3}L}}$$



Case 2

$$\omega = \sqrt{\frac{g}{\frac{7}{12}L}}$$

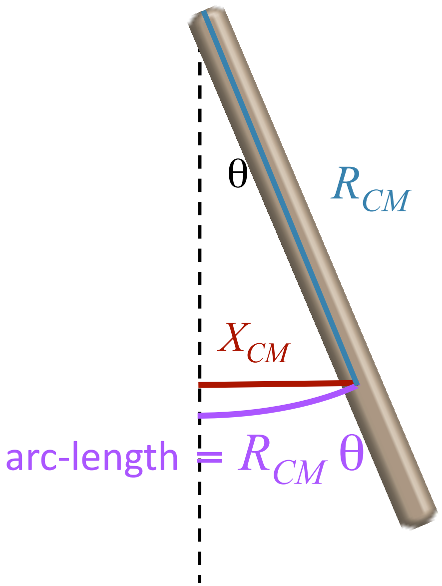
In which case is the period longest?

A) Case 1

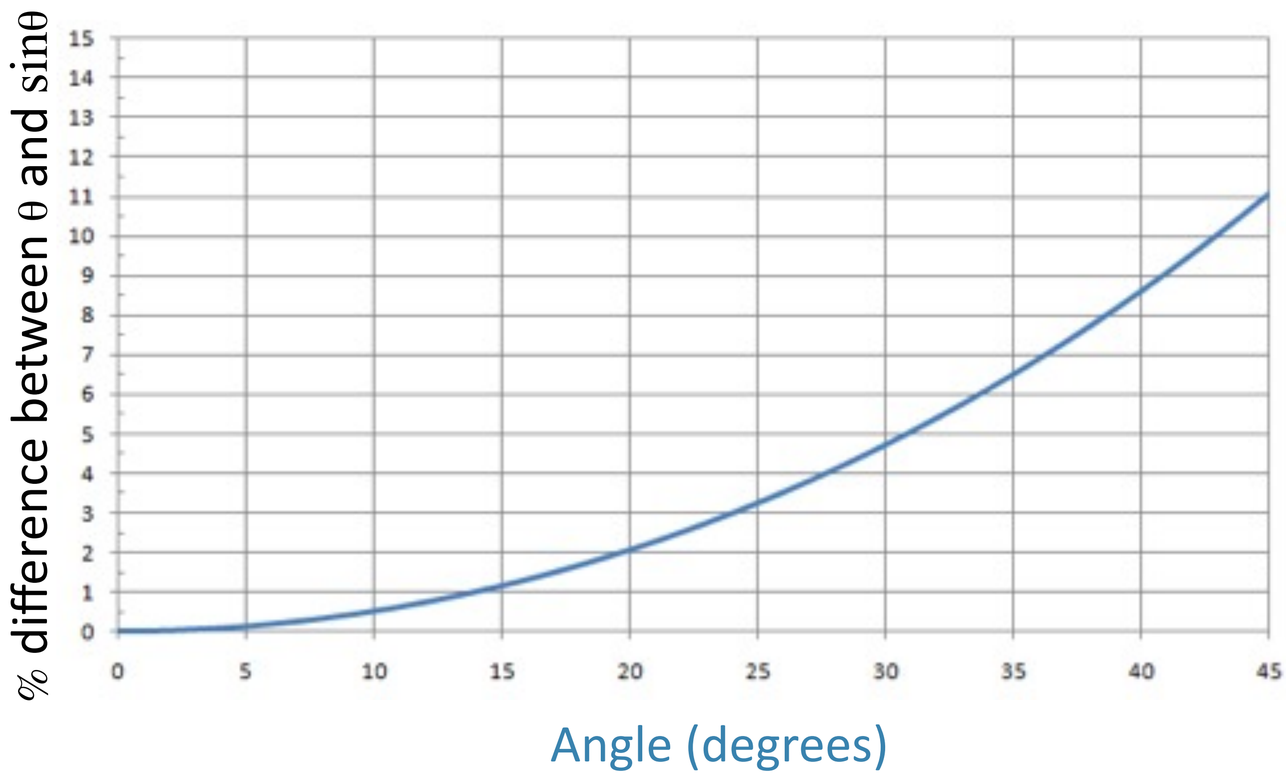
B) Case 2

C) They are the same

The Small Angle Approximation



$$\frac{d^2\theta}{dt^2} = -\omega^2 \sin\theta$$



$$\sin\theta = \theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 - \frac{1}{7!}\theta^7 + \ldots = \frac{1}{6}\theta^3 + \frac{1}{120}\theta^5 - \frac{1}{5040}\theta^7 + \ldots$$

Clicker Question

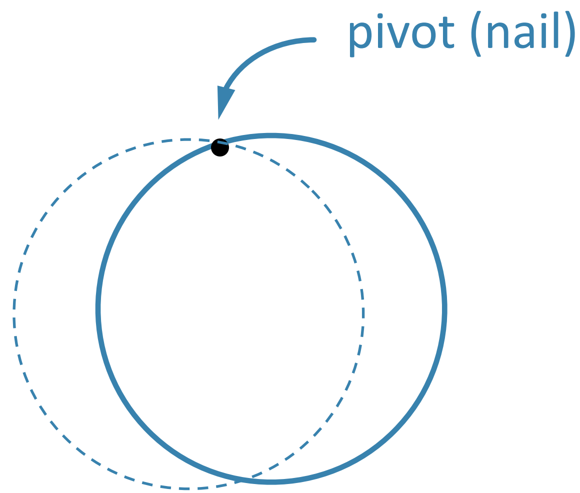
A pendulum is made by hanging a thin hoola-hoop of diameter D on a small nail.

What is the angular frequency of oscillation of the hoop for small displacements? ($I_{CM} = mR^2$ for a hoop)

A) $\boxed{\omega = \sqrt{\frac{g}{D}}}$

B) $\omega = \sqrt{\frac{2g}{D}}$

C) $\omega = \sqrt{\frac{g}{2D}}$



The angular frequency of oscillation of the hoop for small displacements will be given by $\omega = \sqrt{\dfrac{mgR_{CM}}{I}}$

Use parallel axis theorem: $I = I_{CM} + mR^2$

$$= mR^2 + mR^2 = 2mR^2$$

$$\omega = \sqrt{\frac{mgR}{2mR^2}} = \sqrt{\frac{g}{2R}} = \sqrt{\frac{g}{D}}$$

$$\boxed{\text{So } \omega = \sqrt{\frac{g}{D}}}$$