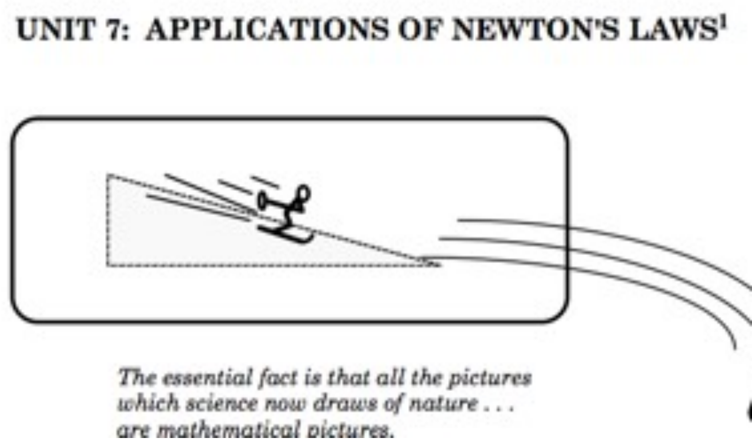
Classical Mechanics
Relative and Circular Motion


UNIT 7: APPLICATIONS OF NEWTON'S LAWS[1]

The essential fact is that all the pictures which science now draws of nature . . . are mathematical pictures.

Today's Concepts:

a) Relative motion

b) Centripetal acceleration

Stuff you asked about:

- I dislike Circular motion and I bet you cant make me like it :p
- I studied this in high school, but the expression of angular velocity is new to me, and I haven't learned about centripetal acceleration in terms of derivatives before
- Centripetal force? Centrifugal? Centra- what are all these words and what do they mean?
- Relative motion is still a really difficult concept for me, and it would be nice to have more practice with it.
- It was stated we will not use accelerated frames of motion, but a single example to demonstrate the difficulty of dealing in those frames would be interesting to see, if only once for my first semester in college.
- I do not 100% understand how you know that the acceleration vector in a case of centripetal acceleration is pointed towards the center of the circle of it's rotation.
- I find the voice in the lecture quite charismatic
- can we discuss the angular velocity? I don't really understand what the angle d theta represents, is it the horizontal factor?
- seems like a simple concept, but going over reference frames while you are moving and there is an object coming towards you would help.

Latin Lesson

Latin:

Centrum: Centre, center, middle

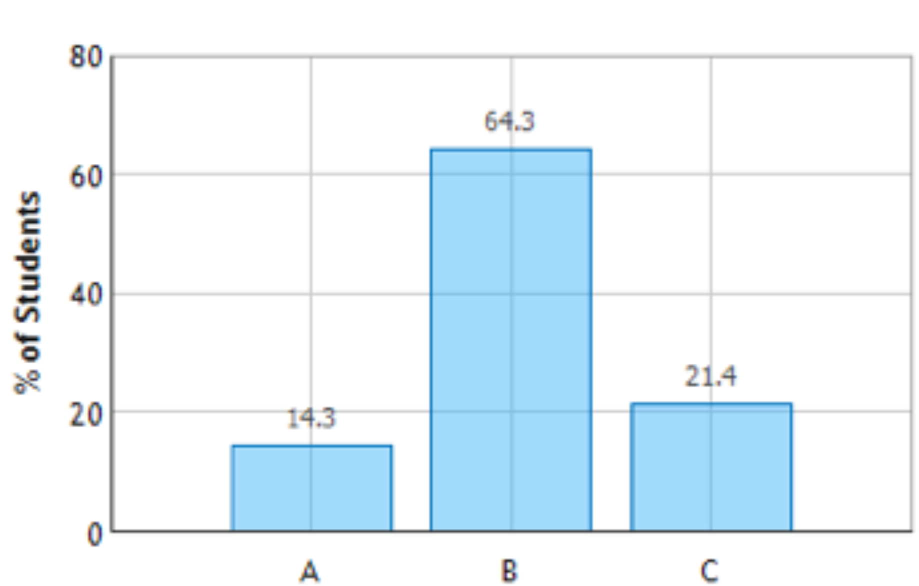
Fugare= To flee [French: fuir— to flee]

Petere= To aim at, to desire [French: péter— to explode]

How familiar are you with the concepts of relative motion and centripetal acceleration from your high school course.

A) I already know this stuff
B) It seems familiar, but I need a review
C) We didn't learn this in high school

58)



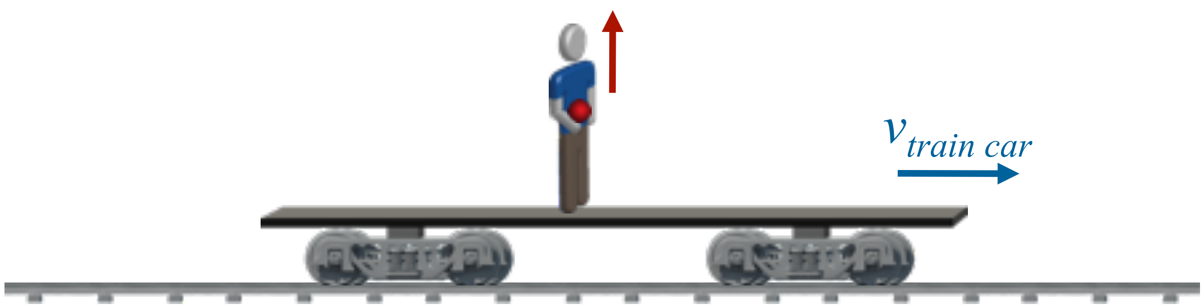
Train Demo Clicker Question

A flatbed railroad car is moving along a track at constant velocity. A passenger at the center of the car throws a ball straight up. Neglecting air resistance, where will the ball land?

A) Forward of the center of the car

B) At the center of the car ← correct

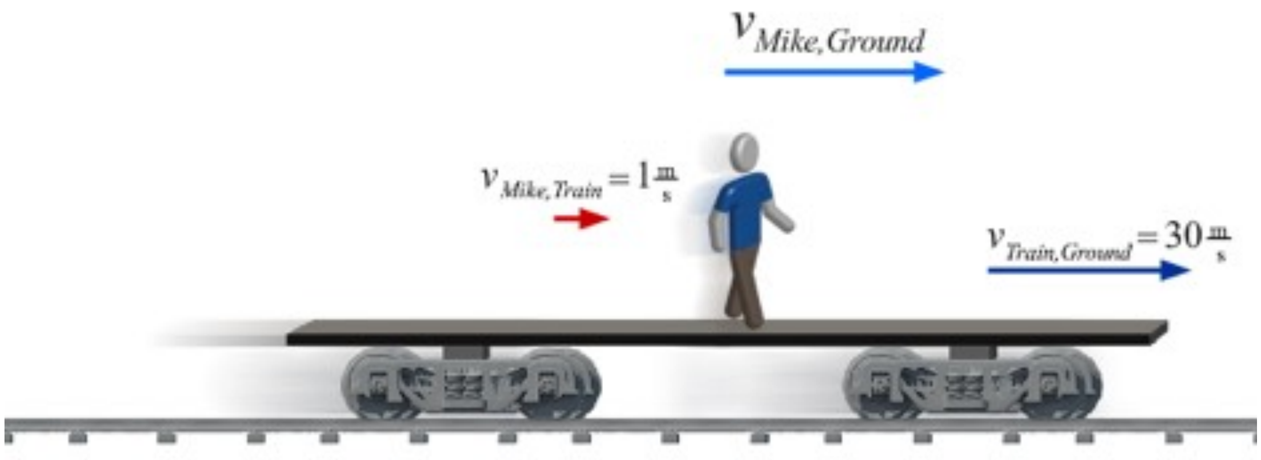
C) Backward of the center of the car



Ball and car start with same x position and x velocity,
Since $a = 0$ they always have same x position.

Demo - train

Relative Motion

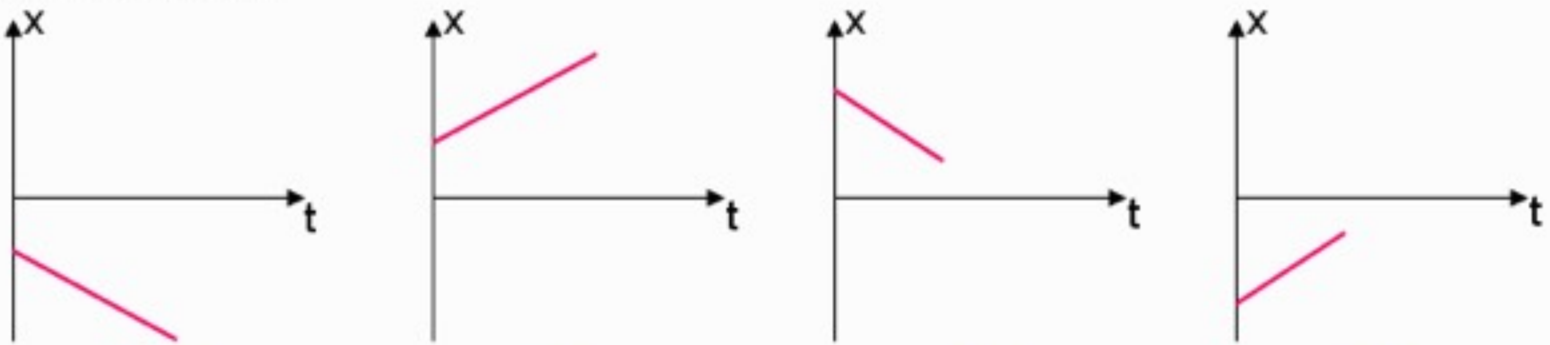


What you just did $\vec{v}_{ac} = \vec{v}_{ab} + \vec{v}_{bc}$

PreLecture 4, Question 1

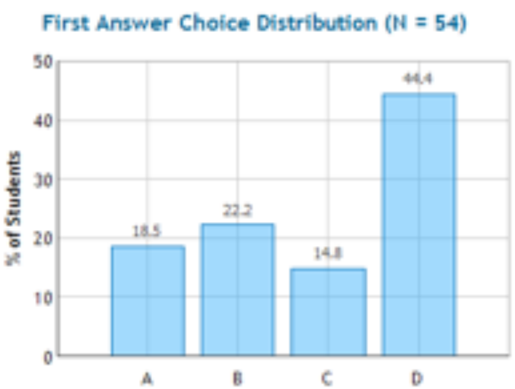
The diagram shows a snapshot at time t = 0 of two balls on a collision course. At t = 0, the balls are separated by a distance D = 12 m. The green ball moves with constant velocity v_G = 6 m/s due East, while the red ball moves with constant velocity v_R = 2 m/s due West.

Which of the following graphs correctly describes the motion of the green ball in the reference frame of the red ball? Take the origin to be the position of the red ball and the positive direction to be East.



(a) (b) (c) (d)

Submit



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Which of the following graphs correctly describes the motion of the green ball in the reference frame of the red ball? Take the origin to be the position of the red ball and the positive direction to be East.

Recall that the $$\vec{v}_{ac} = \vec{v}_{ab} + \vec{v}_{bc}$$

velocity of a with respect to c

=

velocity of a with respect to b + velocity of b with respect to c

so

velocity of green with respect to red

=

velocity of green with respect to Lab + velocity of Lab with respect to red.

=

velocity of green with respect to Lab – velocity of red with respect to Lab

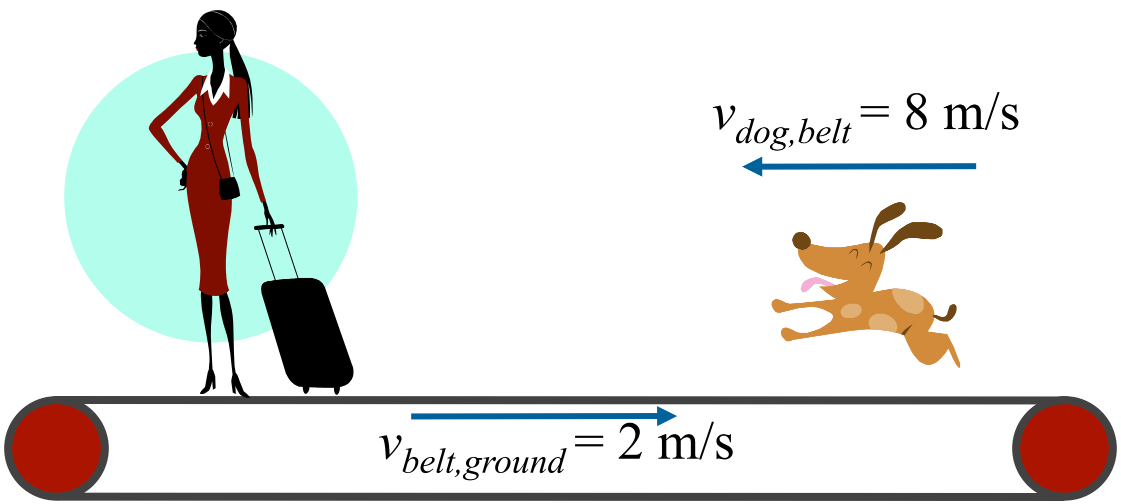
$$v_{gr} = v_{gL} + v_{Lr} = v_{gL} - v_{rL}$$

+ 6 m/s – (–2 m/s) = +8 m/s

CheckPoint

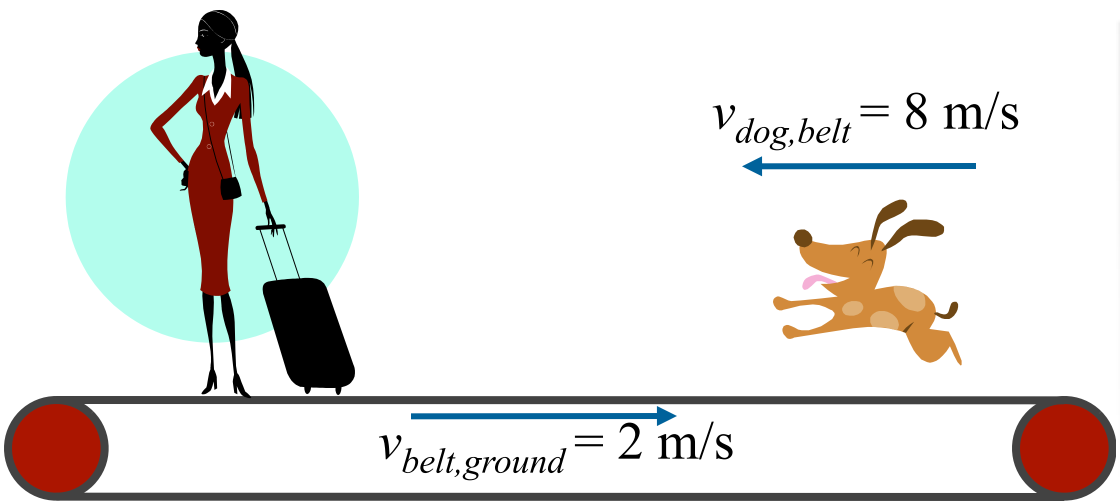
A girl stands on a moving sidewalk that moves to the right at 2 m/s relative to the ground. A dog runs toward the girl in the opposite direction along the sidewalk at a speed of 8 m/s relative to the sidewalk.

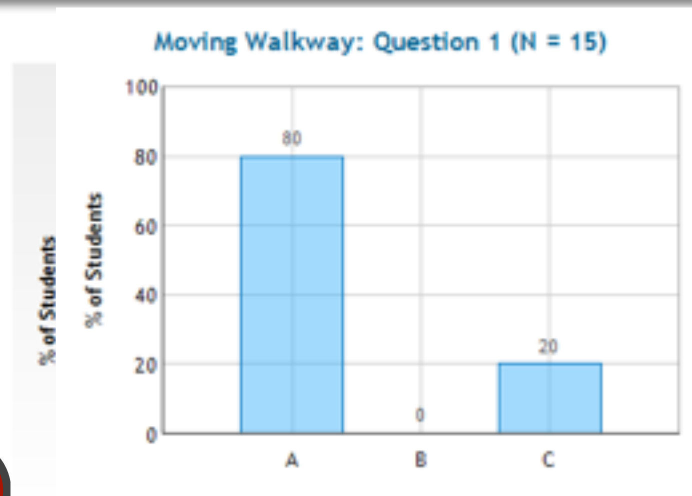
What is the speed of the dog relative to the ground?



A) 6 m/s   B) 8 m/s   C) 10 m/s

What is the speed of the dog relative to the ground?





A) 6 m/s

B) 8 m/s

C) 10 m/s

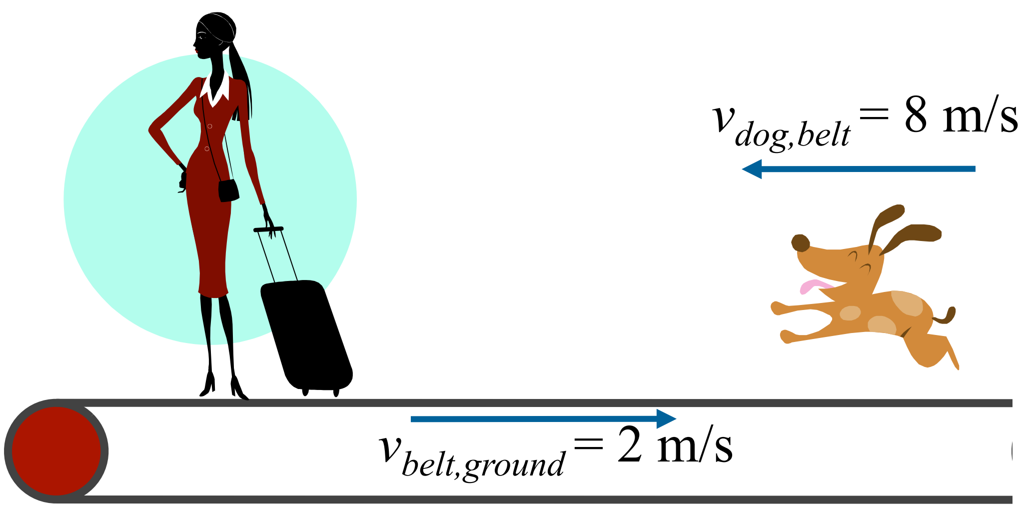
$\longrightarrow +x$

$$v_{dog,\ ground} = v_{dog,\ belt} + v_{belt,\ ground}$$
$$= (-8 \text{ m/s}) + (2 \text{ m/s}) \ = -6 \text{ m/s}$$

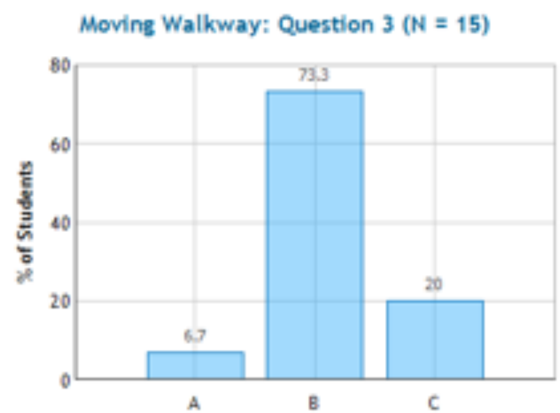
CheckPoint

A girl <u>stands</u> on a moving sidewalk that moves to the right at 2 m/s relative to the ground. A dog runs toward the girl in the opposite direction along the sidewalk at a speed of 8 m/s relative to the sidewalk.

What is the speed of the dog relative to the <u>girl</u>?

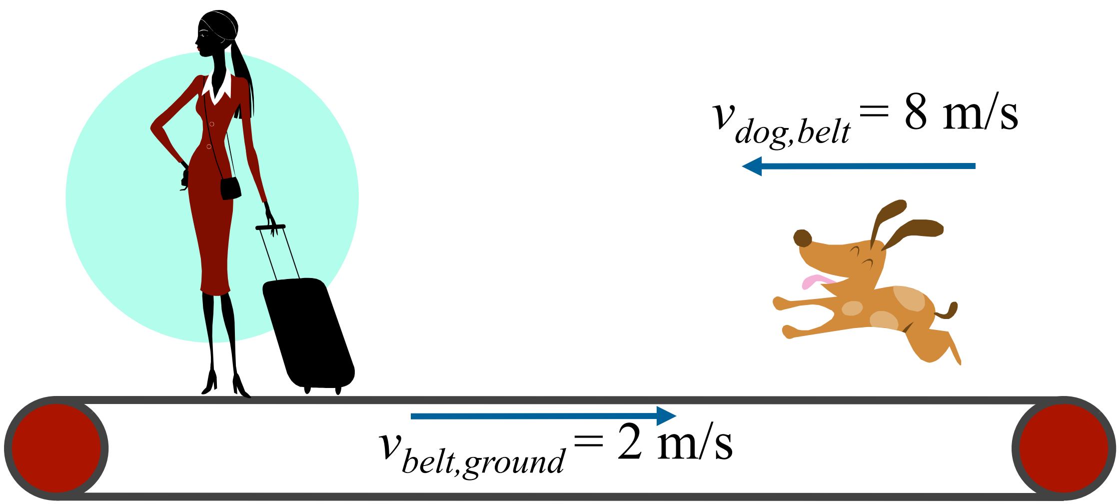


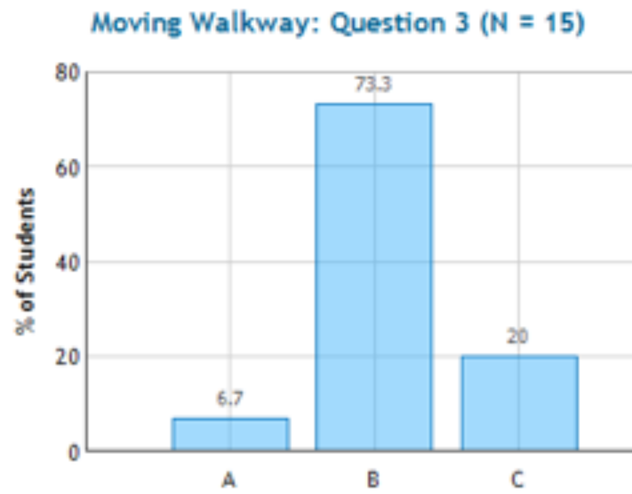
A) 6 m/s B) 8 m/s C) 10 m



Most of you got this.

What is the speed of the dog relative to the girl?





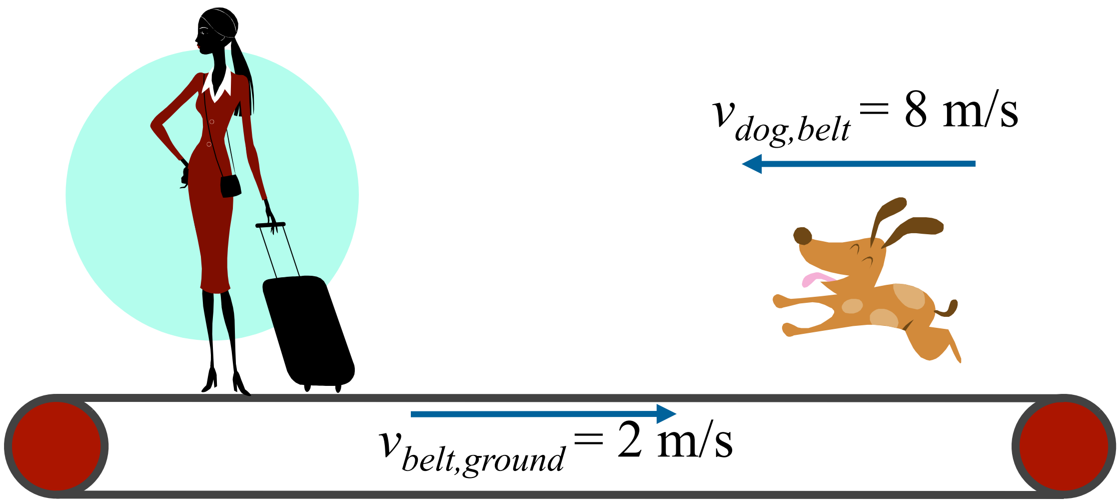
A) 6 m/s B) 8 m/s C) 10 m/s

A) Because the girl is actually moving and the two vectors are opposite, so together they make 6 m/s

B) Because the girl is not moving relative to the belt, and the dog is going 8 m/s relative to the belt, the dog is also moving 8 m/s relative to the girl..

C) The dog and girl are running towards each other so when you add the two velocities together it would be 8+2.

What is the speed of the dog relative to the girl?



A) 6 m/s B) 8 m/s C) 10 m/s

B) Because the girl is not moving relative to the belt, and the dog is going 8 m/s relative to the belt, the dog is also moving 8 m/s relative to the girl.

Using the velocity formula:

$$\begin{aligned} v_{dog,\ girl} &= v_{dog,\ belt} + v_{belt,\ girl} \\ &= -8 \text{ m/s} + 0 \text{ m/s} \\ &= -8 \text{ m/s} \end{aligned}$$

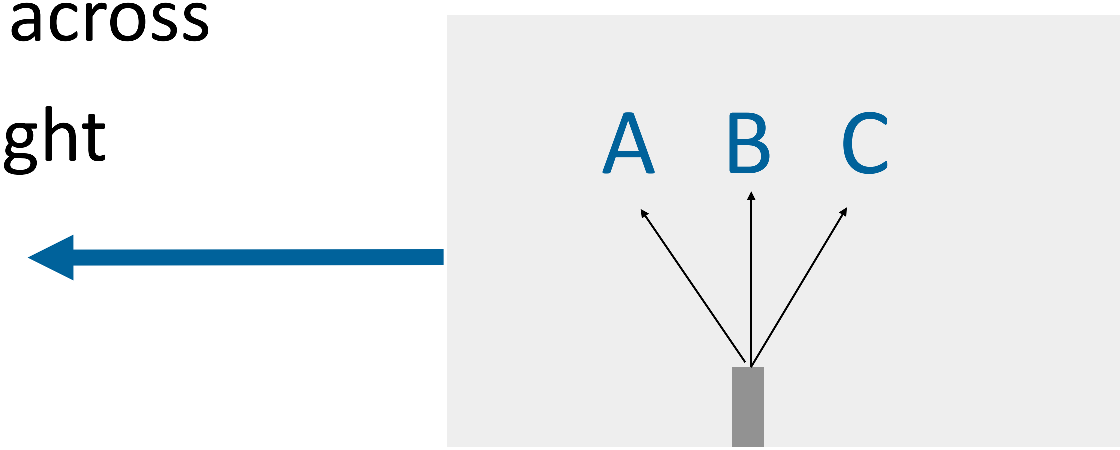
Tractor Demo (moving cardboard)

Which direction should I point the toy bulldozer to get it across the cardboard fastest?

A) To the left

B) Straight across

C) To the right



Clicker Question

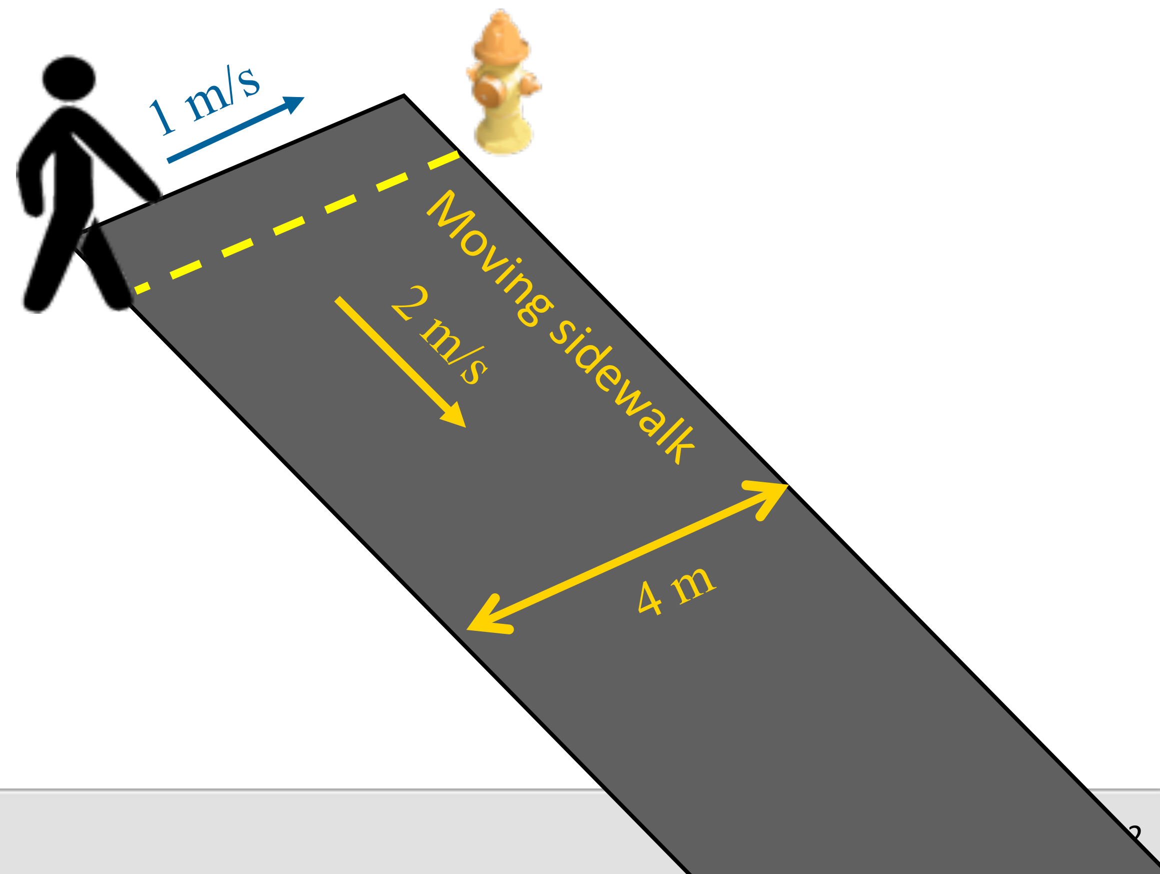
A man starts to walk along the dotted line painted on a moving sidewalk toward a fire hydrant that is directly across from him. The width of the walkway is 4 m, and it is moving at 2 m/s relative to the fire-hydrant. If his walking speed is 1 m/s, how far away will he be from the hydrant when he reaches the other side?

A) 2 m

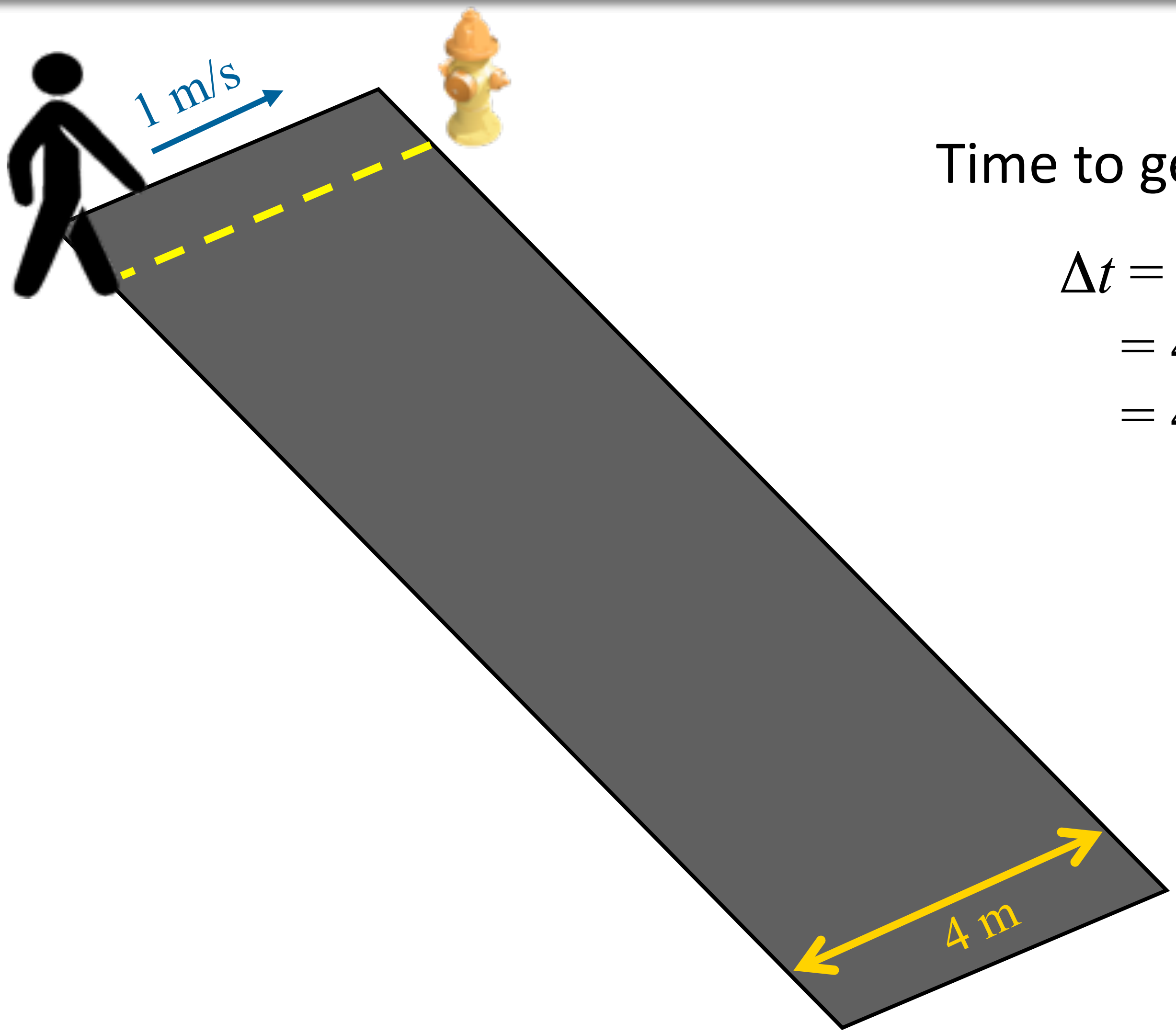
B) 4 m

C) 6 m

D) 8 m



If the sidewalk wasn't moving:

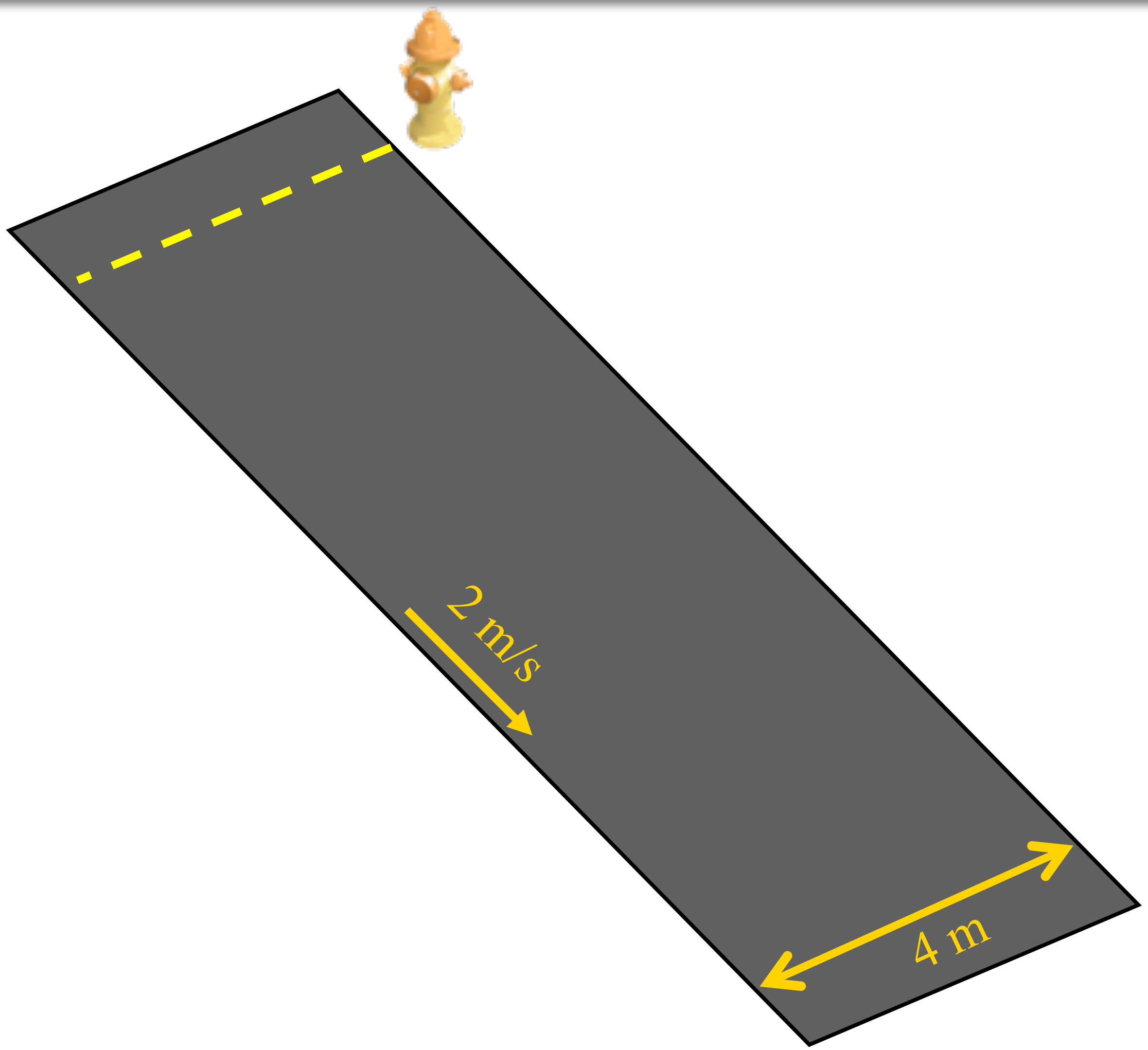


$\vec{v}_{man,sidewalk}$

Time to get across:

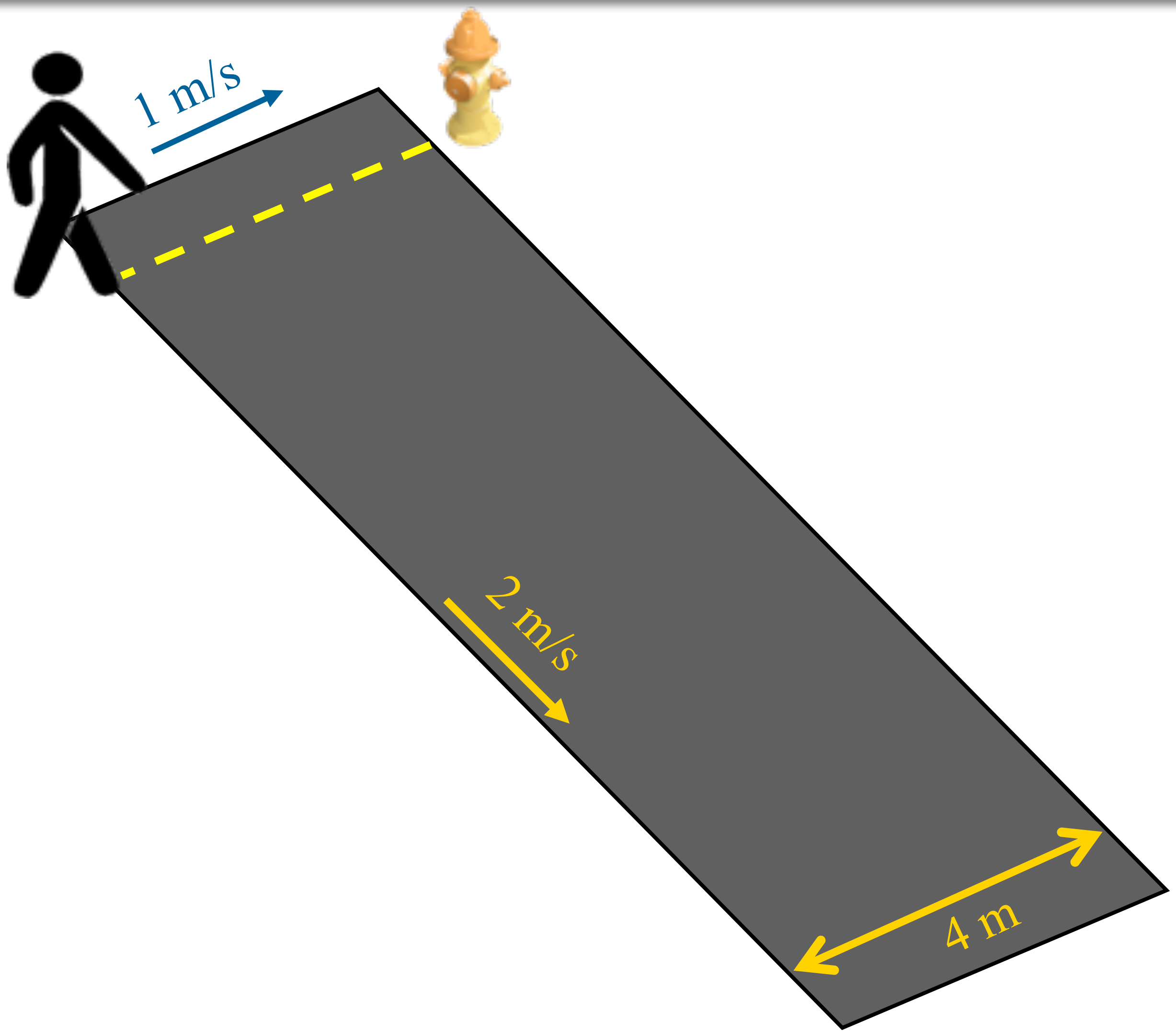
$$\Delta t = \text{distance / speed}$$
$$= 4\text{m} / 1\text{m/s}$$
$$= 4 \text{ s}$$

Just the sidewalk:

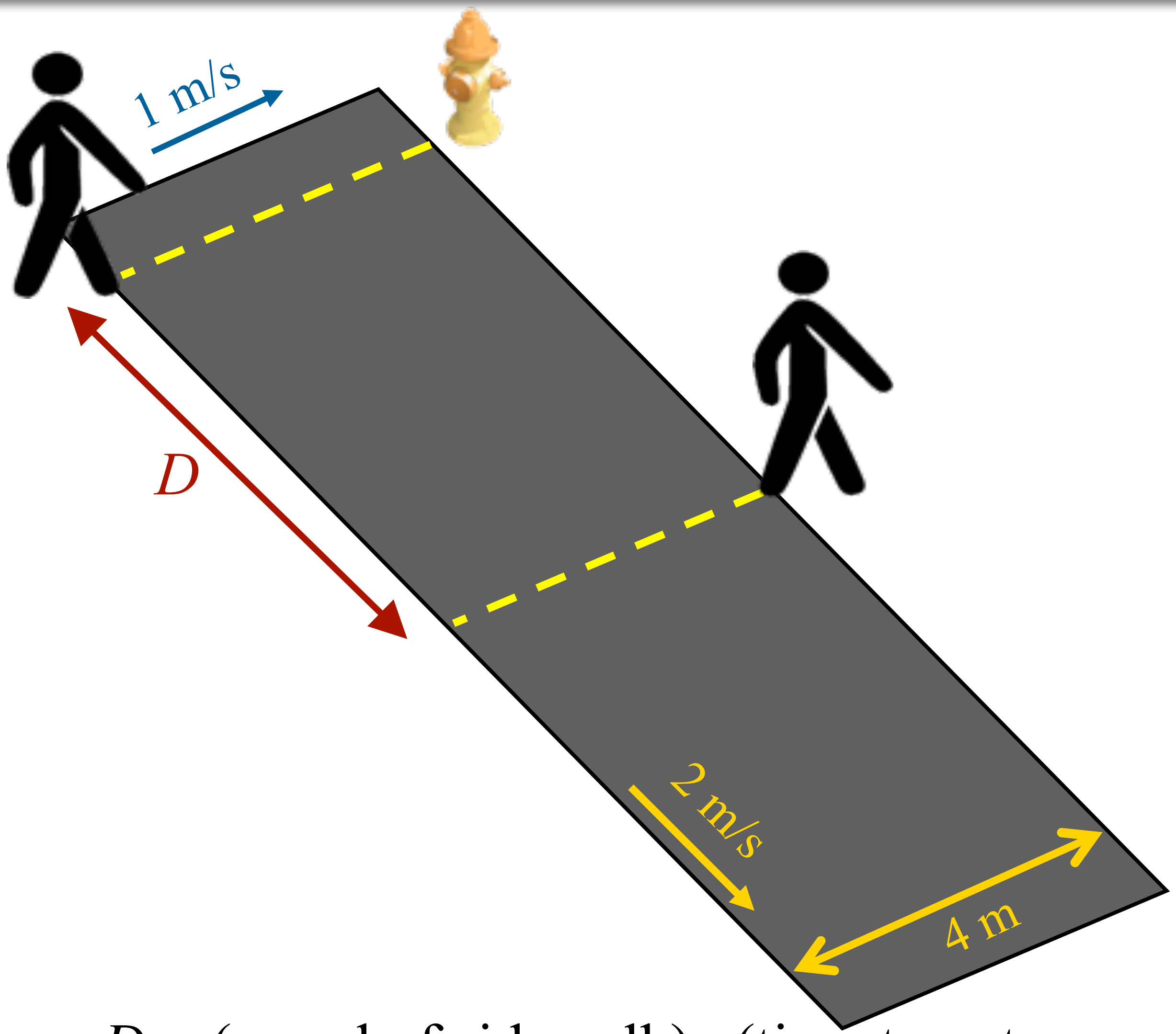


$$\vec{v}_{sidewalk,hydrant}$$

Combination of motions:



$$\vec{v}_{sidewalk,hydrant} = \vec{v}_{man,sidwalk} + \vec{v}_{sidewalk,hydrant}$$



D = (speed of sidewalk) · (time to get across)

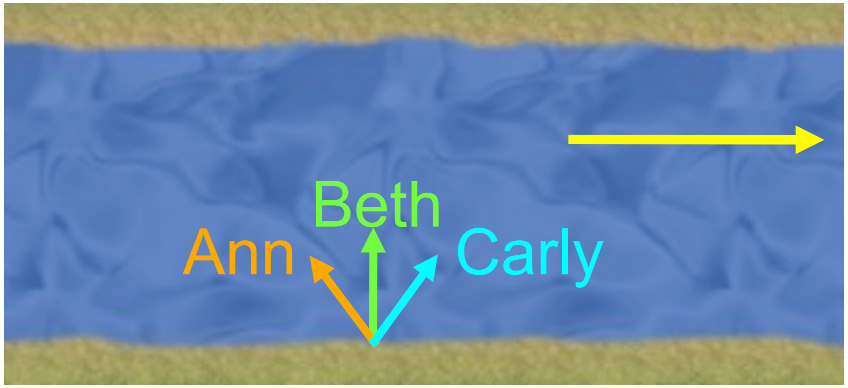
= (2 m/s) · (4 s) = 8 m

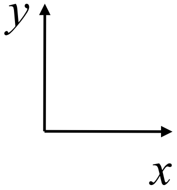
Clicker Question

Three swimmers can swim equally fast relative to the water. They have a race to see who can swim across a river in the least time. Relative to the water, Beth swims perpendicular to the flow, Ann swims upstream at 30 degrees, and Carly swims downstream at 30 degrees.

Who gets across the river first?

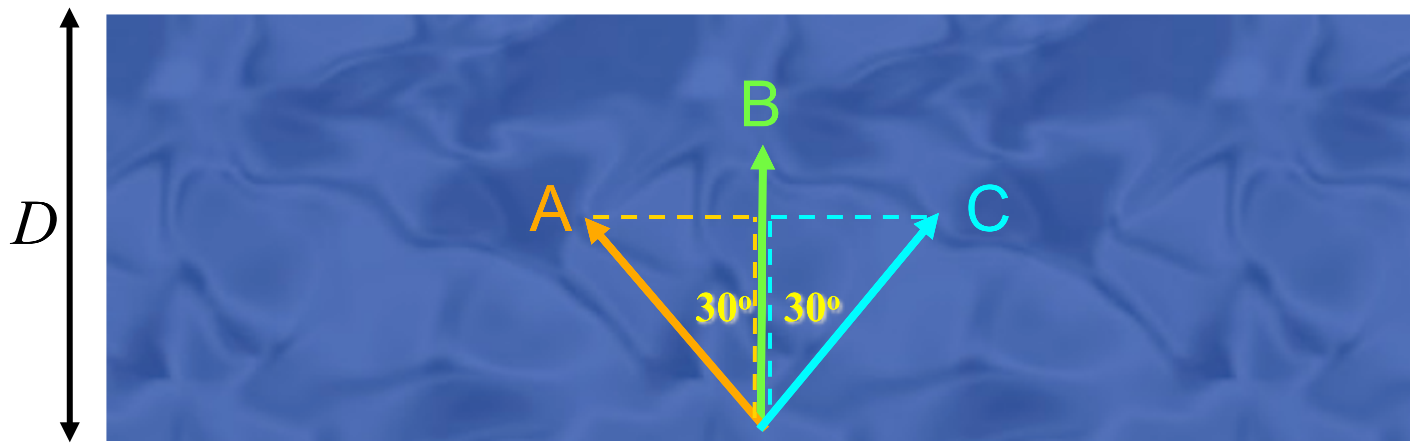
A) Ann   B) Beth   C) Carly

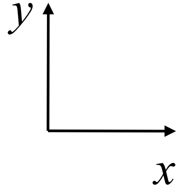




Look at just water & swimmers

Time to get across = D / V_y





$$V_{y,Beth} = V_o$$

$$V_{y,Ann} = V_o \cos 30^o$$

$$V_{y,Carly} = V_o \cos 30^o$$

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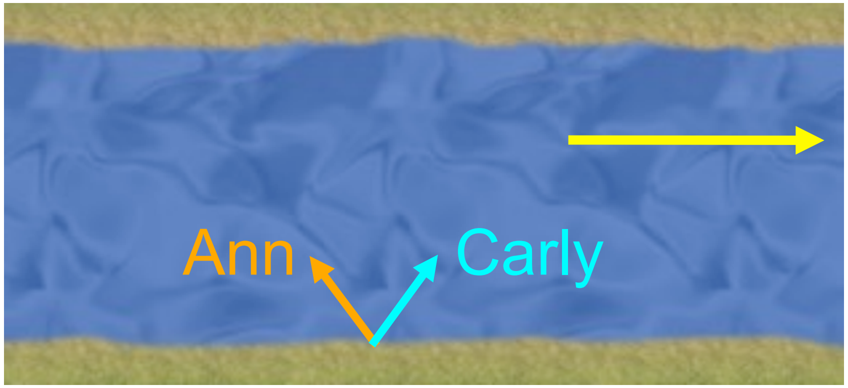
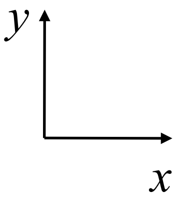
Who gets across the river second?

A) Ann   B) Carly   C) Both same

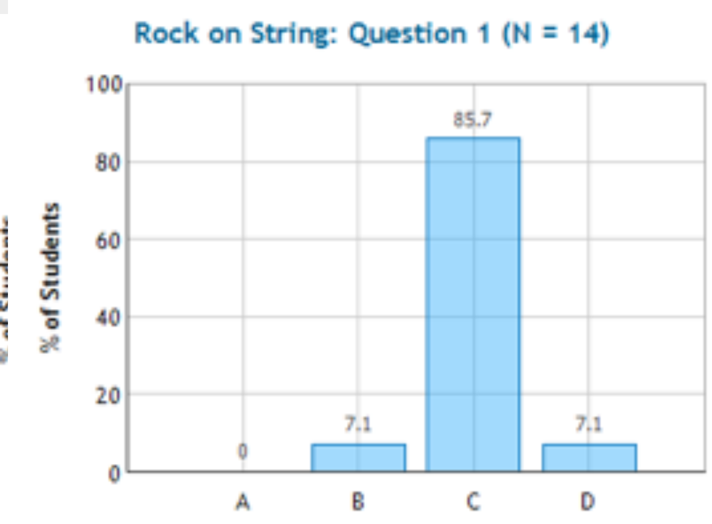


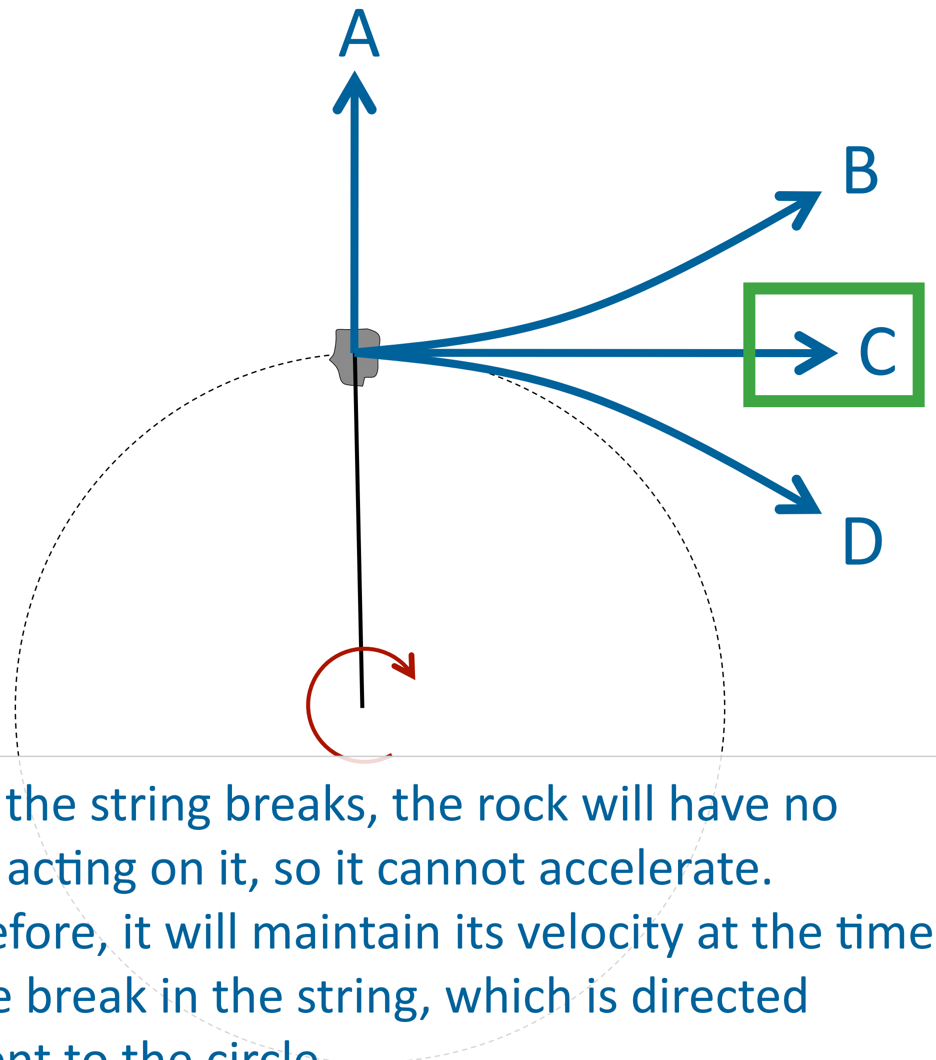


CheckPoint (& Demo)

A girl twirls a rock on the end of a string around in a horizontal circle above her head as shown from above in the diagram.

If the string brakes at the instant shown, which of the arrows best represents the resulting path of the rock?

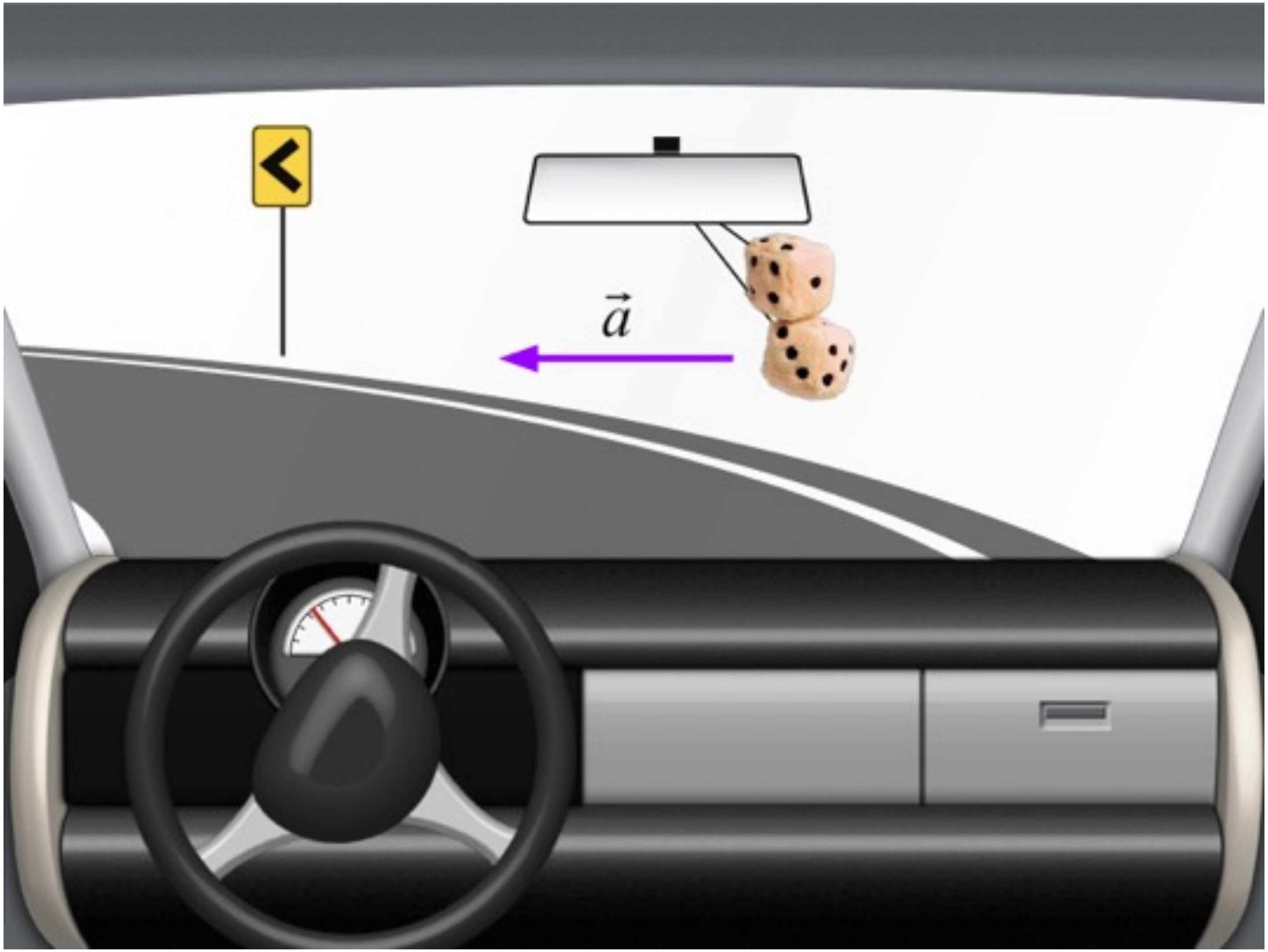




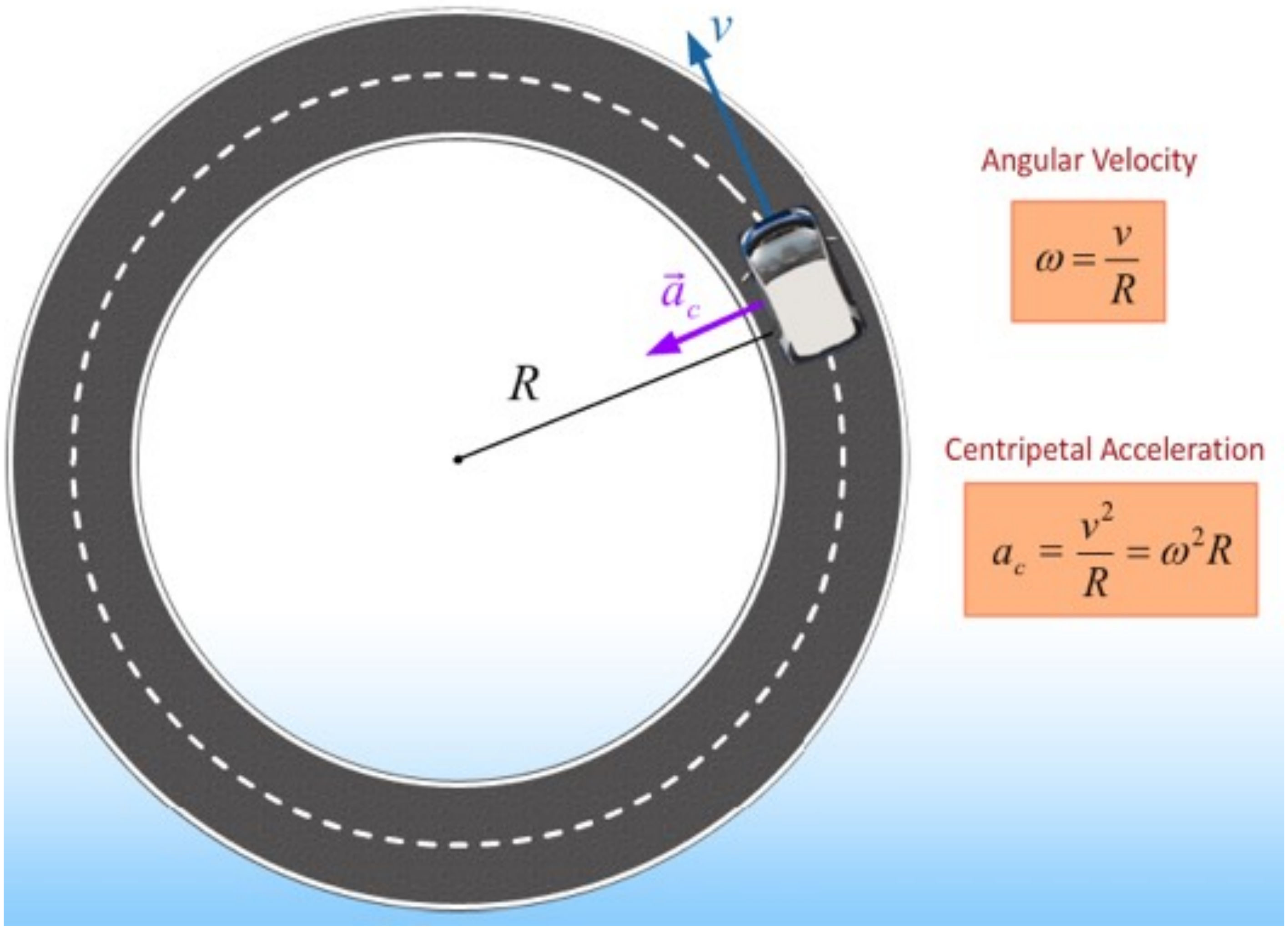
After the string breaks, the rock will have no force acting on it, so it cannot accelerate. Therefore, it will maintain its velocity at the time of the break in the string, which is directed tangent to the circle.

Top view looking down

Show Prelecture

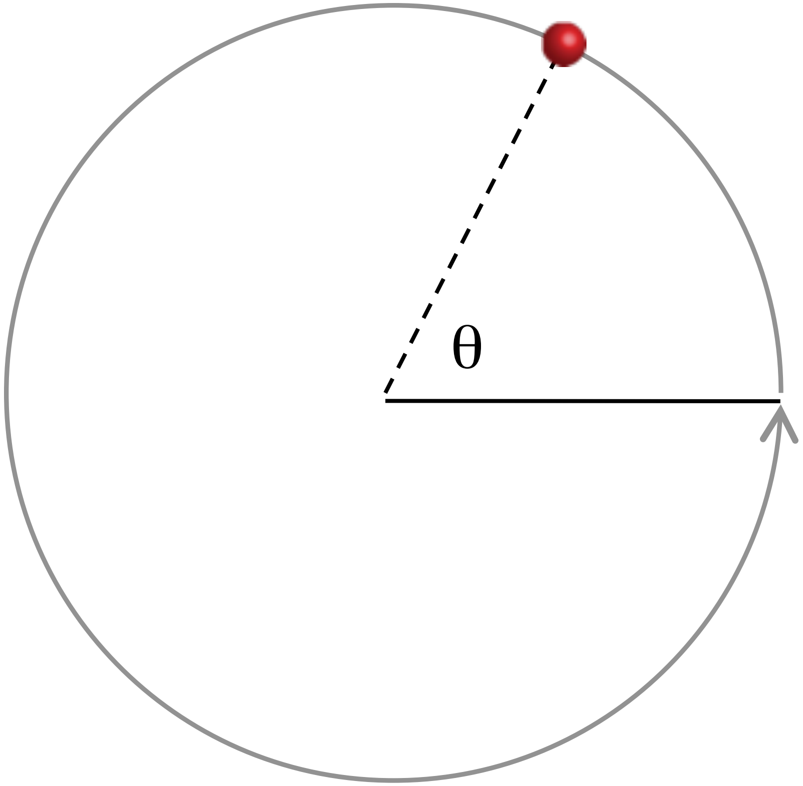


Centripetal Acceleration



$$v = \omega R$$

ω is the rate at which the angle θ changes:



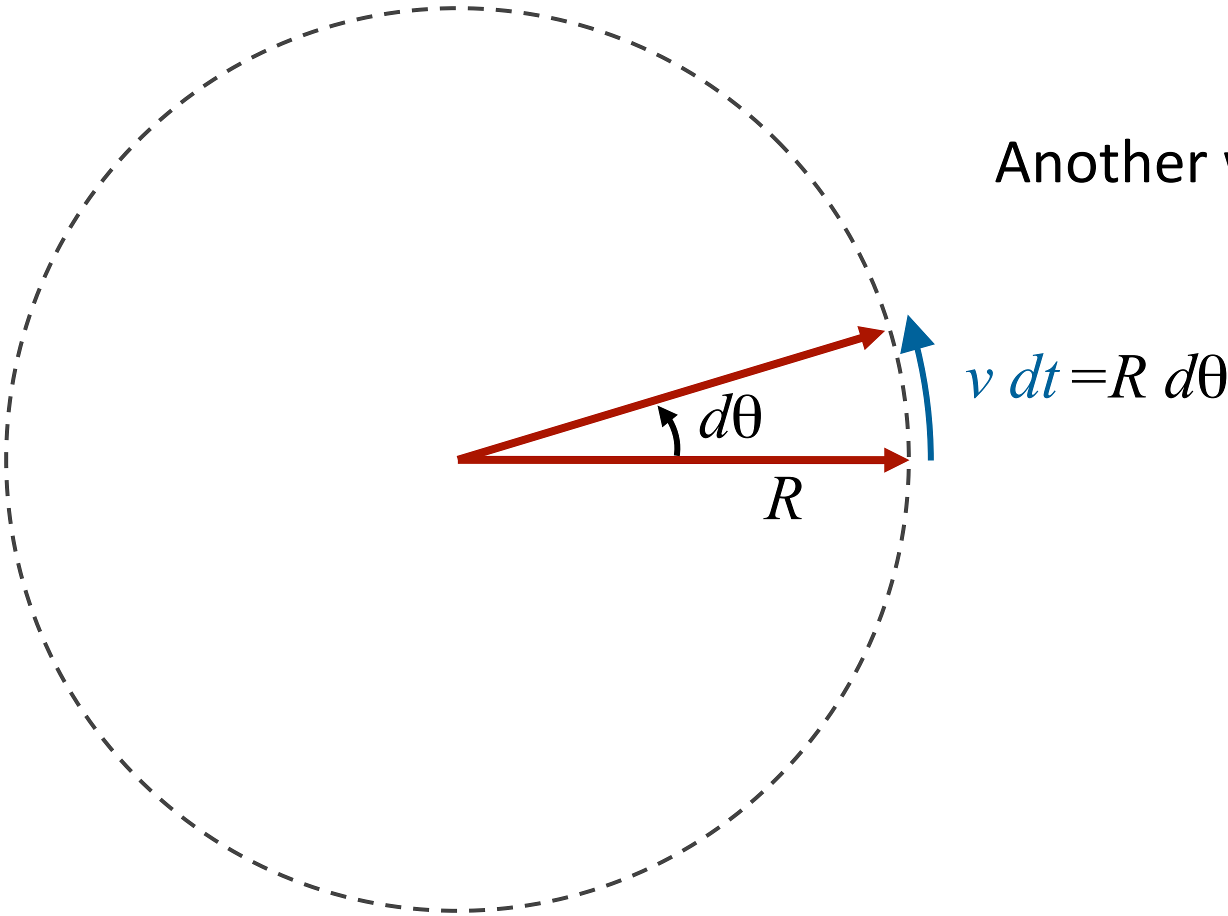
$$\Rightarrow \quad \omega = \frac{d\theta}{dt}$$

Once around:

$$v = \Delta x / \Delta t = 2\pi R / T$$

$$\omega = \Delta\theta / \Delta t = 2\pi / T$$

$$v = \omega R$$

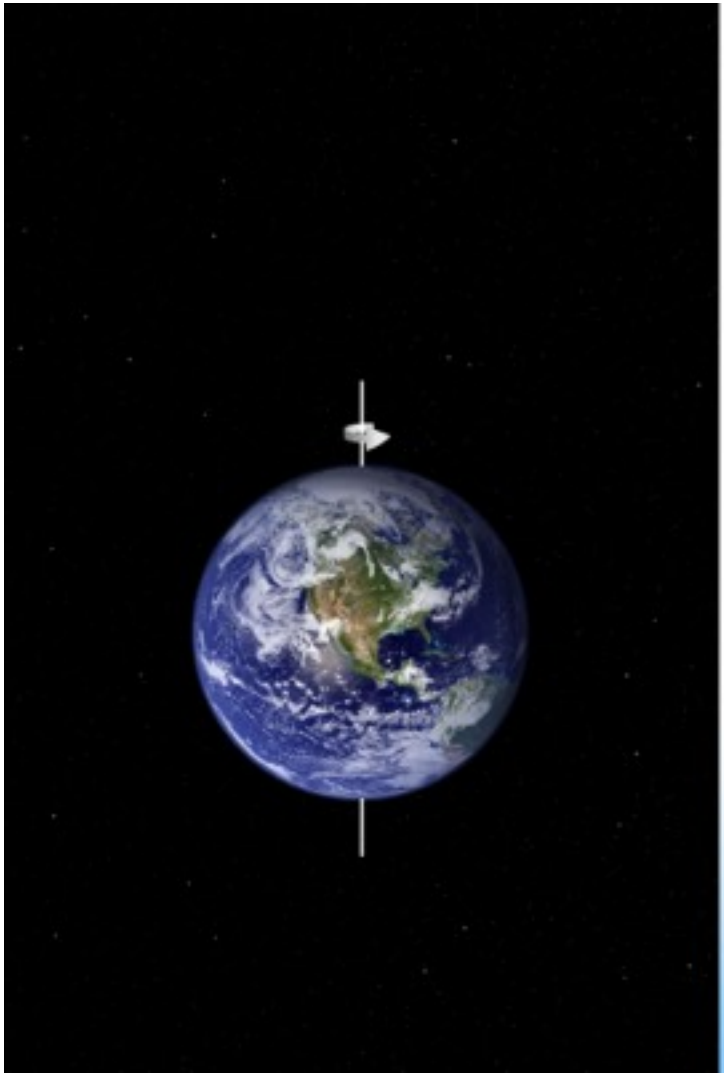


Another way to see it:

$$v = R\frac{d\theta}{dt}$$

$$v = R\omega$$

We can ignore this acceleration due to Earth's rotation since its small



$$v_{equator} = 465\,\tfrac{\text{m}}{\text{s}} \approx 1700\ \text{km/h}$$

$$a_{equator} = \frac{v_{equator}^2}{R_{Earth}}$$

$$a_{equator} = \frac{\left(465\,\tfrac{\text{m}}{\text{s}}\right)^2}{6.4\times10^6\ \text{m}}$$

$$a_{equator} = 3.3\times10^{-2}\,\tfrac{\text{m}}{\text{s}^2}$$