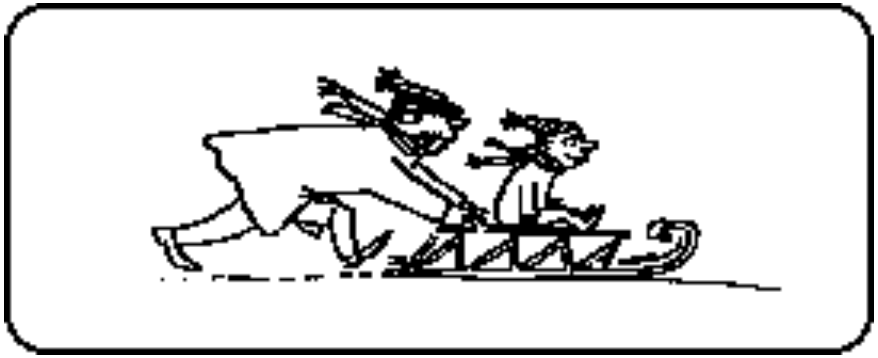
Classical Mechanics
Lecture 7

UNIT 10: WORK AND ENERGY



"Knowing is not enough; we must apply. Willing is not enough; we must do."

Bruce Lee

Today's Concepts:

Work & Kinetic Energy

Karate

Will not do Session 3 of Unit 10.

It is a Karate thing.

You can do it as a “project” in Unit 14 instead of the assigned project on the pendulum.

Stuff you asked about:

WHAT IS GOING ON WITH ALL THE INTEGRALS?????? HELP :(

Why is it necessary to use integrals in the equation for work?

everything. everything is difficult

Everything is fine except the work done by Gravity far from earth

The slides had a lot of information that was confusing and it flew by. I didn't have time to understand what was going on.

The dot product is ocnfusing

Pretty much everything. This prelecture sort of went over my head. I'll definitely have to watch it again.

The concept of positive and negative work is very confusing. please explain

Uh oh. Hotdog.

I hope you have enough funny quotes to fit into your slides.

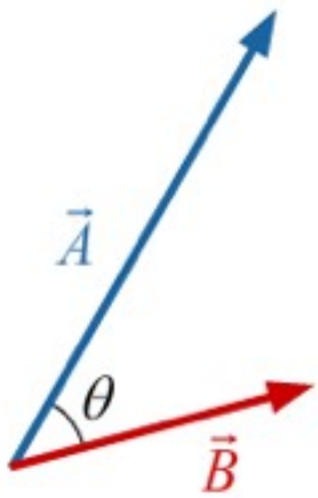
Stuff you asked about:

Does work also define a change in potential energy? For the last checkpoint question, the potential energy is changing as the car moves up the slope. Would this effect the quantity of work?

Can we go over and work out some actual problems using dot product and the equations used in the prelecture next class? Not just concept questions but ones that would be similar to our homework questions as well.

Can we go over Question 1 in the prelecture? I think I'm missing something in the question or something, but it doesn't settle well with me.

The Dot Product



Dot Product

$$\vec{A} \cdot \vec{B} \equiv AB\cos\theta$$

$\vec{A} \cdot \vec{B} = AB$ (Parallel Vectors)

$\vec{A} \cdot \vec{B} = 0$ (Perpendicular Vectors)

$\vec{A} \cdot \vec{B} = -AB$ (Anti-Parallel Vectors)

Dot Product using Unit Vectors

$$\hat{\imath} \cdot \hat{\imath} = 1$$

$$\hat{\jmath} \cdot \hat{\jmath} = 1$$

$$\hat{k} \cdot \hat{k} = 1$$

$$\hat{\imath} \cdot \hat{\jmath} = 0$$

$$\hat{\jmath} \cdot \hat{k} = 0$$

$$\hat{k} \cdot \hat{\jmath} = 0$$

etc.

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

Dot Product in Rectangular Coordinates

$$\vec{A} \cdot \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \cdot (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$= (A_xB_x\hat{\imath} \cdot \hat{\imath} + A_xB_y\hat{\imath} \cdot \hat{\jmath} + \cdots + A_zB_y\hat{k} \cdot \hat{\jmath} + A_zB_z\hat{k} \cdot \hat{k})$$

$$= (A_xB_x \times 1 + \cancel{A_xB_y \times 0} + \cdots + \cancel{A_zB_y \times 0} + A_zB_z \times 1)$$

$$\vec{A} \cdot \vec{B} = (A_xB_x + A_yB_y + A_zB_z)$$

Finding the angle between vectors

If we know the xyz components of two vectors we can find the angle between them:

$$\vec{A} \cdot \vec{B} = (A_x B_x + A_y B_y + A_z B_z)$$

$$= |\vec{A}||\vec{B}| \cos\theta$$

therefore

$$\theta = \arccos\left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}\right)$$

Example

$$\vec{A} = 3\hat{\imath} + 4\hat{\jmath} \qquad |\vec{A}| = |\vec{B}| = 5$$

$$\vec{B} = 4\hat{\imath} + 3\hat{\jmath} \qquad \vec{A}\cdot\vec{B} = 3\cdot 4 + 4\cdot 3 = 24$$

$$\theta = \arccos\left(\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right)$$

$$\theta = \arccos\left(\frac{24}{25}\right) = \arccos 0.96$$

$$= 0.28 \text{ rad} = 16^\circ$$

Work-Kinetic Energy Theorem

The work done by the net force F as it acts on an object that moves between positions r_1 and r_2 is equal to the change in the object's kinetic energy:

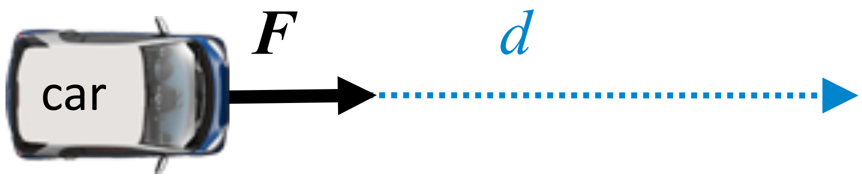
$$W = \Delta K$$

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{l}$$

$$K = \frac{1}{2} m v^2$$

Work-Kinetic Energy Theorem: 1-D Example

If the force is constant and the directions aren't changing then this is very simple to evaluate:



$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{l} = \vec{F} \cdot \vec{d}$$

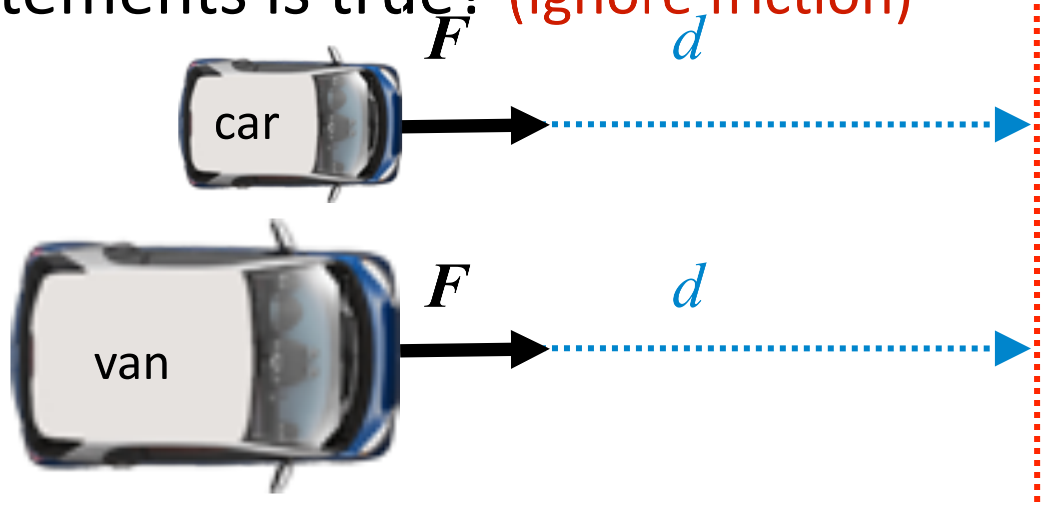
$$\left(\int_{\vec{r}_1}^{\vec{r}_2} d\vec{l} = \vec{d} \right)$$

In this case $= Fd$ since $\cos(0)=1$

This is probably what you remember from High School.
(The notation may be confusing though.)

Clicker Question

A lighter car and a heavier van, each initially at rest, are pushed with the same constant force F. After both vehicles travel a distance d, which of the following statements is true? (Ignore friction)

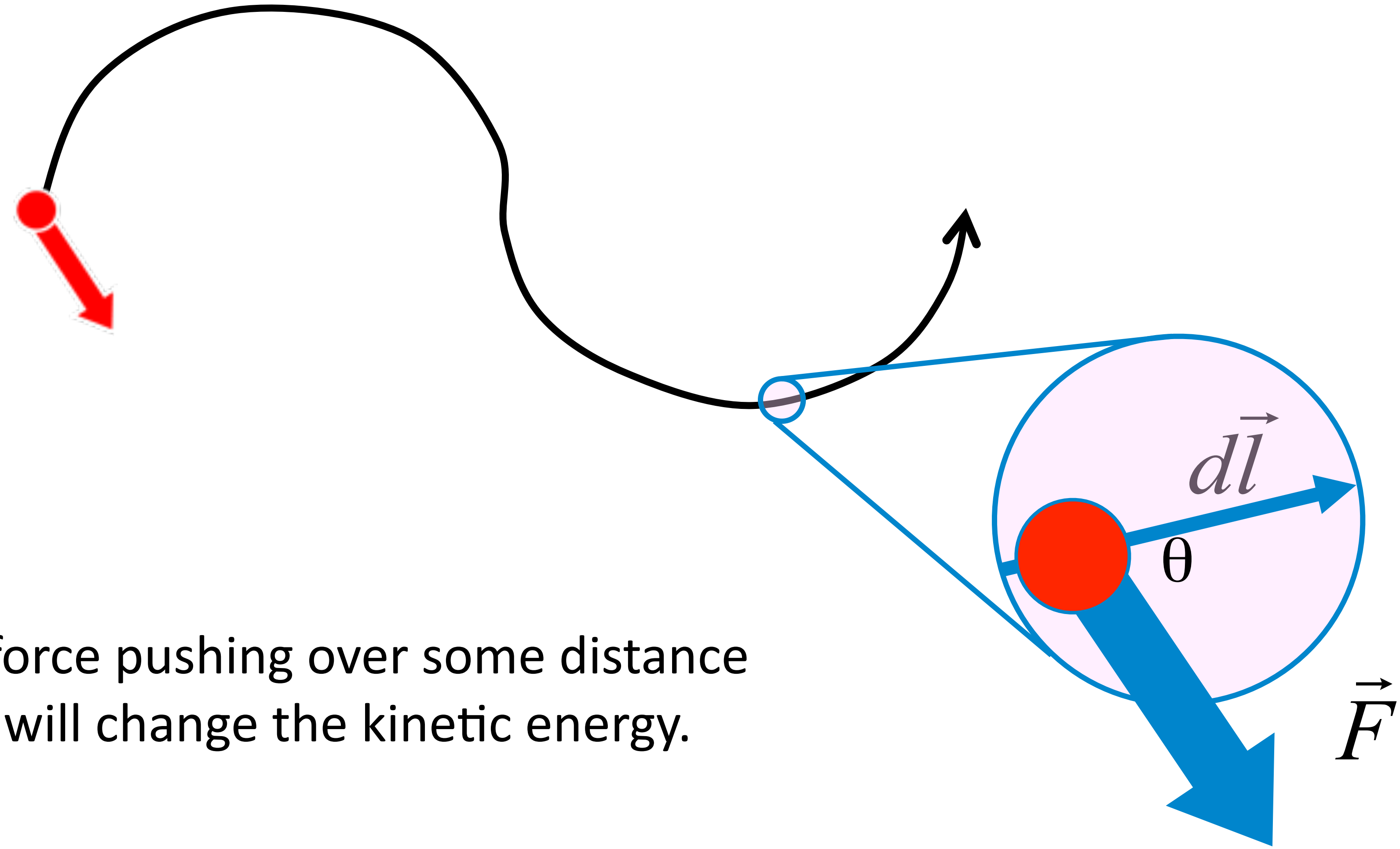


A) They will have the same velocity
B) They will have the same kinetic energy
C) They will have the same momentum

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{l} = \Delta K$$

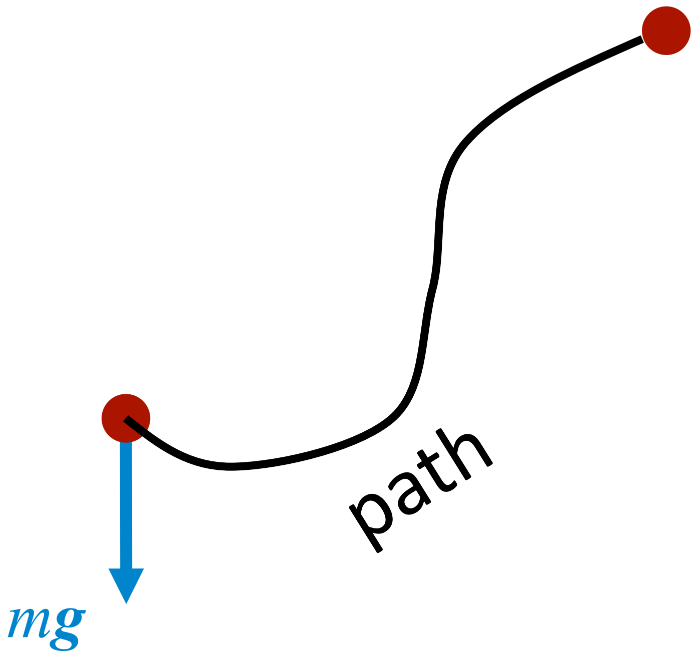
Derivation – not so important

<u>Concept – very important</u>



A force pushing over some distance will change the kinetic energy.

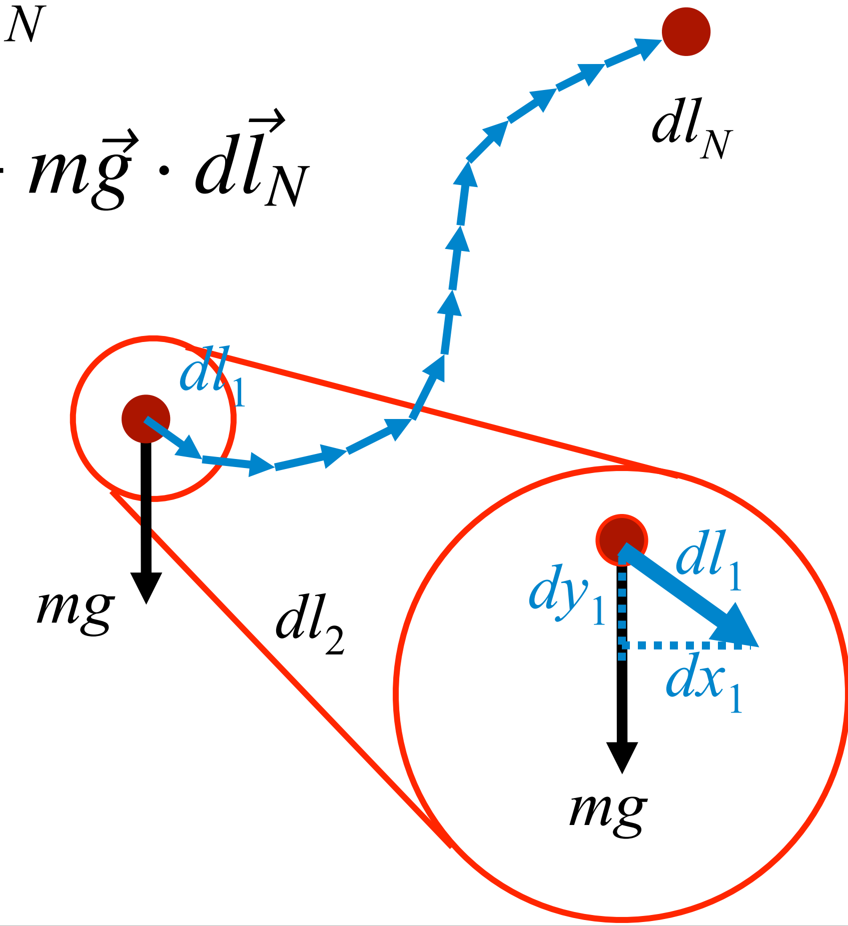
Work done by gravity near the Earth's surface



Work done by gravity near the Earth's surface

$$W_{TOT} = W_1 + W_2 + \ldots + W_N$$

$$= m\vec{g} \cdot d\vec{l}_1 + m\vec{g} \cdot d\vec{l}_2 + \cdots + m\vec{g} \cdot d\vec{l}_N$$



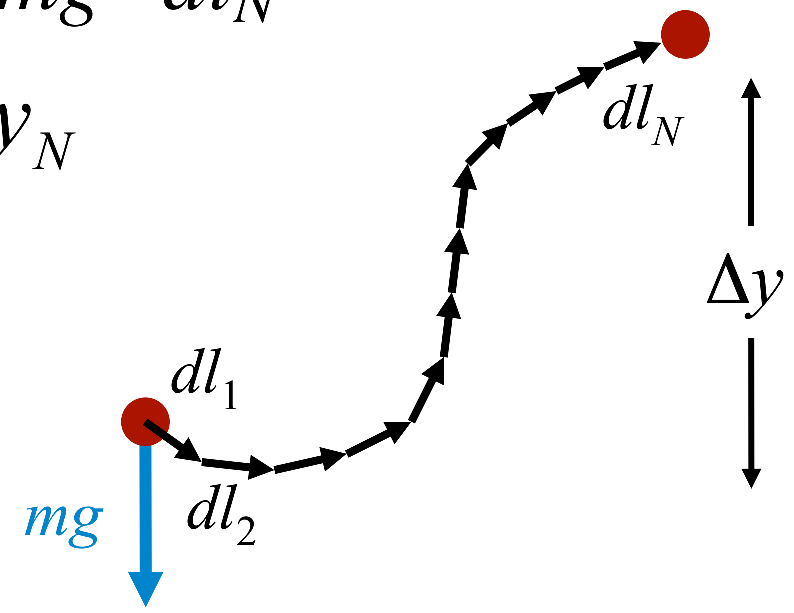
Work done by gravity near the Earth's surface

$$W_{TOT} = W_1 + W_2 + \ldots + W_N$$

$$= m\vec{g} \cdot d\vec{l}_1 + m\vec{g} \cdot d\vec{l}_1 + \cdots + m\vec{g} \cdot d\vec{l}_N$$

$$= -mgdy_1 - mgdy_2 \ldots - mgdy_N$$

$$= -mg\Delta y$$



$$W_g = -mg\Delta y$$

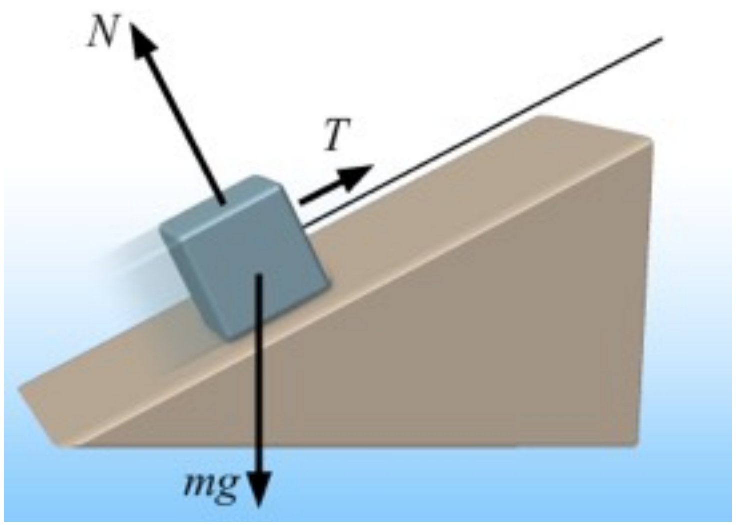
Work-Kinetic Energy Theorem

If there are several forces acting then W is the work done by the net (total) force:

$$W_{NET} = \Delta K$$

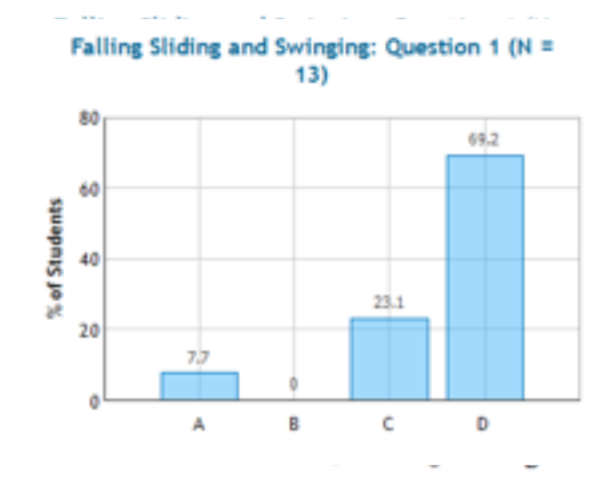
$$= W_1 + W_2 + \ldots$$

You can just add up the work done by each force

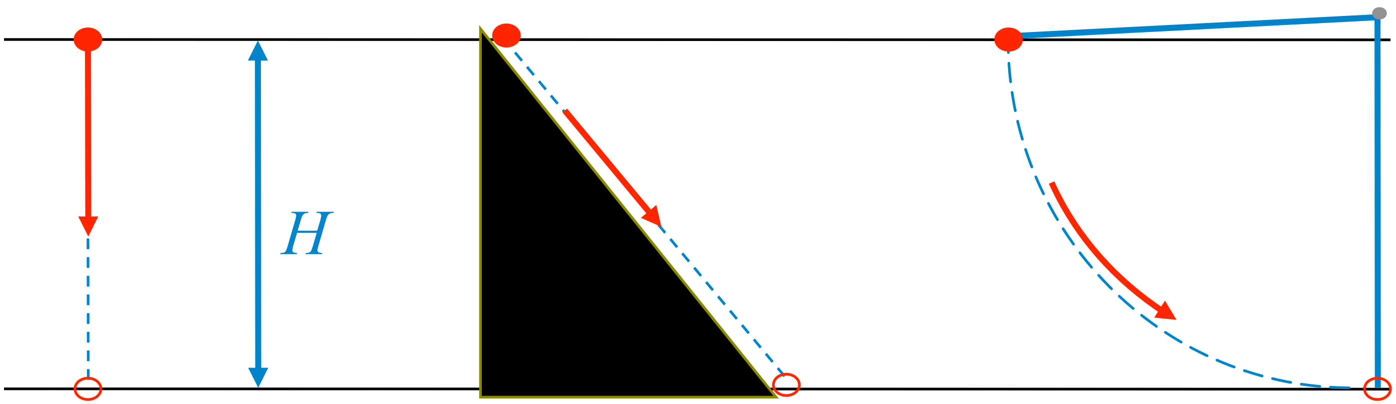


$$W_{NET} = W_{TOT}$$

CheckPoint



Three objects having the same mass begin at the same height, and all move down the same vertical distance H. One falls straight down, one slides down a frictionless inclined plane, and one swings on the end of a string.



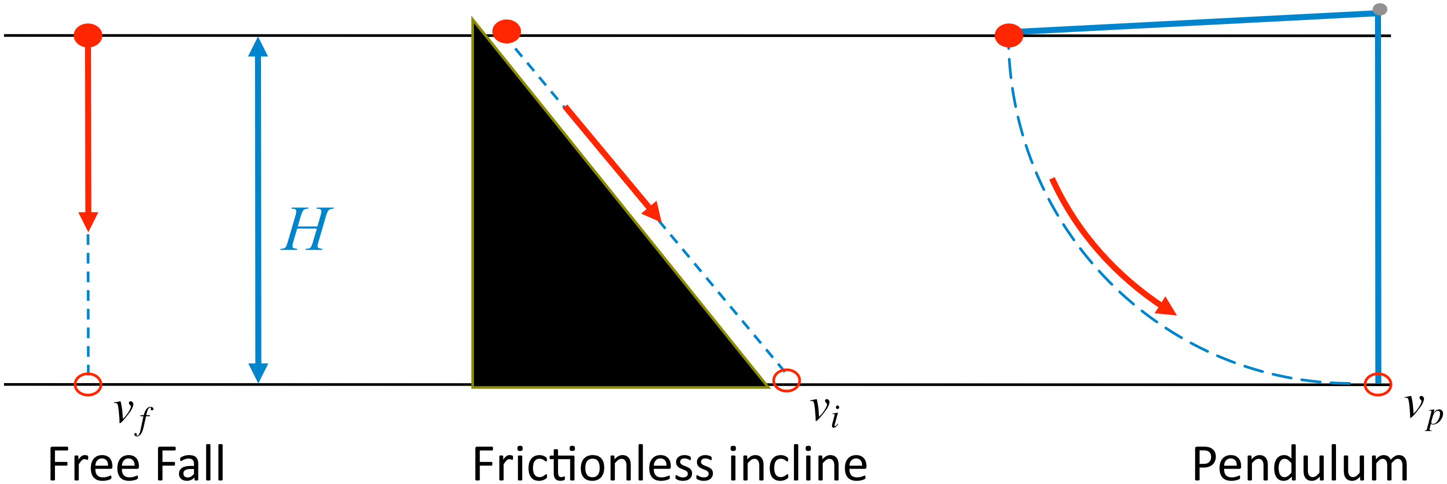
In which case does the object have the biggest net work done on it by all forces during its motion?

A) Free Fall B) Incline C) String D) All the same

Clicker Question

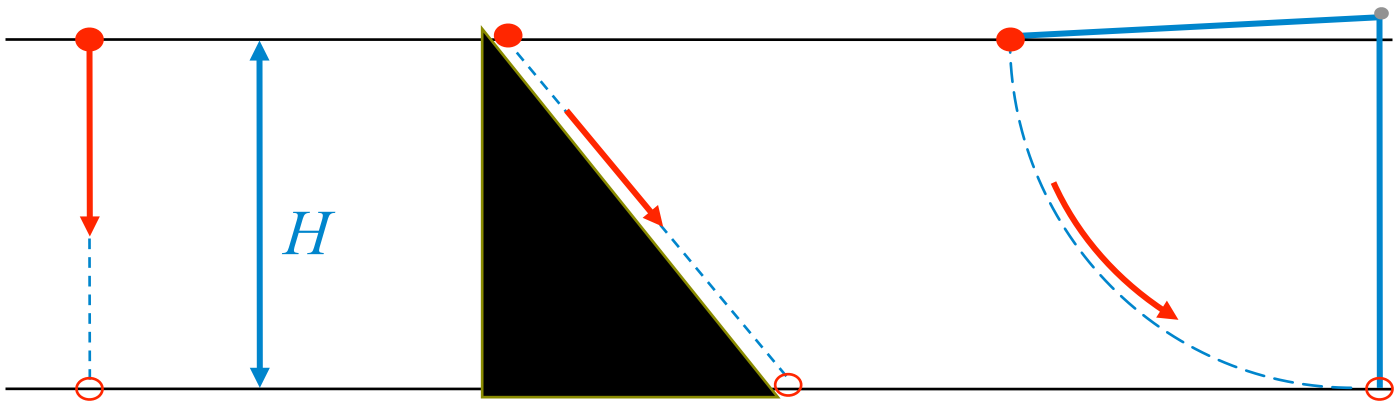
Three objects having the same mass begin at the same height, and all move down the same vertical distance H. One falls straight down, one slides down a frictionless inclined plane, and one swings on the end of a string.

What is the relationship between their speeds when they reach the bottom?



A) $v_f > v_i > v_p$ B) $v_f > v_p > v_i$ C) $v_f = v_p = v_i$

CheckPoint / Clicker Question



Free Fall   Frictionless incline   String

A) $v_f > v_i > v_p$

B) $v_f > v_p > v_i$

C) $v_f = v_p = v_i$

Only gravity will do work: $W_g = \Delta K$

$$\left\{ \begin{array}{l} W_g = mgH \\ \Delta K = {}^1/_2\, mv_2^{\,2} \end{array} \right\} \quad v = \sqrt{2gH}$$

$$\rightarrow \quad v_f = v_i = v_p = \sqrt{2gH}$$

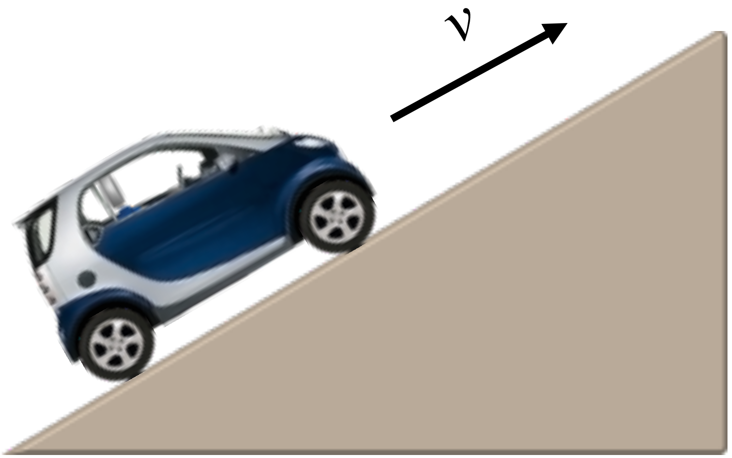
CheckPoint

A car drives up a hill with constant speed. Which statement best describes the total work W_{TOT} done on the car by all forces as it moves up the hill?

A) $W_{TOT} = 0$

B) $W_{TOT} > 0$

C) $W_{TOT} < 0$



Less than 40% got this right…

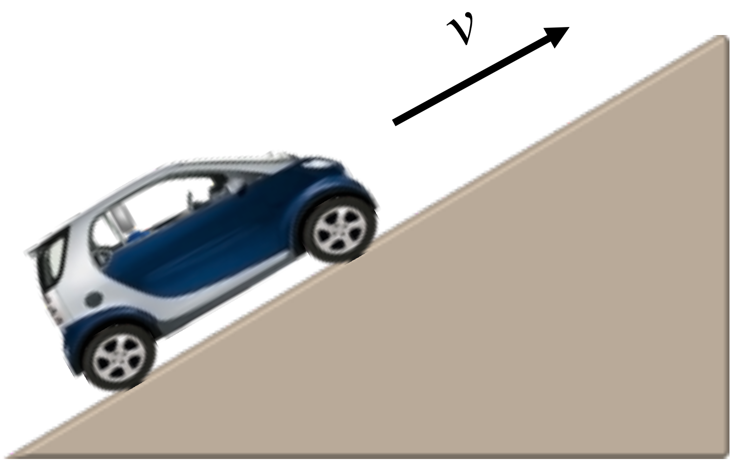
CheckPoint

A car drives up a hill with constant speed. Which statement best describes the total work W_{TOT} done on the car by all forces as it moves up the hill?

A) $W_{TOT} = 0$

B) $W_{TOT} > 0$

C) $W_{TOT} < 0$



A) The change in kinetic energy is zero because the velocity is constant. By the work-kinetic energy theorem, we know that work must also be zero.

B) The force of friction between the car's wheels and the ramp exert a force up the ramp over a particular distance. W=Fd, so the work is positive.

C) gravity is downward and car moves up the hill.

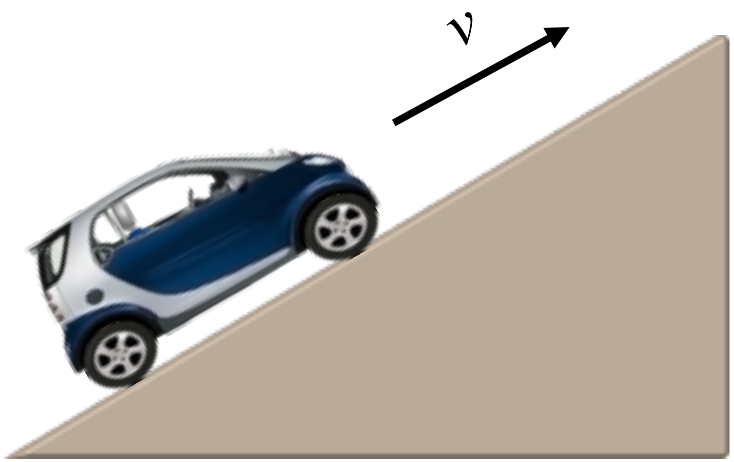
Clicker Question

A car drives up a hill with constant speed. How does the kinetic energy of the car change as it moves up the hill?

A) It increases

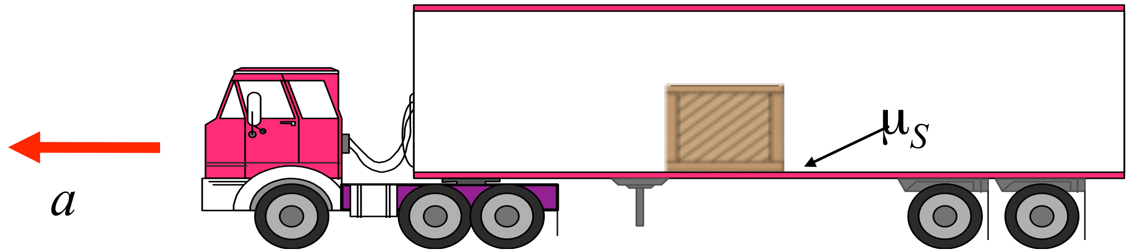
B) It stays the same

C) It decreases



CheckPoint

A box sits on the horizontal bed of a moving truck. Static friction between the box and the truck keeps the box from sliding around as the truck drives.



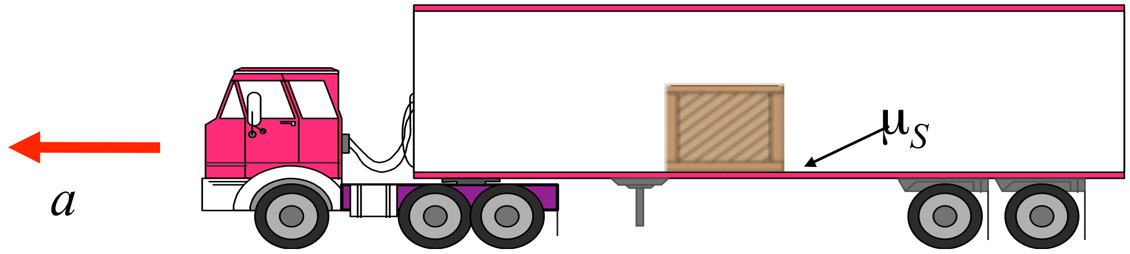
The work done on the box by the static frictional force as the truck moves a distance D is:

A) Positive B) Negative C) Zero

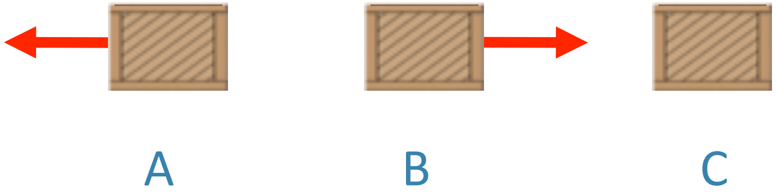
About 60% got this right…

From Before

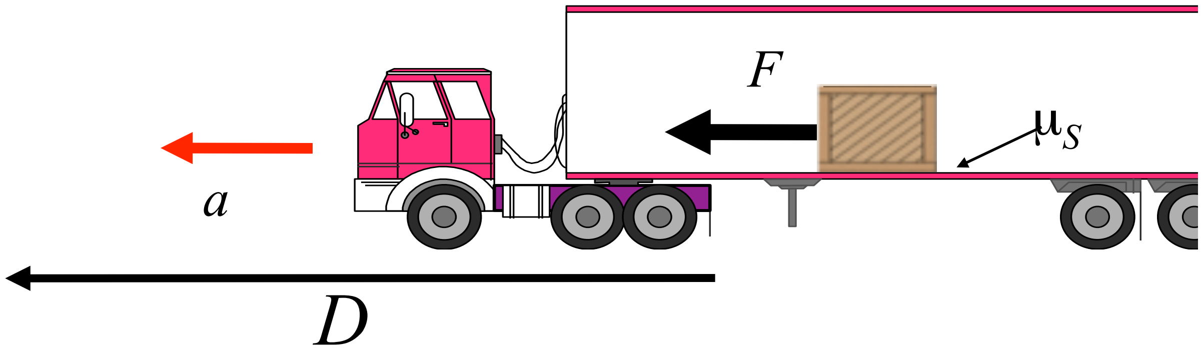
A box sits on the horizontal bed of a moving truck. Static friction between the box and the truck keeps the box from sliding around as the truck drives.

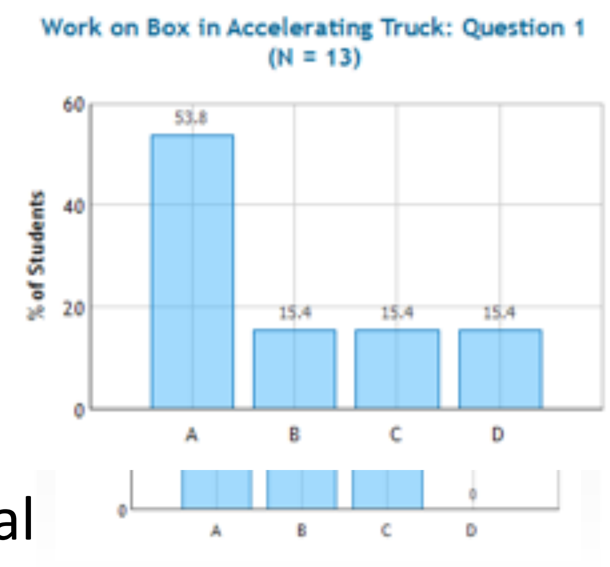


If the truck moves with constant accelerating to the left as shown, which of the following diagrams best describes the static frictional force acting on the box:



CheckPoint





The work done on the box by the static frictional force as the truck moves a distance D is:

A) Positive B) Zero C) Negative D) Depends

A) because the direction of the static frictional force and the direction the box travels are all same.

B) Since the friction force always has opposite direction from the movement direction, the work done is negative

C) Friction keeps the box from sliding, thus D=0, and work done is 0.

Problem

You push a box up a ramp using a constant horizontal 100-N force $\vec{F}$. For each distance of 5.00 m along the ramp, the box gains 3.00 m of height. Find the work done by $\vec{F}$ for each 5.00 m the box moves along the ramp (*a*) by directly computing the scalar product from the components of $\vec{F}$ and $\vec{\ell}$, where $\vec{\ell}$ is the displacement, (*b*) by multiplying the product of the magnitudes of $\vec{F}$ and $\vec{\ell}$ by $\cos\phi$, where ϕ is the angle between the direction of $\vec{F}$ and the direction of $\vec{\ell}$, (*c*) by finding $F_{||}$ (the component of $\vec{F}$ in the direction of $\vec{\ell}$) and multiplying it by ℓ (the magnitude of $\vec{\ell}$), and (*d*) by finding $\ell_{||}$ (the component of $\vec{\ell}$ in the direction of $\vec{F}$) and multiplying it by the magnitude of the force.

PICTURE Draw a sketch of the box in its initial and final positions. Place coordinate axes on the sketch with the x axis horizontal. Express the force and displacement vectors in component form and take the scalar product. Then find the component of the force in the direction of the displacement, and vice versa.

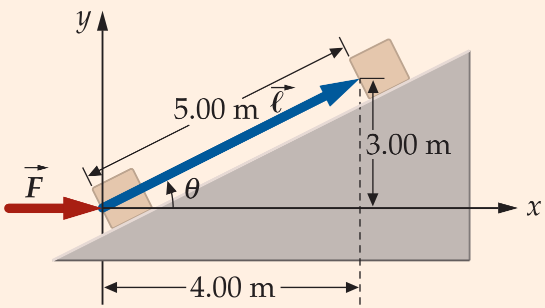


FIGURE 6-19

SOLVE

(*a*) 1. Draw a sketch of the situation (Figure 6-19).

2. Express $\vec{F}$ and $\vec{\ell}$ in component form and take the scalar product:

$$\vec{F} = (100\hat{i} + 0\hat{j})\,\text{N}$$

$$\vec{\ell} = (4.00\hat{i} + 3.00\hat{j})\,\text{m}$$

$$W = \vec{F}\cdot\vec{\ell} = F_x\,\Delta x + F_y\,\Delta y = (100\text{ N})(4.00\text{ m}) + 0(3.00\text{ m})$$

$$= \boxed{4.00 \times 10^2\text{ J}}$$

Solution

(*b*) Calculate $F\ell\cos\phi$, where ϕ is the angle between the directions of the two vectors as shown. Equate this expression with the Part-(*a*) result and solve for $\cos\phi$. Then solve for the work:

$$\vec{F}\cdot\vec{\ell} = F\ell\cos\phi \quad \text{and} \quad \vec{F}\cdot\vec{\ell} = F_x\,\Delta x + F_y\,\Delta y$$

so

$$\cos\phi = \frac{F_x\,\Delta x + F_y\,\Delta y}{F\ell} = \frac{(100\text{ N})(4.00\text{ m}) + 0}{(100\text{ N})(5.00\text{ m})} = 0.800$$

and

$$W = F\ell\cos\phi = (100\text{ N})(5.00\text{ m})\,0.800 = \boxed{4.00\times10^2\text{ J}}$$

(*c*) Find $F_{\|}$ and multiply it by ℓ:

$$F_{\|} = F\cos\phi = (100\text{ N})\,0.800 = 80.0\text{ N}$$

$$W = F_{\|}\ell = (80.0\text{ N})(5.00\text{ m}) = \boxed{4.00\times10^2\text{ J}}$$

(*d*) Multiply F and $\ell_{\|}$, where $\ell_{\|}$ is the component of $\vec{\ell}$ in the direction of $\vec{F}$:

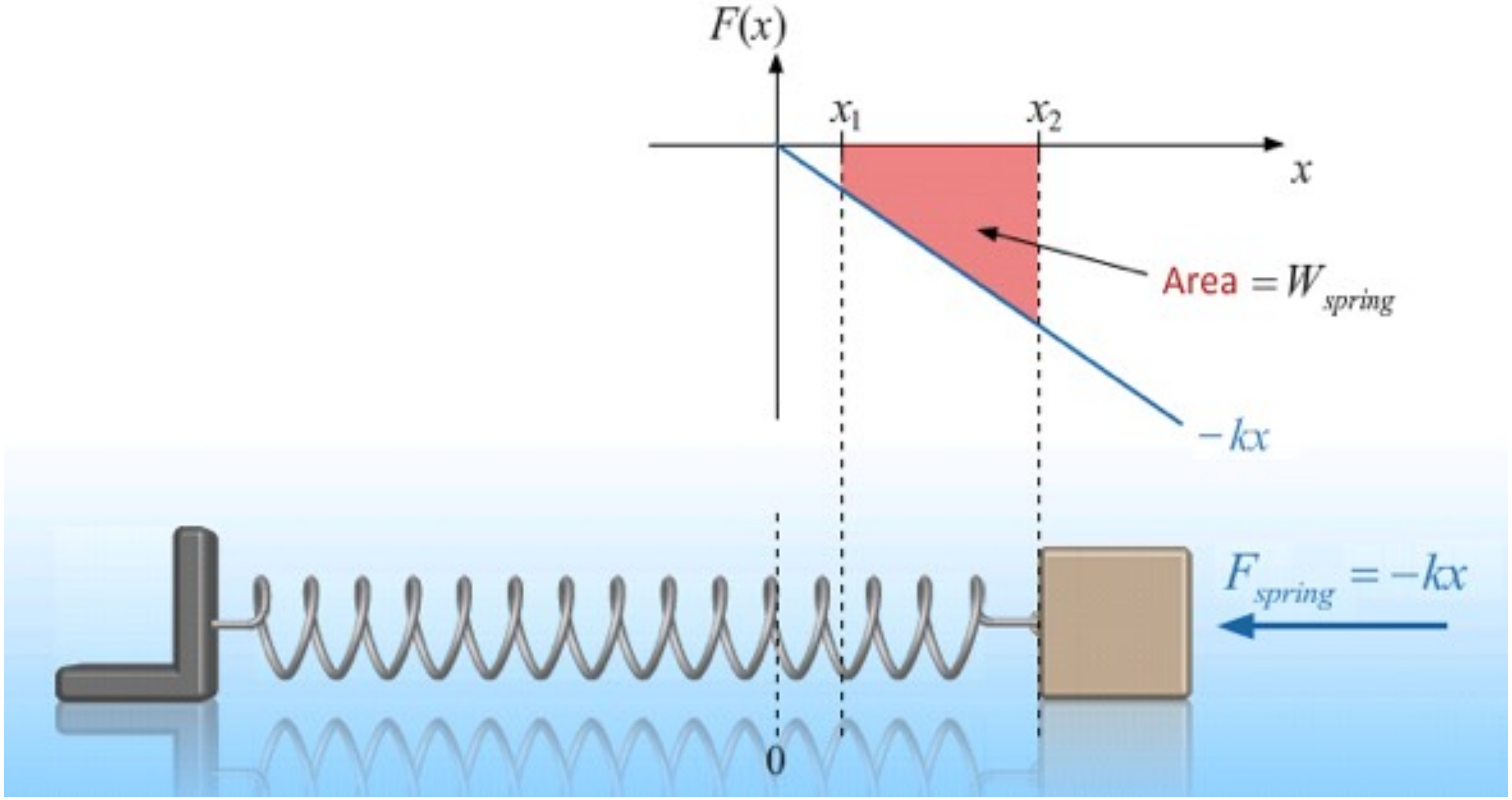
$$\ell_{\|} = \ell\cos\phi = (5.00\text{ m})\,0.800 = 4.00\text{ m}$$

$$W = F\ell_{\|} = (100\text{ N})\,(4.00\text{ m}) = \boxed{4.00\times10^2\text{ J}}$$

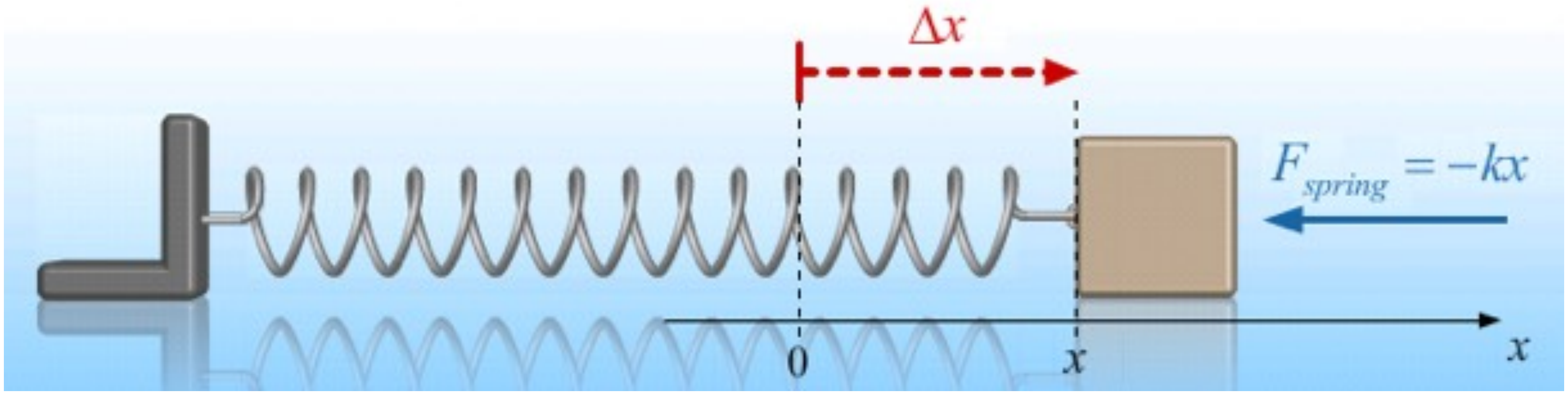
CHECK The four distinct calculations give the same result for the work.

Work done by a Spring

$$W_{1\to 2} = -\frac{1}{2}k(x_2^2 - x_1^2)$$



I am confused about the positive work and negative work and also the positive and negative forces for the spring problems.



Use the formula to get the magnitude of the work

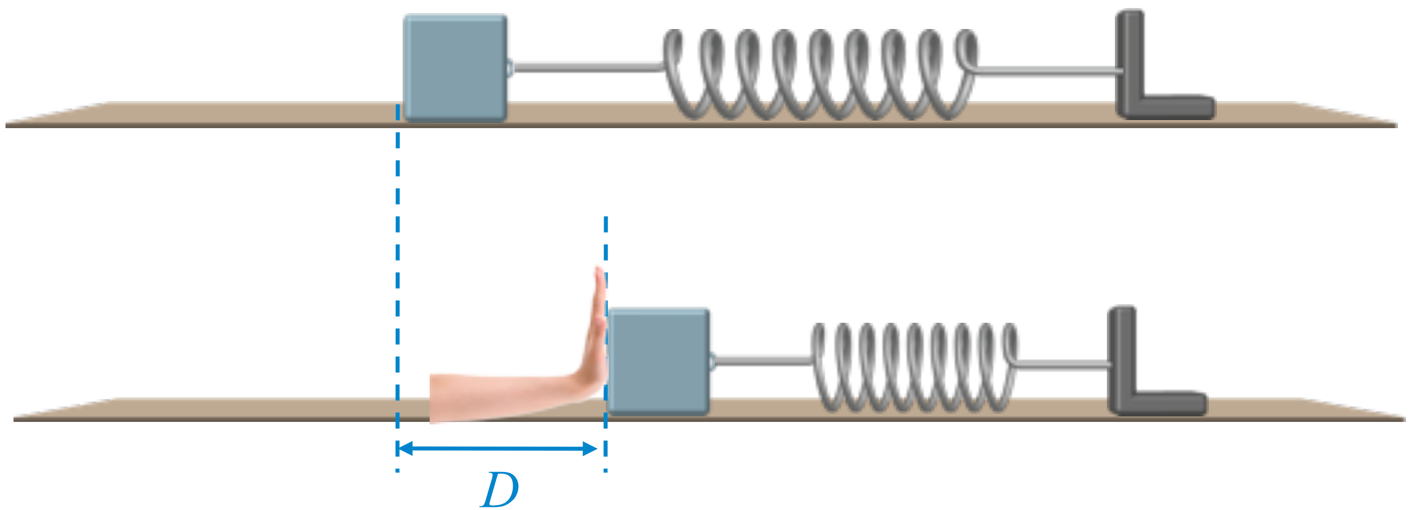
$$W_{1\to 2} = -\frac{1}{2}k(x_2^2 - x_1^2)$$

Use a picture to get the sign (look at directions)

In this example the spring does –ve work since F and Δx are in opposite direction. The axes don't matter.

Clicker Question

A box attached at rest to a spring at its equilibrium length. You now push the box with your hand so that the spring is compressed a distance D, and you hold the box at rest in this new location.

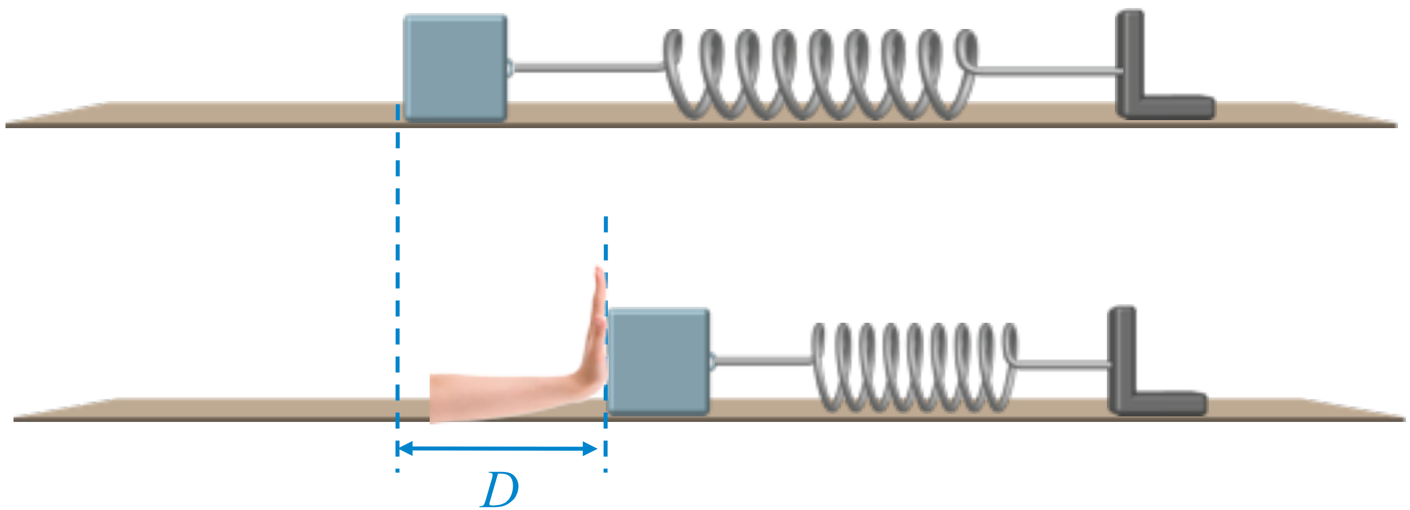


During this motion, the spring does:

A) Positive Work B) Negative Work C) Zero work

Clicker Question

A box attached at rest to a spring at its equilibrium length. You now push the box with your hand so that the spring is compressed a distance D, and you hold the box at rest in this new location.



During this motion, your hand does:

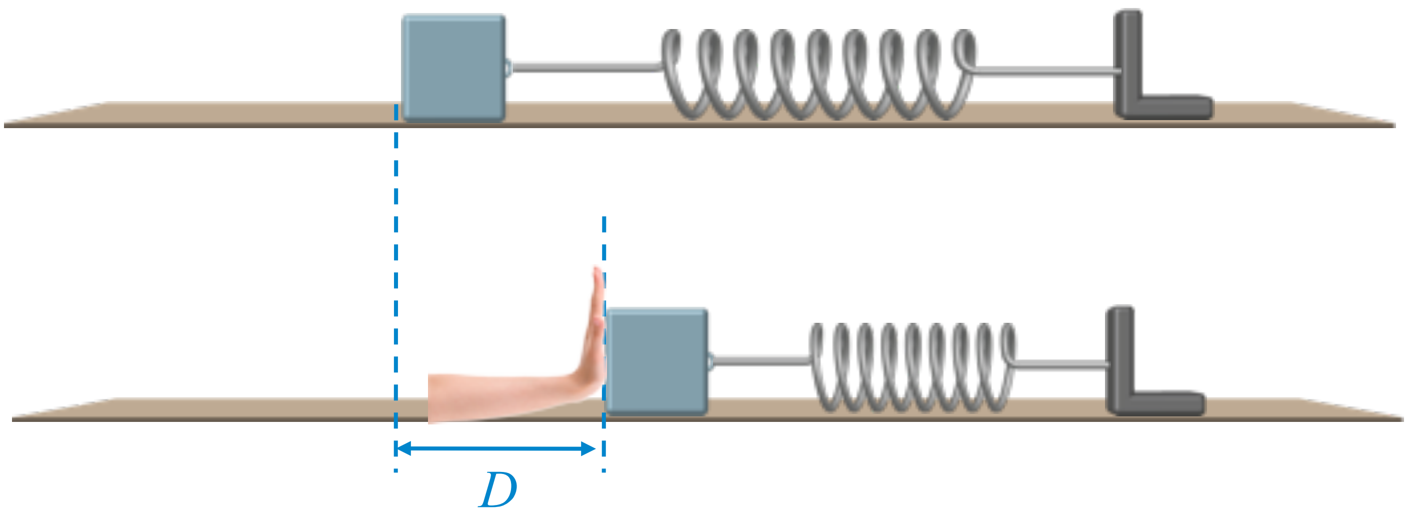
A) Positive Work

B) Negative Work

C) Zero work

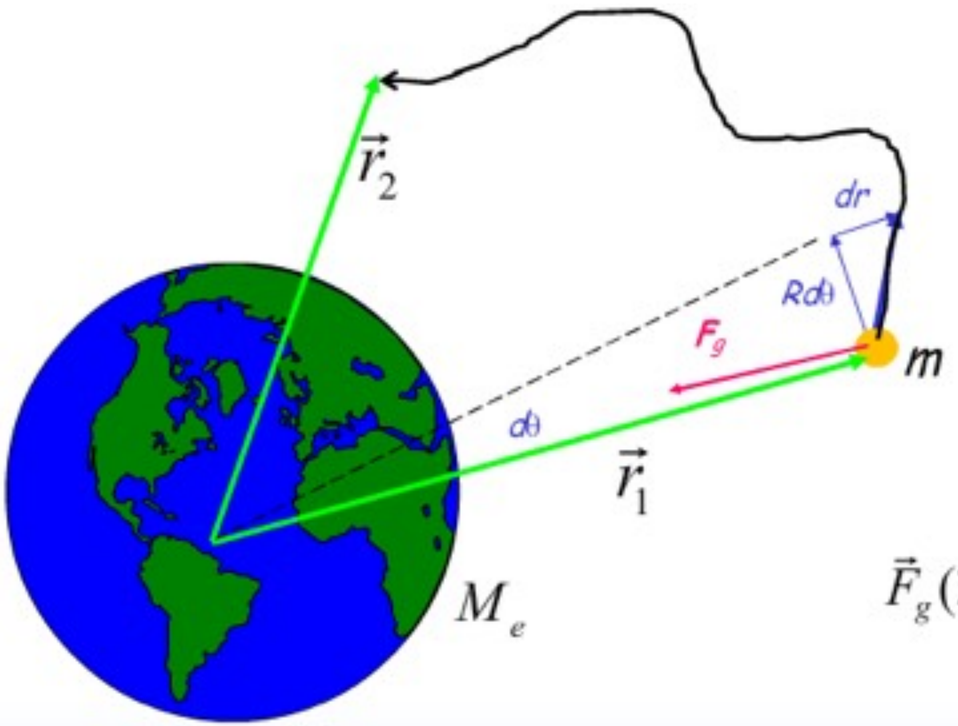
Clicker Question

A box attached at rest to a spring at its equilibrium length. You now push the box with your hand so that the spring is compressed a distance D, and you hold the box at rest in this new location.



During this motion, the total work done on the box is:

A) Positive B) Negative C) Zero



$$dW = -\frac{GM_e m}{r^2}\, dr$$

$$\vec{F}_g(\vec{r}) = -\frac{GM_e m}{r^2}\,\hat{r}$$

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}) \cdot d\vec{r} = -\int_{r_1}^{r_2} \frac{GM_e m}{r^2} dr = \left.\frac{GM_e m}{r}\right|_{r_1}^{r_2} = GM_e m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Clicker Question

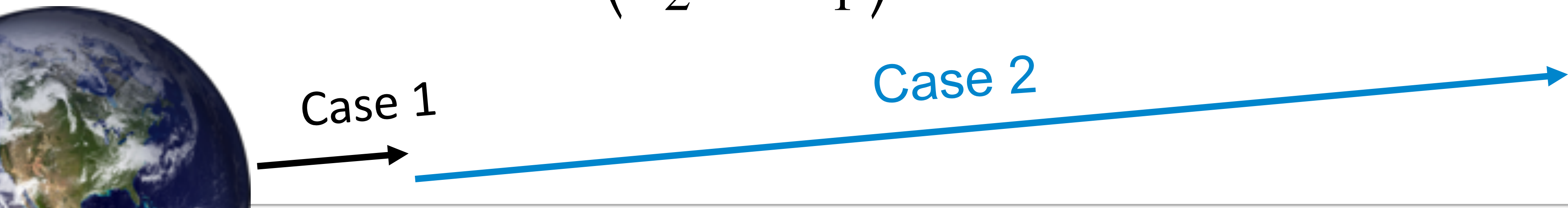
In Case 1 we send an object from the surface of the earth to a height above the earth surface equal to one earth radius.

In Case 2 we start the same object a height of one earth radius above the surface of the earth and we send it infinitely far away.

In which case is the magnitude of the work done by the Earth's gravity on the object biggest?

A) Case 1 B) Case 2 C) They are the same

$$W = GM_e m\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$



Clicker Question Solution

Case 1: $$W = GM_e m\left(\frac{1}{2R_E} - \frac{1}{R_E}\right) = -\frac{GM_e m}{2R_E}$$

Case 2: $$W = GM_e m\left(\frac{1}{\infty} - \frac{1}{2R_E}\right) = -\frac{GM_e m}{2R_E}$$

Same!