

# *Classical Mechanics*

## *Lecture 11*

Today's Concept:

Inelastic Collisions

it is really hard to imagine the frictionless situation since it's just impossible to have absolute frictionless surface on the earth

I find, often, thinking about things in terms of vectors is very helpful. Would it be possible to go through the second part of the cart question in terms of vectors? Because I'm a bit confused. Though I'm happy because inelastic collisions finally make sense! Yay!

Would like some more of the examples like in the prelecture

"There's no challenge in breaking a board.  
Boards don't hit back."

➤ Bruce Lee

That's not really true, Bruce.

Newton's 3rd law works here.



What is the different between conservation of Momentum and conservation of energy? How do I know if the Momentum is conserve or energy is conserve?

Momentum (Prelecture 11)

### Conservation of Momentum

$$\text{When } \vec{F}_{Net, External} = 0, \text{ then } \frac{d\vec{P}_{Total}}{dt} = 0 \longrightarrow \vec{P}_{Total} = \text{Constant}$$

Energy (Prelecture 8)

$$\Delta E_{Mechanical} = W_{NC}$$

Conservation of Mechanical Energy  $\Delta E_{Mechanical} = 0$  (When work by non-conservative forces is zero)

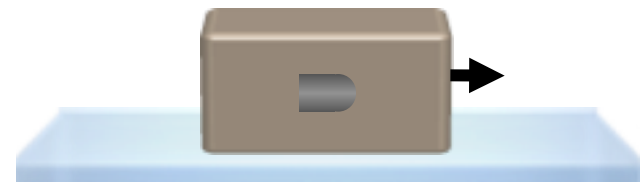
# Clicker Question



A wood block rests at rest on a table. A bullet shot into the block stops inside, and the bullet plus block start sliding on the frictionless surface. The momentum of the bullet plus block remains constant



Before



After

- A) Before the collision.
- B) During the collision
- C) After the collision
- D) All of the above
- E) Only A and C above

As long as there are no **external** forces acting on the system

## *Center-of-mass problem*

A railroad car is 20 m long with mass of 25 tonnes.  
( $2.5 \times 10^4$  kg)

A tank of water 7m x 2m x 2m is in the car. It is 0.5 m from the end of the car.

The water leaks.

If the wheels are frictionless how much does the railroad car move after the water has leaked out?

# Center-of-mass problem

$$m_c = 25 \times 10^4 \text{ kg}$$

$$m_w = 7 \times 2 \times 2 \times \rho_{\text{water}}$$

**before**

$$x_{c,\text{cm}} = 10 \text{ m}$$

$$x_{w,\text{cm}} = 4 \text{ m}$$

**after**

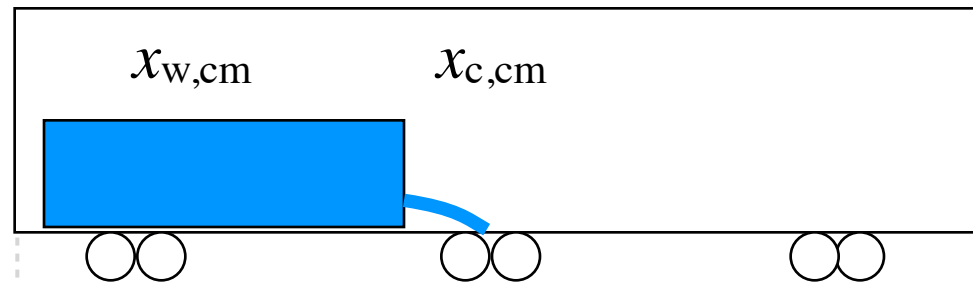
$$x'_{c,\text{cm}} = x'_{w,\text{cm}}$$

$$= x'_{\text{total},\text{cm}}$$

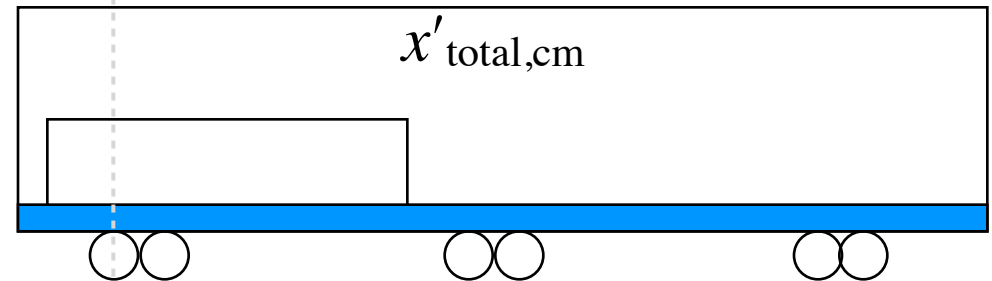
**always**

$$x_{\text{total},\text{cm}} = x'_{\text{total},\text{cm}}$$

**before**



**after**



$x=0$

$$x_{\text{total},\text{cm}} = \frac{m_c x_{c,\text{cm}} + m_w x_{w,\text{cm}}}{m_c + m_w} = \frac{(m_c + m_w) x'_{c,\text{cm}}}{m_c + m_w}$$

$$x_{\text{total,cm}} = \frac{m_c x_{c,\text{cm}} + m_w x_{w,\text{cm}}}{m_c + m_w} = \frac{\cancel{(m_c + m_w)} x'_{c,\text{cm}}}{\cancel{m_c + m_w}} = x'_{c,\text{cm}}$$

$$x_{\text{total,cm}} = (25000 \cdot 10 + 28000 \cdot 4) / (25000 + 28000) \\ = 6.83 \text{ m}$$

$$\text{Distance moved is } x'_{c,\text{cm}} - x_{c,\text{cm}} = 6.83 \text{ m} - 10 \text{ m} = -3.17 \text{ m}$$



# Tipler & Mosca Ch 5

“I find that pre lecture materials are insufficient when explaining concepts in applied situations because I'm not 100% sure about how the concept will be applied in certain situations.”

## Example 5-17

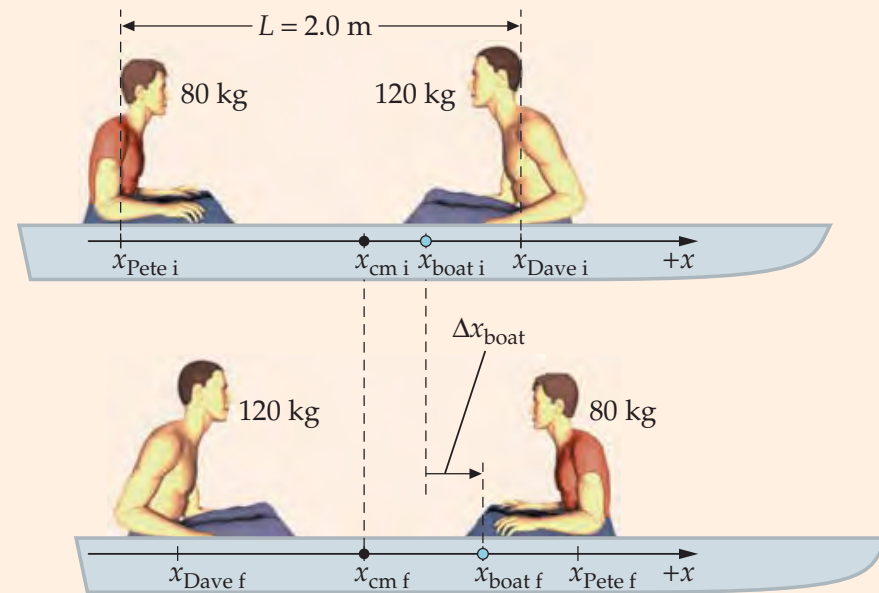
### Changing Places in a Rowboat

Pete (mass 80 kg) and Dave (mass 120 kg) are in a rowboat (mass 60 kg) on a calm lake. Dave is near the bow of the boat, rowing, and Pete is at the stern, 2.0 m from the center. Dave gets tired and stops rowing. Pete offers to row, so after the boat comes to rest they change places. How far does the boat move as Pete and Dave change places? (Neglect any horizontal force exerted by the water.)

**PICTURE** Let the system be Dave, Pete, and the boat. There are no *external* forces in the horizontal direction, so the center of mass does not move horizontally relative to the water. Flesh out Equation 5-15 ( $Mx_{\text{cm}} = \sum m_i x_i$ ) both before and after Pete and Dave change places.

### SOLVE

1. Make a sketch of the system in its initial and final configurations (Figure 5-52). Let  $L = 2.0$  m and let  $d = \Delta x_{\text{boat}}$ , the distance the boat moves forward when Pete and Dave switch places:



**FIGURE 5-52** Pete and Dave changing places viewed from the reference frame of the water. The blue dot is the center of mass of the boat and the black dot is the center of mass of the Pete–Dave–boat system.

# Recap

Total Momentum

$$\vec{P}_{Total} = M_{Total} \vec{V}_{CM}$$

$$\frac{d\vec{P}_{Total}}{dt} = \vec{F}_{Net, External}$$

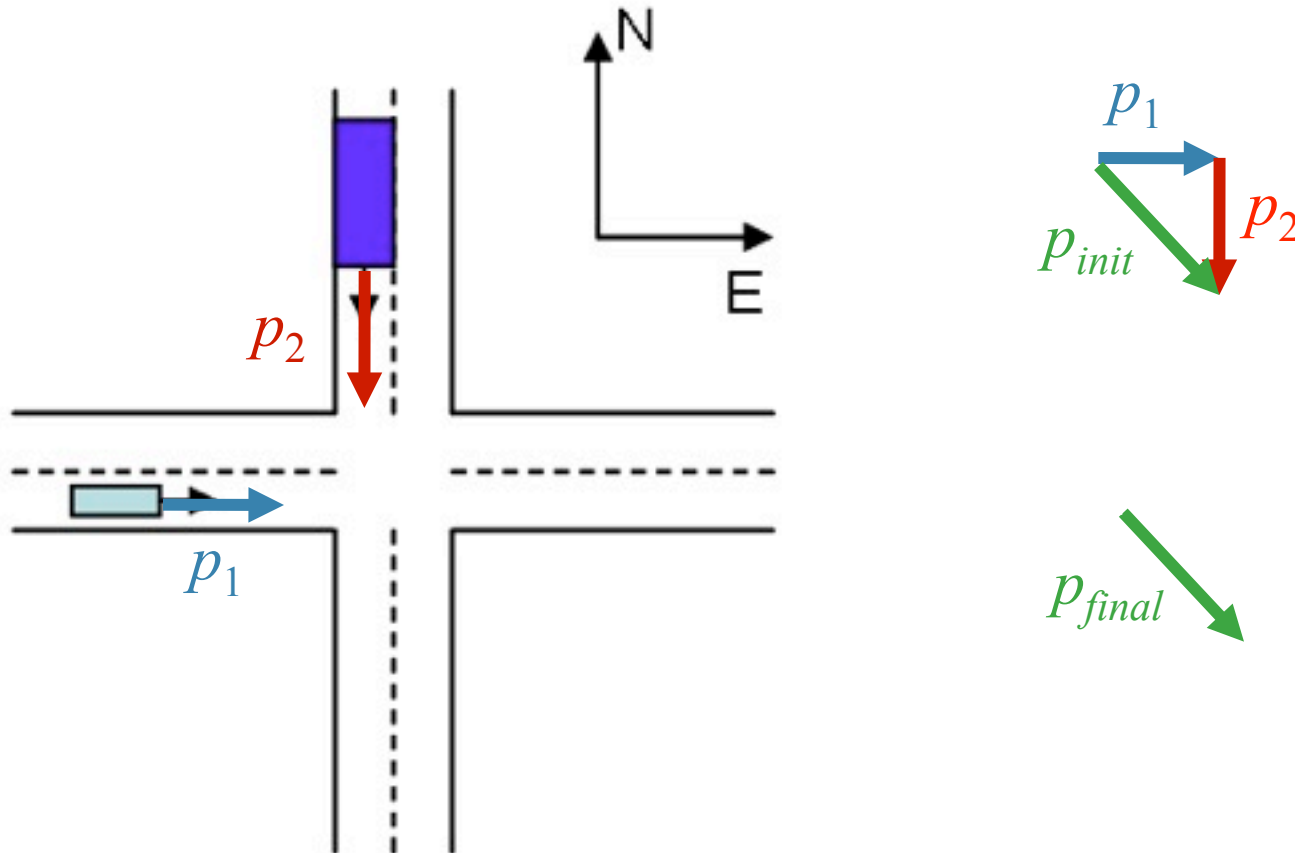
Conservation of Momentum

$$\text{When } \vec{F}_{Net, External} = 0, \text{ then } \frac{d\vec{P}_{Total}}{dt} = 0 \longrightarrow \vec{P}_{Total} = \text{Constant}$$

$$\vec{P}_{tot} = \sum_i m_i \vec{v}_i$$

# Homework Problem

## Car-Truck Collision

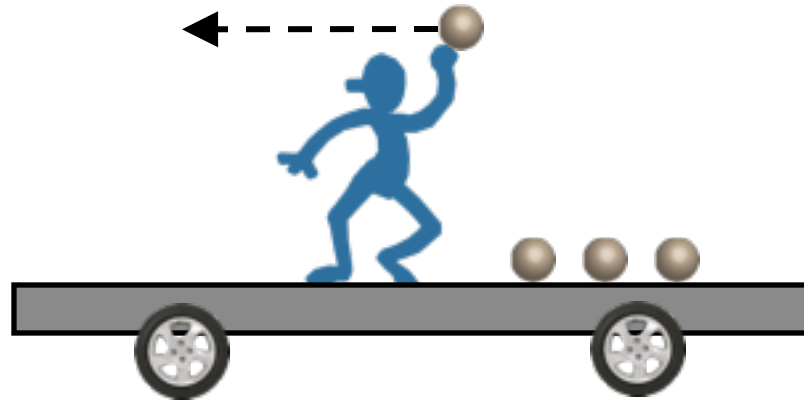


A blue car with mass  $m_c = 438 \text{ kg}$  is moving east with a speed of  $v_c = 23 \text{ m/s}$  and collides with a purple truck with mass  $m_t = 1237 \text{ kg}$  that is moving south with a speed of  $v_t = 10 \text{ m/s}$ . The two collide and lock together after the collision.

# CheckPoint

Suppose you are on a cart initially at rest that rides on a frictionless track. If you throw a ball off the cart towards the left, will the cart be put into motion?

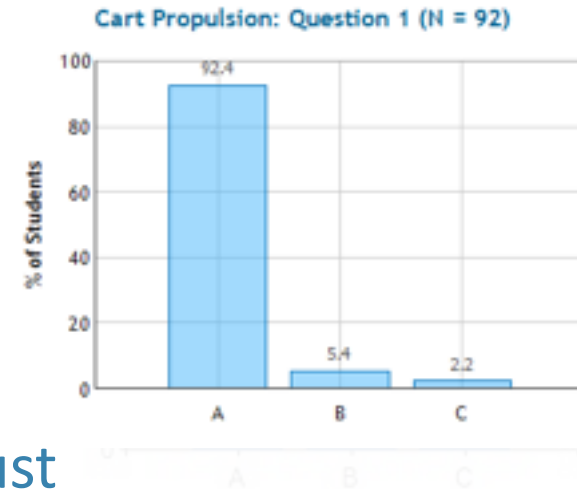
Left



Right

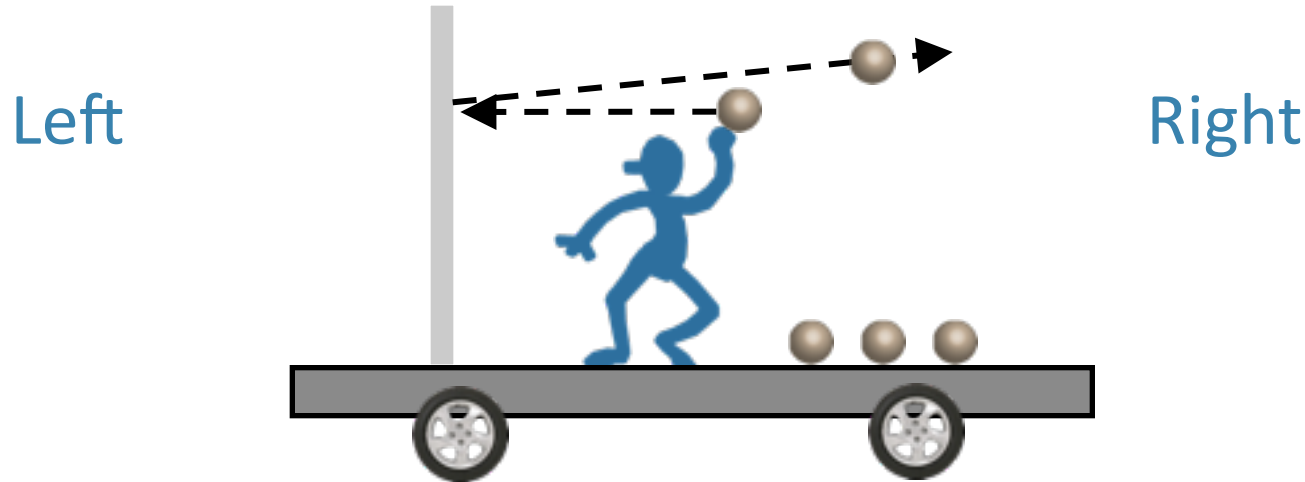
A) Yes, and it moves to the right.

Conservation of momentum means the cart must move to the right since the ball moves to the left.



# CheckPoint

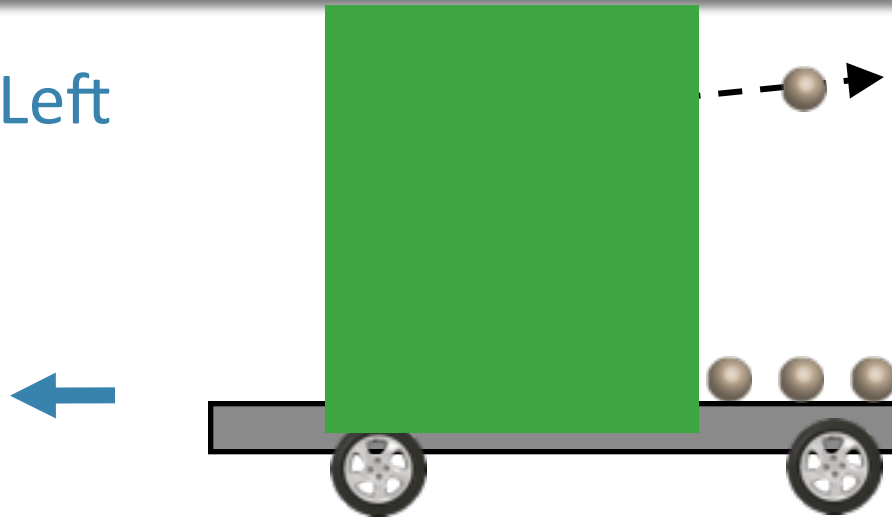
Suppose you are on a cart which is initially at rest that rides on a frictionless track. You throw a ball at a vertical surface that is firmly attached to the cart. If the ball bounces straight back as shown in the picture, will the cart be put into motion after the ball bounces back from the surface?



- A) Yes, and it moves to the right.
- B) Yes, and it moves to the left.
- C) No, it remains in place.

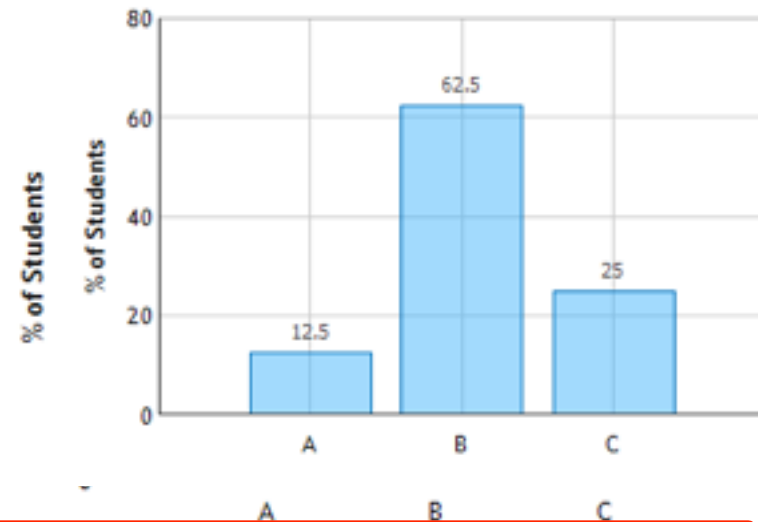
# Checkpoint

Left



Right

Cart Propulsion: Question 3 (N = 8)



- A) Yes, and it moves to the right.
- B) Yes, and it moves to the left.
- C) No, it remains in place.

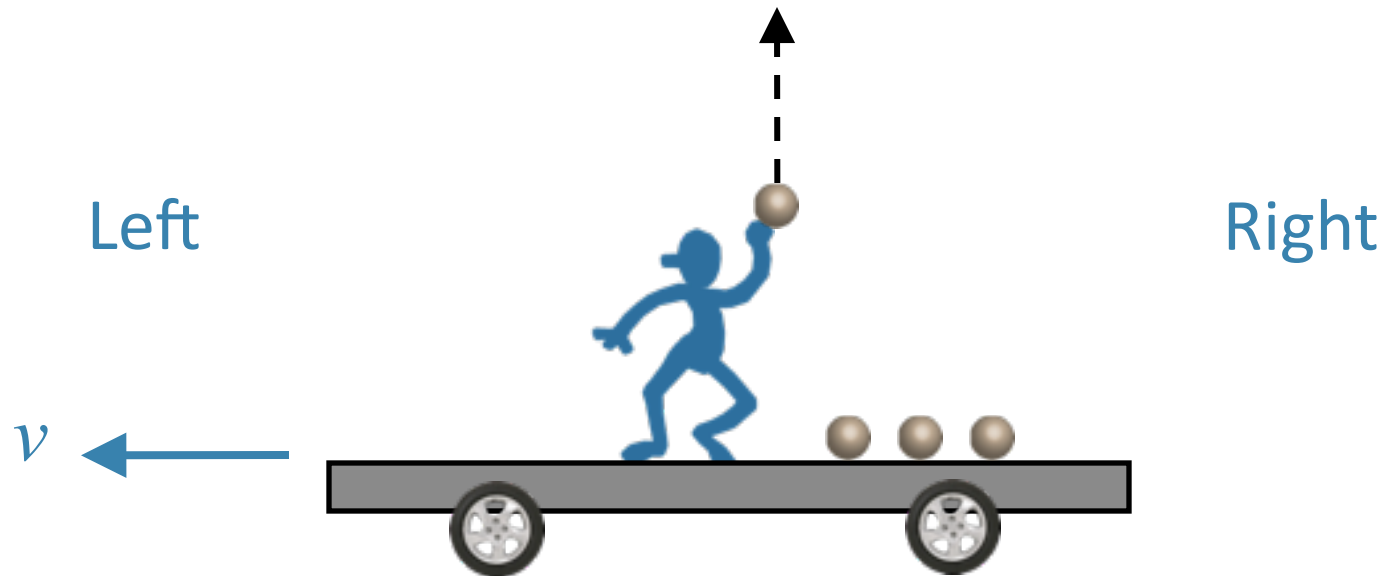
B) There are still no external forces, so the final velocity of the cart must be to the left in order to account for the final velocity of the ball to the right.

C) After the ball has bounced off, the reaction force of the bounce will stop the motion caused by the reaction force from the throwing.

# Clicker Question



Suppose you are on a cart that is moving at a constant speed  $v$  toward the left on a frictionless track. If you throw a massive ball straight up (relative to the cart), how will the speed of the cart change?



- A) Increase
- B) Decrease
- ☒ C) Will not change

As long as there are no external forces acting on the system,  $P_{total}$  is conserved

# CheckPoint

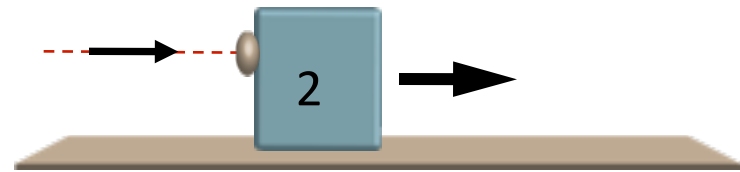
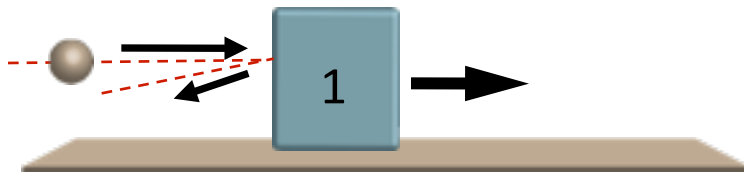
Two balls of equal mass are thrown horizontally with the same initial velocity. They hit identical stationary boxes resting on a frictionless horizontal surface. The ball hitting box 1 bounces back, while the ball hitting box 2 gets stuck.

Which box ends up moving faster?

A) Box 1

B) Box 2

C) same





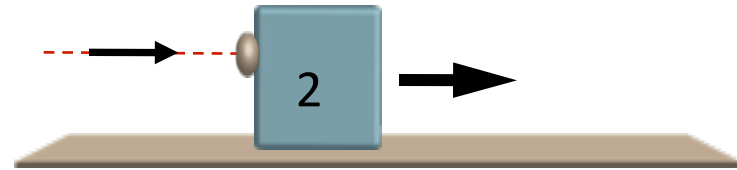
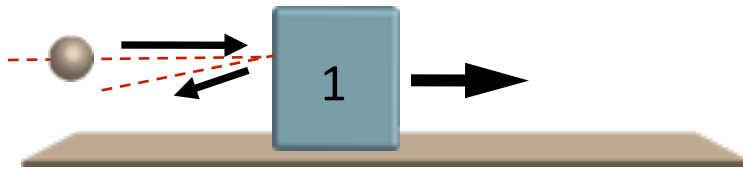
# CheckPoint

Which box ends up moving faster?

A) Box 1

B) Box 2

C) same



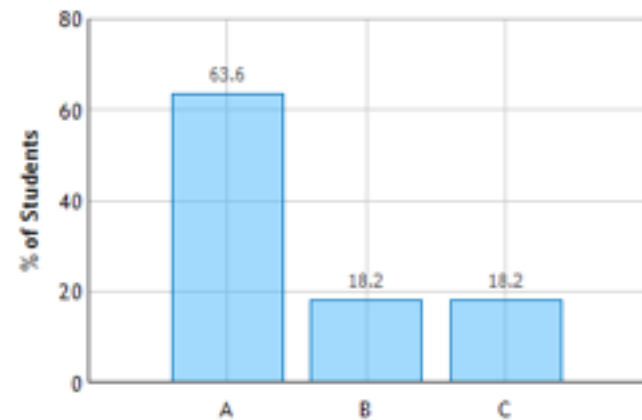
A) As the ball bounces to the left, cart 1 moves faster to the right to conserve momentum.

B) Some of the energy is not transferred to box 1, since the ball bounces back

C) The starting velocity and mass is all the same so the momentum will be the same in both cases

Think of a 2-step “bounce”

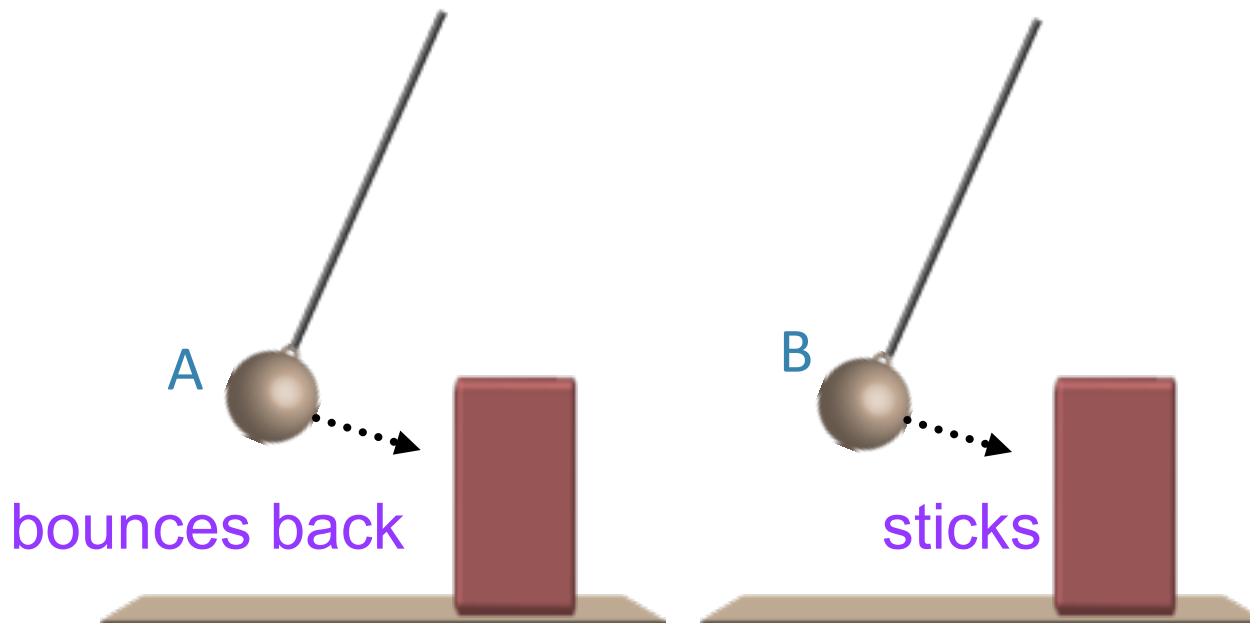
Box and Ball Collisions: Question 1 (N = 11)



# Clicker Question



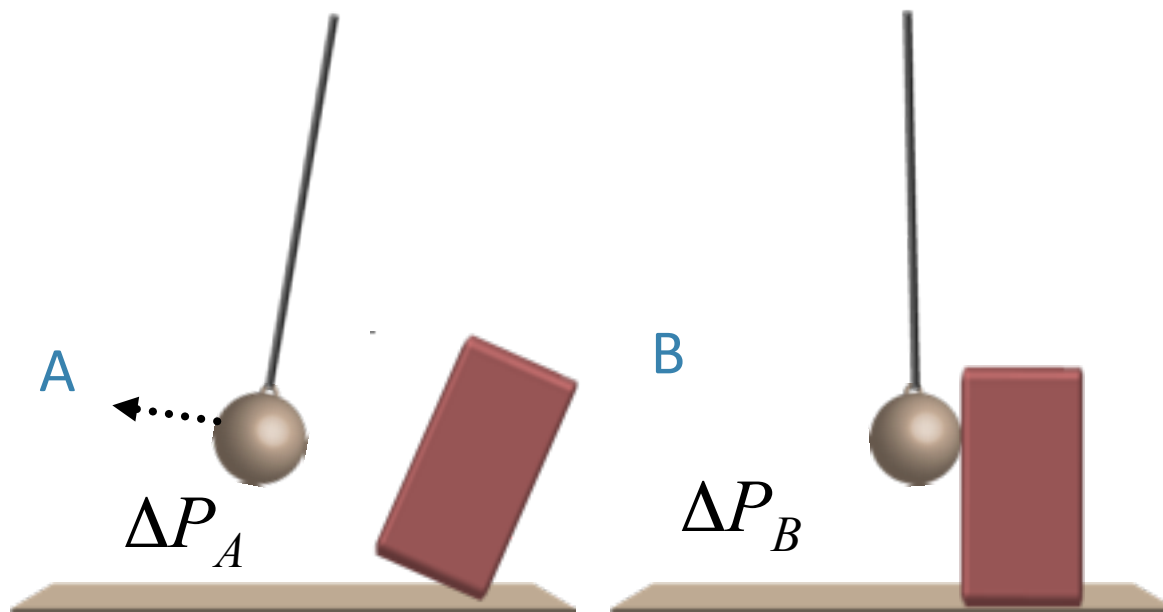
Two equal-mass balls swing down and hit identical bricks while traveling at identical speeds. Ball A bounces back, but ball B just stops when it hits the brick. Which ball is more likely to knock the brick over?



A) A

B) B

C) They both have the same chance.



$$\Delta P_A > \Delta P_B$$

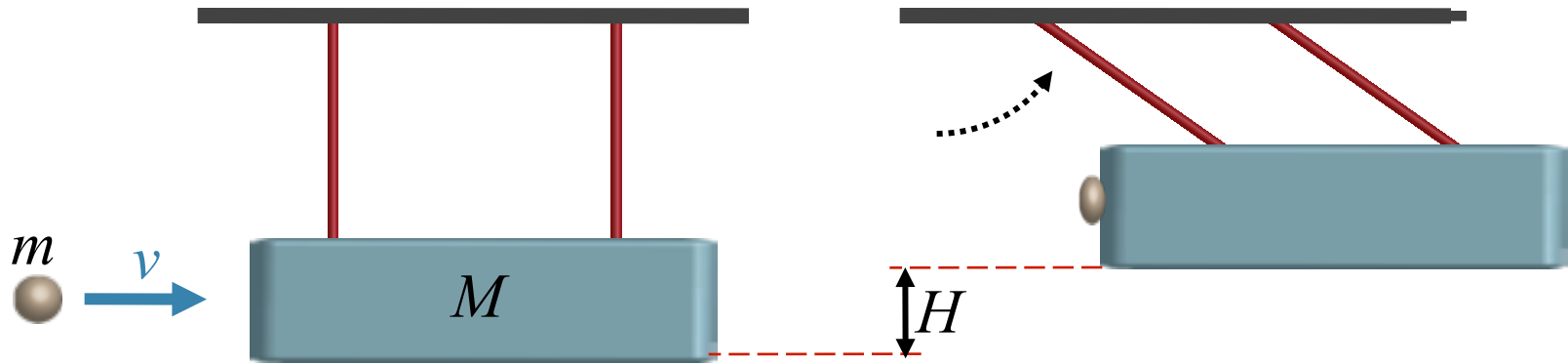
The change in the momentum of the ball is bigger in **A**

I am in dire need of more cool demos, as without them I may be tempted to like math more than physics. You wouldn't want that would you?

[http://youtu.be/GO\\_Gmfsib6M](http://youtu.be/GO_Gmfsib6M)

<http://youtu.be/7sIE03Ac3t0>

# Ballistic Pendulum



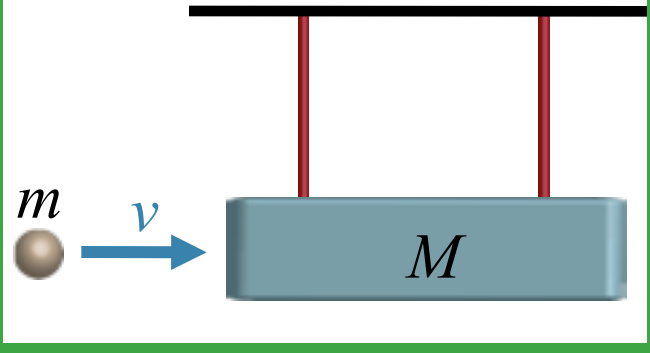
A projectile of mass  $m$  moving horizontally with speed  $v$  strikes a stationary mass  $M$  suspended by strings of length  $L$ . Subsequently,  $m + M$  rise to a height of  $H$ .

Given  $H$ , what is the initial speed  $v$  of the projectile?

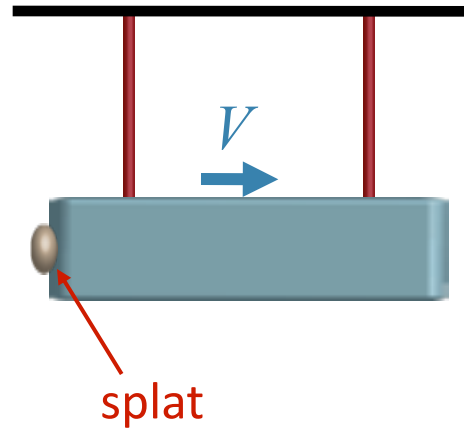
# Breaking it down into steps



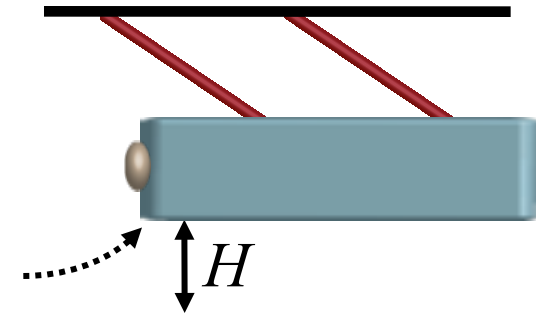
before



during



after

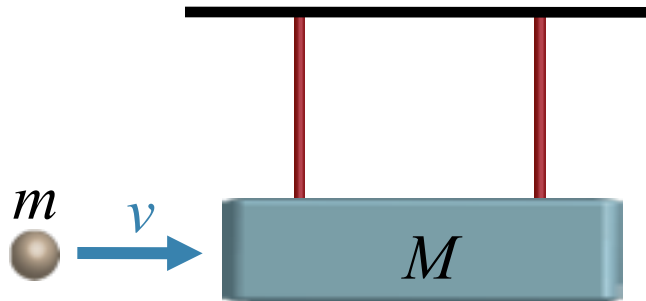


Which quantities are conserved **before** the collision?

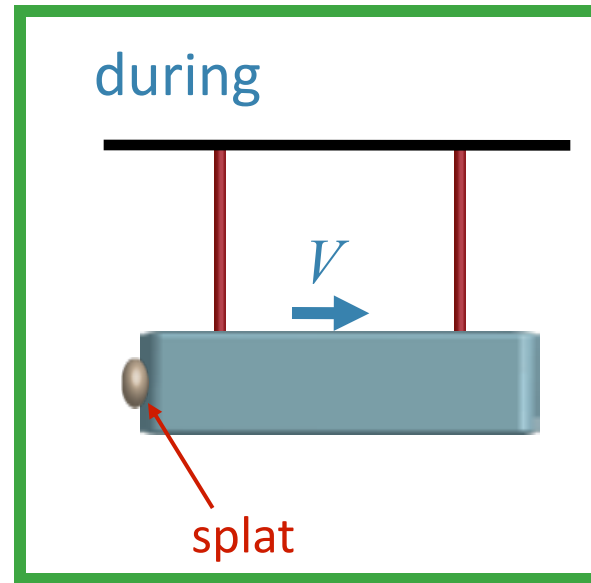
- A) momentum
- B) mechanical energy
- C) both momentum and mechanical energy**

# Breaking it down into steps

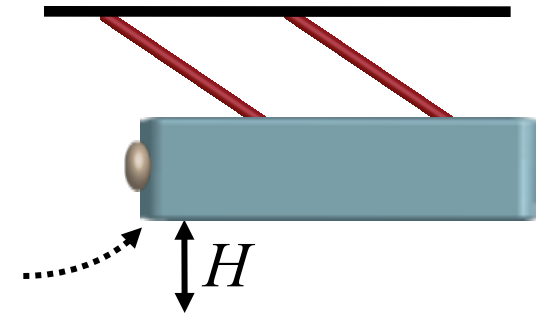
before



during



after



Which quantities are conserved **during** the collision?

A) momentum

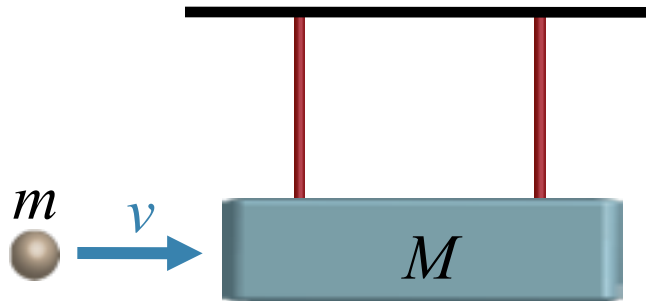
B) mechanical energy

C) both momentum and mechanical energy

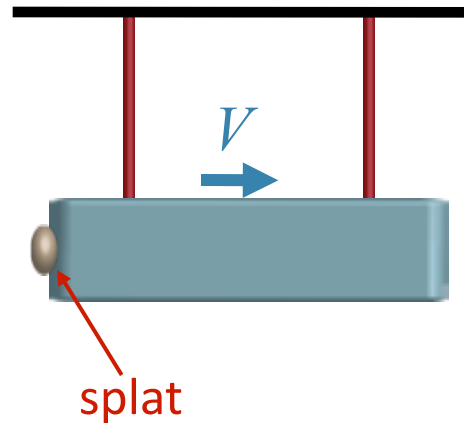
$$mv = (m + M)V$$

# Breaking it down into steps

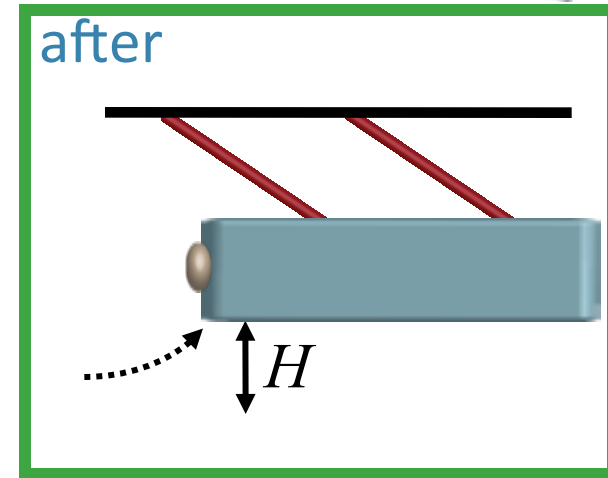
before



during



after



Which quantities are conserved **after** the collision

A) momentum

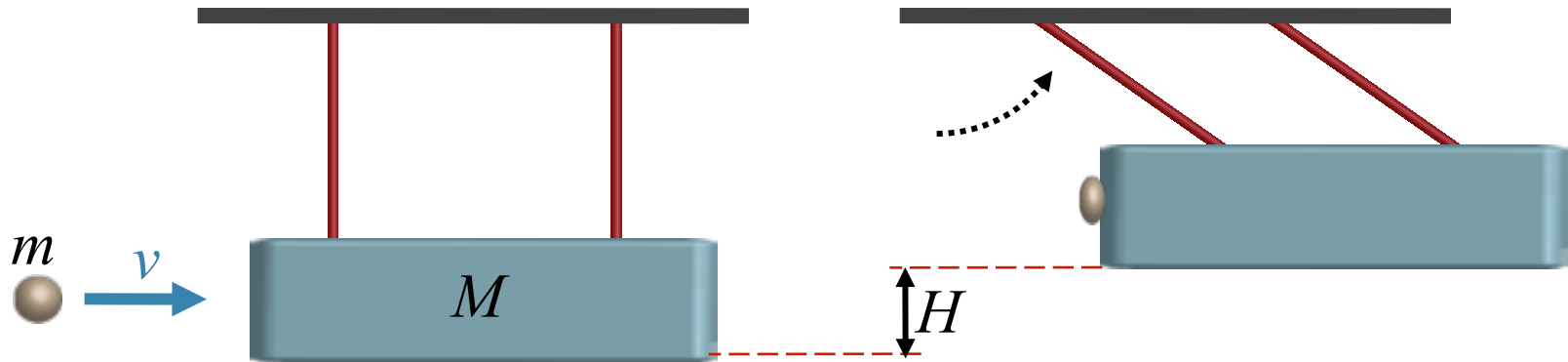
B) mechanical energy

C) both momentum and mechanical energy

$$V^2 = 2gH$$



# Ballistic Pendulum

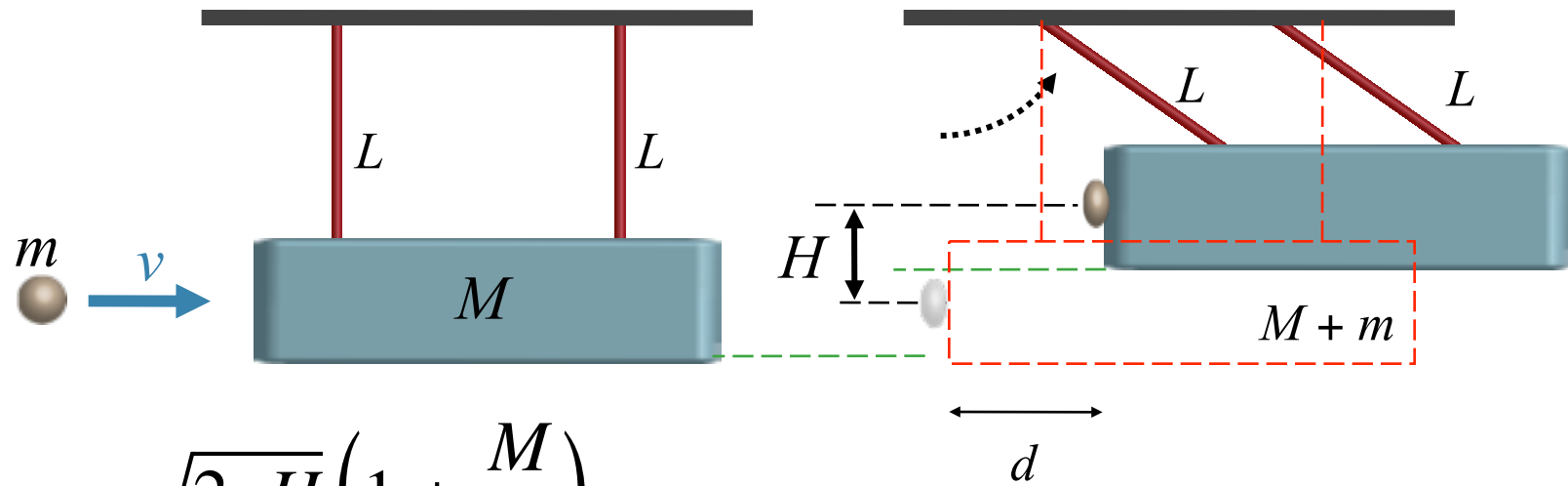


$$mv = (m + M)V \qquad V = \frac{m}{(m + M)}v$$

$$V^2 = 2gH$$

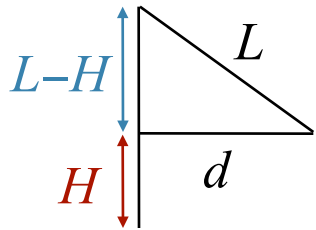
$$\rightarrow v = \sqrt{2gH} \left( \frac{m + M}{m} \right) \rightarrow v = \sqrt{2gH} \left( 1 + \frac{M}{m} \right)$$

# Ballistic Pendulum Demo



$$v = \sqrt{2gH} \left( 1 + \frac{M}{m} \right)$$

In the demo we measure forward displacement  $d$ , not  $H$ :



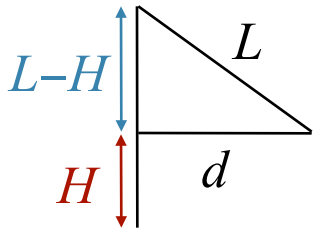
$$L^2 = d^2 + (L - H)^2$$

$$H = L - \sqrt{L^2 - d^2}$$

$$v = d \sqrt{g/L} \left( 1 + \frac{M}{m} \right) \approx (50 \text{ s}^{-1}) d$$

for  $d \ll L$

*because you asked*



$$L^2 = d^2 + (L - H)^2$$

$$H = L - \sqrt{L^2 - d^2}$$

$$v = \sqrt{2gH} \left( 1 + \frac{M}{m} \right)$$

$$v = \sqrt{2g \frac{d^2}{2L}} \left( 1 + \frac{M}{m} \right)$$

$$H = L - L \left( 1 - \frac{d^2}{2L^2} \right)$$

$$H = L - L + \frac{d^2}{2L} = \frac{d^2}{2L}$$

$$v = d \sqrt{\frac{g}{L}} \left( 1 + \frac{M}{m} \right)$$

$$m = 0.325 \text{ kg}, M = 4.216 \text{ kg}$$

$$L = 0.68 + 0.055 \text{ m} = 0.740 \text{ m}$$

$$\sqrt{g/L} = 3.56 \text{ s}^{-1}, (1 + M/m) = 13.70$$

$$\sqrt{g/L} (1 + M/m) = 50 \text{ s}^{-1}$$

$$\sqrt{1 - x} \approx 1 - \frac{x}{2}$$

# Review questions from previous midterm1

1. There are five cats for every four dogs in Felandia. If  $C$  represents the number of cats and  $D$  the number of dogs which equation best represents this relationship?

(a)  $CD = 20$

(b)  $5D = 4C$

(c)  $4D = 5C$

Use ratios. Proportional reasoning.

$$\frac{C}{D} = \frac{5}{4}$$

$$5D = 4C$$

2. The following motion diagram represents acceleration of  $+5 \text{ m/s}^2$ .



What acceleration does the next diagram represent?



(a)  $+5 \text{ m/s}^2$

(b)  $+10 \text{ m/s}^2$

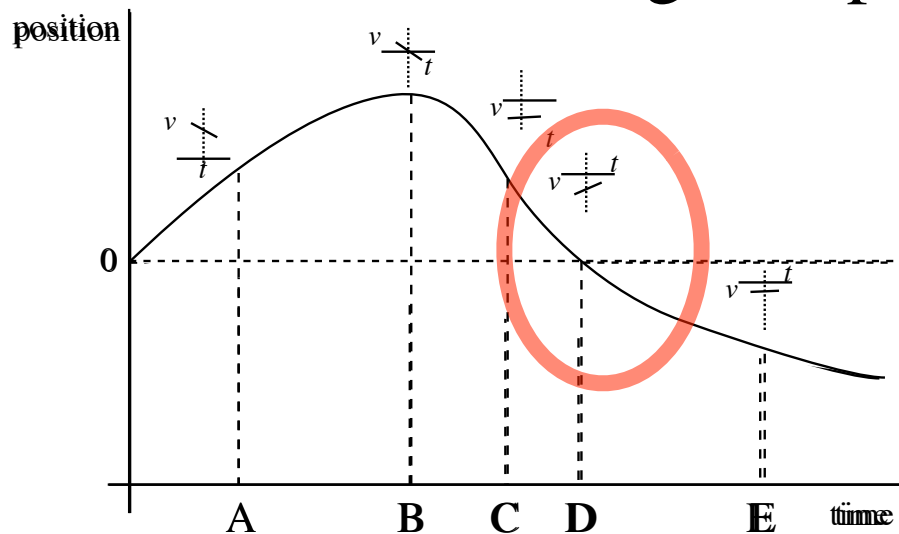
(c)  $-5 \text{ m/s}^2$

(d)  $-10 \text{ m/s}^2$

(e) None of the above

The first diagram represents negative velocity getting less negative, the second represents positive velocity getting more positive. Both represent positive acceleration. Also notice that the smallest and largest steps are the same in both cases.

6. Consider the position-time graph for the motion of a car. At which time (A,B,C,D or E) does the car have the highest positive acceleration?



# Midterm 1 #8

8. Ball A is dropped from the top of a building and, simultaneously, ball B is thrown vertically upward from  $y = 0$  with initial velocity  $v_0$ . The two balls collide at a height  $h/3$  above the ground while traveling in opposite directions. You may neglect the sizes of the balls. What is the initial velocity  $v_0$  of ball B?

For the ball that's dropped:

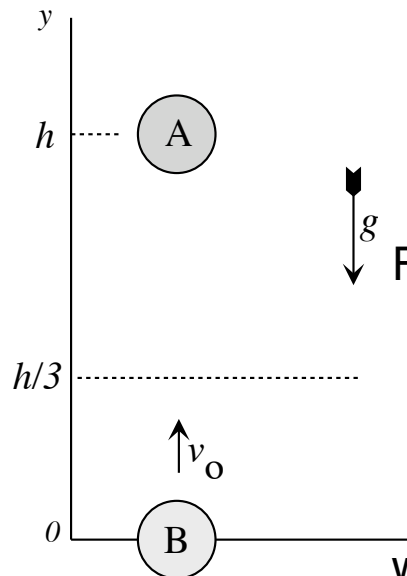
(a)  $v_0 = \sqrt{3gh}$

(b)  $v_0 = \sqrt{6gh}$

(c)  $v_0 = \frac{1}{2} \sqrt{3gh}$

(d)  $v_0 = \frac{1}{3} \sqrt{gh}$

(e) none of the above



$$h - \frac{1}{2}gt_c^2 = h/3$$

For the ball that's thrown upwards:

$$v_0 t_c - \frac{1}{2}gt_c^2 = h/3$$

where  $t_c$  is the time at which they collide. These are two equations with two unknowns. Eliminate  $t_c$  and solve for  $v_0$ . (32%/36%)