Classical Mechanics
Lecture 16

Today's Concepts:

a) Rolling Kinetic Energy

b) Angular Acceleration

Thoughts from the Past

Why is the velocity(CM) of a sphere rolling down an incline less than the velocity(CM) of a block sliding without friction. BUT!! The final kinetic energy of both are equal?

I feel the prelecture has a bit of a tendency to over complicate things. I get that they are trying to introduce integrals and such, but would a simpler explanation also really be that bad? A formula sheet would also be nice....

wow... like wow man.. what just happened.

What unmerciful doom doth rot upon the error of my soul?

What does the center of mass have to do with this section?

A little brush up on last lectures material would help, just to reinforce what has been taught

moments of inertia still throw me it takes me a couple tries to think through them and with really rotation period I am finding it more challenging despite the fact I had trouble learning it last year as well

just stop teaching these things plz -

Very confusing at first, but after I closed facebook it started to make sense.

Your Thoughts

The translational stuff and the rotational stuff added together is very confusing.

"May the NET force be with you."

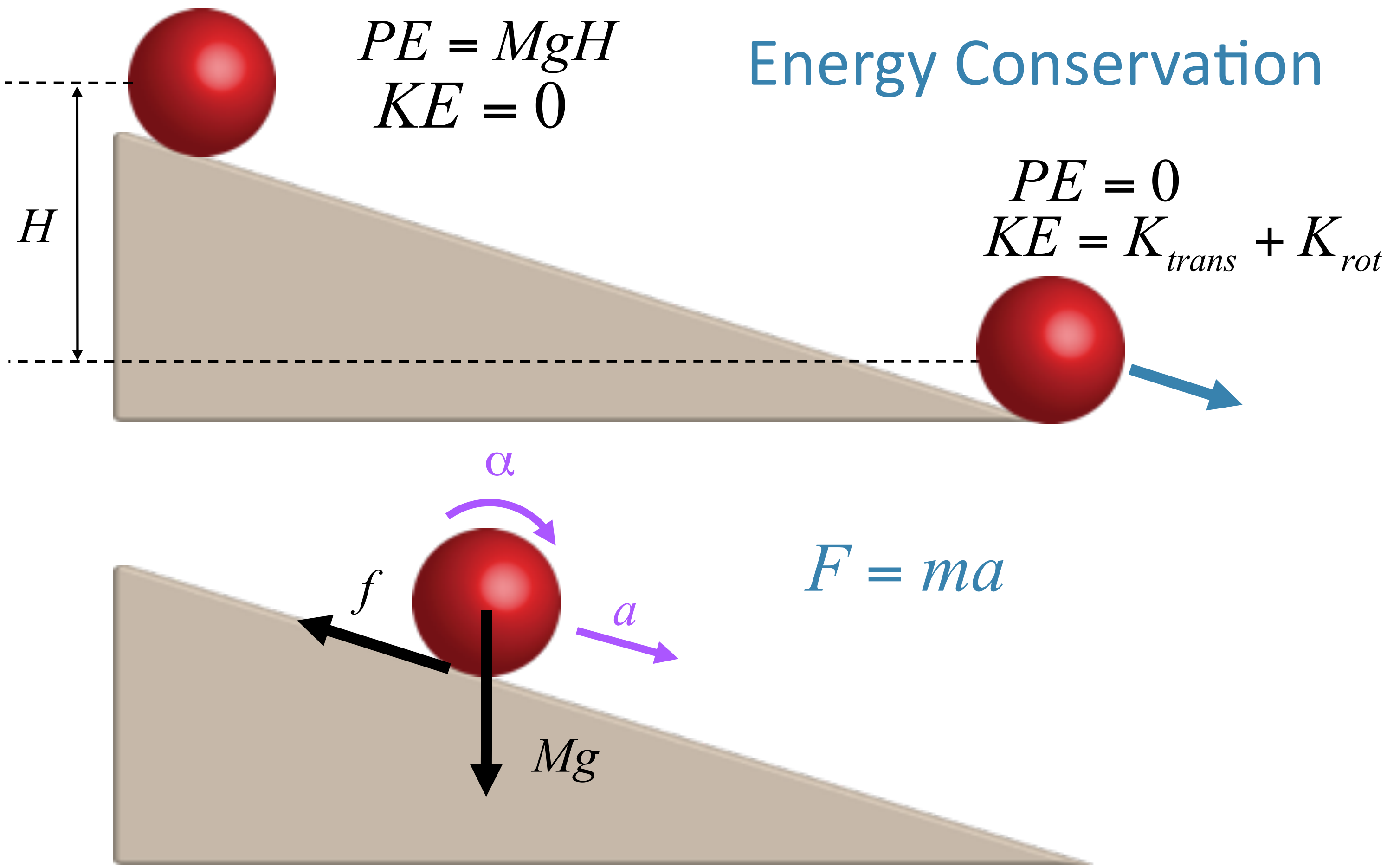
It's really weird that a block would have more acceleration than a ball

why does the mass cancel out for the 2 cylinder on ramp question? And I don't understand how they derived the acceleration of a solid sphere down a plane....

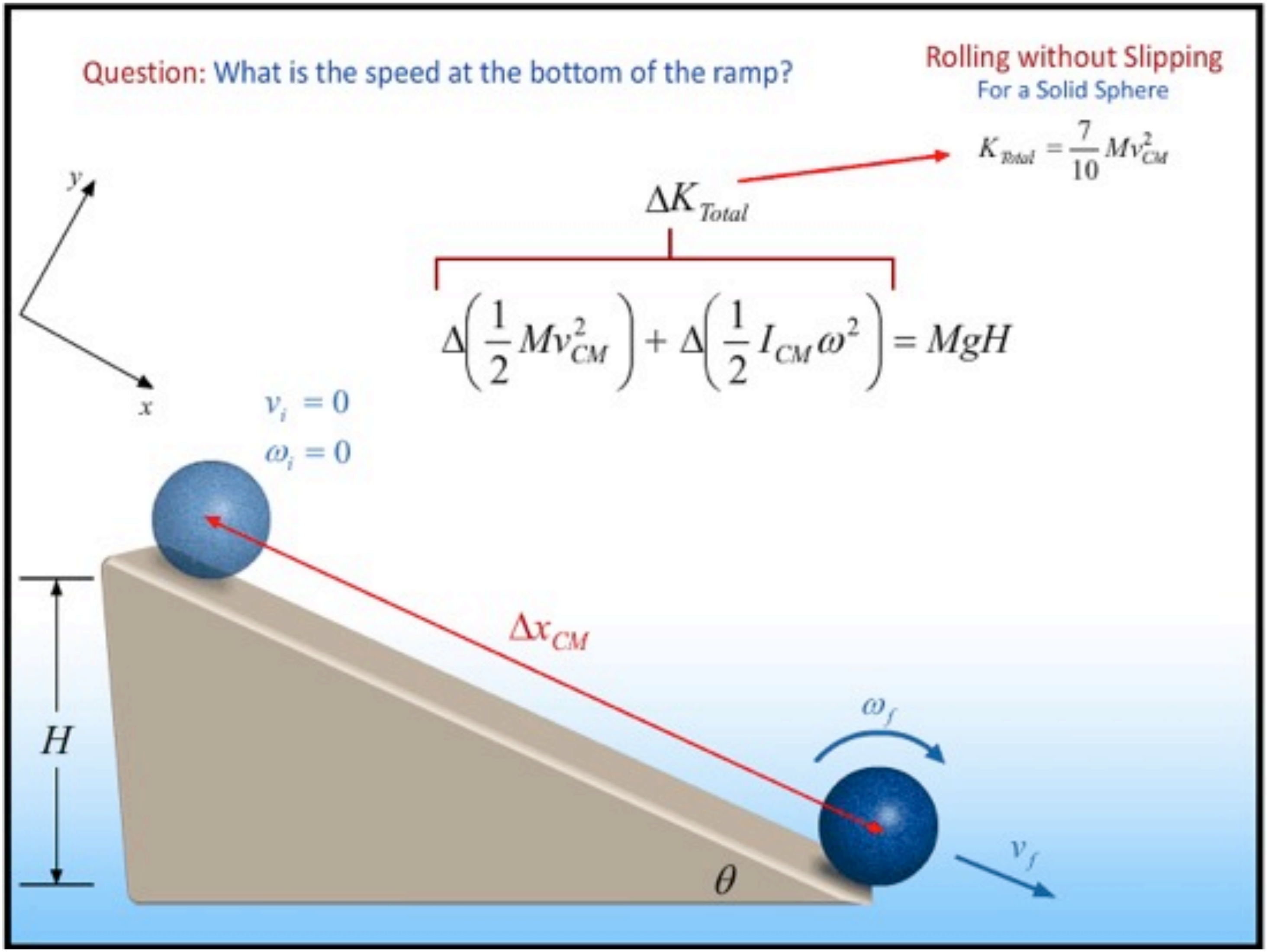
if the objects don't depend on mass, what was with the cookie tin dealio that was weighted unequally?

Rotational Dynamics is going to be the cause of my grey hair

I felt like every slide just had a ton of equations just being used to find new equations. Other than that the actual use of the equations doesn't seem so bad, but I still don't really know what to do for the CheckPoints.

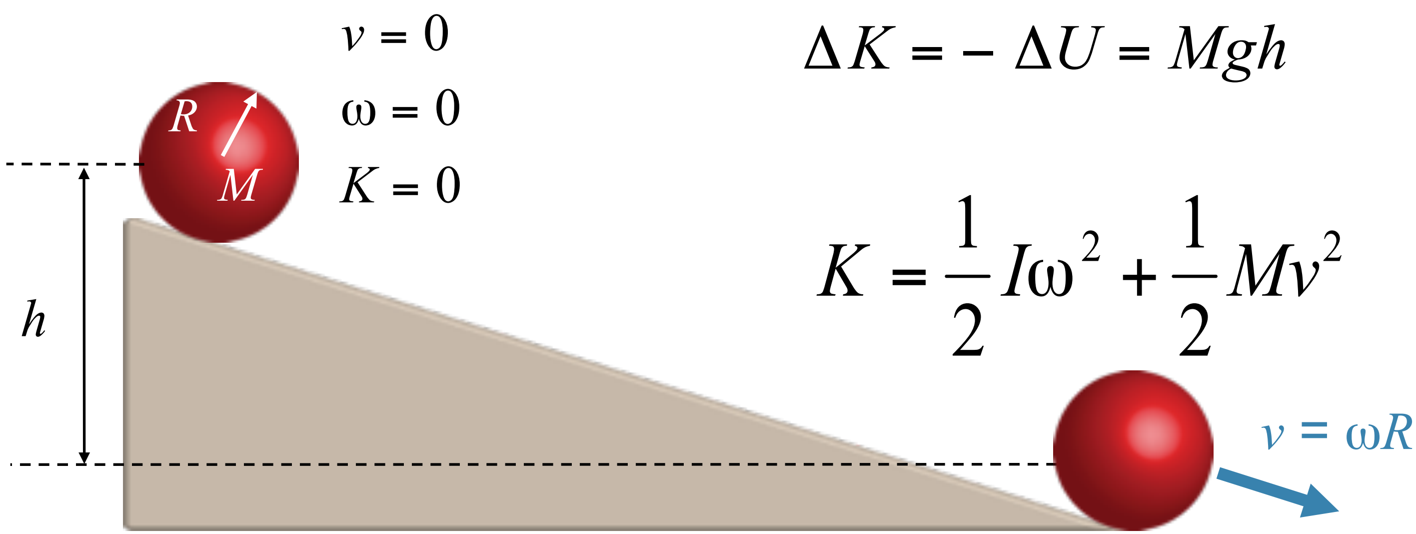


Let's work through this again.



Rolling Motion

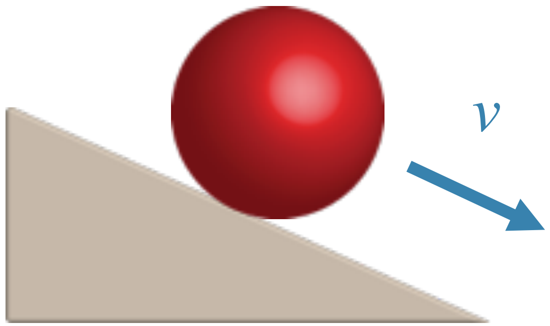
Objects of different I rolling down an inclined plane:



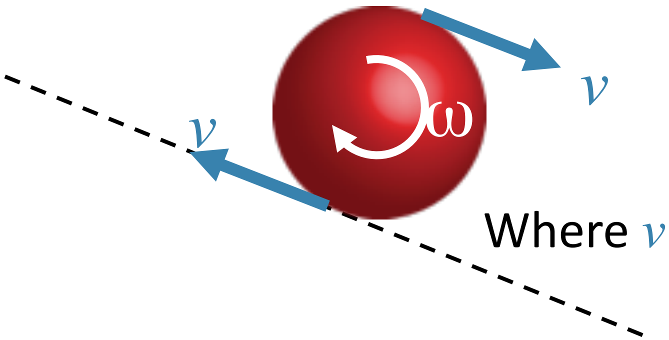
$$\Delta K = -\Delta U = Mgh$$

$$K = \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2$$

If there is no slipping:



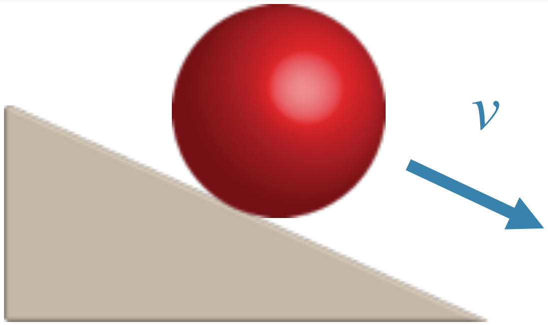
In the lab reference frame



Where $v = \omega R$

In the CM reference frame

Rolling



$$K = \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2$$ Use $v = \omega R$ and $I = cMR^2$.

$$K = \frac{1}{2} cMR^2\omega^2 + \frac{1}{2} Mv^2 = \frac{1}{2}(c+1)Mv^2$$

Hoop: $c = 1$

Disk: $c = 1/2$

Sphere: $c = 2/5$

etc...

So: $$\frac{1}{2}(c+1)Mv^2 = Mgh \quad \longrightarrow \quad \boxed{v = \sqrt{2gh}\sqrt{\frac{1}{c+1}}}$$

Doesn't depend on M or R, just on c (the shape)

Ramp demo

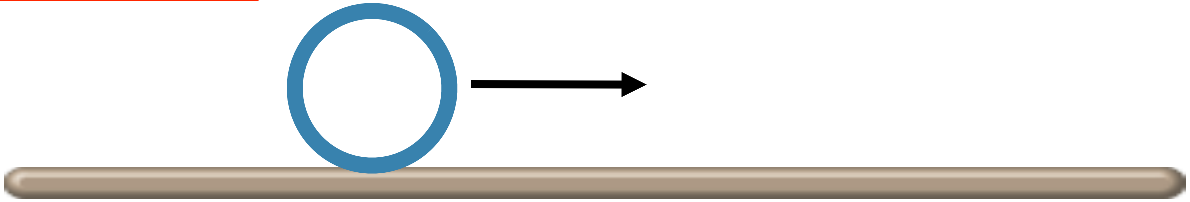
Clicker Question

A hula-hoop rolls along the floor without slipping. What is the ratio of its rotational kinetic energy to its translational kinetic energy?

A) $\dfrac{K_{rot}}{K_{trans}} = 1$

B) $\dfrac{K_{rot}}{K_{trans}} = \dfrac{3}{4}$

C) $\dfrac{K_{rot}}{K_{trans}} = \dfrac{1}{2}$



Recall that $I = MR^2$ for a hoop about

an axis through its CM.

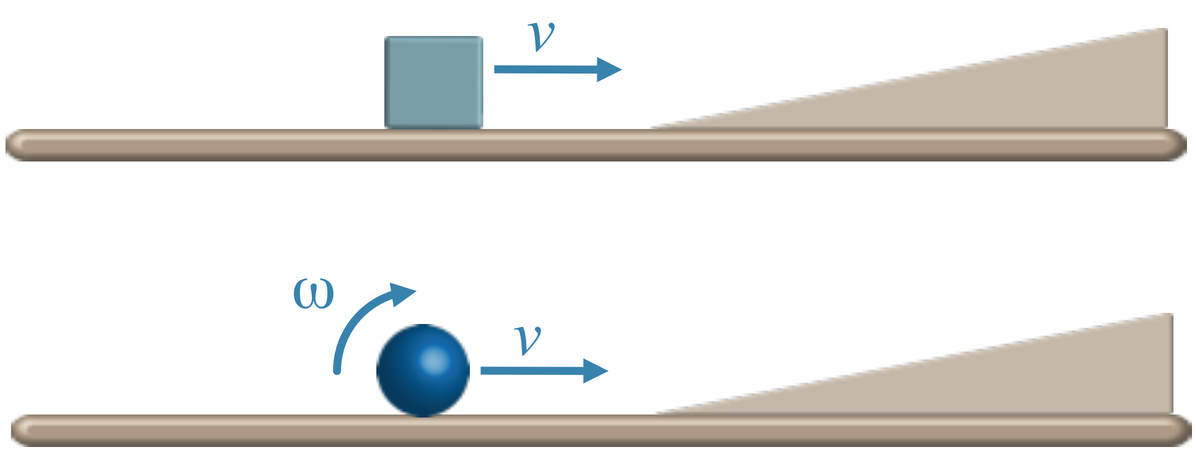
CheckPoint

A block and a ball have the same mass and move with the same initial velocity across a floor and then encounter identical ramps. The block slides without friction and the ball rolls without slipping. Which one makes it furthest up the ramp?

A) Block

B) Ball

C) Both reach the same height.



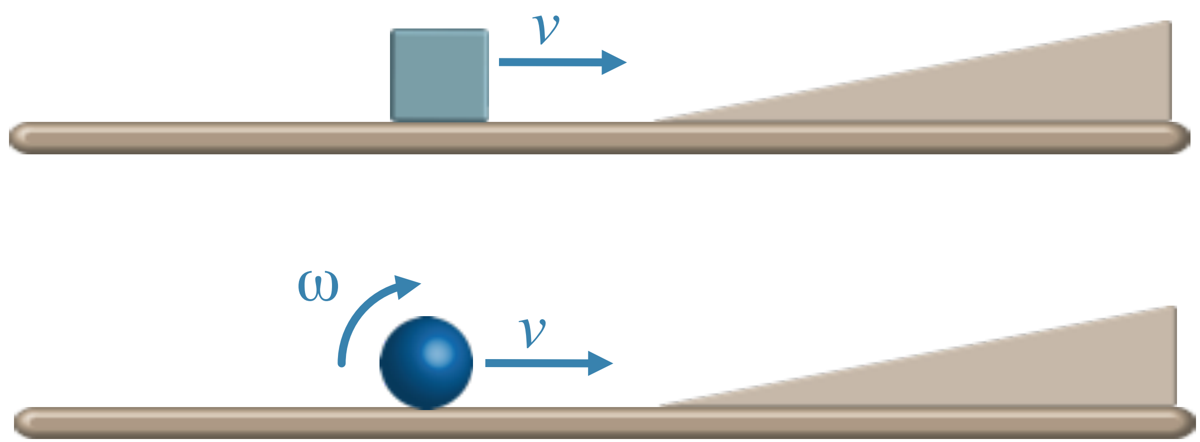
CheckPoint

The block slides without friction and the ball rolls without slipping. Which one makes it furthest up the ramp?

A) Block

B) Ball

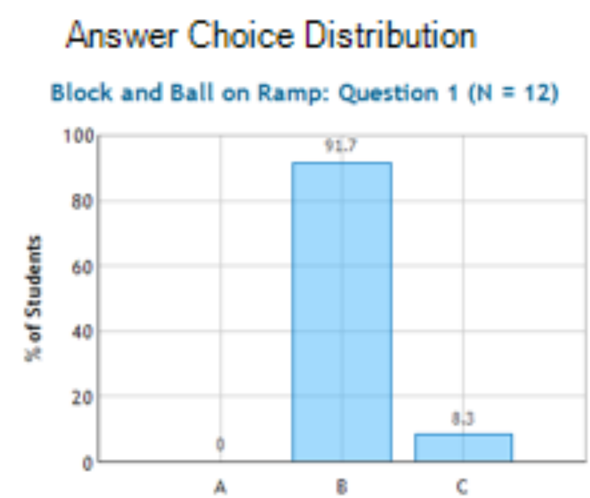
C) Same



A) The ball losses energy to rotation

B) The ball has more total kinetic energy since it also has rotational kinetic energy. Therefore, it makes it higher up the ramp.

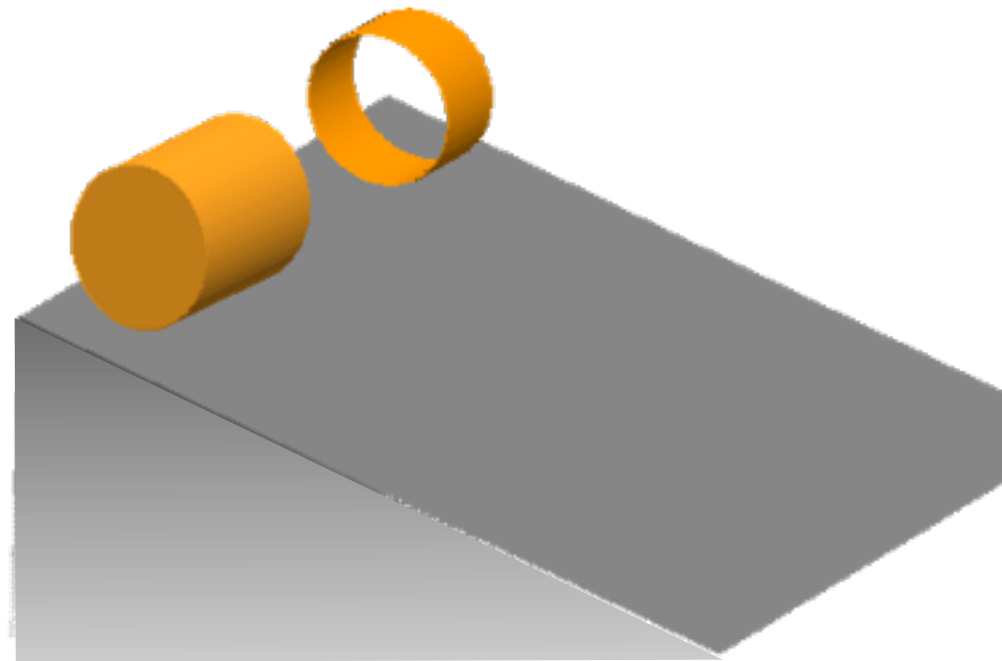
C) If they have the same velocity then they should go the same height. The rotational energy should not affect the ball.



CheckPoint

A cylinder and a hoop have the same mass and radius. They are released at the same time and roll down a ramp without slipping. Which one reaches the bottom first?

A) Cylinder
B) Hoop
C) Both reach the bottom at the same time



Which one reaches the bottom first?

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B) Hoop

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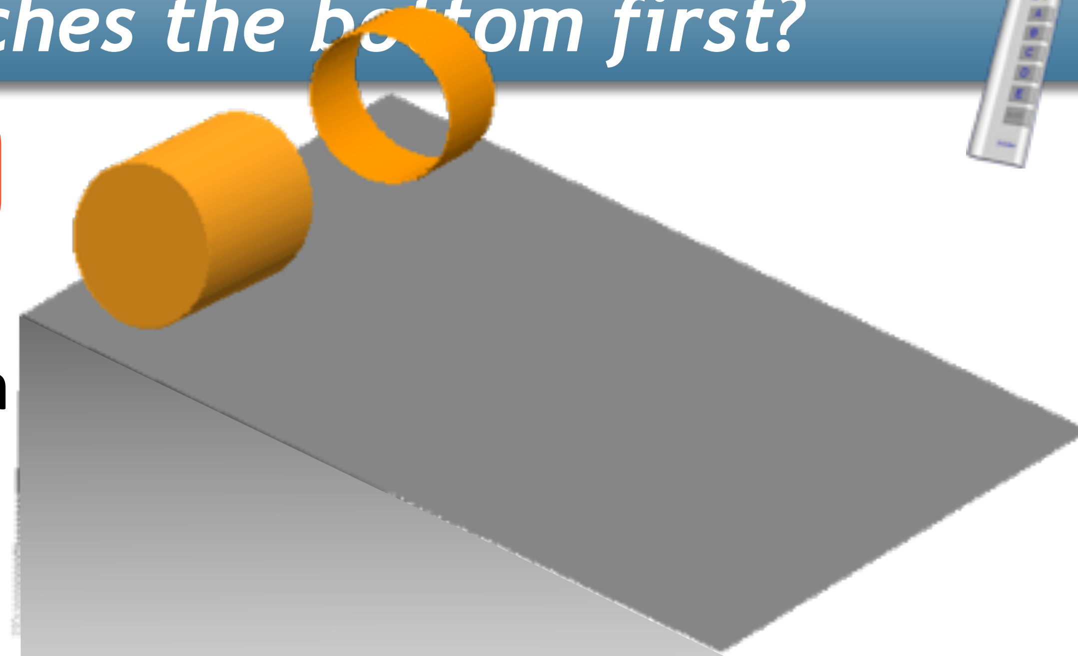
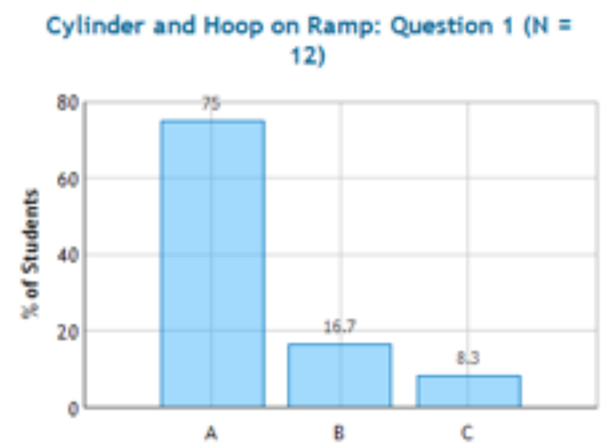


A) same PE but the hoop has a larger rotational inertia so more energy will turn into rotational kinetic energy, thus cylinder reaches it first.

B) It has more kinetic energy because it has a bigger moment of inertia.

C) Both have the same PE so they will have the same KE and reach the ground at the same time.



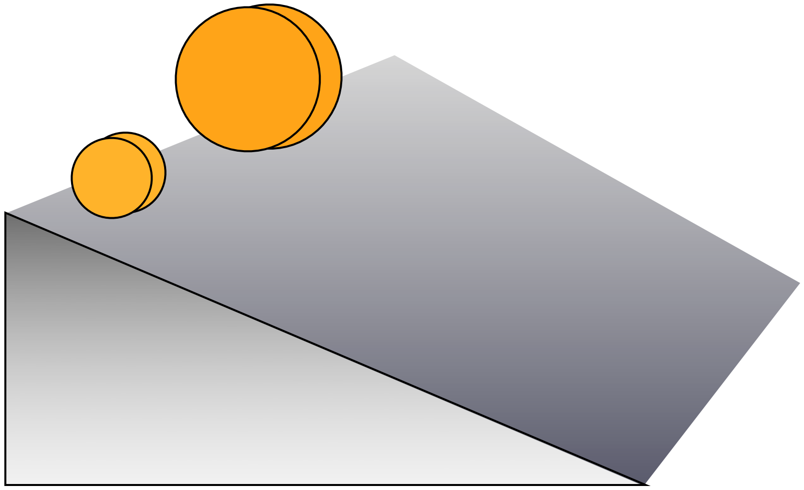
CheckPoint

A small light cylinder and a large heavy cylinder are released at the same time and roll down a ramp without slipping. Which one reaches the bottom first?

A) Small cylinder

B) Large cylinder

C) Both reach the bottom at the same time



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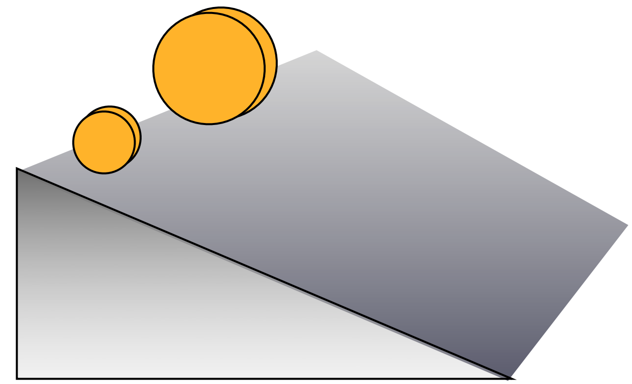
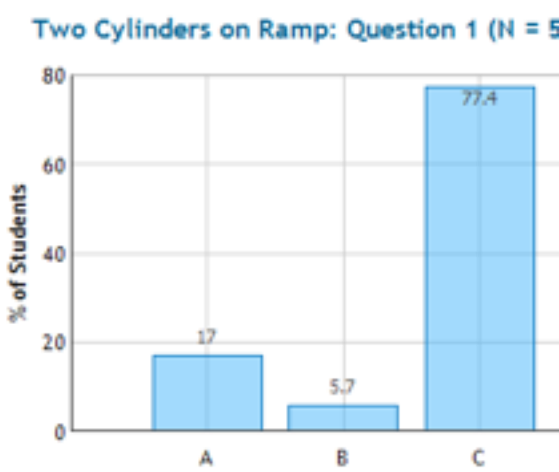


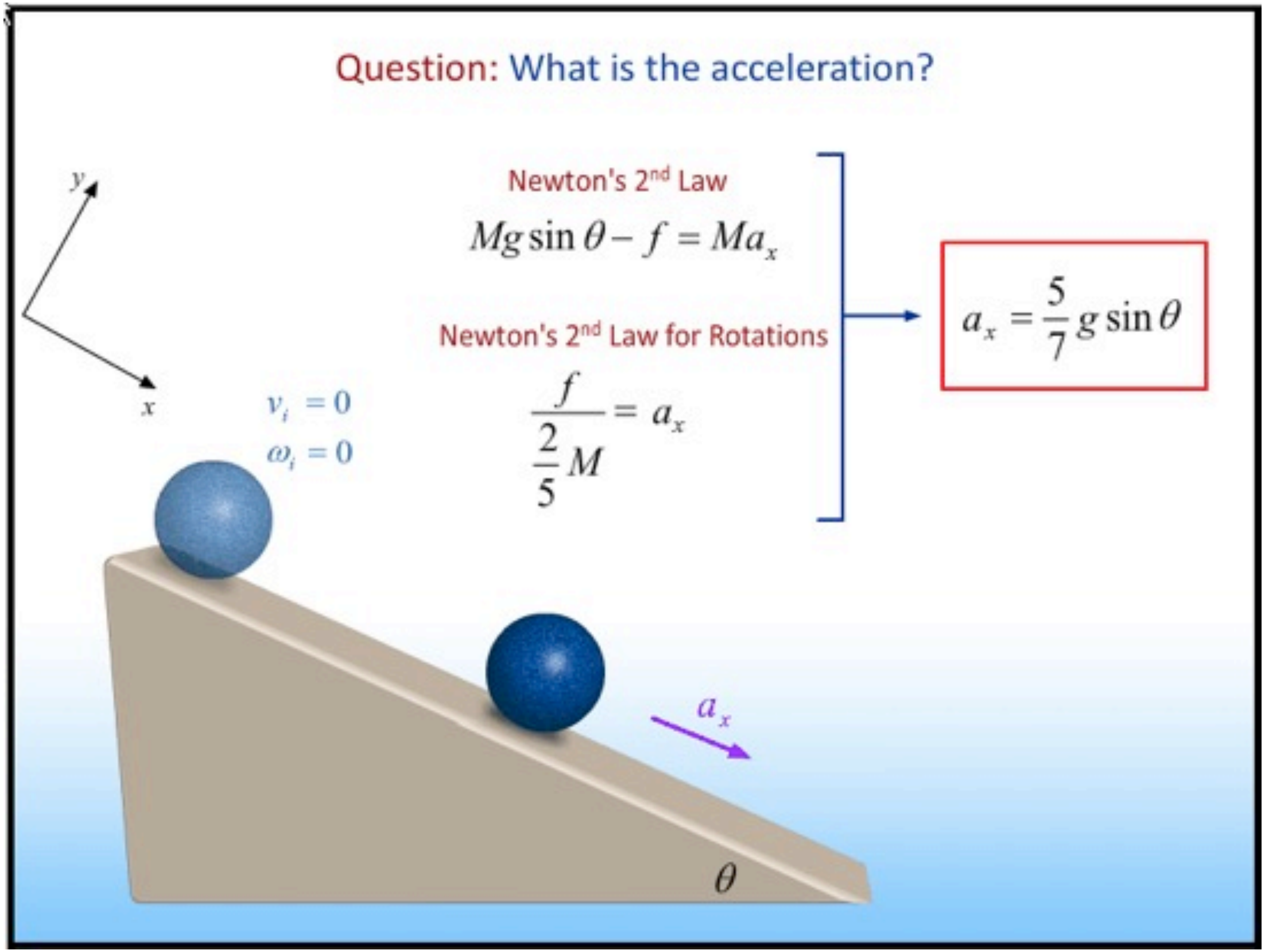
A) Because the smaller one has a smaller moment of inertia.

B) The large cylinder has a larger moment of inertia, and therefore, ends up with more energy.

C) The mass is canceled out in the velocity equation and they are the same shape so they move at the same speed. Therefore, they reach the bottom at the same time.



What you saw in your Prelecture:



Acceleration depends only on the shape, not on mass or radius.

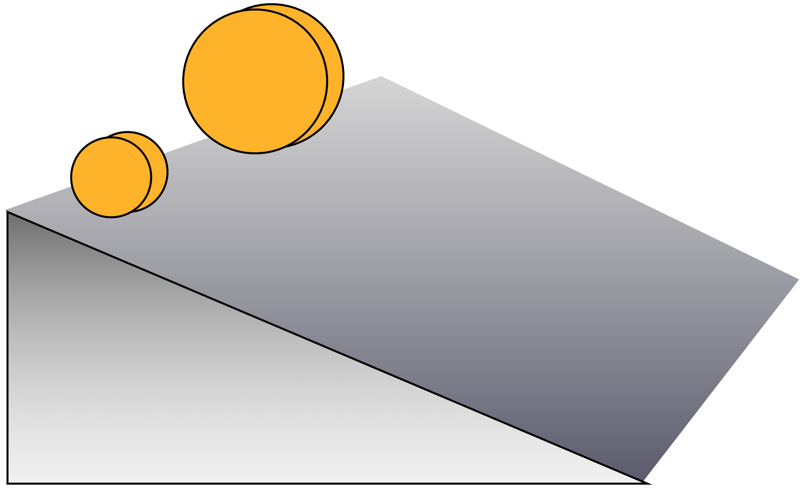
Clicker Question

A small light cylinder and a large heavy cylinder are released at the same time and slide down the ramp without friction. Which one reaches the bottom first?

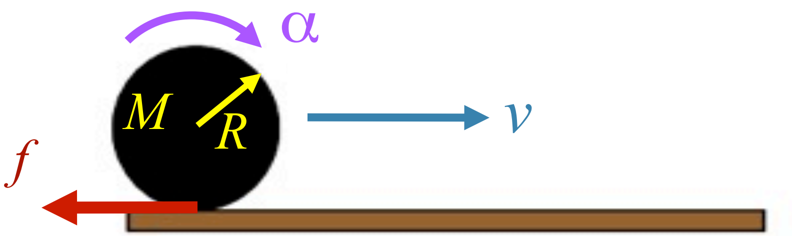
A) Small cylinder

B) Large cylinder

C) Both reach the bottom at the same time



Bowling Ball



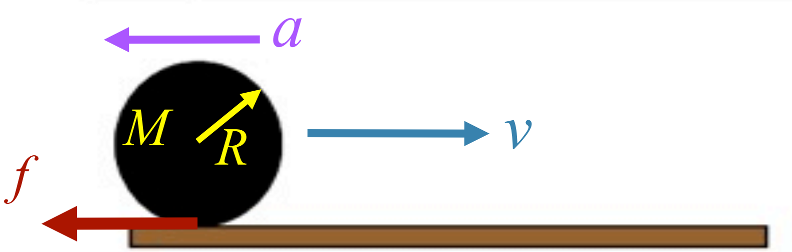
A spherical bowling ball with mass m = 4.2 kg and radius R = 0.114 m is thrown down the lane with an initial speed of v = 8.7 m/s. The coefficient of kinetic friction between the sliding ball and the ground is μ = 0.32. Once the ball begins to roll without slipping it moves with a constant velocity down the lane.

1) What is the magnitude of the angular acceleration of the bowling ball as it slides down the lane?

rad/s^2 Submit

$$\tau = I\alpha \;\rightarrow\; \alpha = \frac{\tau}{I} = \frac{fR}{I} = \frac{\mu MgR}{\frac{2}{5}MR^2} = \frac{5\mu g}{2R}$$

Bowling Ball



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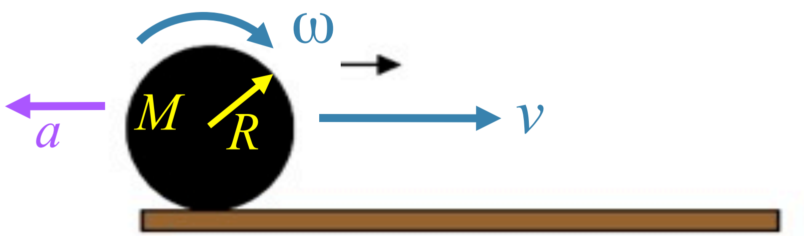
2) What is magnitude of the linear acceleration of the bowling ball as it slides down the lane?

[ ] m/s^2 Submit

$$F = Ma \rightarrow a = \frac{F}{M} = \frac{\mu M g}{M} = \mu g$$

Bowling Ball



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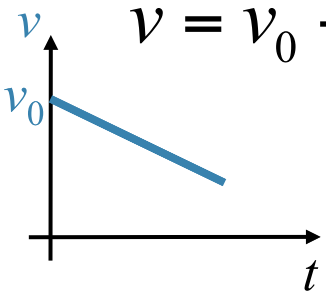
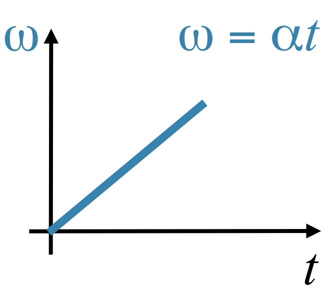
3) How long does it take the bowling ball to begin rolling without slipping?

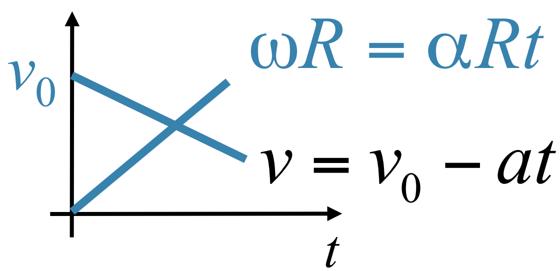
[ ] s Submit

$$v = v_0 - at$$





Once $v = \omega R$ it rolls without slipping



$$a = \mu g$$

$$a = \frac{5\mu g}{2R}$$

$$v = \omega R$$

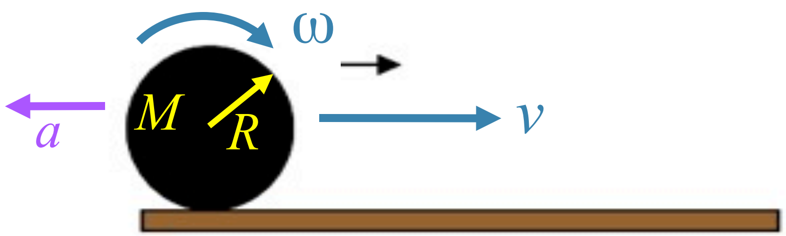
$$v_0 - at = \alpha t R$$

$$v - \mu g t = \frac{5\mu g}{2R} t$$

$$v_0 = \frac{7\mu g}{2} t$$

$$t = \frac{2}{7\mu g} v_0$$

Bowling Ball



$$t = \frac{2}{7\mu g} v_0$$

$$a = \mu g$$

4) How far does the bowling ball slide before it begins to roll without slipping?

m Submit

5) What is the magnitude of the final velocity?

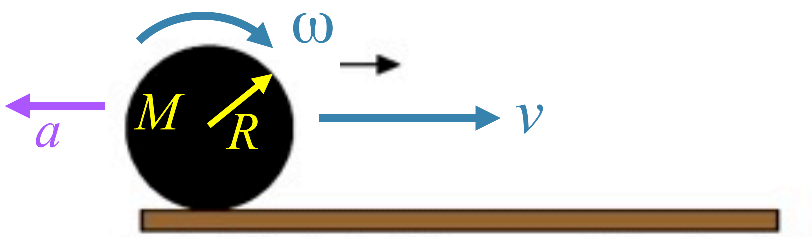
m/s Submit

$$x = v_0 t - \frac{1}{2} a t^2$$

$$v = v_0 - at$$

Plug in a and t found in parts 2) & 3)

Bowling Ball



$$a = \mu g$$

$$t = \frac{2}{7\mu g} v_0$$

Interesting aside: how v is related to v_0 :

$$v = v_0 - at$$

$$v = v_0 - \mu g \left(\frac{2v_0}{7\mu g} \right)$$

$$v = v_0 - \frac{2}{7} v_0$$

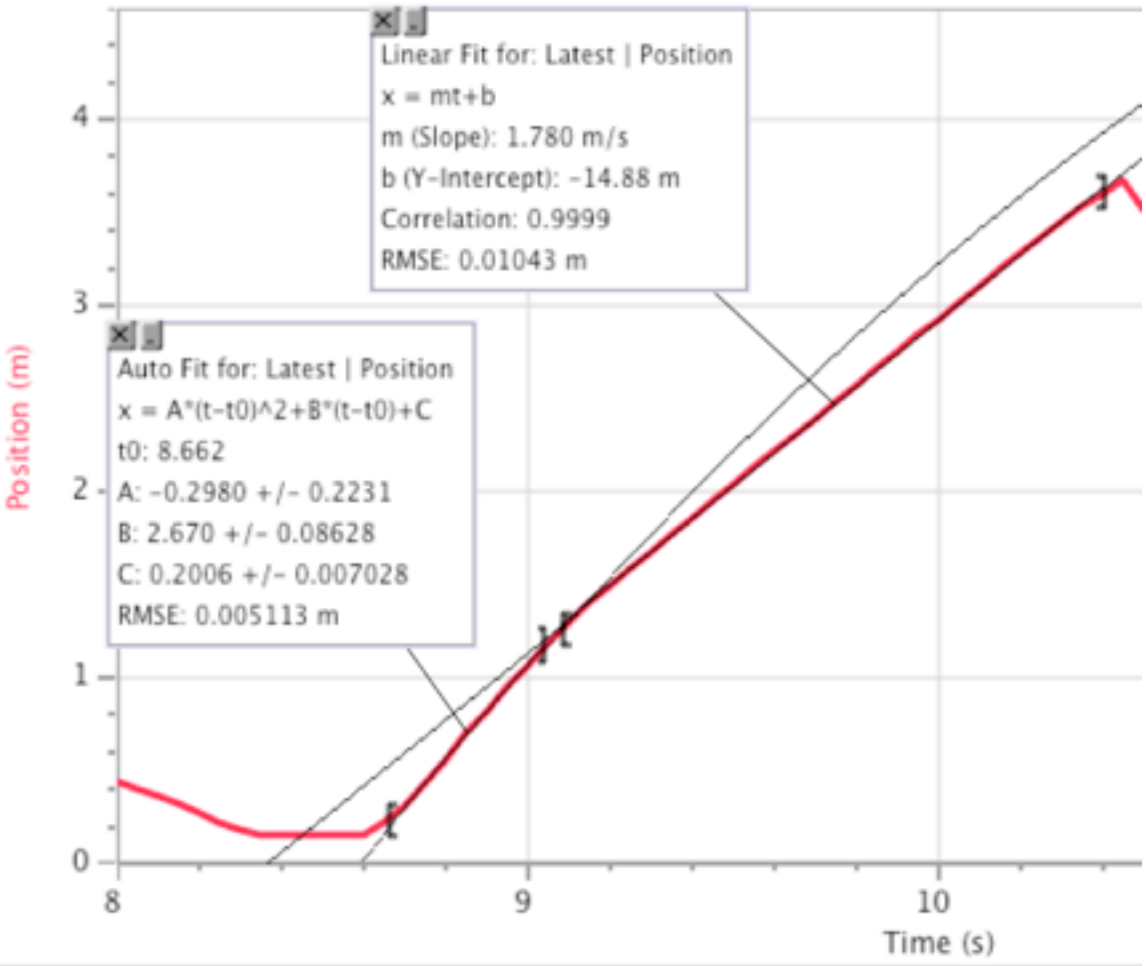
$$v = \frac{5}{7} v_0$$

Doesn't depend on μ

$$v = (0.714) v_0$$

We can try this…

Position vs time for Bowling Ball



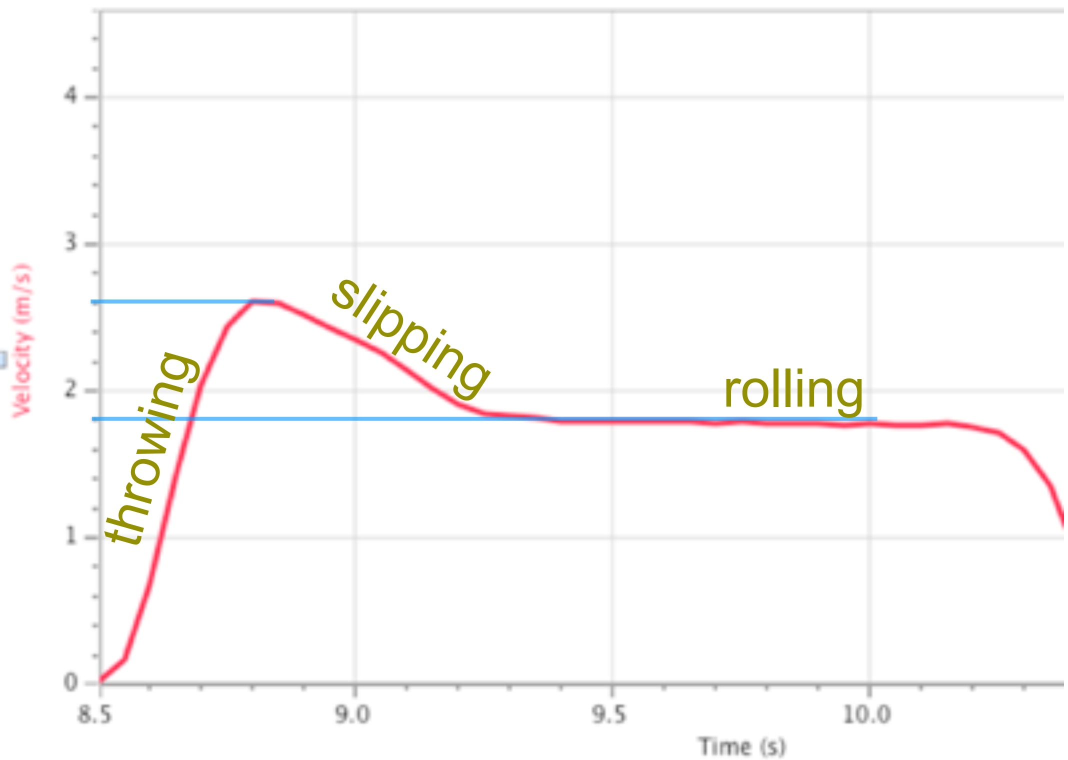
Quadratic curve fit to region 8.66 s to 9.05 s:

$v_0 = 2.67$ m/s

Linear fit to region 9.1 s to 10.3 s

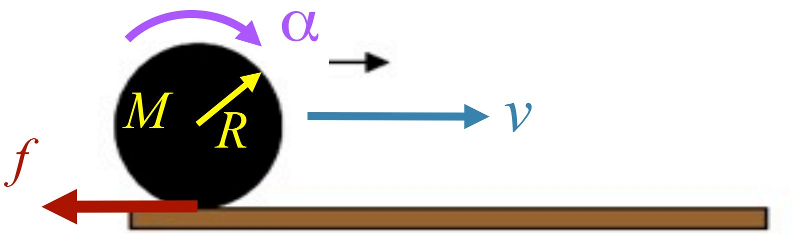
$v = 1.78$ m/s

Linear velocity vs time



the velocity vs time graph shows that clearly v is nearly (5/7) v_0

Bowling Ball



6) After the bowling ball begins to roll without slipping, compare the rotational and translational kinetic energy of the bowling ball:

- $KE_{rot} < KE_{tran}$
- $KE_{rot} = KE_{tran}$
- $KE_{rot} > KE_{tran}$

$$K_{tran} = \frac{1}{2}Mv^2$$

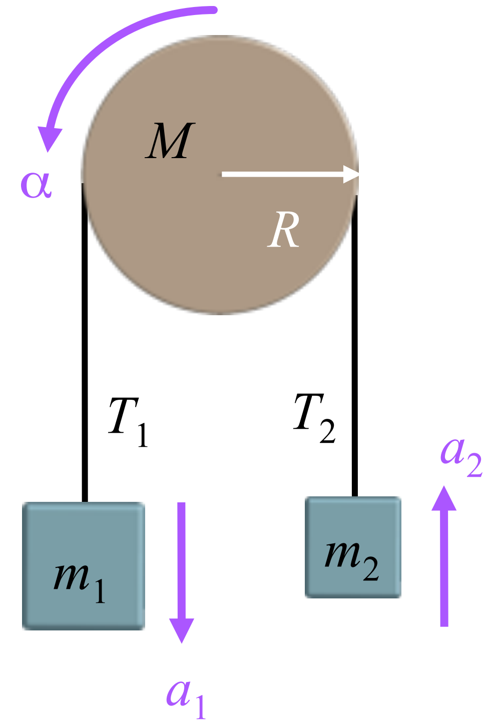
$$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{5}Mv^2$$

Clicker Question

Suppose a cylinder (radius R, mass M) is used as a pulley. Two masses ($m_1 > m_2$) are attached to either end of a string that hangs over the pulley, and when the system is released it moves as shown. The string does not slip on the pulley.

Compare the magnitudes of the acceleration of the two masses:

A) $a_1 > a_2$
B) $a_1 = a_2$
C) $a_1 < a_2$



Clicker Question

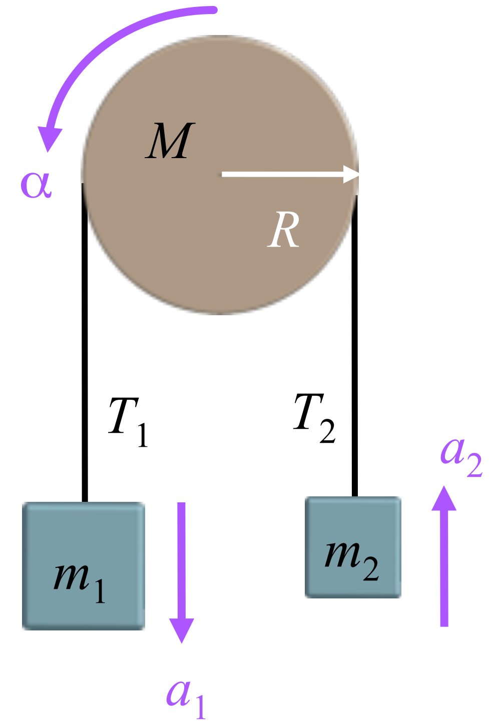
Suppose a cylinder (radius R, mass M) is used as a pulley. Two masses ($m_1 > m_2$) are attached to either end of a string that hangs over the pulley, and when the system is released it moves as shown. The string does not slip on the pulley.

How is the angular acceleration of the wheel related to the linear acceleration of the masses?

A) $\alpha = Ra$

B) $\alpha = a/R$

C) $\alpha = R/a$

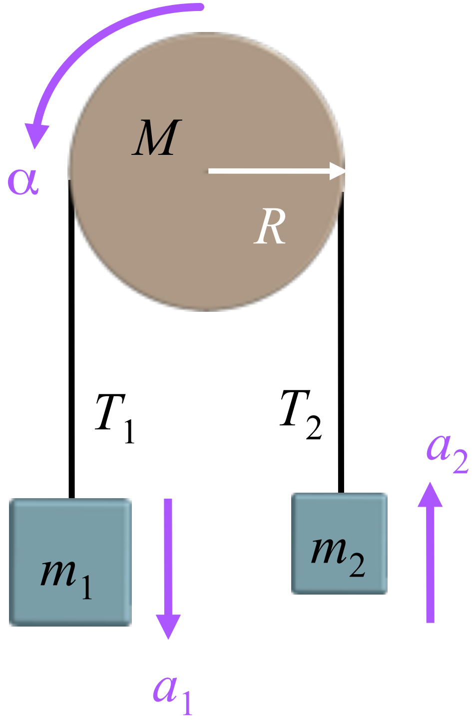


Clicker Question

Suppose a cylinder (radius R, mass M) is used as a pulley. Two masses ($m_1 > m_2$) are attached to either end of a string that hangs over the pulley, and when the system is released it moves as shown. The string does not slip on the pulley.

Compare the tension in the string on either side of the pulley:

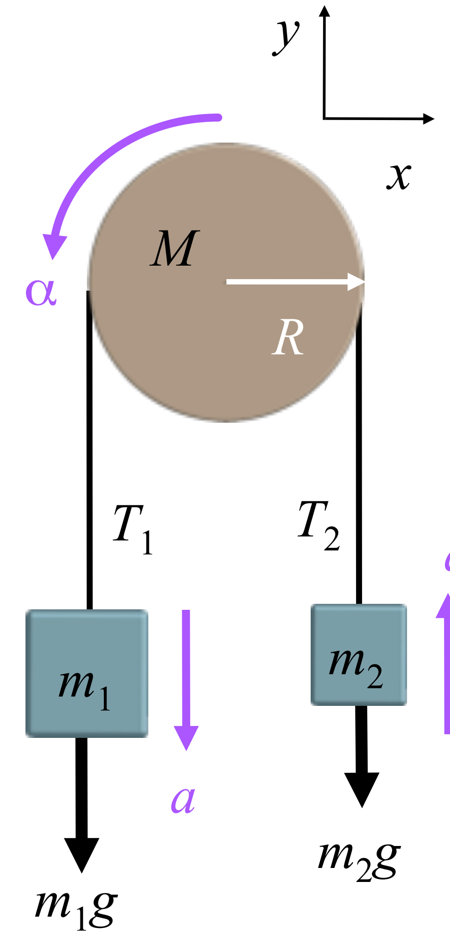
A) $T_1 > T_2$
B) $T_1 = T_2$
C) $T_1 < T_2$



Atwood's Machine with Massive Pulley:

A pair of masses are hung over a massive disk-shaped pulley as shown.

- Find the acceleration of the blocks.



For the hanging masses use $F = ma$

$$-m_1 g + T_1 = -m_1 a$$

$$-m_2 g + T_2 = m_2 a$$

For the pulley use $\tau = I\alpha = I\frac{a}{R}$

$$T_1 R - T_2 R = I\frac{a}{R} = \frac{1}{2}MRa$$

(Since $I = \frac{1}{2}MR^2$ for a disk)

Atwood's Machine with Massive Pulley:

We have three equations and three unknowns (T_1, T_2, a).

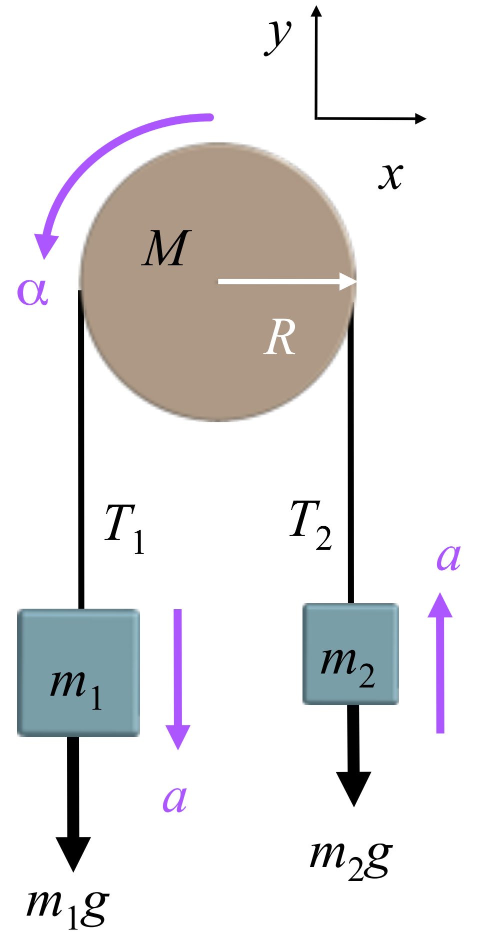
Solve for a.

$$-m_1 g + T_1 = -m_1 a \quad (1)$$

$$-m_2 g + T_2 = m_2 a \quad (2)$$

$$T_1 - T_2 = \frac{1}{2} M a \quad (3)$$

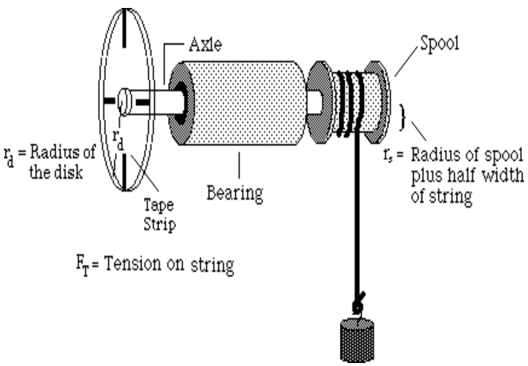
$$a = \left(\frac{m_1 - m_2}{m_1 + m_2 + M/2} \right) g$$



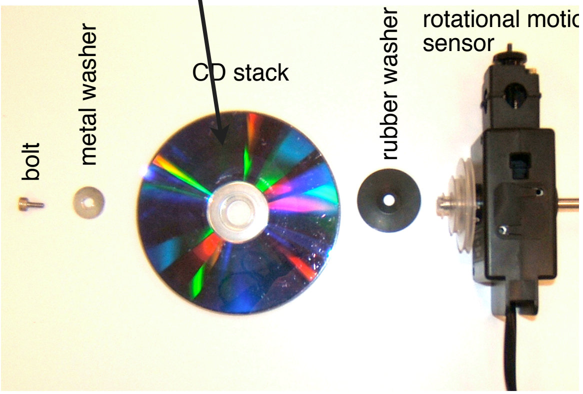
Yo-Yo

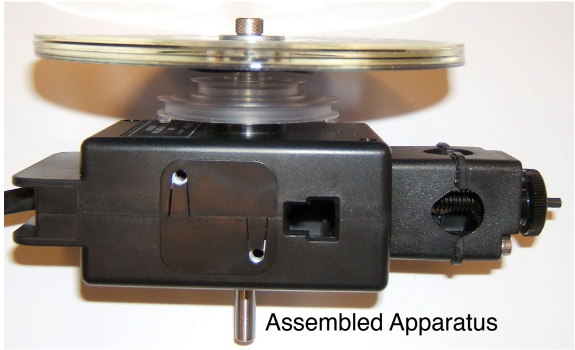
Solution coming later.....

rubber washer maybe not needed.



use 12” LP





Deanna and the Machine

http://alberding.phys.sfu.ca/wordpress/?page_id=465