Classical Mechanics Lecture 17

Today's Concept:

Angular Momentum

Your Comments

So does L=r x p or L= Iω?

Maybe its not about today's lecture ,,,, but can you make more explanations about those vector notations , the "i hat" "j hat" thing ,,, we are confused about how to use them , we saw them on mid terms but we didn't know what to do :)

How does an object not moving in a circle have an angular momentum?

It was pretty understandable. However, if I throw a playdo at a rotating object and friction allows my clay to stick to it and go at the same angular speed, is there loss of momentum? I know energy will lost

Having an "L" represent angular momentum is confusing me :(

I don't underst ... rotation. I u ... n the velocity

I feel that the 2 ... y tight time f

I really like how

The wheels ... and roun ... y to town.

Yahoo answers: This comes from the "right hand rule" of vector cross products. The vector for the radius, r-> and the vector for the translational momentum p-> when made into a cross product :
r × p-> = L
Since r and p are in the x-y plane, they are perpendicular to each other and form an "L" shape when made to be part of a cross product.

Source: I'm an Engineering/Physics major.

Angular Momentum

We have shown that for a system of particles

$$\vec{F}_{EXT} = \frac{d\vec{p}}{dt}$$

→

Momentum is conserved if

$$\vec{F}_{EXT} = 0$$

What is the rotational version of this?

The rotational analogue of force F is torque $\vec{\tau} = \vec{r} \times \vec{F}$

Define the rotational analogue of momentum p to be

Angular Momentum: $\vec{L} = \vec{r} \times \vec{p}$

For a symmetric solid object $\vec{L} = I\vec{\omega}$

Torque & Angular Momentum

$$\vec{\tau}_{EXT} = \frac{d\vec{L}}{dt}$$

where

$$\vec{L} = \vec{r} \times \vec{p}$$
$$\vec{L} = I\vec{\omega}$$

and

$$\vec{\tau}_{ext} = \vec{r} \times \vec{F}_{EXT}$$

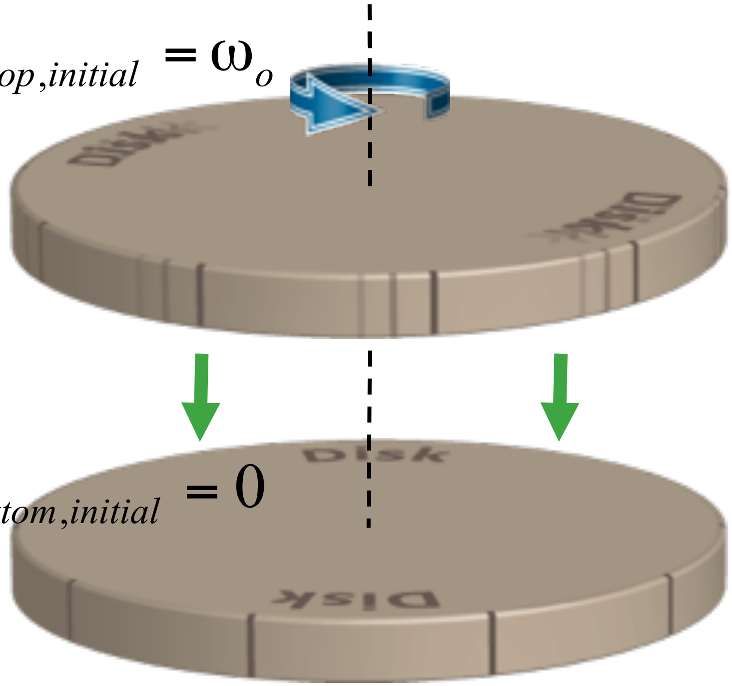
In the absence of external torques $\vec{\tau}_{EXT} = \frac{d\vec{L}}{dt} = 0$

↓

Total angular momentum is conserved

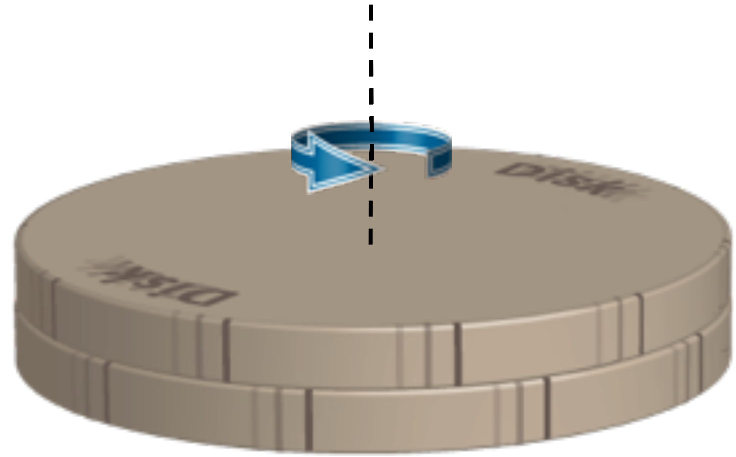
Example - Disk dropped on Disk

$$\omega_{top,initial} = \omega_o$$



$$\omega_{bottom,initial} = 0$$

$$\omega_{top,final} = \omega_{bottom,final} \equiv \omega_f$$

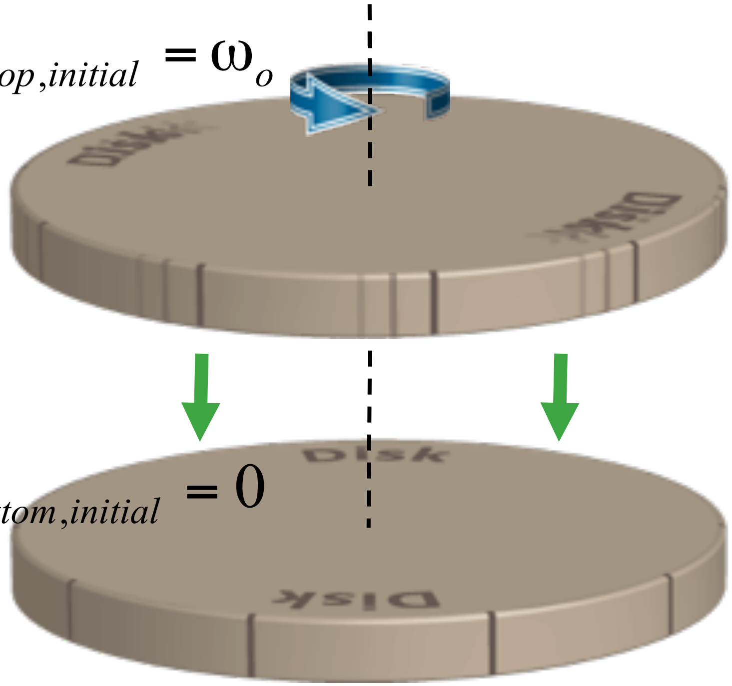


$$L_{initial} = \frac{1}{2} M R^2 \omega_o + 0 \quad = \quad L_{final} = \frac{1}{2} M R^2 \omega_f + \frac{1}{2} M R^2 \omega_f$$

$$\frac{1}{2} M R^2 \omega_o = M R^2 \omega_f \quad \rightarrow \quad \boxed{\omega_f = \frac{1}{2}\omega_0}$$

What about Kinetic Energy?

$$\omega_{top,initial} = \omega_o$$



$$\omega_{bottom,initial} = 0$$

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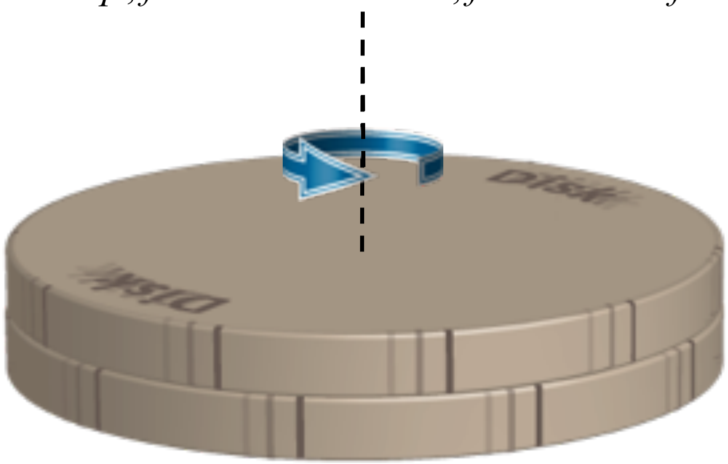


$$K = \frac{1}{2} I\omega^2 = \frac{L^2}{2I}$$

$$K_{before} = \frac{L^2}{2I_{before}}$$

$$K_{after} = \frac{L^2}{2I_{after}}$$

same

x2

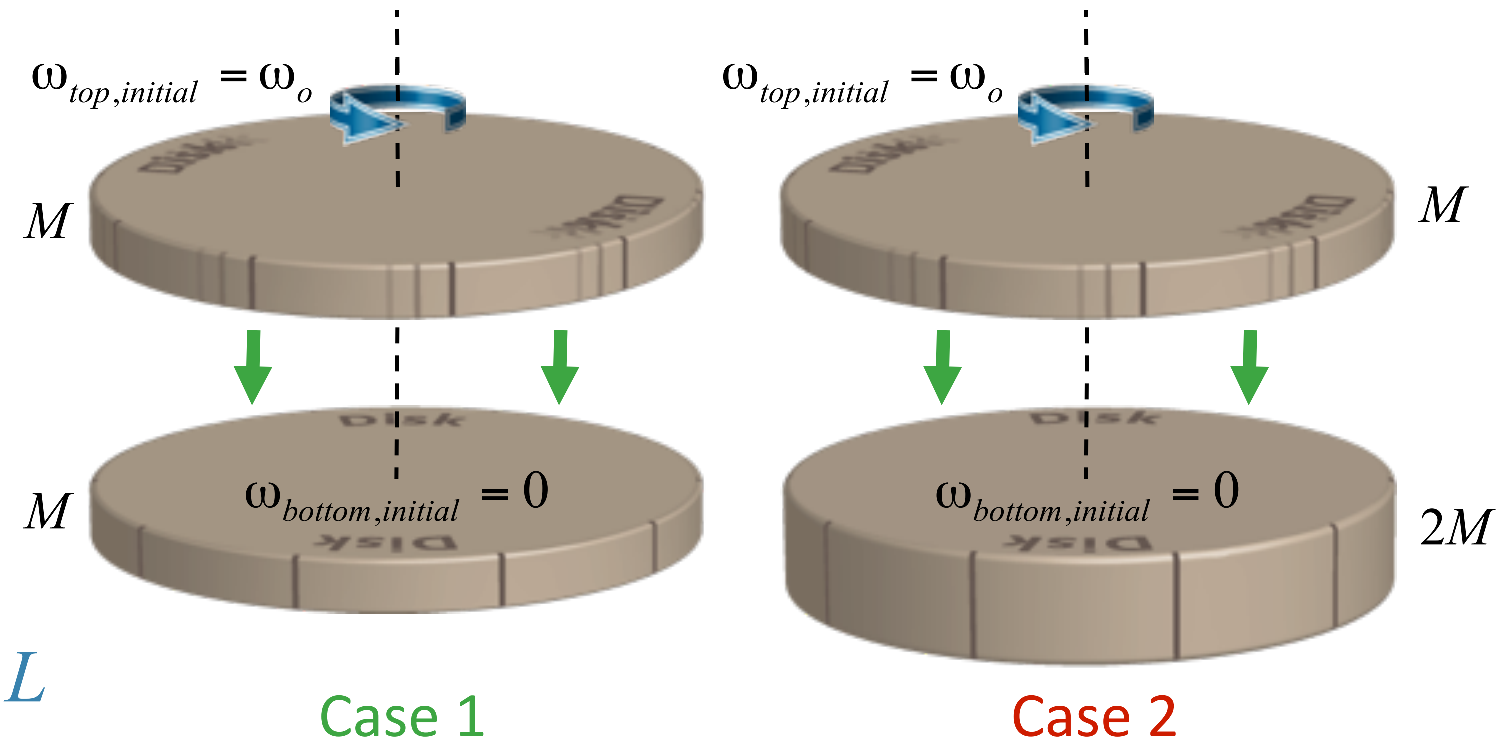
CheckPoint

In both cases shown below a solid disk of mass M, radius R, and initial angular velocity ω_o is dropped onto an initially stationary second disk having the same radius. In Case 2 the mass of the bottom disk is twice as big as in Case 1. If there are no external torques acting on either system, in which case is the final kinetic energy of the system biggest?

A) Case 1

B) Case 2

C) Same



Same initial L

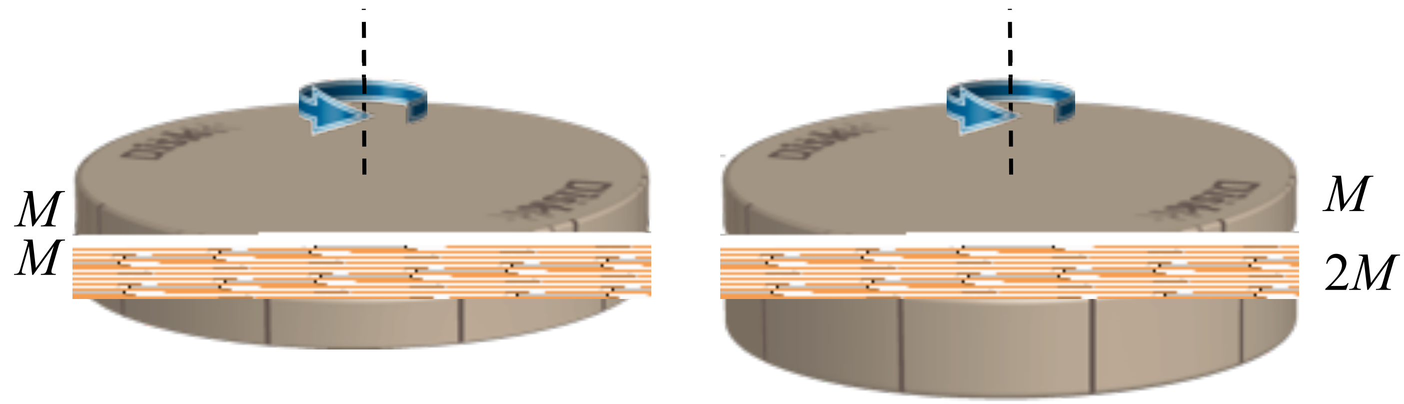
CheckPoint

In which case is the final kinetic energy of the system biggest?

A) Case 1

B) Case 2

C) Same



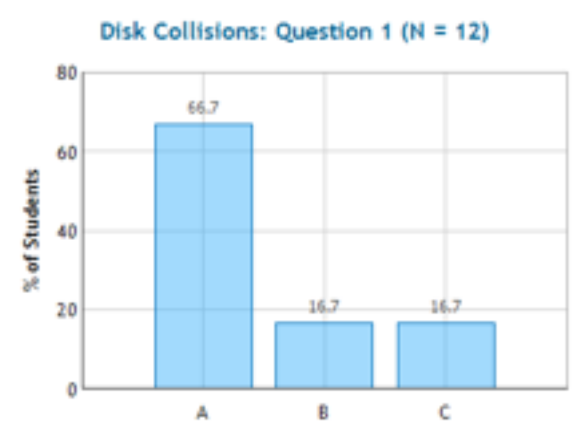
Case 1 Case 2

$$K = \frac{L^2}{2I}$$

A) L is equal but bigger mass in case 2

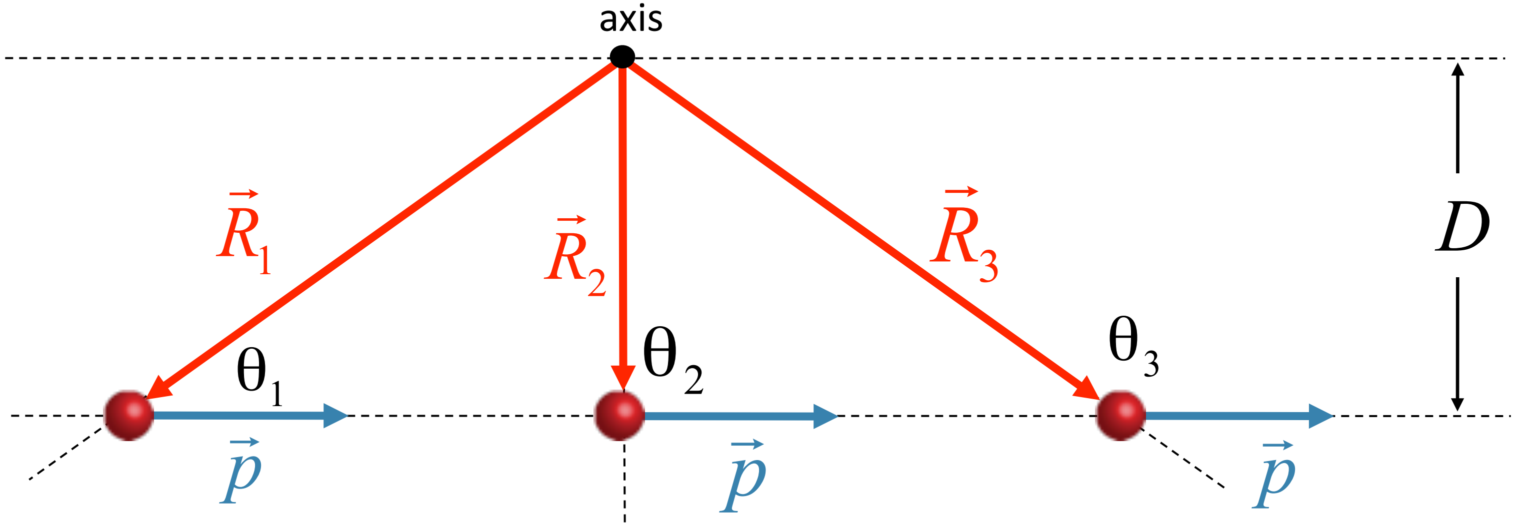
B) Case 2 has greater mass

C) Conservation of angular momentum



Point Particle moving in a Straight Line

$$\vec{L} = \vec{R} \times \vec{p}$$



$$\vec{L}_1 = \vec{R}_1 \times \vec{p} = R_1 p \sin(\theta_1) = pD$$

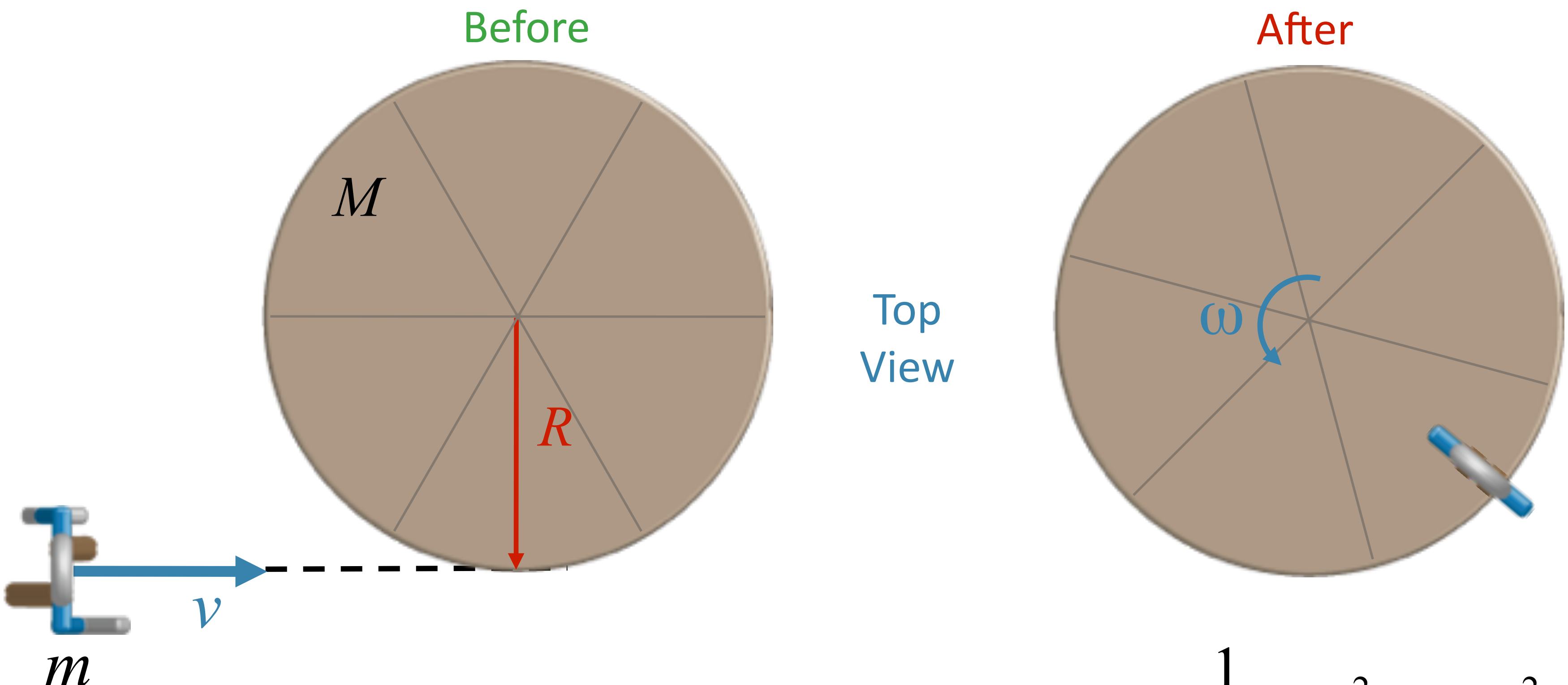
$$\vec{L}_2 = \vec{R}_2 \times \vec{p} = R_2 p \sin(\theta_2) = pD$$

$$\vec{L}_3 = \vec{R}_3 \times \vec{p} = R_3 p \sin(\theta_3) = pD$$

$$\vec{L} = \vec{R} \times \vec{p} \quad \longrightarrow \quad L = mvD = pD$$

Direction given by right hand rule
(out of the page in this case)

Playground Example



$$L_{Before} = mvR$$

$$L_{After} = I\omega = \left(\frac{1}{2}MR^2 + mR^2\right)\omega$$

$$L_{Before} = L_{After} \quad \rightarrow \quad \omega = \frac{v}{R}\frac{1}{\left(1 + M/2m\right)}$$

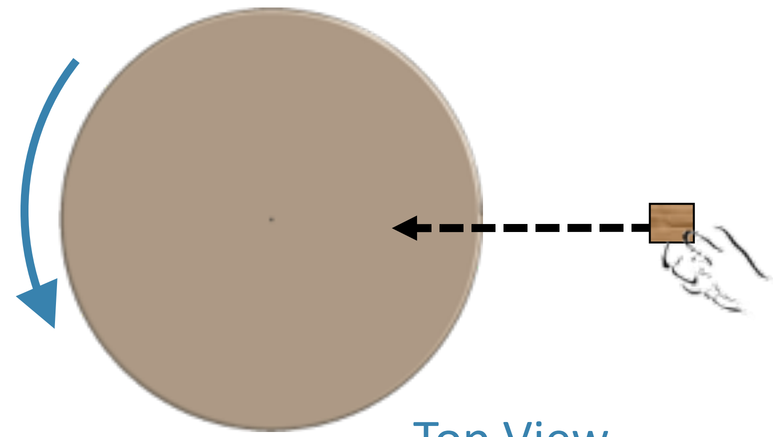
CheckPoint

The magnitude of the angular momentum of a freely rotating disk around its center is L. You toss a heavy block onto the disk along the direction shown. Friction acts between the disk and the block so that eventually the block is at rest on the disk and rotates with it. What is the magnitude of the final angular momentum of the disk-block system:

A) $> L$

B) $= L$

C) $< L$



Top View

Instead of a block, imagine it's a kid hopping onto a merry go round

CheckPoint

What is the magnitude of the final angular momentum of the disk-block system:

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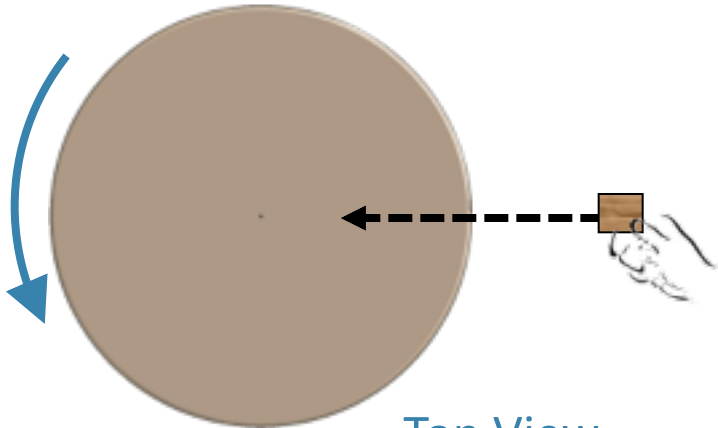
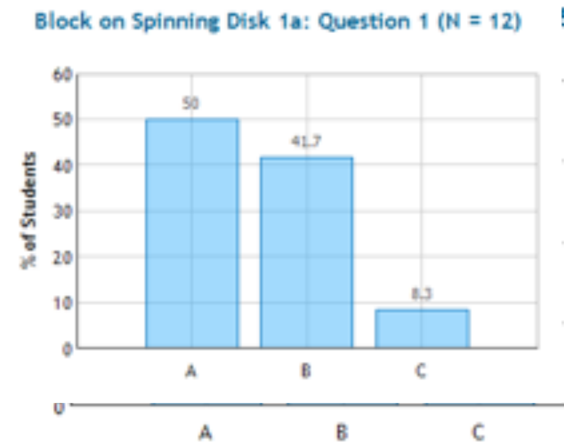
C) $< L$



Top View



A) With the added weight the momentum must be greater.

B) Initial total momentum is L (L + 0*m*v) Because there is no external force, this momentum is conserved.

C) Friction decrease the speed of disk

CheckPoint

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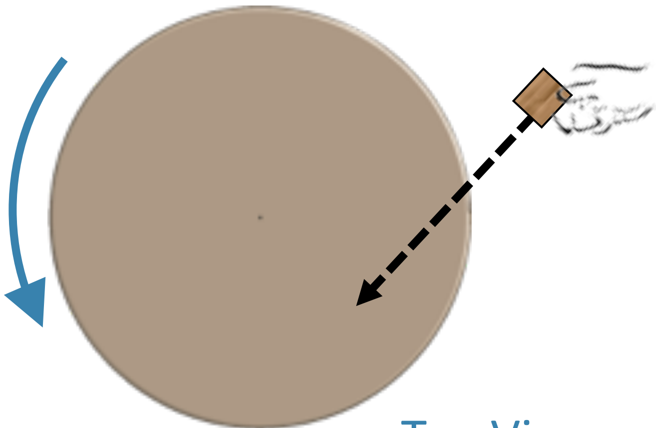

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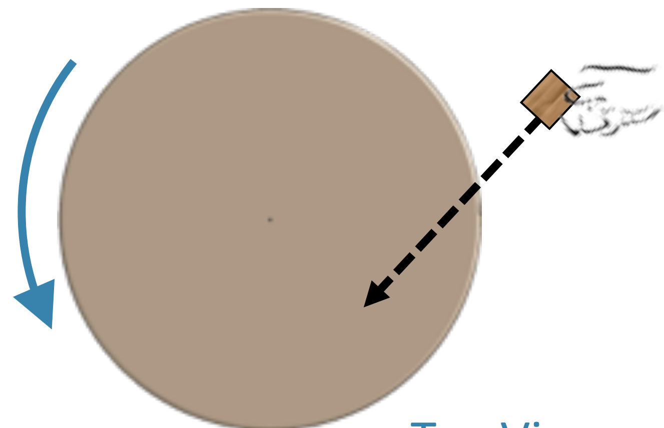
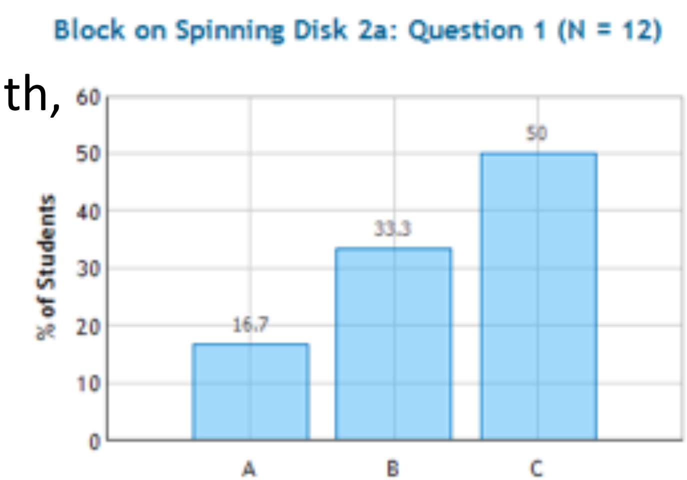


Top View



A) Since the block has angular momentum to begin with, that is added into L.

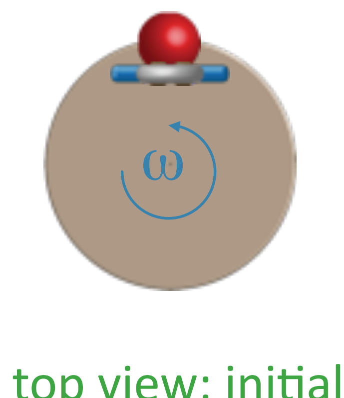
B) L is conserved so $L_{initial}=L_{final}$

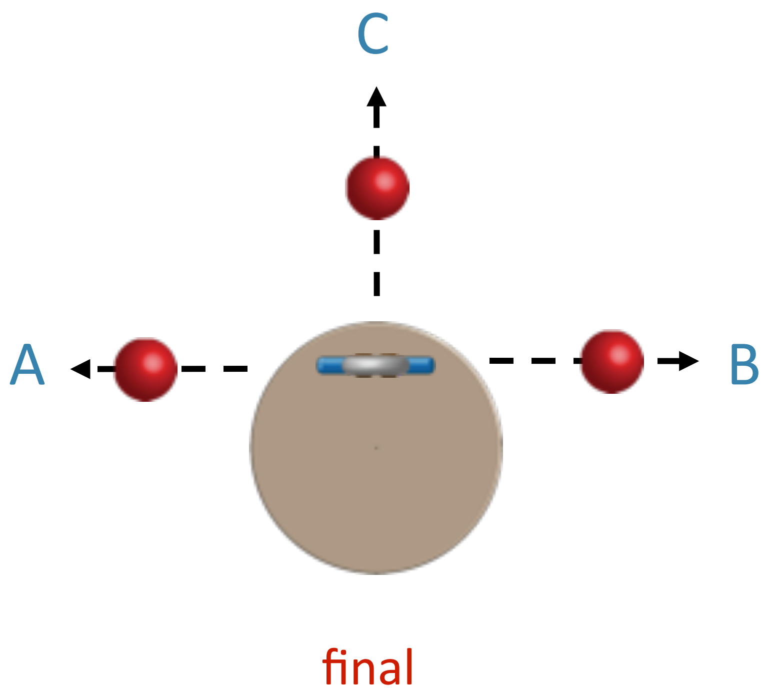
C) The block adds momentum going in the opposite direction, so the total momentum is smaller than the momentum of the disk.

Clicker Question

A student holding a heavy ball sits on the outer edge a merry go round which is initially rotating counterclockwise. Which way should she throw the ball so that she stops the rotation?

A) To her left
B) To her right
C) Radially outward

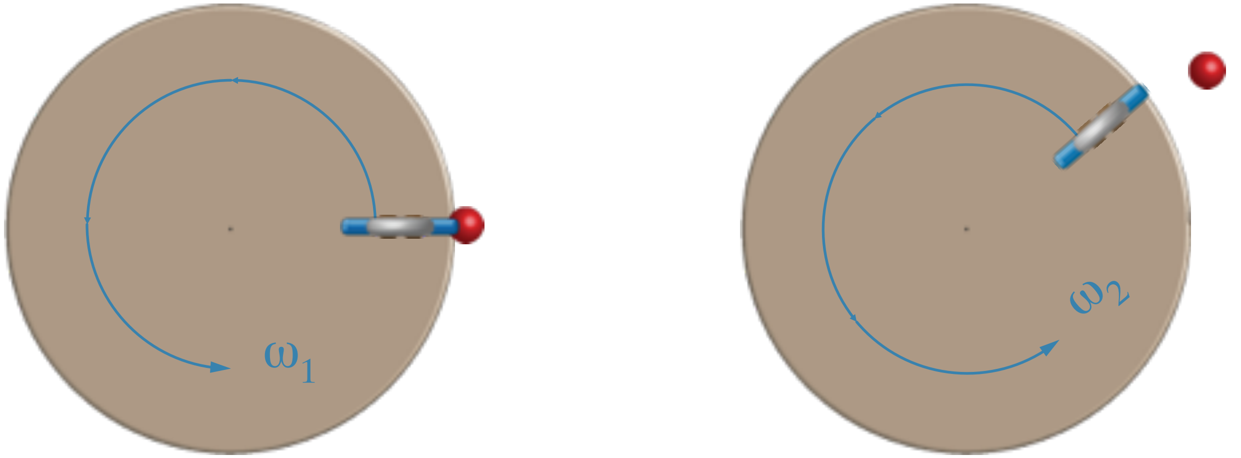




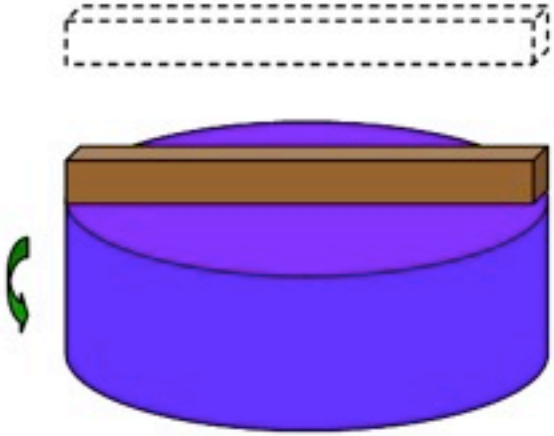
Clicker Question

A student is riding on the outside edge of a merry-go-round rotating about a frictionless pivot. She holds a heavy ball at rest in her hand.   If she releases the ball, the angular velocity of the merry-go-round will:

A) Increase B) Decrease C) Stay the same



Rod and Disk



A solid disk of mass m_1 = 9.3 kg and radius R = 0.23 m is rotating with a constant angular velocity of ω = 32 rad/s. A thin rectangular rod with mass m_2 = 3.2 kg and length L = 2R = 0.46 m begins at rest above the disk and is dropped on the disk where it begins to spin with the disk.

1) What is the initial angular momentum of the rod and disk system?

kg-m^2/s Submit

2) What is the initial rotational energy of the rod and disk system?

J Submit

3) What is the final angular velocity of the disk?

rad/s Submit

4) What is the final angular momentum of the rod and disk system?

kg-m^2/s Submit

5) What is the final rotational energy of the rod and disk system?

J Submit

6) The rod took t = 7 s to accelerate to its final angular speed with the disk, what average torque was exerted on it by the disk?

N-m Submit

A solid disk of mass m_1 = 9.3 kg and radius R = 0.23 m is rotating with a constant angular velocity of ω = 32 rad/s. A thin rectangular rod with mass m_2 = 3.2 kg and length L = 2R = 0.46 m begins at rest above the disk and is dropped on the disk where it begins to spin with the disk.

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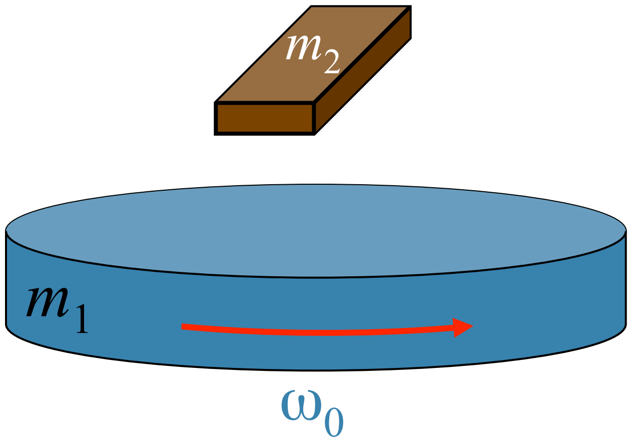

$$I_{disk} = \frac{1}{2} m_1 R^2$$

$$L_{initial} = I_{disk} \omega_o$$

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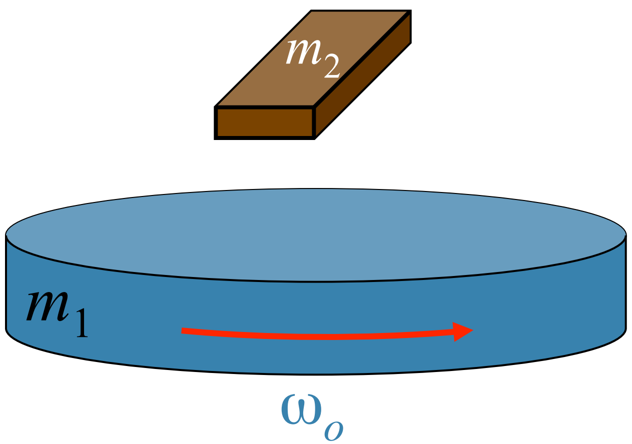
[ ] J Submit



$$I_{disk} = \frac{1}{2} m_1 R^2$$

$$K_{initial} = \frac{1}{2} I_{disk} \omega_o^2$$

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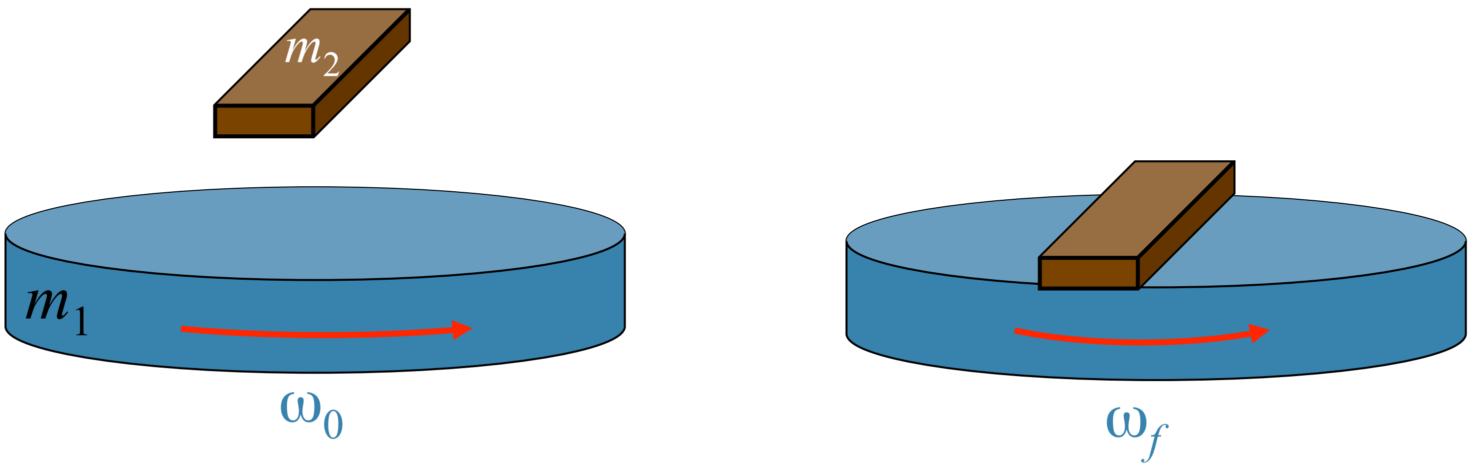
3) What is the final angular velocity of the disk?

rad/s Submit



$$L_{initial} = I_{disk}\omega_0$$

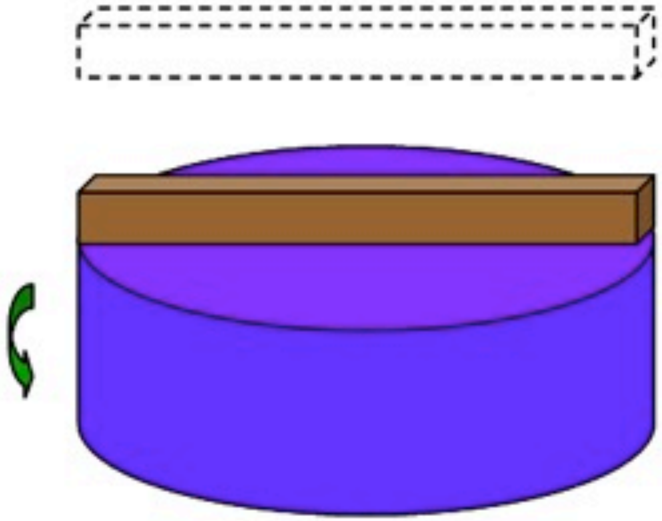
$$L_{final} = (I_{disk} + I_{rod})\omega_f$$

$$L_{final} = L_{initial}$$

$$I_{rod} = \frac{1}{12} m_2 L^2$$

Solve for ω_f

Rod and Disk



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rad/s Submit

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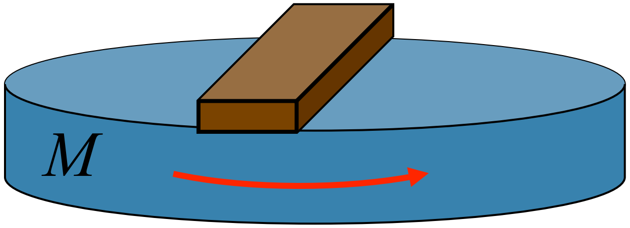
kg-m^2/s Submit

$$L_{final} = L_{initial}$$

A solid disk of mass m_1 = 9.3 kg and radius R = 0.23 m is rotating with a constant angular velocity of ω = 32 rad/s. A thin rectangular rod with mass m_2 = 3.2 kg and length L = 2R = 0.46 m begins at rest above the disk and is dropped on the disk where it begins to spin with the disk.

5) What is the final rotational energy of the rod and disk system?

J Submit



ω_f

$$K_{final} = \frac{1}{2}\left(I_{disk} + I_{rod}\right)\omega_f^2$$

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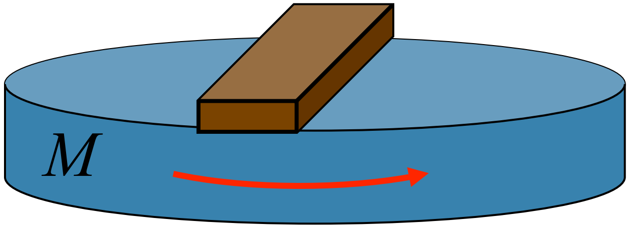
6) The rod took t = 7 s to accelerate to its final angular speed with the disk, what average torque was exerted on it by the disk?

[ ] N-m Submit

Just like

$$F_{average} = \frac{\Delta P}{\Delta t}$$

for linear momentum



ω_f

$$\tau_{average} = \frac{\Delta L}{\Delta t}$$ for angular momentum

$$\Delta L_{rod} = I_{rod}(\omega_f - \omega_0) \quad \tau_{average} = \frac{I_{rod}(\omega_f - \omega_0)}{\Delta t}$$