Classical Mechanics
Lecture 21

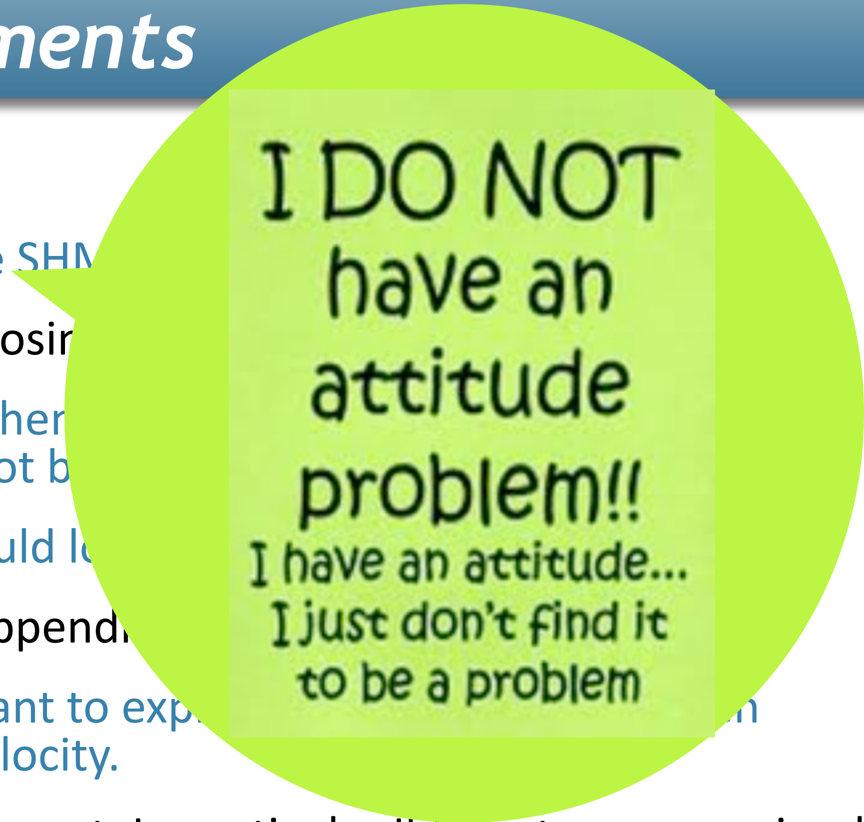
Today's Concept:

Simple Harmonic Motion: *Mass on a Spring*

The Mechanical Universe, Episode 16: Harmonic Motion

http://www.learner.org/vod/login.html?pid=565

Your Comments



Good stuff :) Easier than angular stuff.

Why are we doing springs AGAIN? —— I hate SHM

I did not understand whether to use sine or cosir

This stuff looks much more easier to look at ther
Rotational Harmonic Motion? There better not b

Go over adding 2 graphs together. How it would l

is this part of the main course or is this like append

Not really from this lecture, but you might want to exp n
frequency, angular frequency, and angular velocity.

Most of it was something I need time to figure out. In particular I'd like to go over simpl
harmonic motion with examples using the trigonometric equations.

What is the phase angle and how do you find it? Also, what is the use of the "general solution" equation they gave us? Can't we just use x(t)=Acos(wt+0)? (where 0 = phase angle).

How does physics help bring me closer to discovering the meaning of life?

The answer to the question of life, the universe, and everything:

42

Next time I will explain why!

Next Friday Project

Simple Pendulum

- choose your method: stopwatch, rotary motion detector, etc

Karate

- Measure your fist's energy and energy needed to break board
- break board

ioLab feasibility study

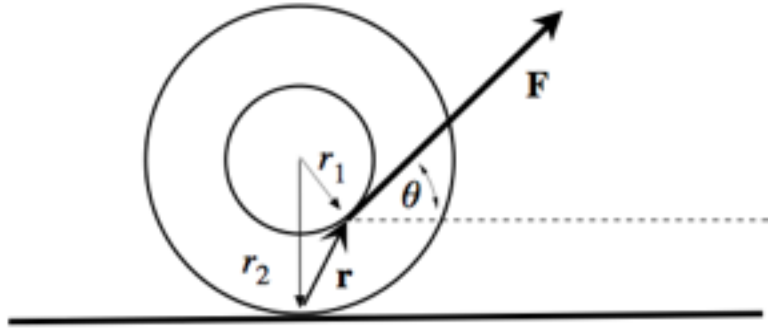
- do an experiment and write it up; e.g., torsional pendulum

Yo Yo Challenge

http://www.sfu.ca/phys/140/1137/Homework/Solutions/YoYo.pdf

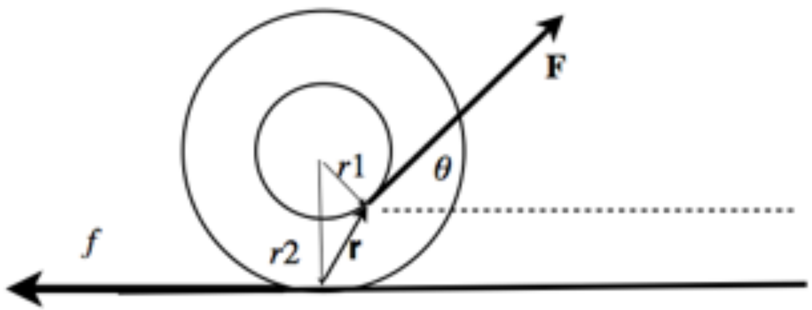
Solution 1:

- rotation about the contact point



Solution 2:

- rotation about centre of mass



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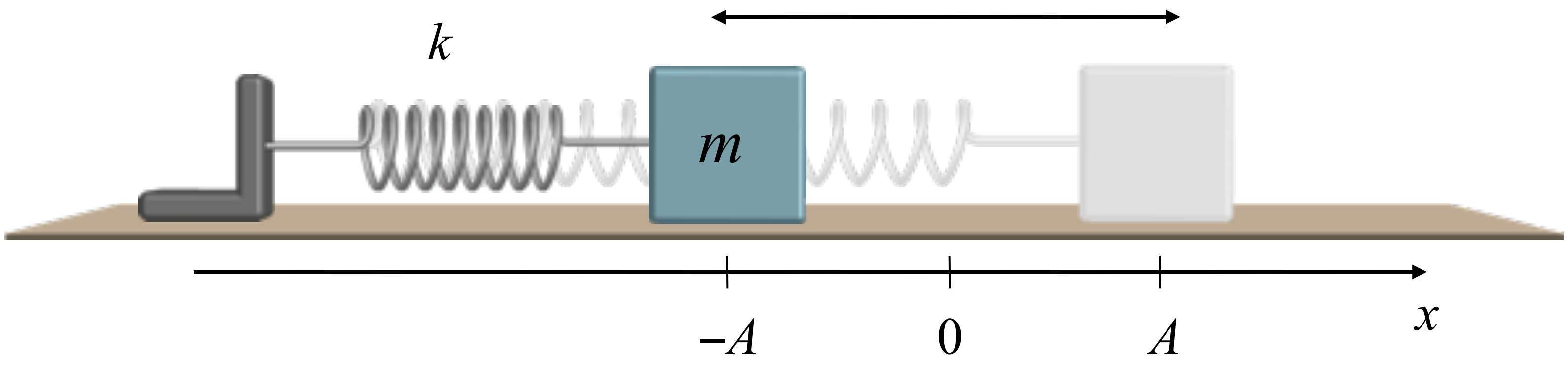
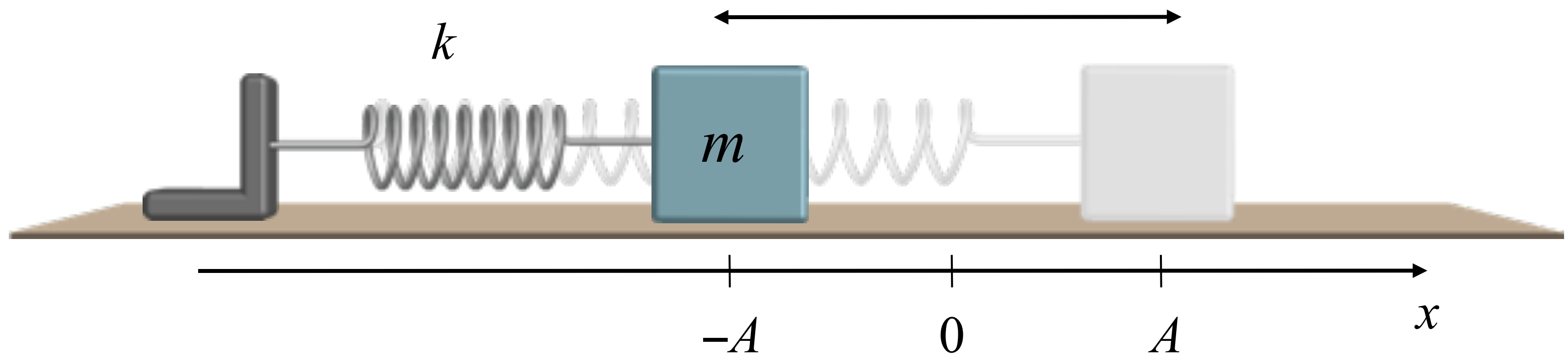


A mass on a spring moves with simple harmonic motion as shown. Where is the acceleration of the mass most positive?

A) $x = -A$ B) $x = 0$ C) $x = A$



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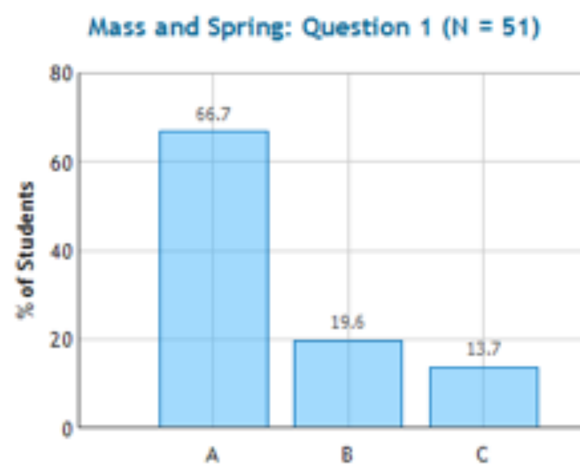


A) The positive force on the mass is greatest wher the spring most compressed.

B) Because at x=A, -A there is no acceleration.

C) Force is greatest at this point so therefore acceleration is too.

Newton's 2[nd] Law

(1-D motion)

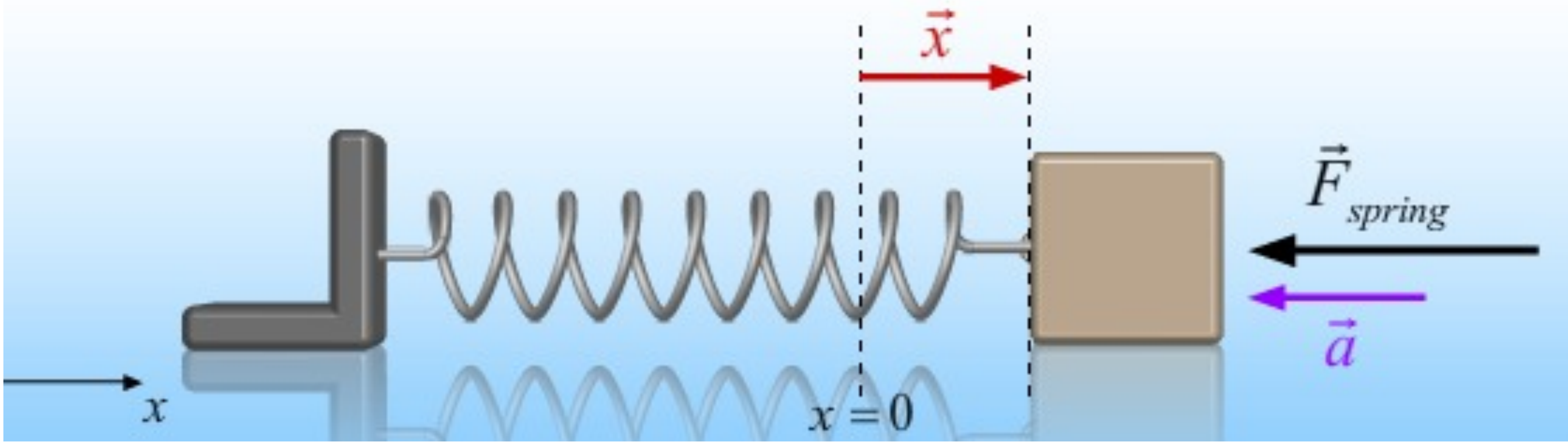
$$-kx = m\frac{d^2x}{dt^2}$$

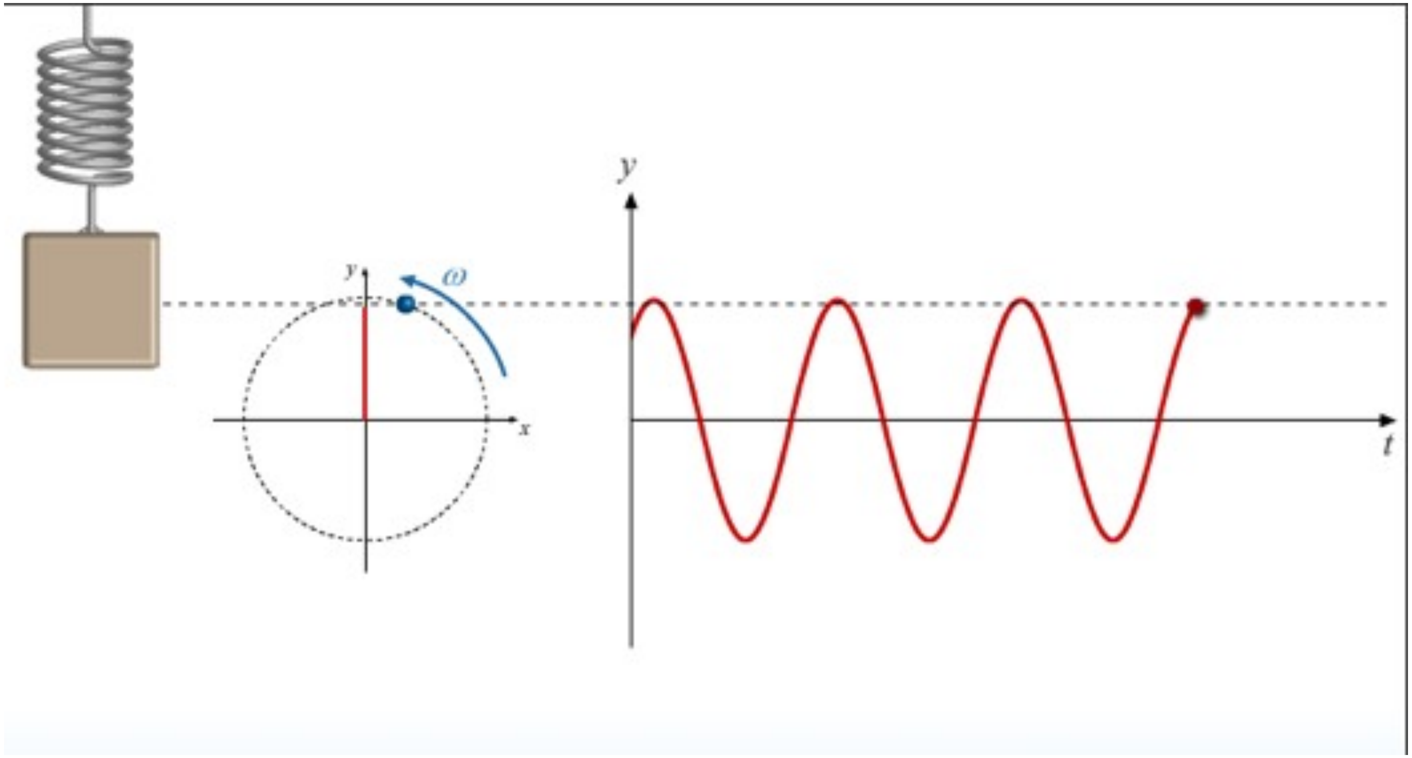
$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad \omega^2 \equiv \frac{k}{m}$$

Most general solution:

$$x(t) = A\cos(\omega t - \phi)$$

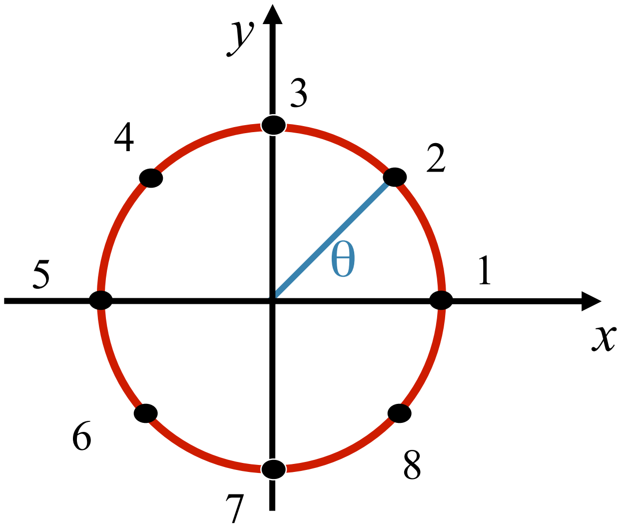


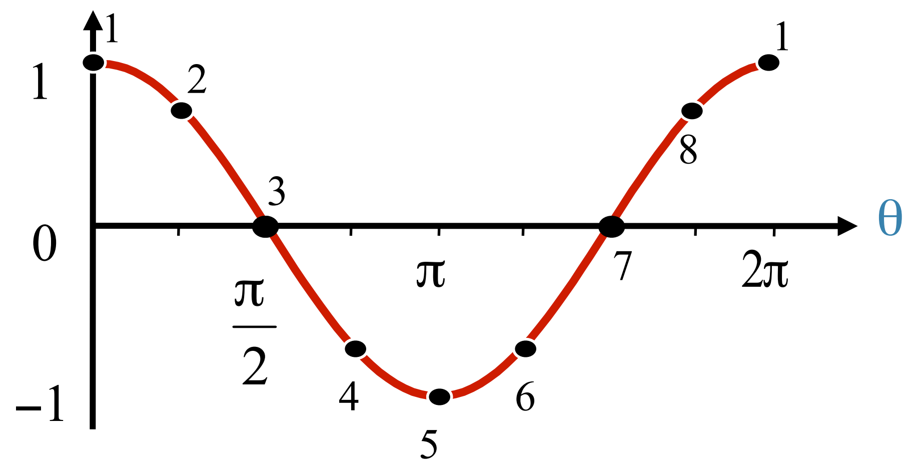


demo

What does *angular frequency* ω have to do with moving back and forth *in a straight line*?

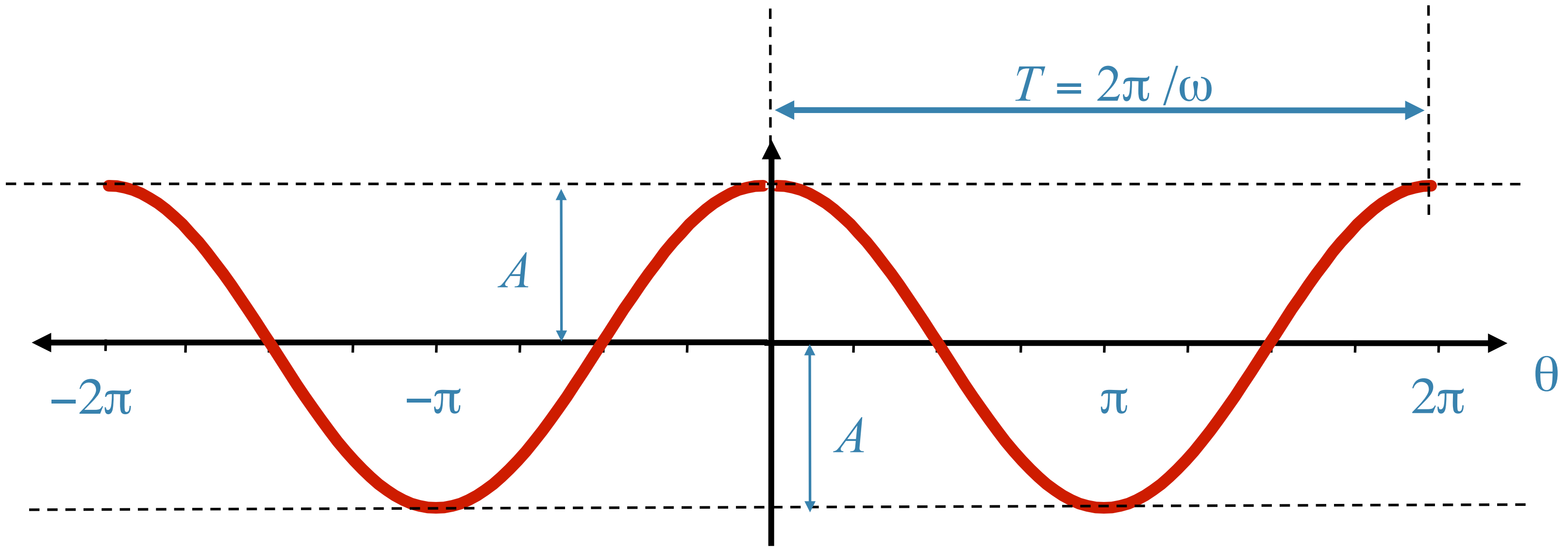
$$y = R\cos\theta = R\cos(\omega t)$$





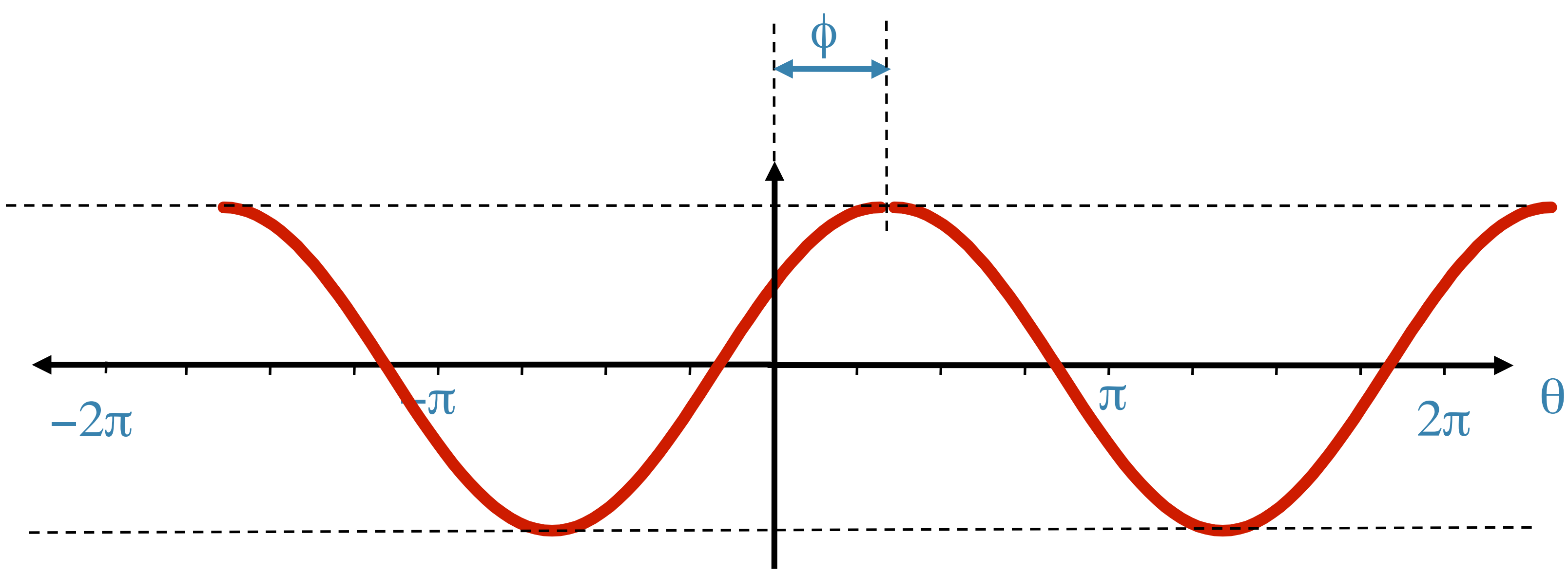
SHM Solution

Drawing of $A\cos(\omega t)$



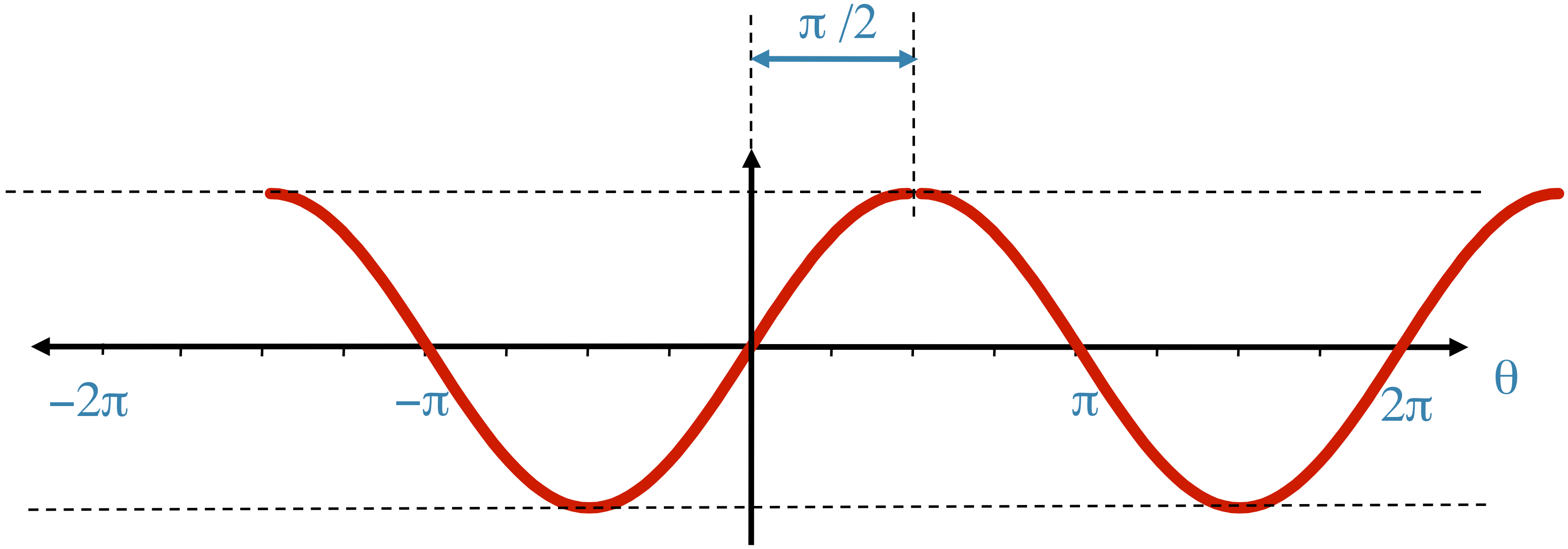
SHM Solution

Drawing of $A\cos(\omega t - \phi)$

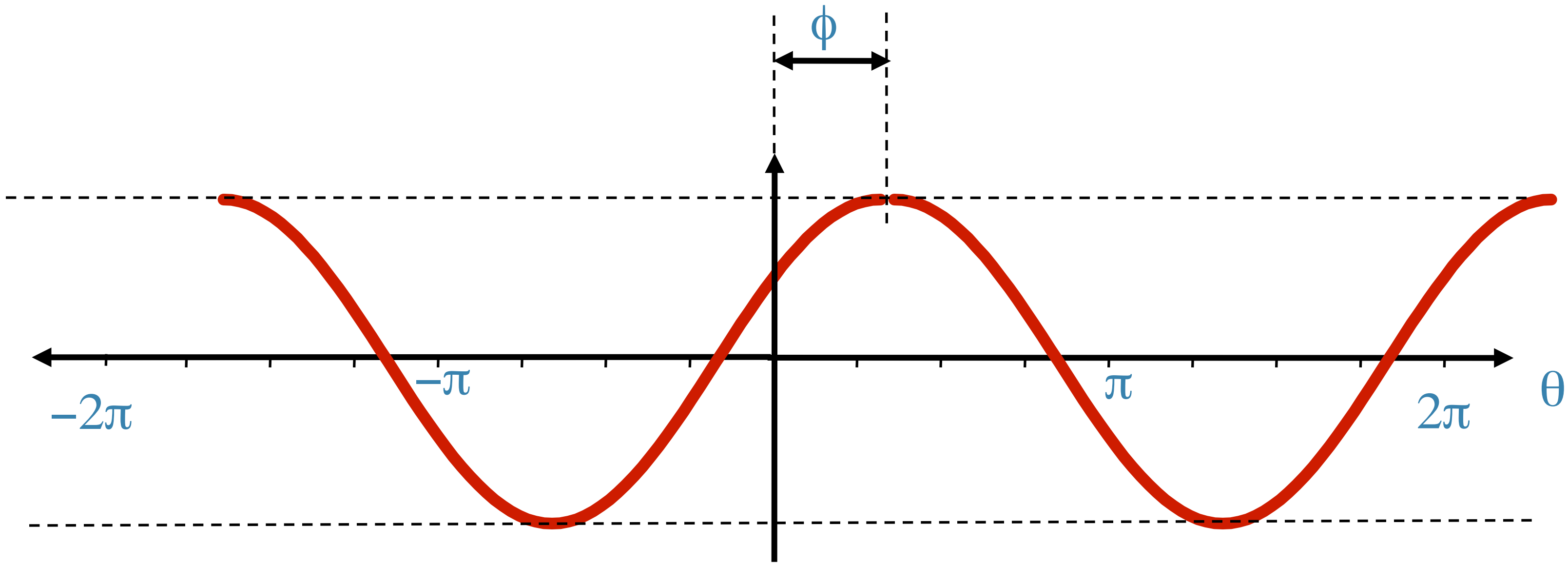


SHM Solution

Drawing of $A\cos(\omega t - \pi/2) = A\sin(\omega t)$

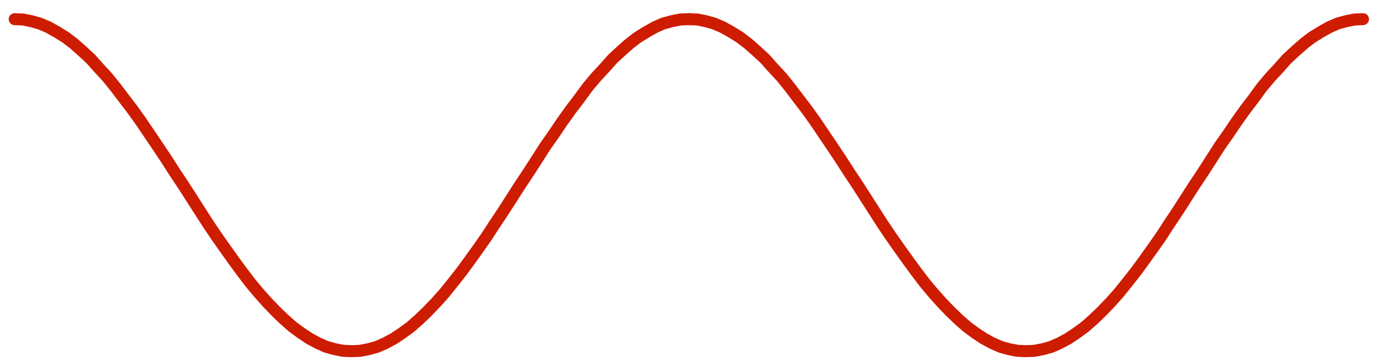


In the slide titled, "example", the decision to choose the equation "A*cos(w*t +'phi') seemed like an arbitrary decision. Please explain why that equation was chosen over the other general solution equations. The description of the phase angle is a little confusing too.

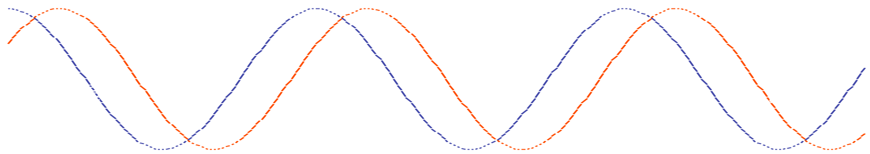


Drawing of $A\cos(\omega t - \phi)$

In the slide titled, "example", the decision to choose the equation "A*cos(w*t +'phi') seemed like an arbitrary decision. Please explain why that equation was chosen over the other general solution equations. The description of the phase angle is a little confusing too.

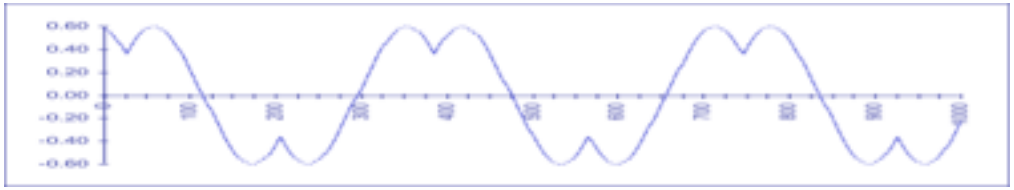


Is this a sine or a cosine?

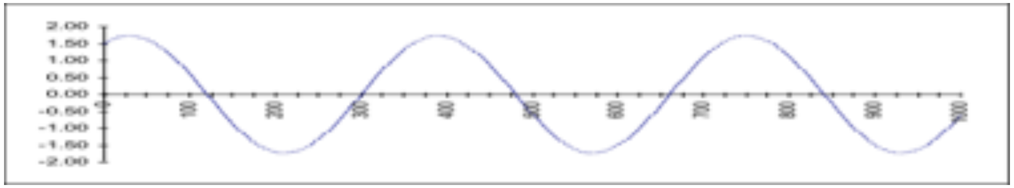


Suppose the two sinusoidal curves shown above are added together. Which of the plots shown below best represents the result?

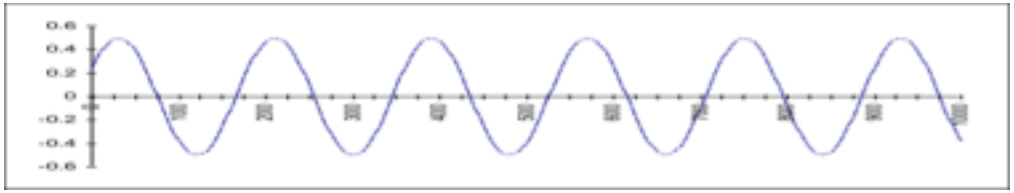
A)

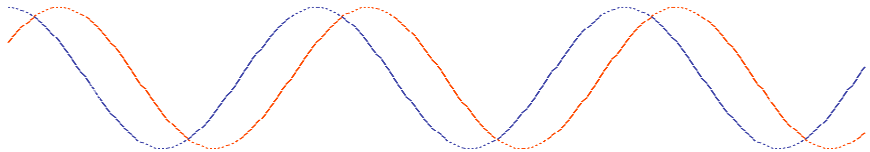


B)



C)

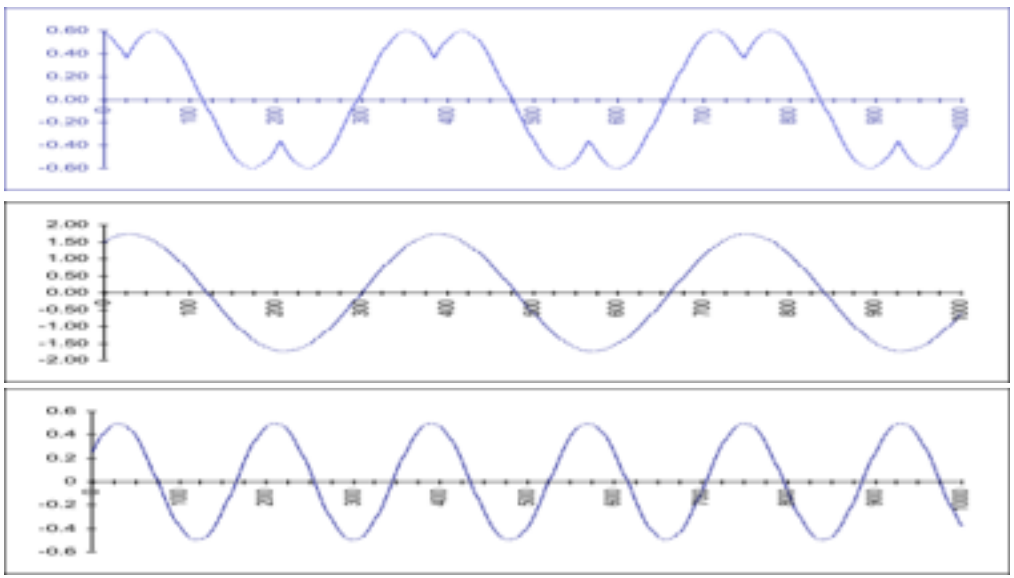


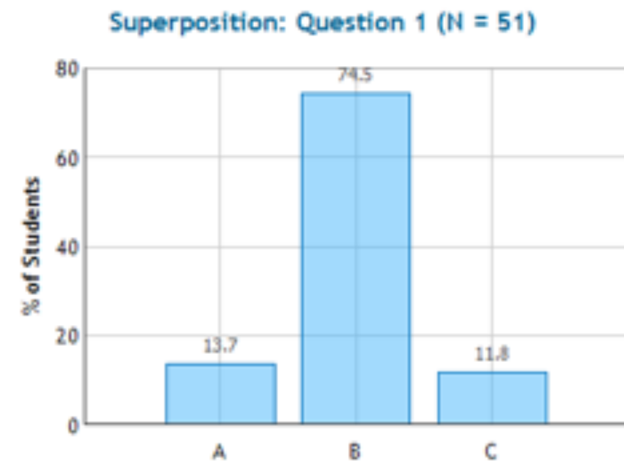


A)

B)

C)





A) The SUM of them would be a little irregular.

B) The sum of two waves with equal frequency gives another signal with the same frequency and different amplitude/phase.

C) When adding the two curves together the aplitude decreases and the frequency increases.

Newton's 2nd Law

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

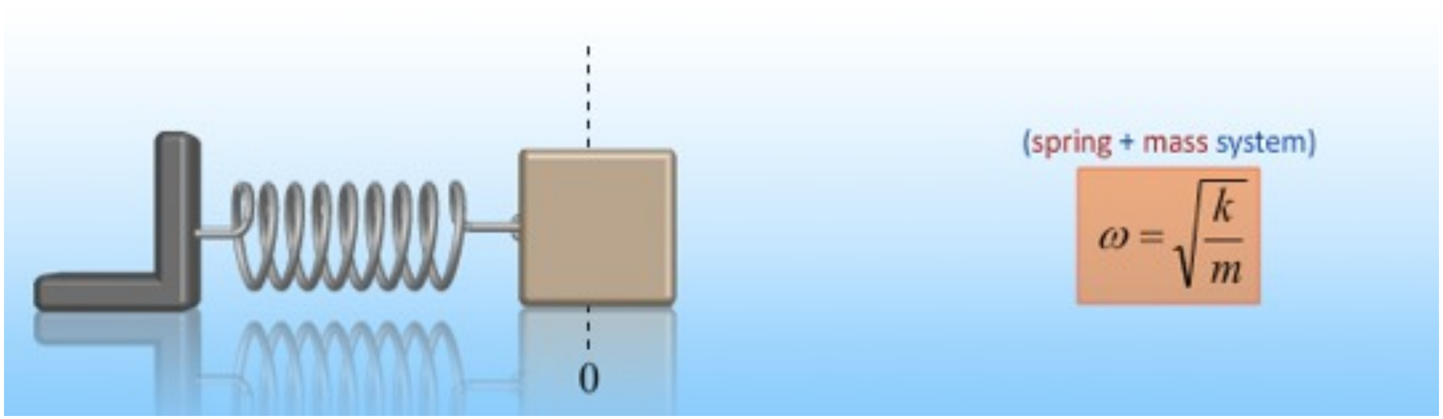
→

Possible Solutions

1) $x(t) = A_1 \sin(\omega t) + A_2 \cos(\omega t)$

2) $x(t) = A\cos(\omega t + \phi)$

3) $x(t) = A\sin(\omega t + \phi)$



Any linear combination of sines and cosines having the same frequency will result in a sinusoidal curve with the same frequency.

Can we talk more about the trigonometrical functions? Like in question #2.

$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

Suppose

$$a = \omega t + \alpha$$

$$b = \omega t + \beta$$

$$\sin(\omega t + \alpha) + \cos(\omega t + \beta) = 2\sin\left(\omega t + \frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$= A\sin\left(\omega t + \phi\right)$$

Question: What is the position of the mass as a function of time?

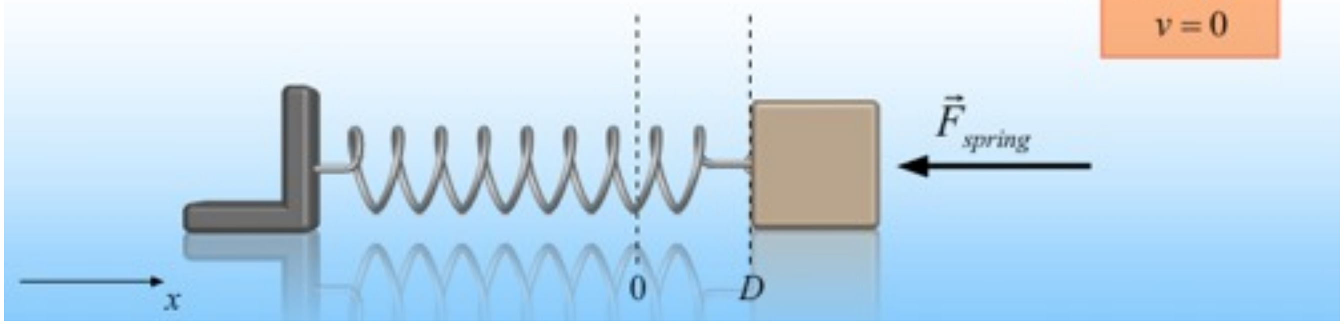
Answer: $$x(t) = D\cos(\omega t)$$ where $\omega = \sqrt{\frac{k}{m}}$

$$v(t) \equiv \frac{dx}{dt} = -\omega D\sin(\omega t)$$

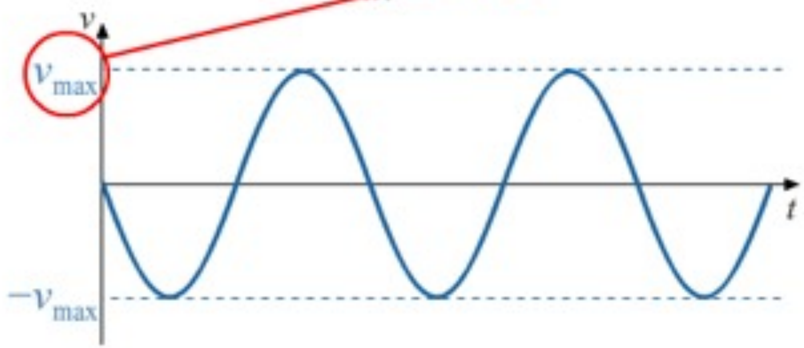
$$a(t) \equiv \frac{dv}{dt} = -\omega^2 D\cos(\omega t)$$

Initial Conditions

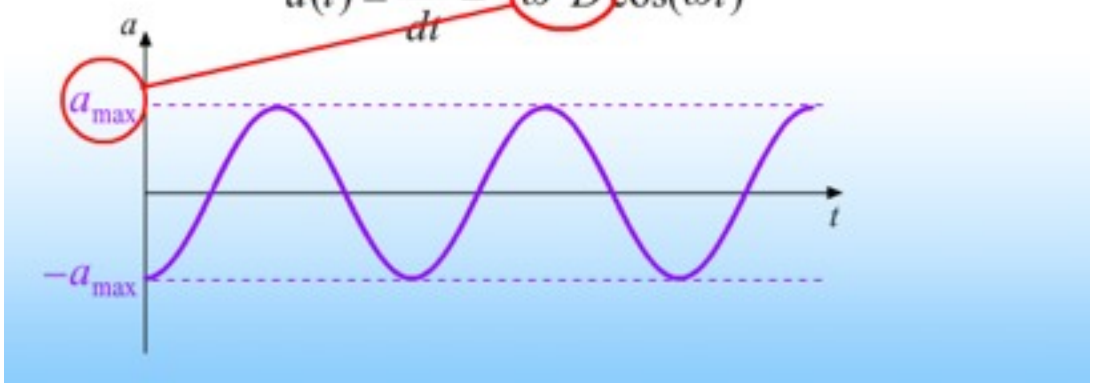
$t = 0$
$x = D$ $v = 0$



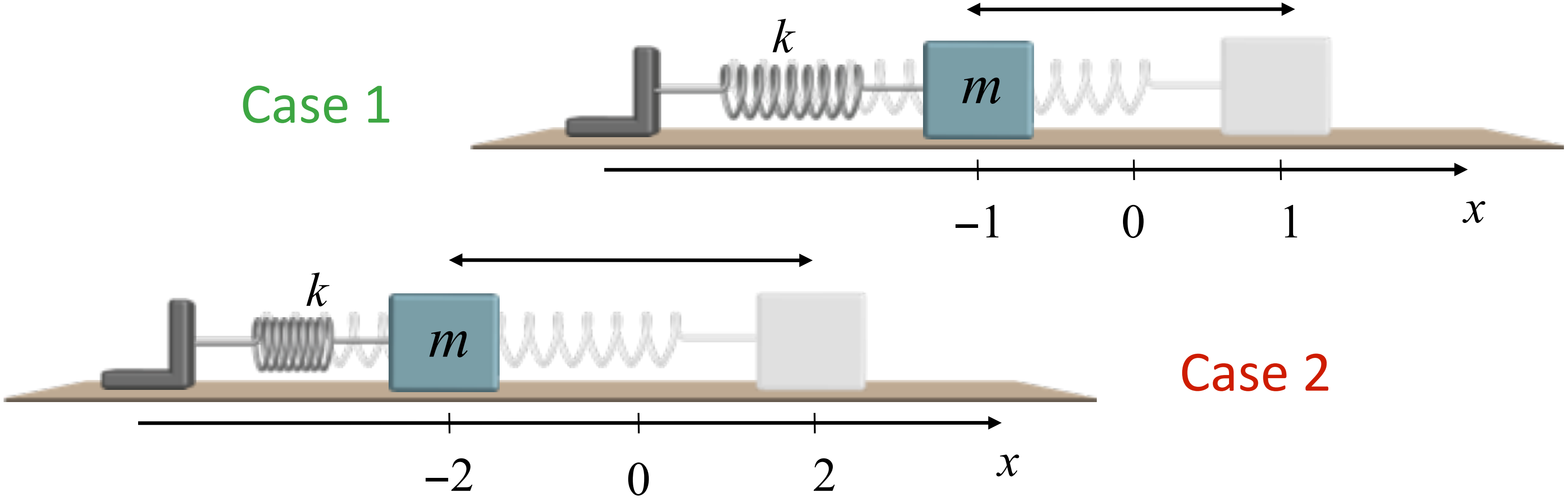
$$v(t) \equiv \frac{dx}{dt} = -\omega D \sin(\omega t)$$



$$a(t) \equiv \frac{dv}{dt} = -\omega^2 D \cos(\omega t)$$



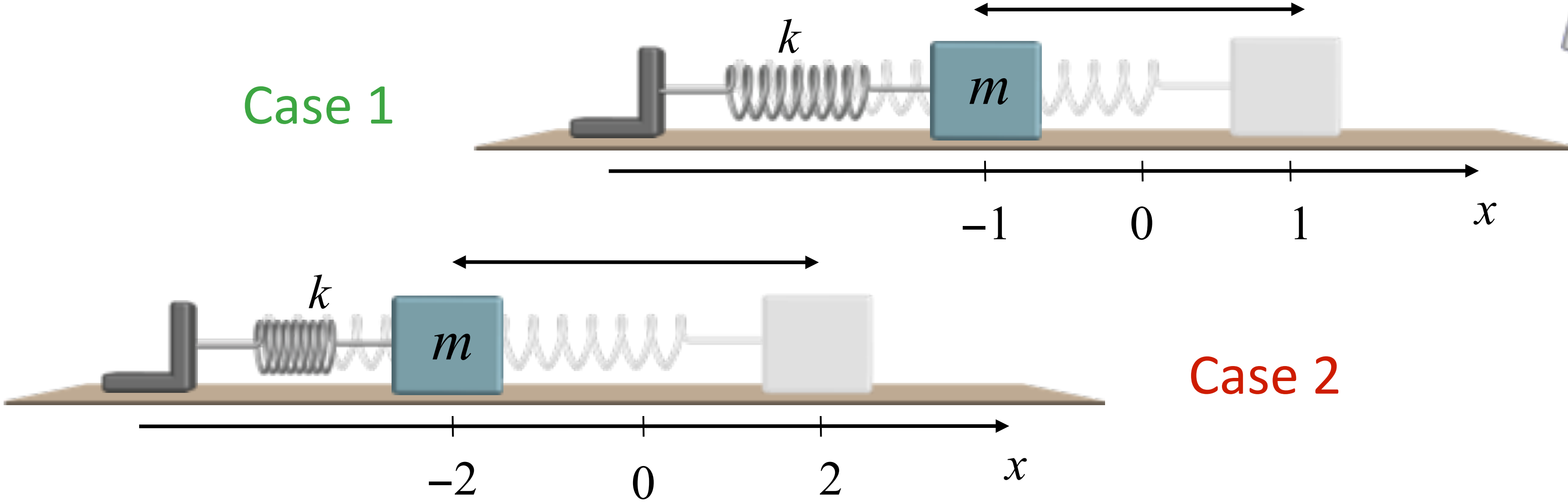
CheckPoint



In the two cases shown the mass and the spring are identical but the amplitude of the simple harmonic motion is twice as big in Case 2 as in Case 1.

How are the maximum velocities in the two cases related:

A) $v_{max,2} = v_{max,1}$ B) $v_{max,2} = 2v_{max,1}$ C) $v_{max,2} = 4v_{max,1}$



$$\rightarrow \quad v_{\max} = \omega A$$

How are the maximum velocities in the two cases related:

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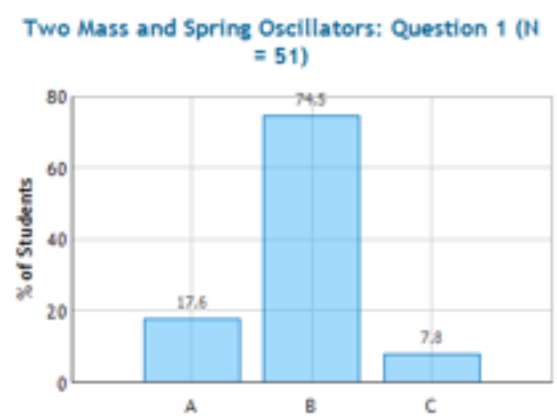


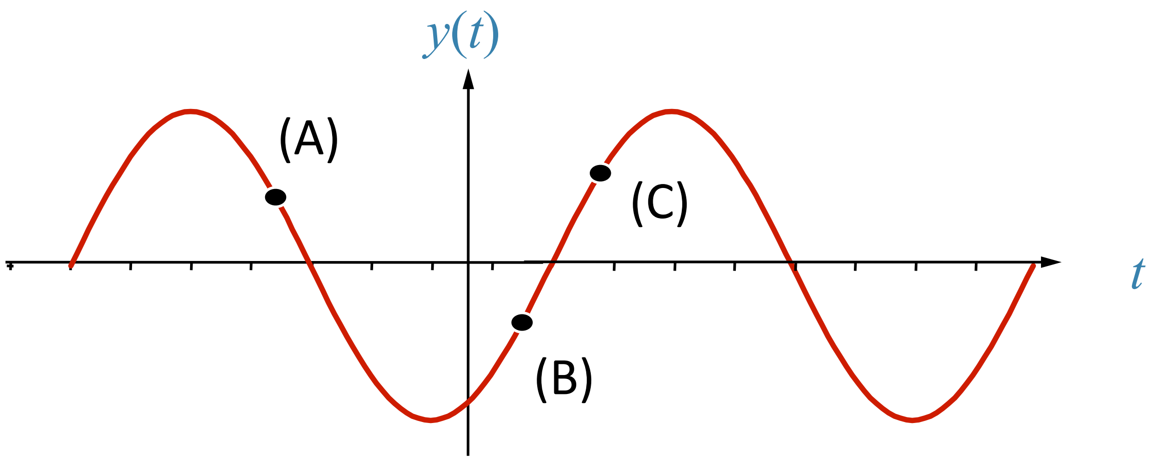
A) The velocity does not depend on the amplitude.

B) Vmax=ω*D so twice the amplitude means twice the velocity

C) KE will depend on spring energy, which has an (x)^2 term. So doubling that x term will quadruple the KE.

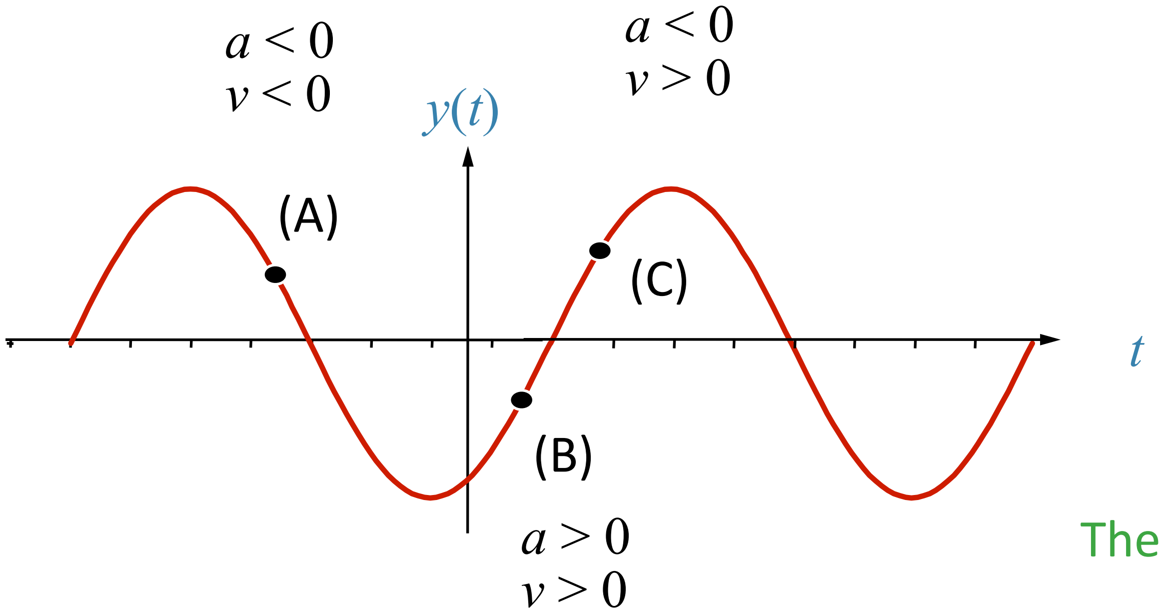
Clicker Question

A mass oscillates up & down on a spring. Its position as a function of time is shown below. At which of the points shown does the mass have positive velocity and negative acceleration?



The slope of $y(t)$ tells us the sign of the velocity since $v_y = \frac{dy}{dt}$

$y(t)$ and $a(t)$ have the opposite sign since $a(t) = -\omega^2 y(t)$



The answer is (C).

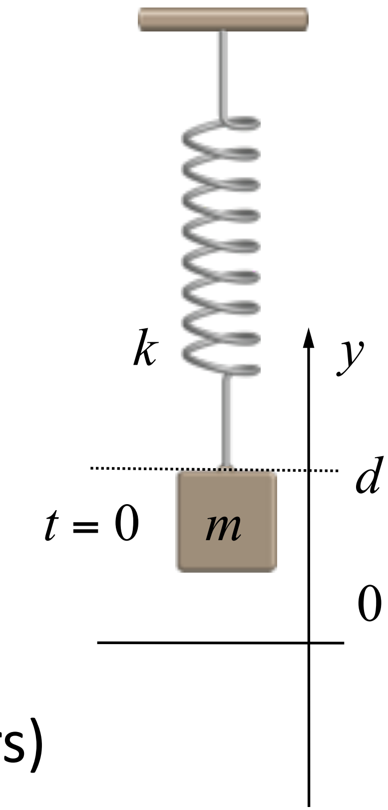
Clicker Question

A mass hanging from a vertical spring is lifted a distance d above equilibrium and released at $t = 0$. Which of the following describes its velocity and acceleration as a function of time?

A) $v(t) = -v_{max}\sin(\omega t) \qquad a(t) = -a_{max}\cos(\omega t)$

B) $v(t) = v_{max}\sin(\omega t) \qquad a(t) = a_{max}\cos(\omega t)$

C) $v(t) = v_{max}\cos(\omega t) \qquad a(t) = -a_{max}\cos(\omega t)$



(both v_{max} and a_{max} are positive numbers)

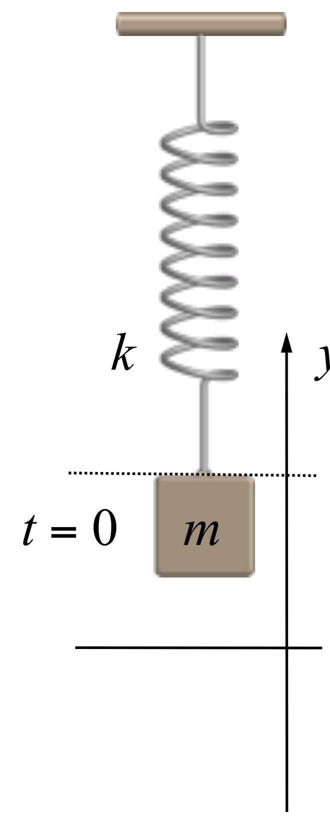
Clicker Question

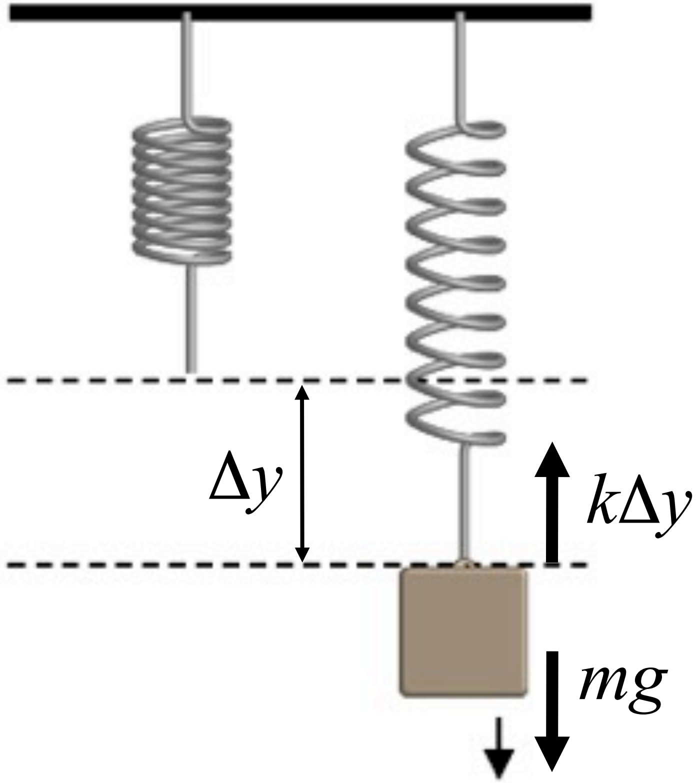
Since we start with the maximum possible displacement at $t = 0$ we know that:

$$y = d\cos(\omega t)$$

$$v_y = \frac{dy}{dt} = -\omega d \sin(\omega t) = -v_{max}\sin(\omega t)$$

$$a_y = \frac{dv_y}{dt} = -\omega^2 d\cos(\omega t) = -a_{max}\cos(\omega t)$$





A block with mass m =6.4 kg is hung from a vertical spring. When the mass hangs in equilibrium, the spring stretches x = 0.27 m. While at this equilibrium position, the mass is then given an initial push downward at v = 4.8 m/s. The block oscillates on the spring without friction.

1) What is the spring constant of the spring?

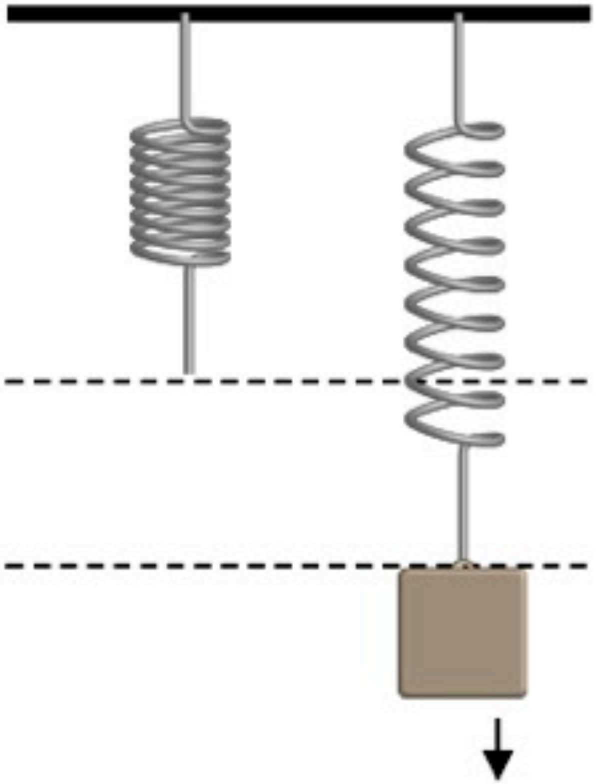
[ ] N/m Submit

$$mg = k\Delta y \rightarrow k = \frac{mg}{\Delta y}$$

2) What is the oscillation frequency?

[ ] Hz Submit

$$\omega = \sqrt{\frac{k}{m}}$$



At $t = 0$, $y = 0$, moving down

$$y(t) = -A\sin(\omega t)$$

$$v(t) = -\omega A\cos(\omega t)$$

$$a(t) = \omega^2 A\sin(\omega t)$$

Use energy conservation to find A

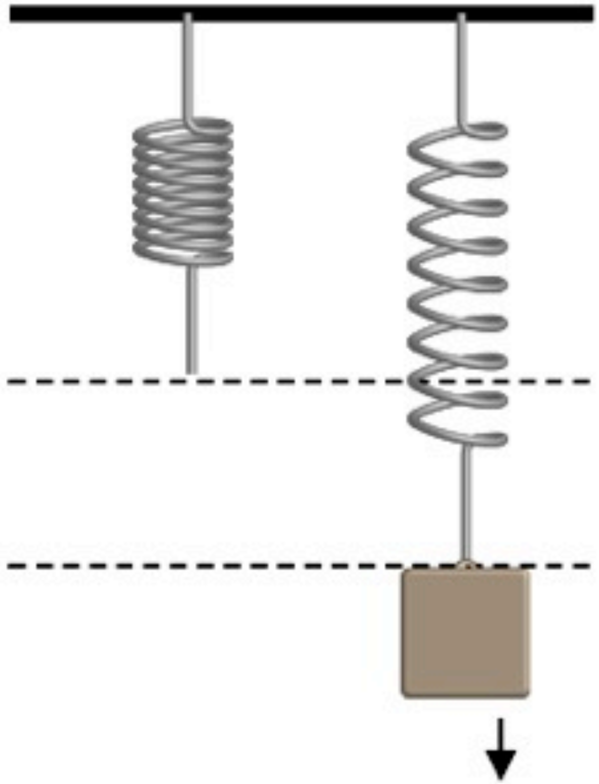
$$\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2 \rightarrow A = v_{max}\sqrt{\frac{m}{k}}$$

A block with mass m =6.4 kg is hung from a vertical spring. When the mass hangs in equilibrium, the spring stretches x = 0.27 m. While at this equilibrium position, the mass is then given an initial push downward at v = 4.8 m/s. The block oscillates on the spring without friction.

3) After t = 0.32 s what is the speed of the block?

m/s Submit

$$v(t) = -\omega A\cos(\omega t)$$



$$a(t) = \omega^2 A \sin(\omega t)$$

$$a_{max} = \omega^2 A$$

Or similarly

$$a_{max} = \frac{F_{max}}{m} = \frac{k\Delta y_{max}}{m} = \frac{kA}{m}$$

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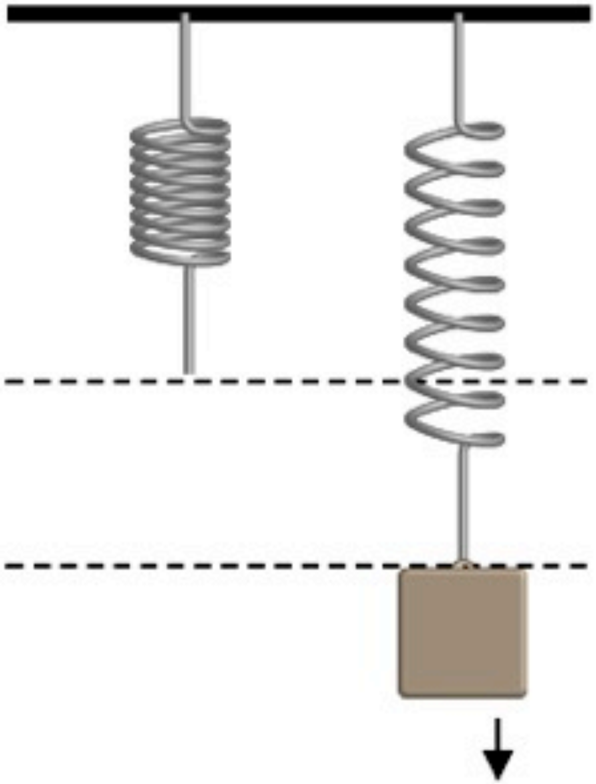
4) What is the magnitude of the maximum acceleration of the block?

[ ] m/s^2 Submit



$$|F(t)| = |ky(t)| = kA\sin(\omega t)$$

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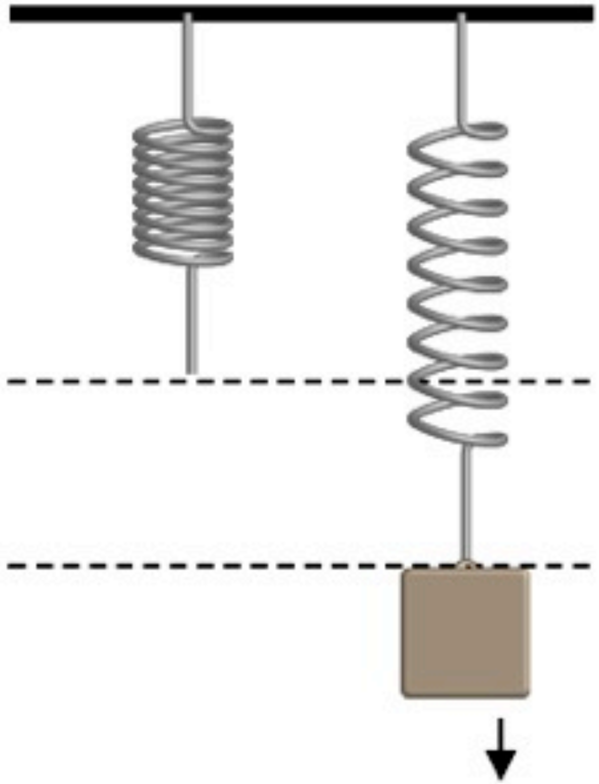
5) At t = 0.32 s what is the magnitude of the net force on the block?

N Submit



$$U = \frac{1}{2} k y^2$$

$$y(t) = -A \sin(\omega t)$$

A block with mass m =6.4 kg is hung from a vertical spring. When the mass hangs in equilibrium, the spring stretches x = 0.27 m. While at this equilibrium position, the mass is then given an initial push downward at v = 4.8 m/s. The block oscillates on the spring without friction.

6) Where is the potential energy of the system the greatest?

- ☐ At the highest point of the oscillation.
- ☐ At the new equilibrium position of the oscillation.
- ☐ At the lowest point of the oscillation.