Classical Mechanics Lecture 23

Today's Concept:

Harmonic Waves

Your comments are very important to us…

is the final project mandatory?

Will harmonic waves be on the final exam?

The wave equation is a little confusing. I dont get why P is now period, where in other sections capital T is the period. and we also have T for Tension, and a small t for time aswell. so many T's!

When is the frequency and the velocity not dependent on the amplitude and period etc? Or is it always the same?

Your comments are still very important to us...

So, the tension in a rope is represented with the symbol "T", instead of a force with a subscript like "F_t". Well that makes sense, because there's no other quantity symbolized with "T" that comes up all the time when dealing with waves ... OH WAIT, YES THERE IS, IT'S PERIOD!!! Seriously, who is making these decisions?! Is it the same guy who decided to use omega for angular frequency AND angular velocity? That guy is overdue for a good lynching ...

Final Exam: April 25. Do you want to have it in this room?

My Exam Schedule > 2015 Spring > Simon Fraser University

Class	Class Title	Exam Date	Exam Time	Enrolled	Exam Room
PHYS 140-D100 (5018)	Mechanics and Modern Physics (Lecture)	2015/4/25, Saturday	3:30PM - 6:30PM	15	SUR 5240 Surrey

What is a Wave?

A wave is a traveling disturbance that transports energy but not matter.

Examples:

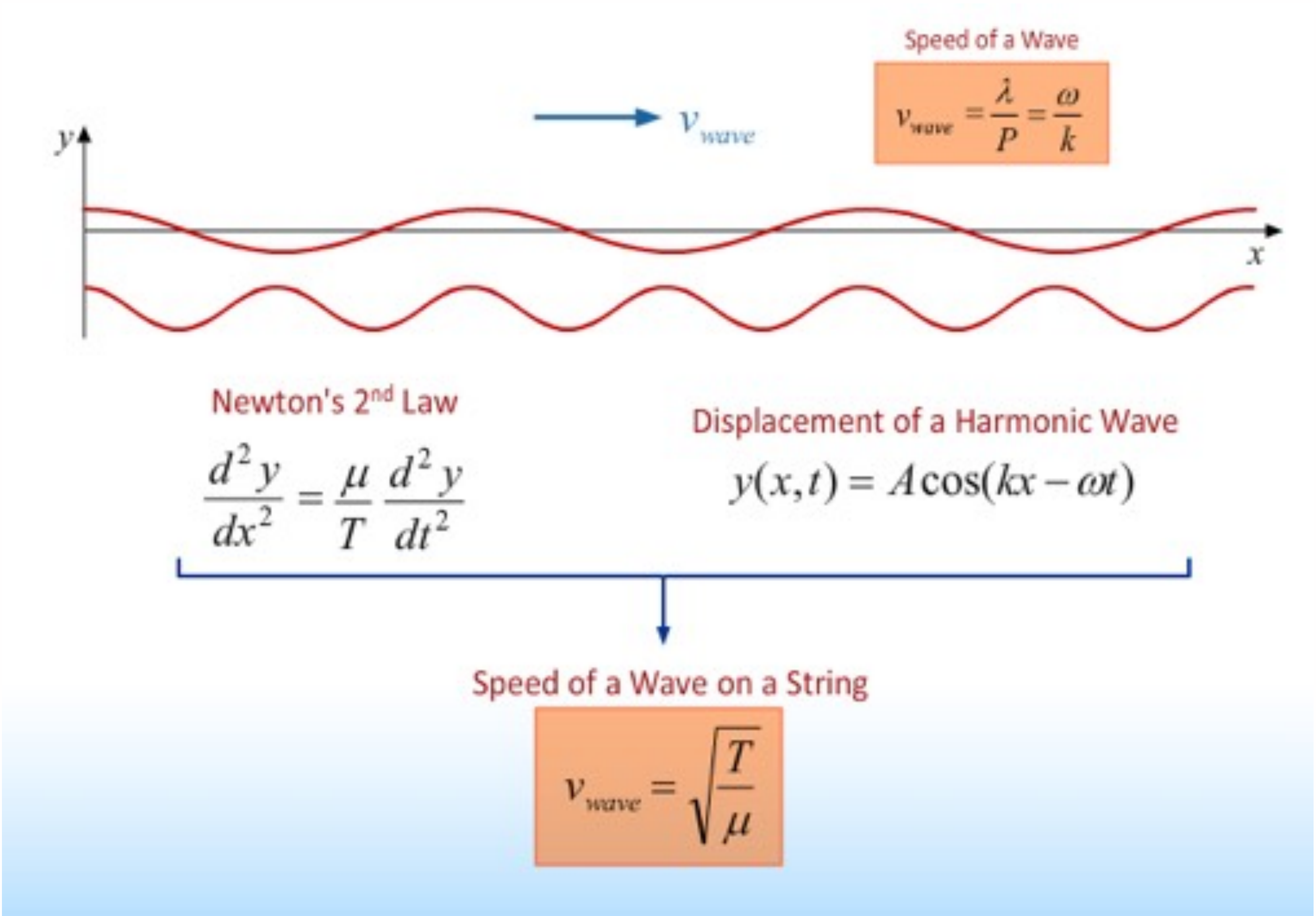
- Sound waves (air moves back & forth)
- Stadium waves (people move up & down)
- Water waves (water moves up & down)
- Light waves (what moves?)

Types of Waves

Transverse: The medium oscillates perpendicular to the direction the wave is moving. This is what we study in Physics 140.

Movie (*t*wave vs *l*wave)

Longitudinal: The medium oscillates in the same direction as the wave is moving.


Speed of a Wave
$v_{wave} = \frac{\lambda}{P} = \frac{\omega}{k}$
v_{wave}
y
x
Newton's 2nd Law
$\frac{d^2 y}{dx^2} = \frac{\mu}{T} \frac{d^2 y}{dt^2}$
Displacement of a Harmonic Wave
$y(x,t) = A\cos(kx - \omega t)$
Speed of a Wave on a String
$v_{wave} = \sqrt{\frac{T}{\mu}}$

Wave Properties

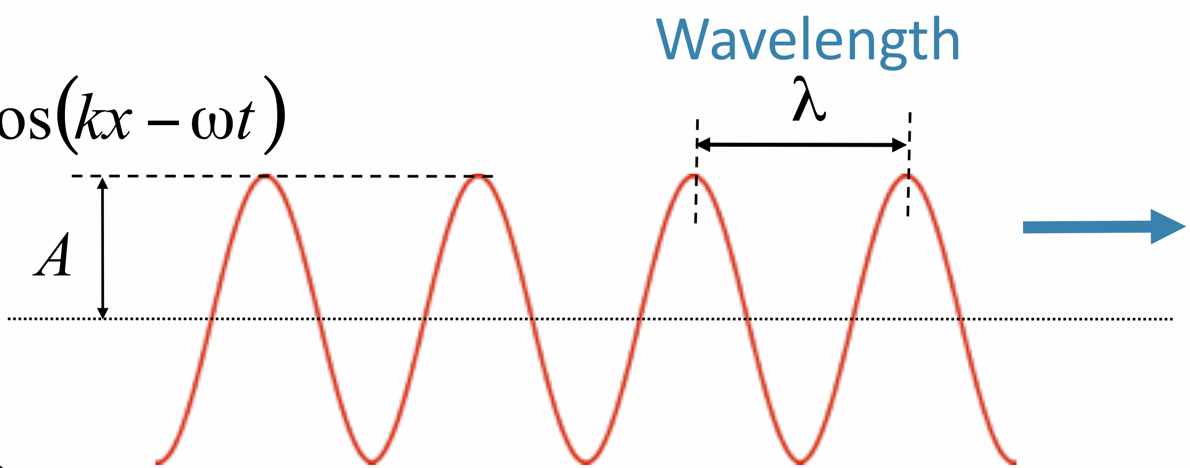
Wavelength:
The distance λ between identical points on the wave.

Movie (tspeed)

Amplitude:
The maximum displacement A of a point on the wave.

Period:
The time P it takes for an element of the medium to make one complete oscillation.

$$y = A\cos(kx - \omega t)$$

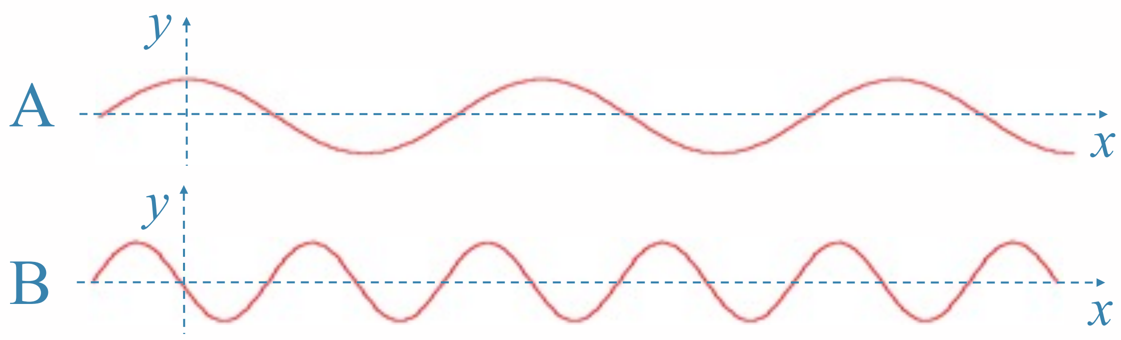


$$v = \frac{\lambda}{P} = \frac{\omega}{k}$$

$$\omega = \frac{2\pi}{P} = 2\pi f \qquad k = \frac{2\pi}{\lambda}$$

String: $v = \sqrt{\frac{T}{\mu}}$

CheckPoint

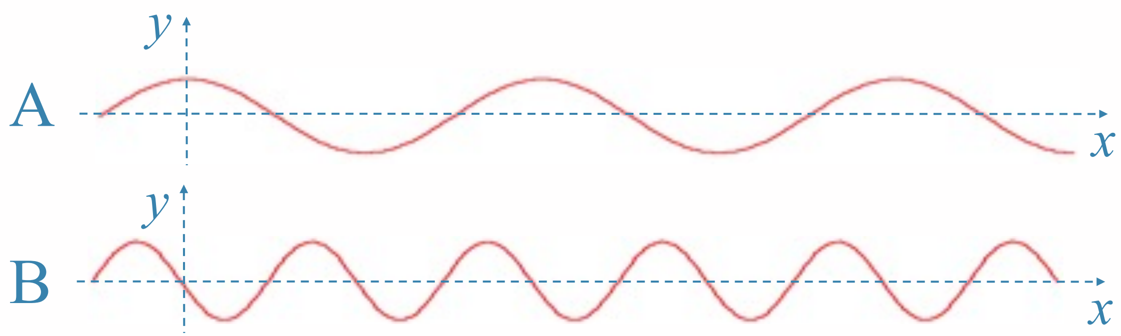


Waves A and B shown above are propagating in the same medium.
How do their frequencies compare?

A) $\omega_A > \omega_B$

B) $\omega_A < \omega_B$

C) $\omega_A = \omega_B$



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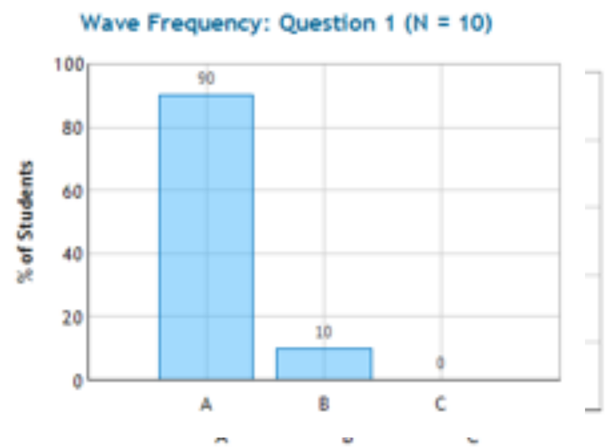


A) It's period is shorter.

B) As they both have the same velocities, and the wavelength of A is larger, the frequency must be smaller than that of B.

C) The tension and mass density are the same in both strings so they have the same frequency.

Clicker Question

The speed of sound in air is a bit over 300 m/s, and the speed of light in air is about 300,000,000 m/s.

Suppose we make a sound wave and a light wave that both have a wavelength of 3 meters. What is the ratio of the frequency of the light wave to that of the sound wave?

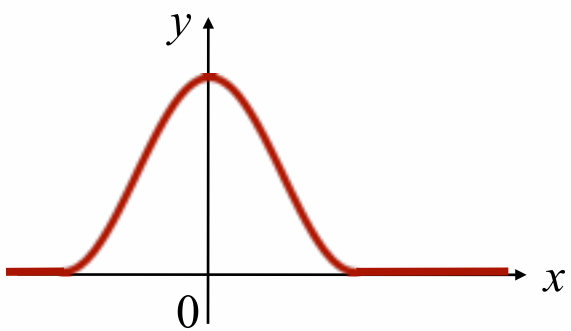
A) About 1,000,000

B) About 0.000001

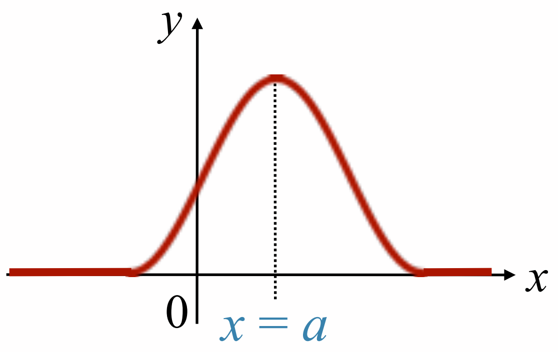
C) About 1000

How to make a Function Move

Suppose we have some function $y = f(x)$:

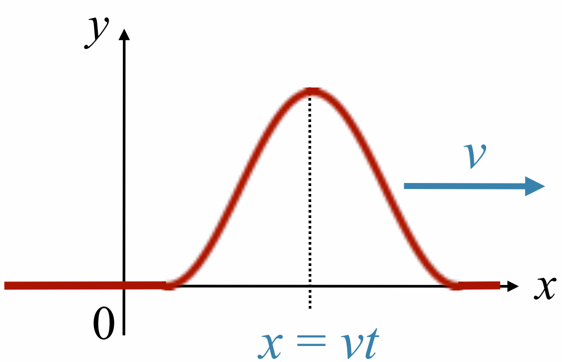


$f(x - a)$ is just the same shape moved a distance a to the right:



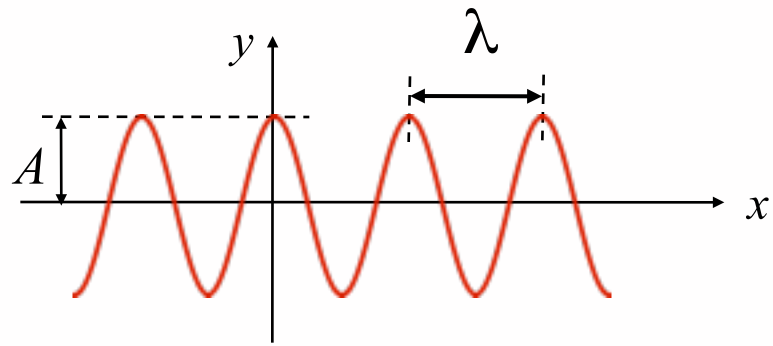
Let $a = vt$ Then

$f(x - vt)$ will describe the same shape moving to the right with speed v.



Harmonic Wave

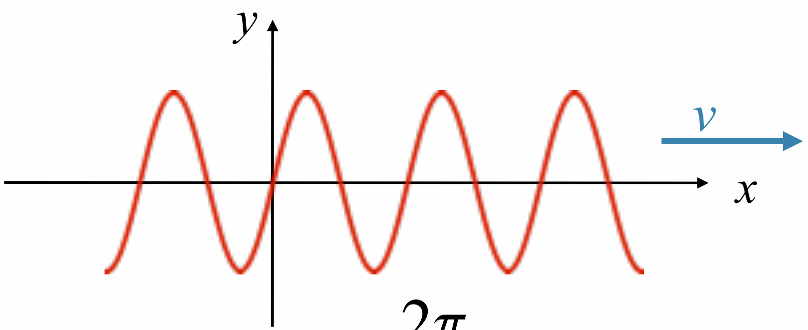
Consider a wave that is harmonic in x and has a wavelength of λ.



If the amplitude is maximum at $x = 0$ this has the functional form:

$$y(x) = A\cos\left(\frac{2\pi}{\lambda}x\right)$$

Now, if this is moving to the right with speed v it will be described by:

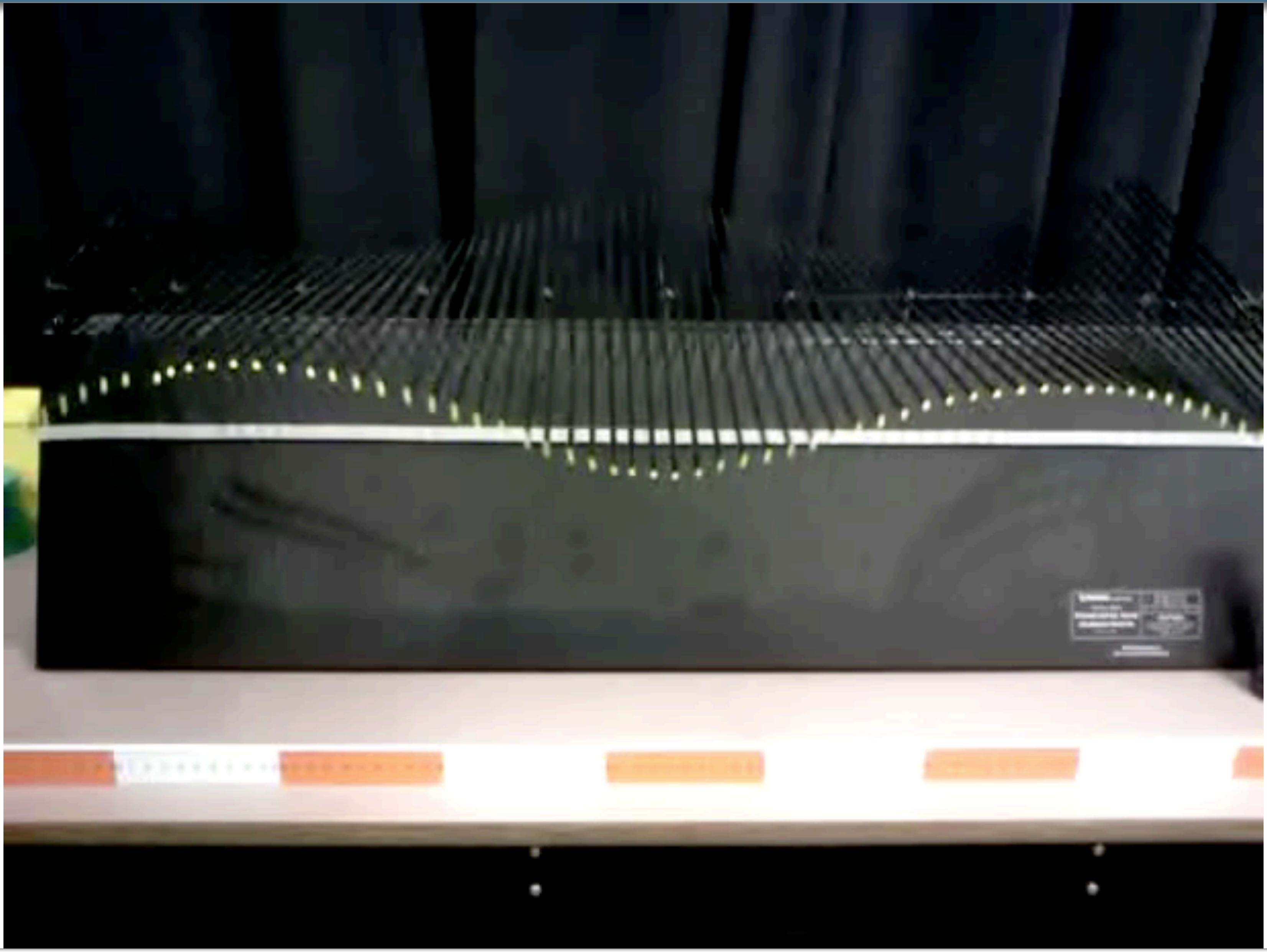


$$y(x) = A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right) = A\cos(kx - \omega t)$$

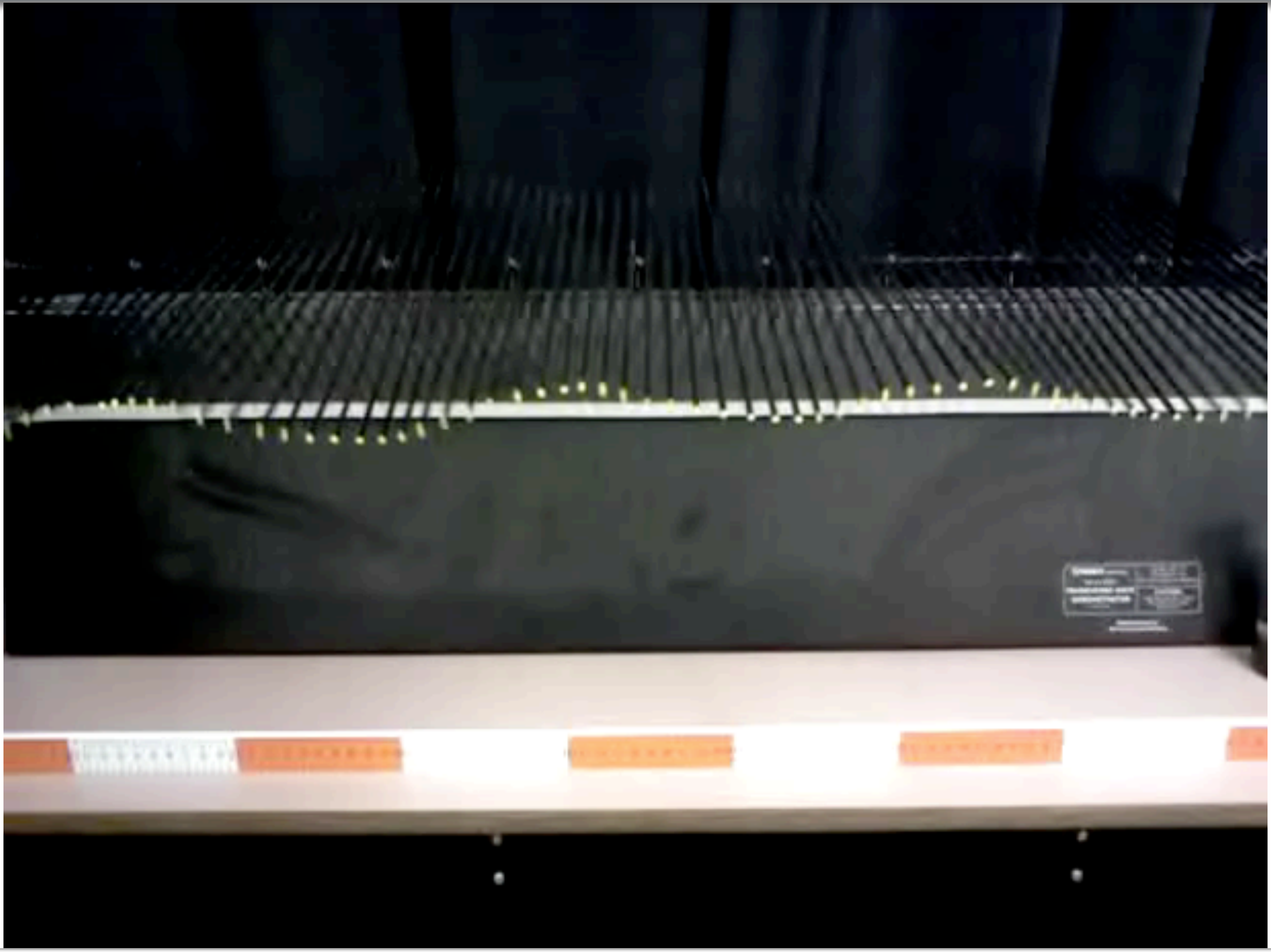
$$k = \frac{2\pi}{\lambda}$$ "wavenumber"

$$\omega = \frac{2\pi v}{\lambda}$$

1 cycle per second

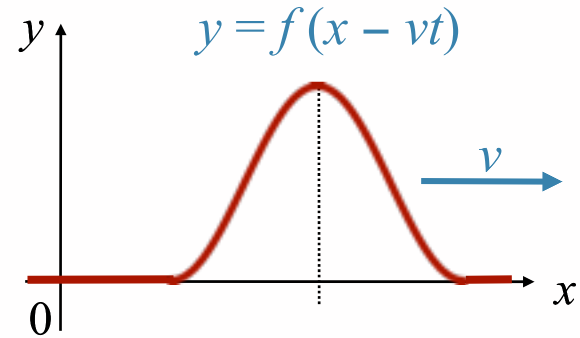


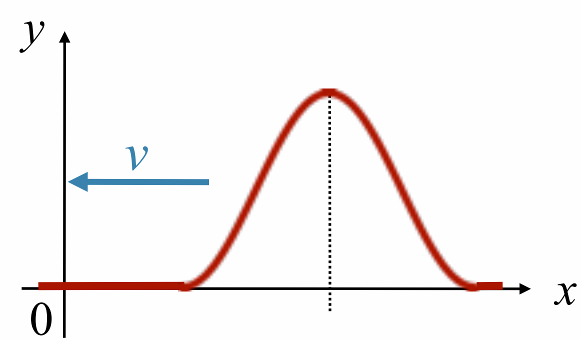
2 cycles per second



Clicker Question

If a function moving to the right with speed v is described by $f(x - vt)$ then what describes the same function moving to the left with speed v?





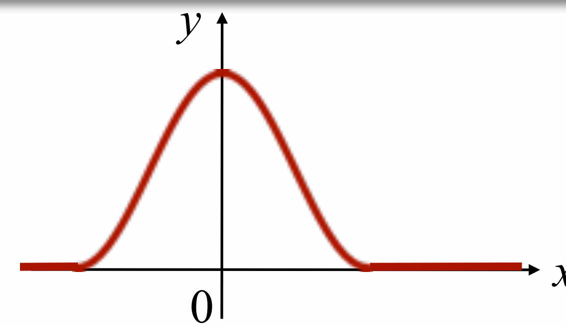
A) $y = -f(x - vt)$

B) $y = f(x + vt)$

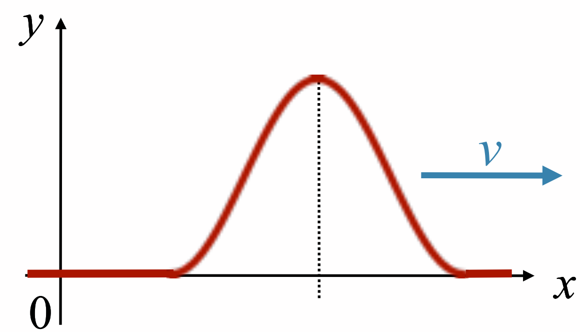
C) $y = f(-x + vt)$

Suppose the function has its maximum at $f(0)$.

$y = f(x - vt)$ $\quad x - vt = 0 \quad x = vt$ $\quad v \rightarrow$

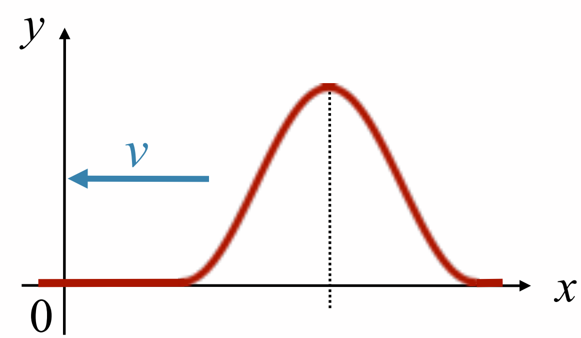


A) $y = -f(x - vt)$ $\quad x - vt = 0 \quad x = vt$ $\quad v \rightarrow$



B) $y = f(x + vt)$ $\quad x + vt = 0 \quad x = -vt$ $\quad v \leftarrow$

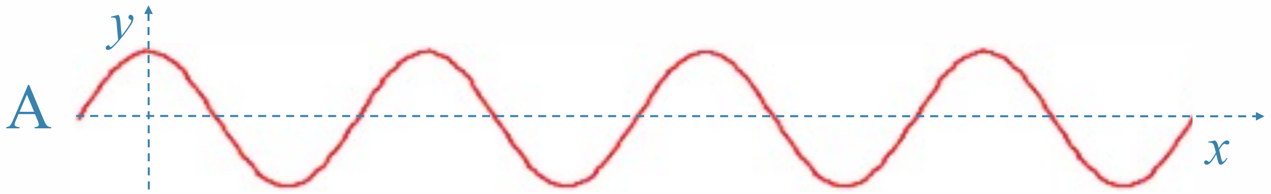
C) $y = f(-x + vt)$ $\quad -x + vt = 0 \quad x = vt$ $\quad v \rightarrow$



Moves to the right when the signs in front of the x and t terms are different

Moves to the left when the signs in front of the x and t terms are the same

CheckPoint



We have shown that the functional form $y(x,t) = A\cos(kx - \omega t)$ represents a wave moving in the $+x$ direction.

Which of the following represents a wave moving in the $-x$ direction?

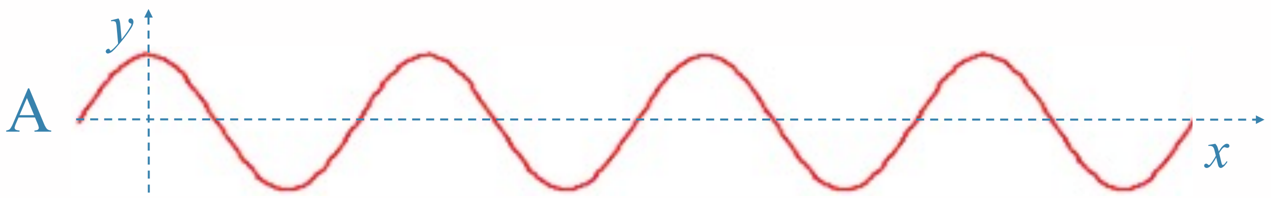
A) $y(x,t) = A\cos(\omega t - kx)$

B) $y(x,t) = A\sin(kx - \omega t)$

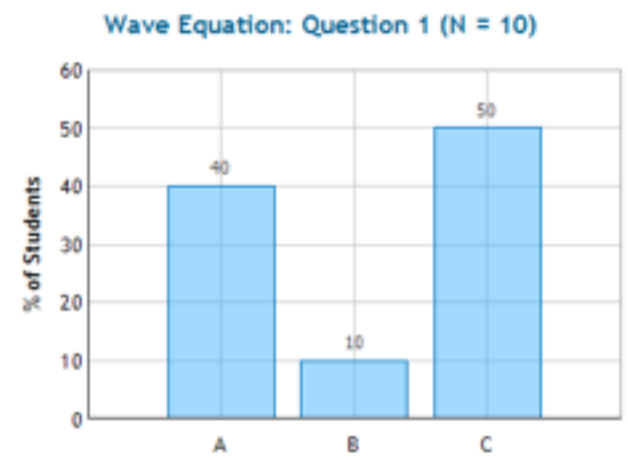
C) $y(x,t) = A\cos(kx + \omega t)$

CheckPoint

We have shown that the functional form $y(x,t) = A\cos(kx - \omega t)$ represents a wave moving in the $+x$ direction.



Which of the following represents a wave moving in the $-x$ direction?

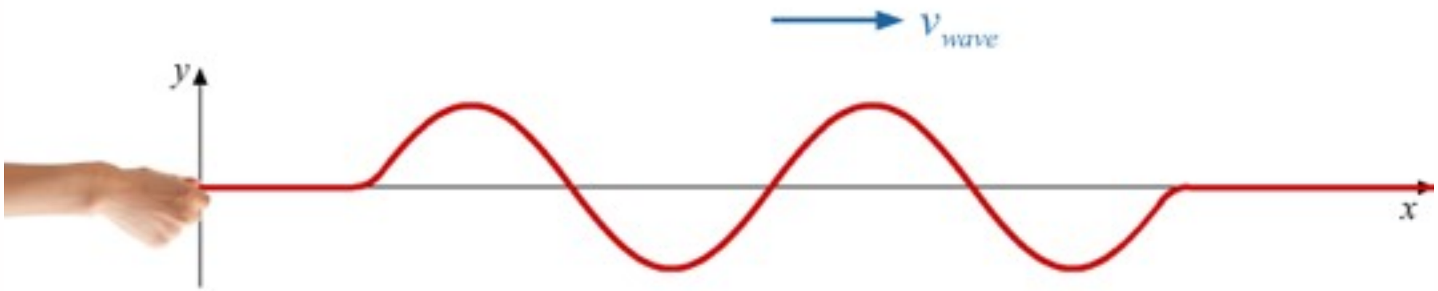


A) $y(x,t) = A\cos(\omega t - kx)$
Cosine is an odd function, so reversing kx and wt will reverse the sign of the function from positive to negative

C) $y(x,t) = A\cos(kx + \omega t)$
With parabolas, (x-a), where a is any number, results in a graph shifted to the right/or positive x direction. Therefore, the opposit[e] (x+a) shi[f]ts it to the left.

Energy

Energy Transmission



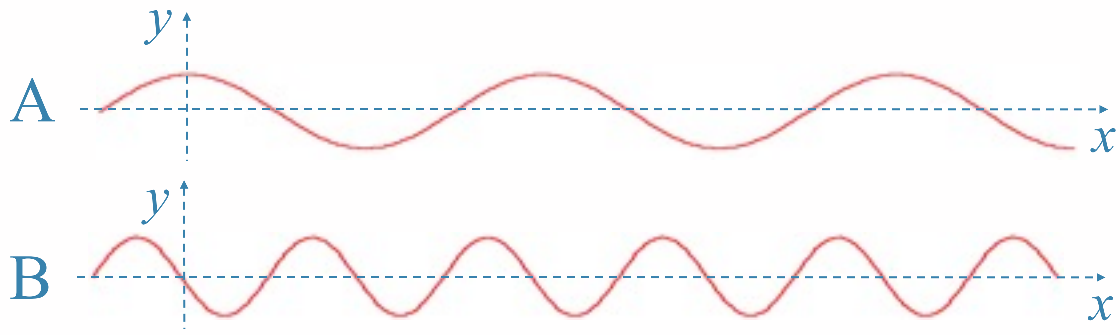
Displacement of a String Element $y(x,t) = A\cos(kx - \omega t)$

Velocity of a String Element $v_y(x,t) = \omega A \sin(kx - \omega t)$

Max Velocity of a String Element $v_{\max} = \omega A$

Max Kinetic Energy of a String Element $K_{\max} \propto \omega^2 A^2$

CheckPoint



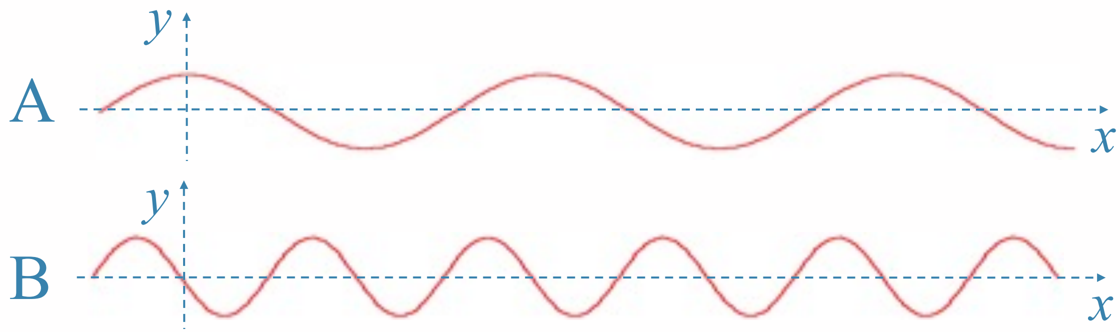
Waves A and B shown above are propagating in the same medium with the same amplitude. Which one carries the most energy per unit length?

A) A

B) B

C) They carry the same energy per unit length

CheckPoint



Which one carries the most energy per unit length?

A) A B) B C) Same

A) Since A has a larger wavelength, it carries more energy per unit.

B) kinetic energy is directly proportional to (ωA^2), since their amplitude are the same and B has a larger ω, B has more energy

C) Assuming that C was supposed to be A=B, their energies are equal since their amplitudes are equal.

Homework Problem

$$y = A\sin(kx + \omega t)$$

Wave Equation

A transverse harmonic wave travels on a rope according to the following expression:

y(x,t) = 0.13sin(3x + 18t)

The mass density of the rope is μ = 0.101 kg/m. x and y are measured in meters and t in seconds.

1) What is the amplitude of the wave?

[ ] m Submit

$$A$$

2) What is the frequency of oscillation of the wave?

[ ] Hz Submit

$$f = \frac{\omega}{2\pi}$$

3) What is the wavelength of the wave?

[ ] m Submit

$$\lambda = \frac{2\pi}{k}$$

4) What is the speed of the wave?

[ ] m/s Submit

$$v = f\lambda$$

Homework Problem

Wave Equation

A transverse harmonic wave travels on a rope according to the following expression:

$y(x,t) = 0.13\sin(3x + 18t)$

The mass density of the rope is $\mu = 0.101$ kg/m. x and y are measured in meters and t in seconds.

5) What is the tension in the rope?

N Submit

$$v = \sqrt{\frac{T}{\mu}} \qquad T = v^2\mu$$

Homework Problem

Wave Equation

A transverse harmonic wave travels on a rope according to the following expression:

$y(x,t) = 0.13\sin(3x + 18t)$

The mass density of the rope is $\mu = 0.101$ kg/m. x and y are measured in meters and t in seconds.

6) At x = 4 m and t = 0.5 s, what is the velocity of the rope? (watch your sign)

[ ] m/s Submit

7) At x = 4 m and t = 0.5 s, what is the acceleration of the rope? (watch your sign)

[ ] m/s^2 Submit

$$y = A\sin(kx + \omega t)$$

$$v_y = \frac{dy}{dt} = \omega A\cos(kx + \omega t)$$

$$a_y = \frac{dv_y}{dt} = -\omega^2 A\sin(kx + \omega t)$$

Homework Problem

Wave Equation

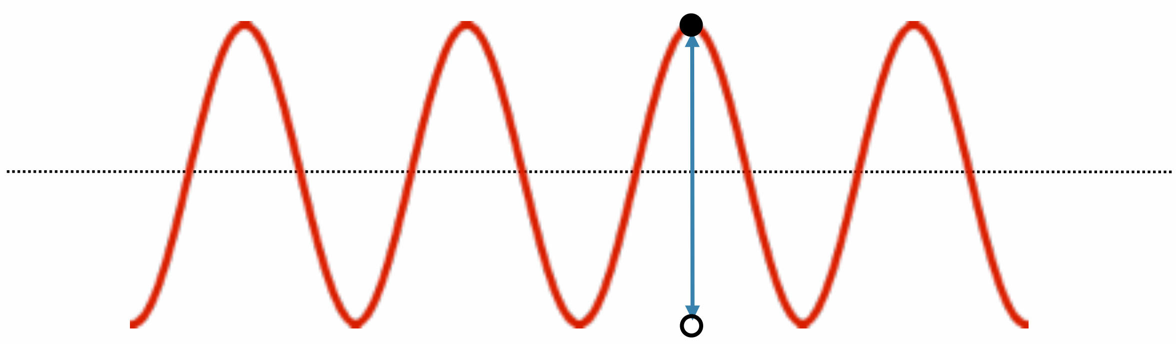
A transverse harmonic wave travels on a rope according to the following expression:

$y(x,t) = 0.13\sin(3x + 18t)$

The mass density of the rope is $\mu = 0.101$ kg/m. x and y are measured in meters and t in seconds.

8) What is the average speed of the rope during one complete oscillation of the rope?

m/s Submit



Average speed = distance traveled / time taken

Distance traveled by a piece of rope during one period is $4A$