Classical Mechanics Lecture 24

Today's Concepts:

A) Superposition

B) Standing Waves

Your Comments

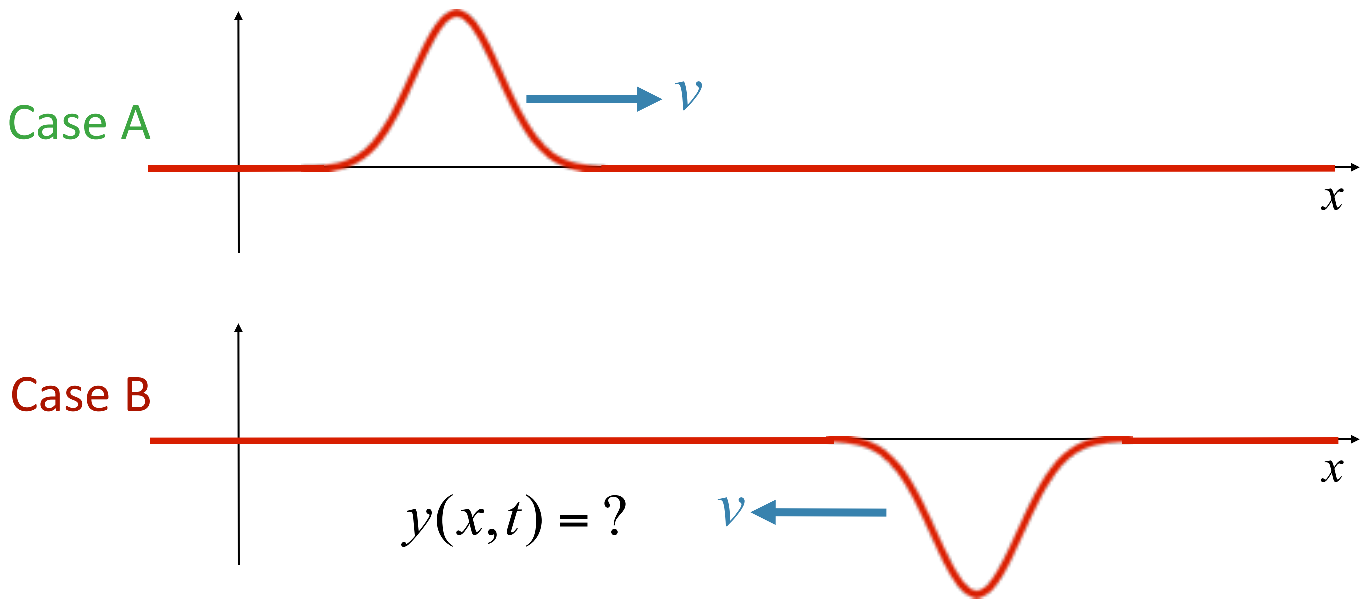
Is the guitar stuff going to be on the final?

Sorry I took 30 seconds on this prelecture...I'm having my first ever semi-emotional exchange with a member of the opposite sex and my brain just can't handle anything at all.

What is lambda fundamental? — Can we talk about the wave number?

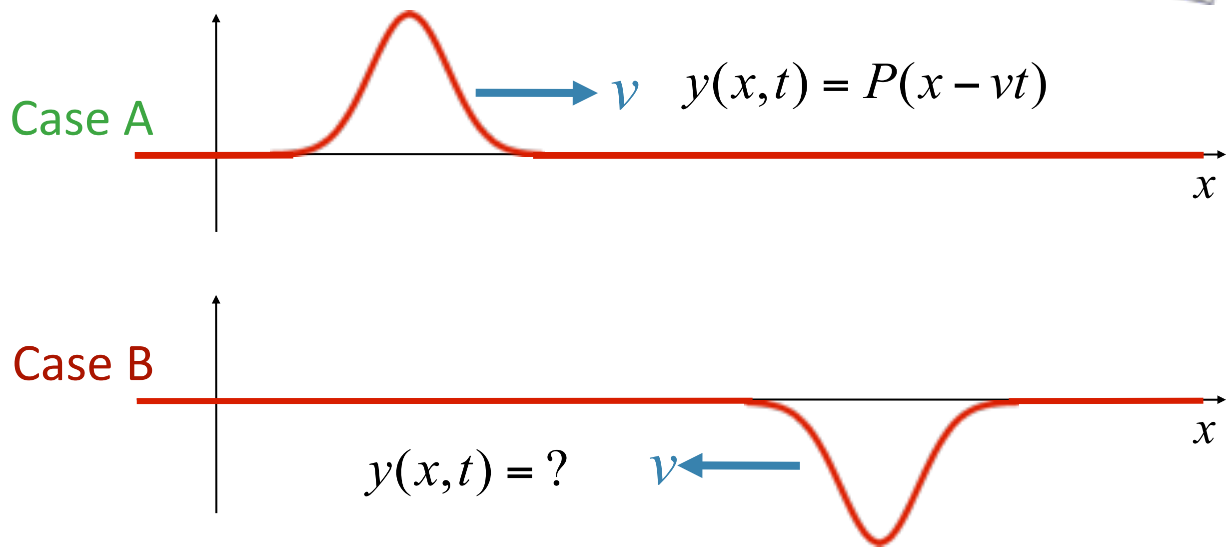
I've finished "watching" all the pre-lectures. Now I have to go back and genuinely watch them.

CheckPoint



Suppose a pulse in Case A described by the function $y(x,t) = P(x-vt)$.
Which of the following functions described the pulse in Case B?

A) $y(x,t) = \quad P(x + vt)$
B) $y(x,t) = -P(x + vt)$
C) $y(x,t) = -P(x - vt)$



A) $y(x,t) = \ \ P(x + vt)$

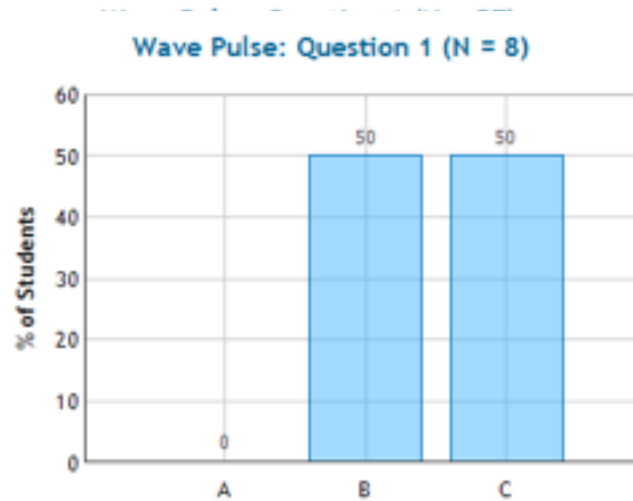
B) $y(x,t) = -P(x + vt)$

C) $y(x,t) = -P(x - vt)$

A) Everything is the same except the direction of the velocity is in the opposite direction.

B) x+vt makes the wave go in the negative x direction, -P flips the wave upside down

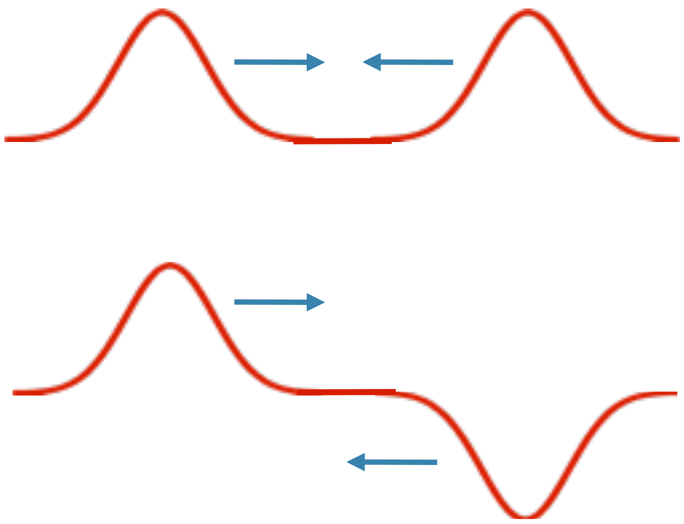
C) This is the only equation that the waves will move to the left.



Superposition

Q: What happens when two waves "collide?"

A: They ADD together



[Movie (super_pulse)](#)

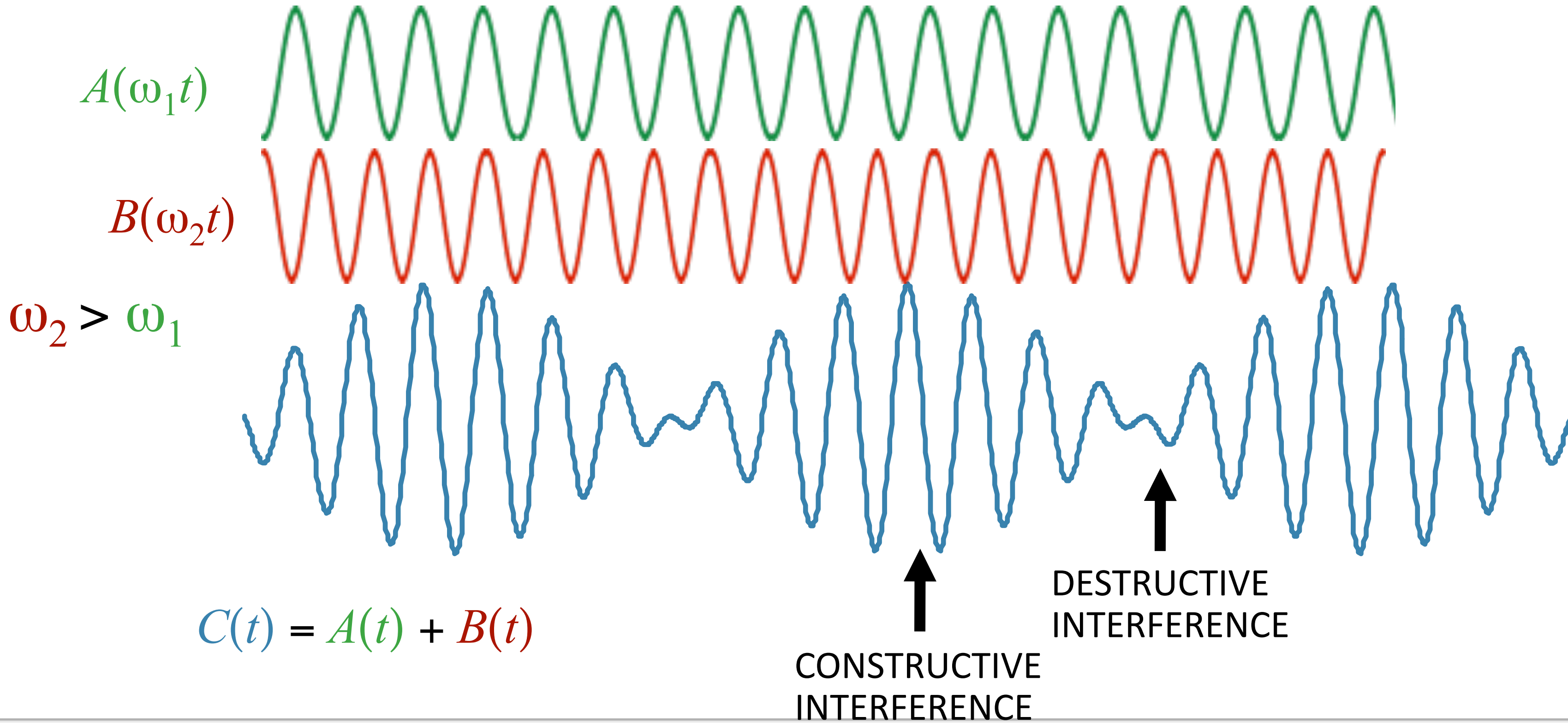
[Movie (super_pulse2)](#)

Superposition

The wave equation we derived last time is linear. (It has no terms where the variables x,t are squared.)

$$\frac{d^2x}{dy^2} = \frac{1}{v^2}\frac{d^2x}{dt^2}$$

For linear equations, if we have two separate solutions, A and B, then $A + B$ is also a solution!



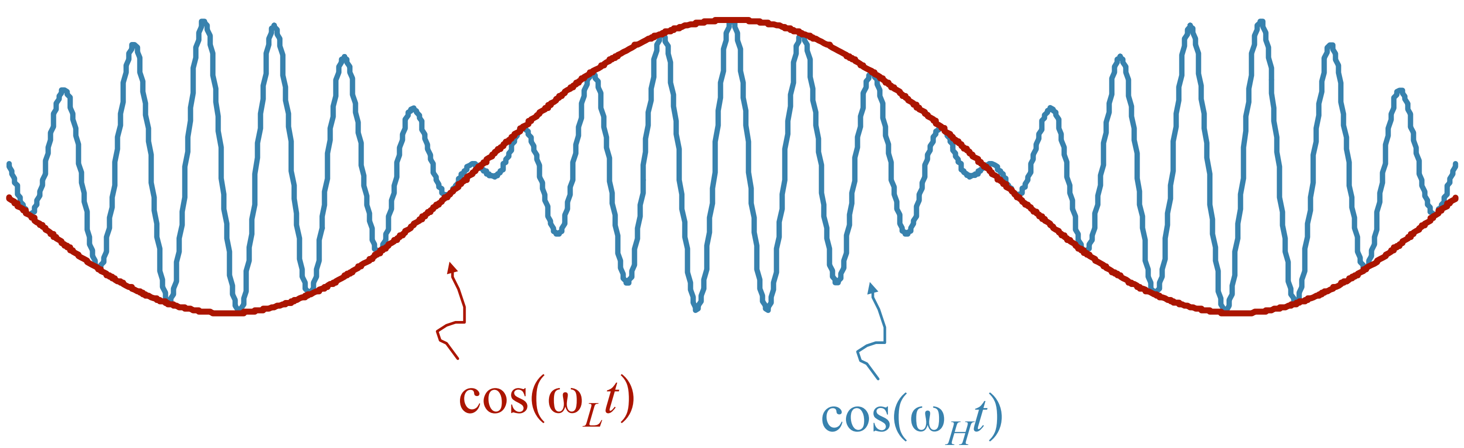
Beats

Can we predict this pattern mathematically? Of course!

Just add two cosines and remember the identity:

$$A\cos(\omega_1 t) + A\cos(\omega_2 t) = 2A\cos(\omega_L t)\cos(\omega_H t)$$

$$\omega_L = \frac{1}{2}(\omega_1 - \omega_2) \qquad \omega_H = \frac{1}{2}(\omega_1 + \omega_2)$$

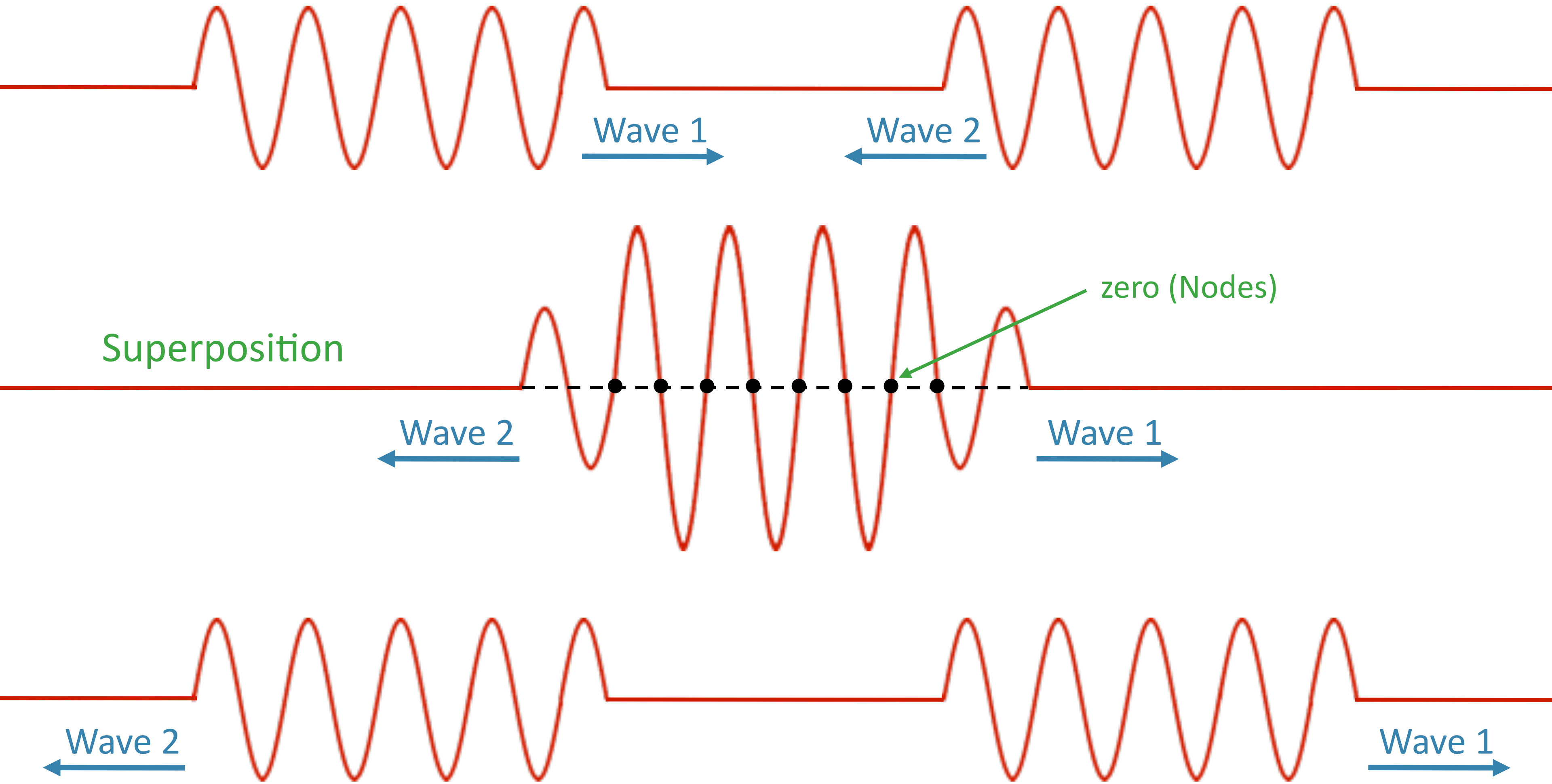


http://www.youtube.com/watch?v=5hxQDAmdNWE

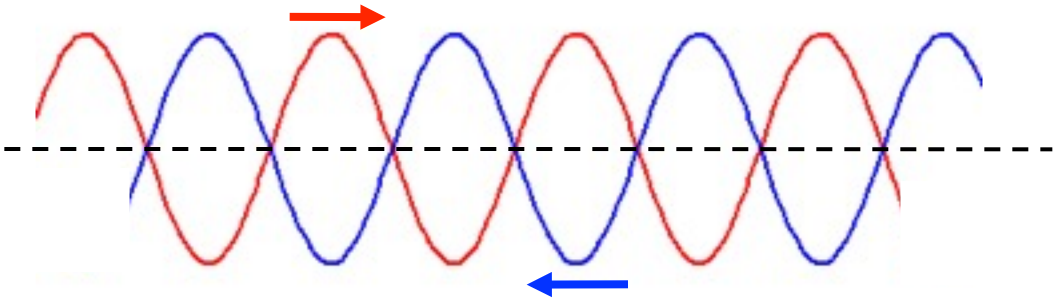
Standing Waves

What happens when two waves having the same frequency but moving in the opposite direction meet?

Movie (super)



How it Works

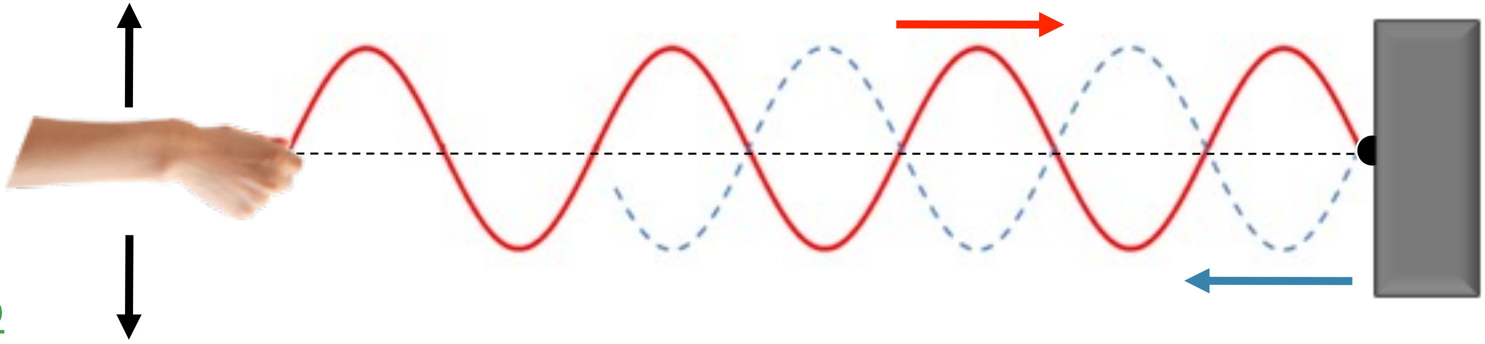


$$A\cos(kx-\omega t)+A\cos(kx+\omega t)=2A\cos(kx)\cos(\omega t)$$

Stationary wave

Changing amplitude

How to make it:



Demo

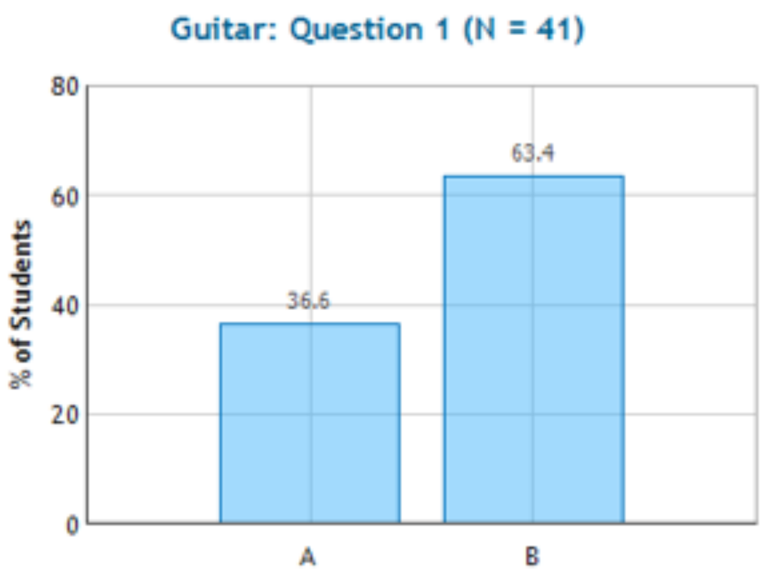
CheckPoint

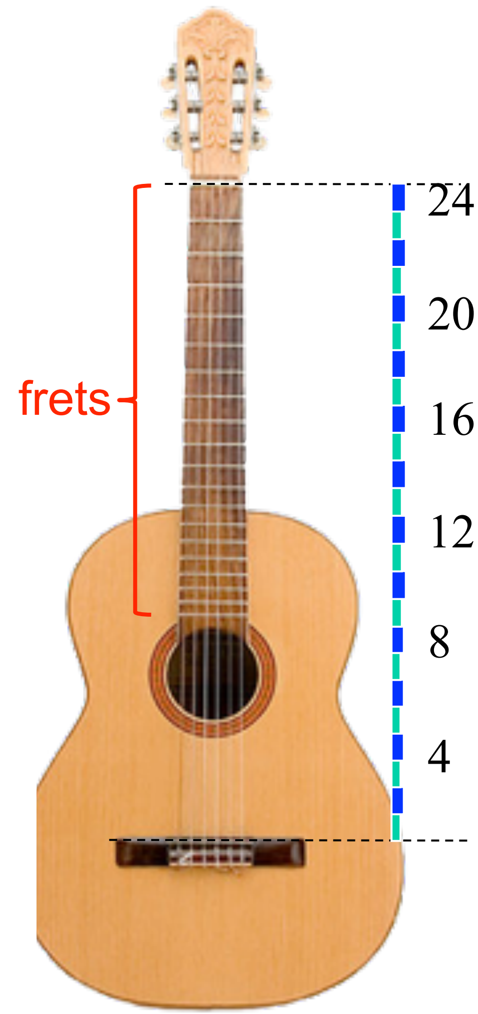
Suppose the strings on your guitar are 24" long as shown. The frets are the places along the neck where you can put your finger to make the wavelength shorter, and appear as horizontal white lines on the picture.

When no frets are being pushed the frequency of the highest string is 4 times higher than the frequency of the lowest string. Is it possible to play the lowest string with your finger on any of the frets shown and hear the same frequency as the highest string?

A) Yes

B) No

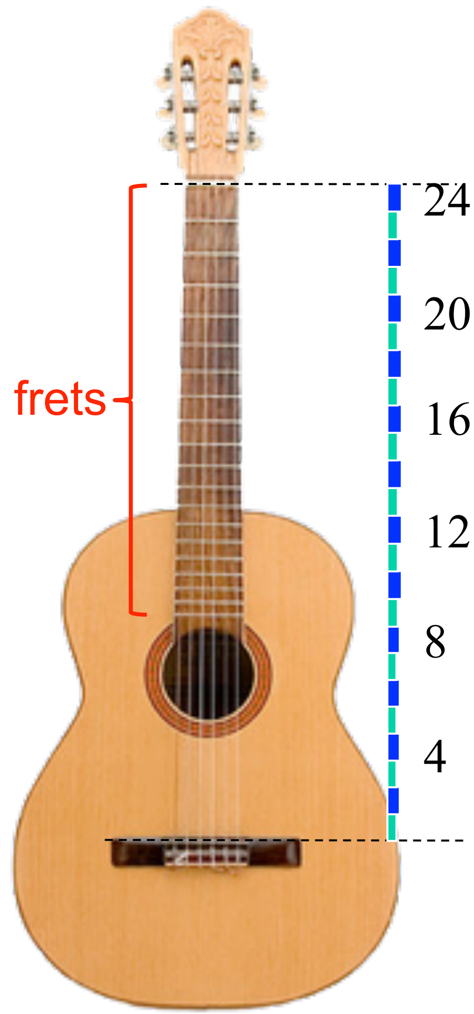




Clicker Question

If you want to increase the frequency of a 24" string by a factor of 4 while keeping the tension the same, how long should the string be?

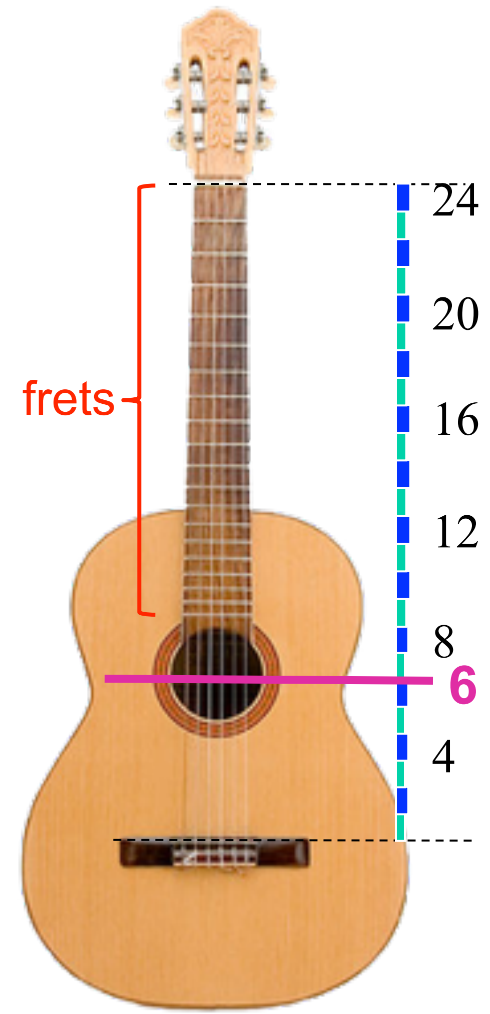
A) 4"   B) 6"   C) 8"   D) 12"



Clicker Question

Is there a fret that you can push on this guitar to make the string 6" long?

A) Yes   B) No



Clicker Question

The highest string vibrates with a frequency that is 4 times that of the lowest string.

Compare the speed of a wave on the high string and the low string:

A) $v_{high} = v_{low}{}^{*}2$

B) $v_{high} = v_{low}{}^{*}4$

C) $v_{high} = v_{low}{}^{*}16$

$$v = \lambda f$$

Clicker Question

The speed of a wave on the high string is 4 times the speed on the lowest string.

If the tensions are the same, how does the mass per unit length of the strings compare:

A) $\mu_{low} = \mu_{high}{}^{*}2$

B) $\mu_{low} = \mu_{high}{}^{*}4$

C) $\mu_{low} = \mu_{high}{}^{*}16$

Recall

$$v^2 = \frac{T}{\mu}$$

$$\mu = \frac{T}{v^2}$$

Clicker Question

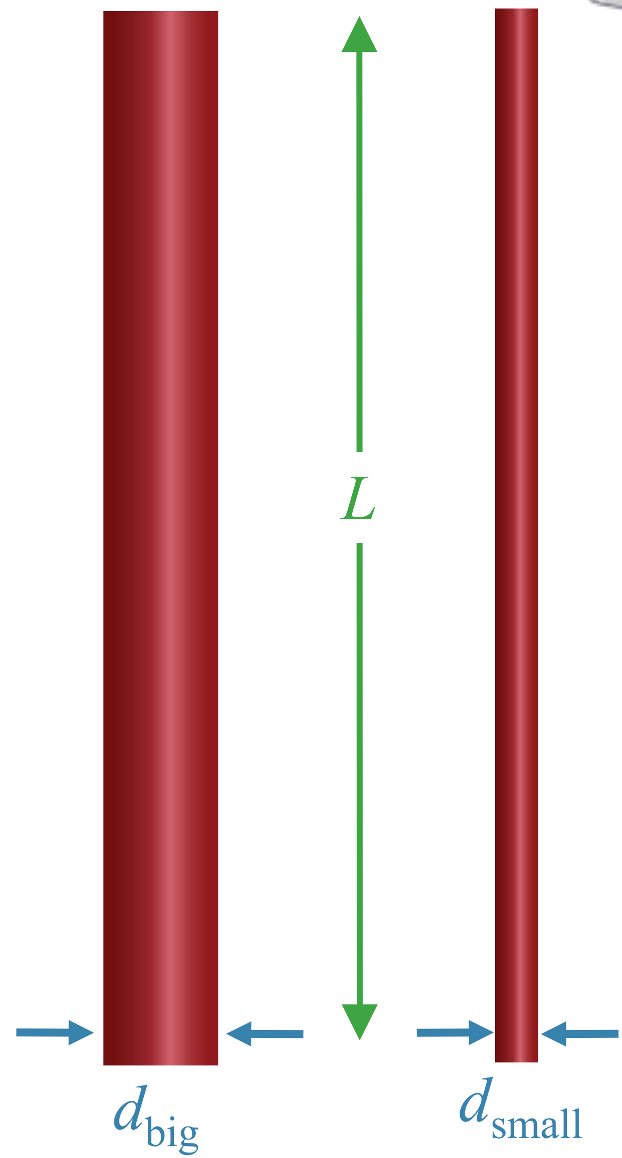
Two cylinders have the same length and are made from the same material. If the mass of the bigger one is 16 times the mass of the smaller one, how do their diameters compare?

A) $d_{low} = d_{high}{}^{*}2$

B) $d_{low} = d_{high}{}^{*}4$

C) $d_{low} = d_{high}{}^{*}16$

$$M = \mu V = \mu L \pi r^2$$



CheckPoint

The 6 strings on a guitar all have about the same length and are stretched with about the same tension. The highest string vibrates with a frequency that is 4 times that of the lowest string.

If the strings are made of the same material, how would you expect the diameters of the lowest and highest strings to compare?

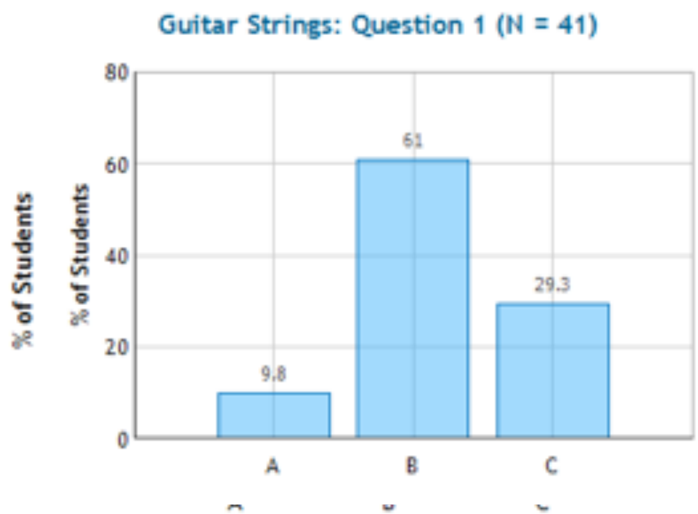
A) $d_{low} = d_{high}*2$
B) $d_{low} = d_{high}*4$
C) $d_{low} = d_{high}*16$

The highest string vibrates with a frequency that is 4 times that of the lowest string. If the strings are made of the same material, how would you expect the diameters of the lowest and highest strings to compare?

A) $d_{\text{low}} = d_{\text{high}} {}^{*}2$

B) $d_{\text{low}} = d_{\text{high}} {}^{*}4$

C) $d_{\text{low}} = d_{\text{high}} {}^{*}16$



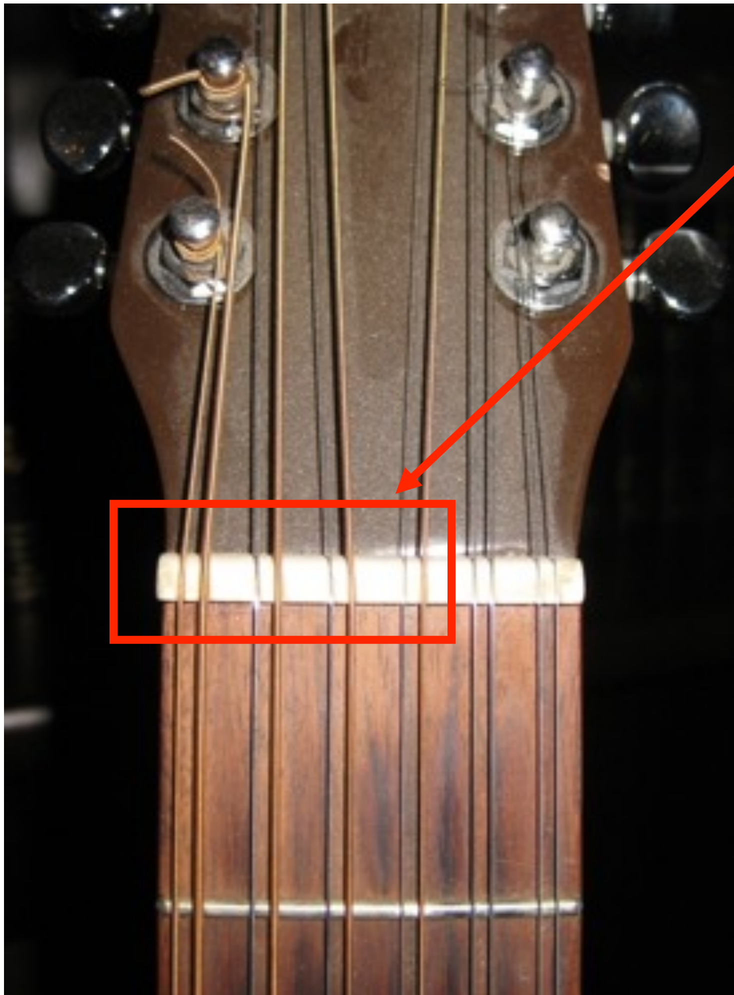
B) Because the frequency is inversely proportional to the square root of the mass density, the mass density would be 16 times greater for the low string. So, the radius would be 4 times greater.

C) Because of the square root sign in the denominator, if diameter is 16 times, mass is 16 times and speed is 1/4 times. So frequency is 1/4 times as wavelength is constant.

We just found that the frequency of a string scales with its diameter if the tension is the same.

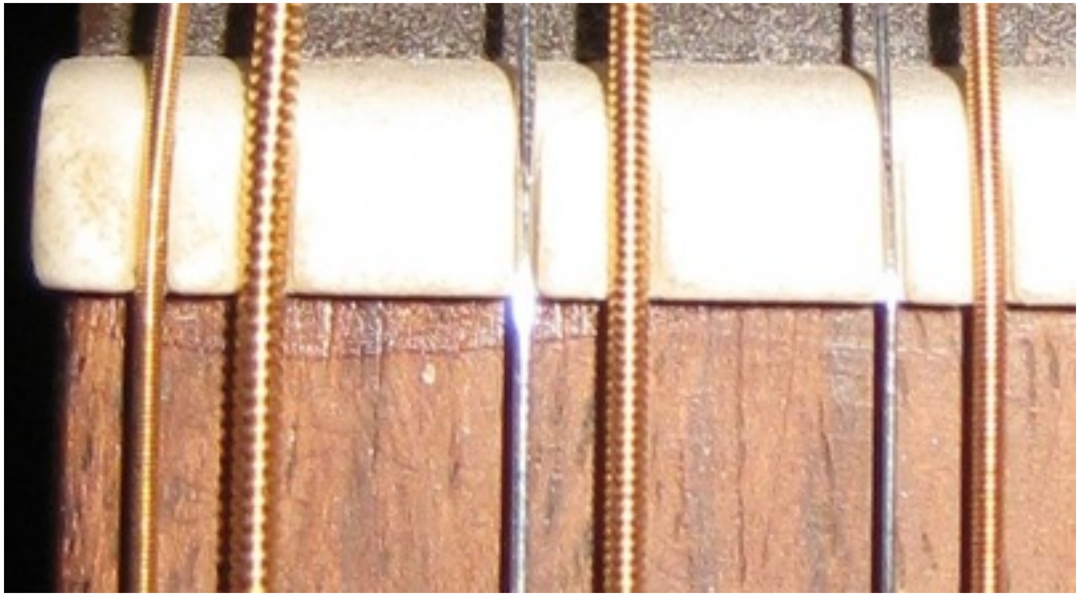
This is really what you find if you examine a guitar:

Different Example – 12 String Guitar



The lighter string in these pairs is one octave higher (2x frequency).

Their diameters are about half of the adjacent strings.



the prelecture claims that the tension in all the strings are the same, but when we tune a guitar isn't the whole process about increasing the tension in different strings or decreasing them to make the string more slack, and thus the note flatter?

Videos

[Tacoma Narrows Bridge Collapse]

[Wave on String Simulation]

Homework Problem

Wave Pulse

A wave pulse travels down a slinky. The mass of the slinky is m = 0.86 kg and is initially stretched to a length L = 6.8 m. The wave pulse has an amplitude of A = 0.2 m and takes t = 0.404 s to travel down the stretched length of the slinky. The frequency of the wave pulse is f = 0.4 Hz.

1) What is the speed of the wave pulse?

m/s Submit

2) What is the tension in the slinky?

N Submit

$$v = L/t$$

$$m = m/L$$

$$v^2 = \frac{T}{\mu} \rightarrow T = v^2 \mu$$

Wave Pulse

A wave pulse travels down a slinky. The mass of the slinky is m = 0.86 kg and is initially stretched to a length L = 6.8 m. The wave pulse has an amplitude of A = 0.2 m and takes t = 0.404 s to travel down the stretched length of the slinky. The frequency of the wave pulse is f = 0.4 Hz.

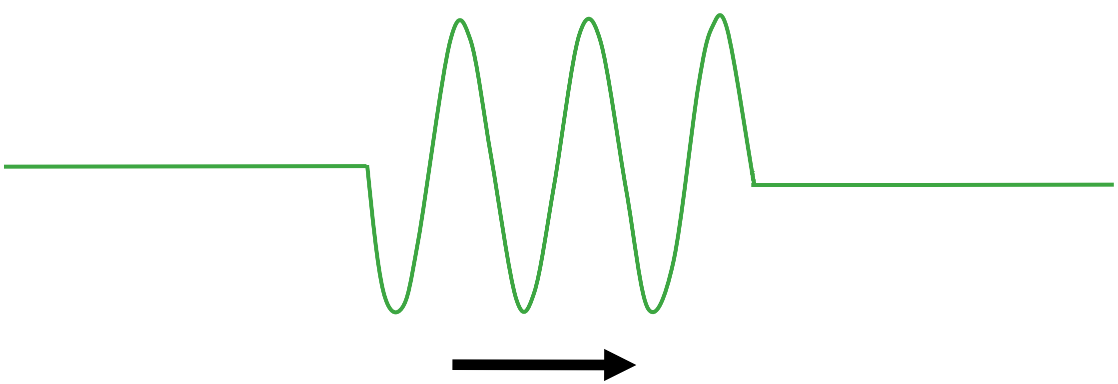
3) What is the average speed of a piece of the slinky as a complete wave pulse passes?

m/s Submit

4) What is the wavelength of the wave pulse?

m Submit

Average speed = distance / time $= 4A/P = 4Af$



$$v = f\lambda \qquad \lambda = \frac{v}{f}$$

Homework Problem

Wave Pulse

A wave pulse travels down a slinky. The mass of the slinky is m = 0.86 kg and is initially stretched to a length L = 6.8 m. The wave pulse has an amplitude of A = 0.2 m and takes t = 0.404 s to travel down the stretched length of the slinky. The frequency of the wave pulse is f = 0.4 Hz.

5) Now the slinky is stretched to twice its length (but the total mass does not change).

What is the new tension in the slinky? (assume the slinky acts as a spring that obeys Hooke's Law)

N Submit

6) What is the new mass density of the slinky?

kg/m Submit

7) What is the new time it takes for a wave pulse to travel down the slinky?

s Submit

Same thing again…

So long, and thanks for all the fish.