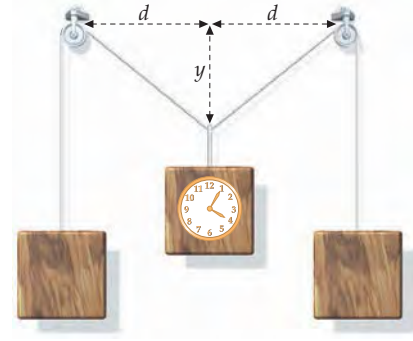
15. You have designed a novelty desk clock, as shown. You are worried that it is not ready for market because the clock itself might be in an unstable equilibrium configuration. You decide to apply your knowledge of potential energies and equilibrium conditions and analyze the situation. The clock (mass m) is supported by two light cables, each of length L, running over the two frictionless pulleys of negligible diameter, which are attached to counterweights that each have mass m.

(a) Find the potential energy of the system as a function of the distance y.



$$U(y) = mgy - 2mg\sqrt{d^2 + y^2}$$

Any constant can be added to this without loss of generality, eg L

(b) For $y = d/\sqrt{3}$ the potential energy is a minimum, and then the system is in equilibrium. Show that the forces on it sum to zero for this value of y.

If $y = d/\sqrt{3}$ and T is the tension on the ropes and T_y the y component on the ropes supporting the clock then

$$\frac{T_y}{T} = \frac{d\sqrt{3}}{\sqrt{d^2 + d^2/3}} = \frac{1}{2}$$

$T = mg$ so $T_y = mg/2$

Thus the two ropes supporting the clock balance the gravitational force downward on the clock.