SP2-1)

Measurements: 21.3 cm
21.5 cm
21.4 cm
21.2 cm
21.4 cm

(a)

Average:

$$\mu = \frac{\sum_{i=1}^{N} x_i}{N}$$
$$= \frac{x_1 + x_2 + x_3 + x_4 + x_5}{N}$$
$$= \frac{21.3 + 21.5 + 21.4 + 21.2 + 21.4}{5}$$
$$= 21.36 \text{ cm}$$

$$\mu = 21.4 \text{ cm}$$

Standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} r_i^{\,2}}{N-1}}$$

$$r_i = \mu - x_i$$

r_1 = 21.4 – 21.3 = 0.1 cm
r_2 = 21.4 – 21.5 = - 0.1 cm
r_3 = 21.4 – 21.4 = 0.0 cm
r_4 = 21.4 – 21.2 = 0.2 cm
r_5 = 21.4 – 21.4 = 0.0 cm

$$\sigma = \sqrt{\frac{0.1^2 + (-0.1)^2 + 0.0^2 + 0.2^2 + 0.0^2}{4}}$$
$$= \sqrt{\frac{0.06}{4}}$$
$$= 0.015 \text{ cm}$$

$$\sigma = 0.1 \text{ cm}$$

(b)

The standard deviation is a way of defining the amount of uncertainty in a set of measurements.

If there were no uncertainty in the measurements, then every measurement would be equal to the mean of the set of data. Then, the residuals (r_i) would all be zero. The standard deviation would then be zero:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} {r_i}^2}{N-1}} = \sqrt{\frac{0}{N-1}} = 0$$

In this problem, the standard deviation is not zero. Therefore, there is uncertainty in the measurements.

Food for thought:

Could you come up with an experiment in which there is no statistical error? Try to come up with one or both of an idealized and realistic experiment. (In an idealized experiment you could assume that the measuring apparatus work perfectly, or that gravity is the only force in existence, for examples.)

(c)

Systematic errors would (generally) change all of the measurements' values away from their true value in the same direction. For example, a systematic error might increase all of the measurements, relative to what the measurements should show.

Since we do not know the true value of the quantity that we are trying to measure (i.e. the size of the paper), we cannot say whether there are any systematic errors.

SP2-3)

More scatter in the data means the data is less precise: the experiment has less precision. Our predictive power also goes down, when the precision goes down: we can not tell where the next dart will fall, since they are spread over such a large range – so, we say our measurements are ***uncertain***. Hence, the term "uncertainty".

Sara's throws land over a larger area, on the target, than do Ryan's. So, Sara's throws are less precise than are Ryan's.

Data being preferentially located off to one side of the desired (or, true) value is being ***systematically*** changed: every time a throw is made, something changes the dart's trajectory by a consistent amount. (If we did not know the true value of what we were trying to measure, we might not be able to identify the presence of a systematic error.) This changes the experiment's accuracy.

Ryan's board shows that the mean value of his throws is not on the centre of the board, whereas Sara's throws have a mean somewhat near the board's centre. Therefore, Ryan's throws are more accurate than are Sara's.

Poor eyesight would cause the board to be blurry; since it is harder to hit a target that you cannot see properly, the scatter would increase. Precision would therefore decrease.

Dart-throwing methods that preferentially send the dart to one side - such as flicking one's hand to the left just before releasing the dart – would decrease the accuracy of the throws.

SP2-4)

For example, let's look at how we can roll a sum of 4. The only three (integer) numbers that add to that sum are (1,1,2). I could roll those numbers a few different ways.

Let's say my dice are coloured: one each of green, red, and blue. Then, I could roll (1,1,2) in any of the following ways:

Green	Red	Blue
1	1	2
1	2	1
2	1	1

So, there are three ways in which I could roll the combination (1,1,2).

Let's look at how many ways in which we could roll (1,2,3) – one way to sum the dice to 6.

Green	Red	Blue
1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

So, we can see that there are six possible ways to roll three dice such that one of the dice shows a 1, another shows a 2, and another shows a 3.

Extrapolating, we see that when we want to roll three dice so that each die shows a different (specified) value, there are six ways to do it.

Similarly, when we want to roll three dice so that two of the dice show the same (specified) value and the third one shows a different (specified) number, then there are three ways to do it.

We can make a similar table for the case of each die showing the same number, as for the roll (1,1,1). We see that there is one way to roll three dice such that all of the dice show the same (specified) value.

With this information, we can make our table.

Sum	Three numbers which add to that sum	Number of ways to roll these numbers	Number of ways to roll this sum	Probability of rolling this sum *
3	1,1,1	1	1	1/216
4	1,1,2	3	3	3/216
5	1,2,2 1,1,3	3 3	6	6/216
6	1,1,4 1,2,3 2,2,2	3 6 1	10	10/216
7	1,1,5 1,2,4 1,3,3 2,2,3	3 6 3 3	15	15/216
8	1,1,6 1,2,5 1,3,4 2,2,4 2,3,3	3 6 6 3 3	21	21/216
9	1,2,6 1,3,5 1,4,4 2,2,5 2,3,4 3,3,3	6 6 3 3 6 1	25	25/216
10	1,3,6 1,4,5 2,2,6 2,3,5 2,4,4 3,3,4	6 6 3 6 3 3	27	27/216

By symmetry, the following sums have the same probability:

3 and 18
4 and 17
5 and 16
6 and 15
7 and 14
8 and 13
9 and 12
10 and 11

(You should show a more detailed argument as to why the probabilities are the same.)

* The following is an elaboration on the calculation of probability, in column 5.

The probability of rolling a given sum (call it sum A) is equal to the number of ways to roll that sum (call it N[A]) divided by the total number of ways to roll the dice (call it T).

$$\mathrm{P[A]} = \frac{\mathrm{N[A]}}{T}$$

We can find the total number of ways to roll the dice in two ways:

- For each possible sum, we know the number of possible ways to roll that sum. Add these numbers of possible rolls, to get the total number of possible rolls. (I.e. sum column 4 in the above table, accounting for the sums which we specified the probability for via a symmetry argument.)

- We know each die can roll the numbers 1 to 6. The probability of rolling a given number on one die is therefore 1/6. Via the rules of probability theory (which you might not have learned, yet), the probability of rolling a specified number on one die, another specified number on another die, and a third specified number on a third die (whew!) is simply the multiplication of the probabilities of rolling each specified number each die. For instance, to roll a 3 on dice one, a 2 on dice 2, and a 6 on dice 3, the probability is (1/6) x (1/6) x (1/6) = 1/216.

There are therefore 216 possible ways to roll the dice: T = 216.