"Knowing is not enough; we must apply. Willing is not enough; we must do."

Bruce Lee

OBJECTIVES

1. To extend the intuitive notion of work as physical effort to a formal mathematical definition of work as a function of force and distance.

2. To develop an understanding of the physical significance of mathematical integration.

3. To understand the concept of power and its relationship to work.

4. To understand the concept of kinetic energy and its relationship to work as embodied in the work-energy theorem.

5. To relate experimentally determined kinetic energy loss to frictional loss.

6. To apply concepts involving the conservation of momentum, work, and energy to the analysis of breaking boards in Karate.
OVERVIEW

In the study of collisions you found that, although momentum always appears to be conserved, very different outcomes are possible. For example, when two carts move toward each other with the same speed they may stop dead in a sticky or inelastic collision, they may bounce off each other and have the same speed after their interaction, or they may explode as a result of springs being released and move even faster than before.

In order to understand the effects of various types of interactions, it is helpful to develop two new concepts — work and energy. These concepts can be related by Newton's laws to force, distance, and velocity. In this unit, you will begin the process of understanding the scientific definitions of work and energy. You will pay particular attention to one special form of mechanical energy, called kinetic energy. For example, in inelastic collisions we know that kinetic energy is lost and hence not conserved. Kinetic energy can be gained in an explosion because stored energy is added to the system.

By relating kinetic energy to work in an idealized situation involving no frictional forces, you can begin to develop an understanding of one of the most powerful principles in both classical and modern physics, that of conservation of energy.

In order to study the relationship between work and kinetic energy for masses that move without much friction, we will study two situations: (1) the motion of a falling mass near the surface of the earth; and (2) the motion of a mass oscillating on a spring. A mass at the end of a spring experiences a force whenever the spring is stretched or contracted and no force whenever the spring is neither stretched nor contracted. Thus, a mass that is oscillating back and forth at the end of a spring experiences forces that are not constant. How much work is needed to stretch a spring with a mass on the end of it? What kinetic energy can be associated with the motion of the mass as it oscillates. How is this kinetic energy related to work?
The Concept of Physical Work
Suppose you are president of the Load 'n' Go Co. A local college has three jobs available and will allow you to choose which one you want before offering the other two jobs to rival companies. All three jobs pay the same amount of money. Which one would you choose for your crew?

Figure 10-1: A description of jobs to bid on

Activity 10-1: Choosing Your Job
Examine the descriptions of the jobs shown in Figure 10-1. Which one would you be most likely to choose? Least likely to choose? Explain the reasons for your answer.

Job 1 and job 2 are essentially the same, since job 2 requires me to move the boxes twice the distance as in job 1 (8m) but I'm only working against half the force of gravity due to the 30° ramp, so I would end up doing the same amount of work. Job 3 is better, though, because while the boxes weigh twice as much, there are half as many of them AND I only have to move them half the distance as in job 1, so I would go with job 3.
You obviously want to do the least amount of work for the most money. Before you reconsider your answers later in this unit, you should do a series of activities to get a better feel for what physicists mean by work and how the president of Load 'n' Go can make top dollar.

In everyday language we refer to doing work whenever we expend effort. In order to get an intuitive feel for how we might define work mathematically, you should experiment with moving your textbook back and forth along a tabletop and a rougher surface such as a carpeted floor.

**Figure 10-2: Doing physical work while "playing".**

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**Activity 10-2: This is Work!**

(a) Pick a distance of a metre or so. Sense how much effort it takes to push a heavy book that distance. How much more effort does it take to push it twice as far?

It takes about twice the effort to push the block twice the distance.

(b) Pile another similar book on top of the original one and sense how much effort it takes to push the two books through the distance you picked.

Again, it takes about twice the effort to push two books the original distance.

(c) From your study of sliding friction, what is the relationship between the mass of a sliding object and the friction force it experiences? On the basis of your experience with sliding friction, estimate how much more force you have to apply to push two books compared to one book.

Sliding friction is proportional to the mass of an object when it is sliding across a horizontal surface, so twice the mass should require twice to force to overcome friction.

(d) If the "effort" it takes to move an object is associated with physical work, guess an equation that can be used to define work mathematically when the force on an object and its displacement (i.e., the distance it moves) lie along the same line.

I imagine it would be something like

\[ \text{Work} = \text{Force} \times \text{Distance} \]
In physics, work is not simply effort. In fact, the physicist’s definition of work is precise and mathematical. In order to have a full understanding of how work is defined in physics, we need to consider its definition in a very simple situation and then enrich it later to include more realistic situations.

**A Simple Definition of Physical Work:** If an object that is moving in a straight line experiences a constant force in the direction of its motion while it is moving, the **work done by the external force,** \( F_{\text{ext}} \), is defined as the product of the force and the displacement of the object.

\[
W = F_{\text{ext}} \Delta x
\]  

where \( W \) represents the work done by the external force, \( F_{\text{ext}} \) is the external force, and \( \Delta x \) the displacement of the object. When the force and displacement are in the same direction the work is *positive*:

\[
W = |F_{\text{ext}}| |\Delta x| > 0
\]

What if the force of interest and the displacement are in the opposite direction? For instance, what about the work done against the force of sliding friction, \( F_f \), when a block slides down an inclined plane as a result of the gravitational force? In this case friction does *negative* work because the signs of \( \Delta x \) and \( F_f \) are opposite. This is given by

\[
W_{\text{friction}} = F_f \Delta x = -|F_f| |\Delta x| < 0
\]  

**Activity 10-3: Applying the Physics Definition of Work**

(a) Does effort necessarily result in physical work? Suppose two guys are in an evenly matched tug of war. They are obviously expending effort to pull on the rope, but according to the definition of *physical work*, are they doing any physical work? Explain.

*They are both exerting a force, but nothing is moving so the displacement \( \Delta x \) is 0, which means the work done is 0, so no work is done if nothing is actually moved by the force.*
(b) A wooden block with a mass of 0.30 kg is pushed along a sheet of ice that has no friction with a constant external force of 10 N which acts in a horizontal direction. After it moves a distance of 0.40 m how much work does the external force do on the block?
\[ W = F \times \Delta x \]
\[ W = (10 \text{ N})(0.40 \text{ m}) \]
\[ W = 4.0 \text{ J} \]

(c) The same wooden block with a mass of 0.30 kg is pushed along a table with a constant external force of 10 N which acts in a horizontal direction. It moves a distance of 0.40 m. However, there is a friction force opposing its motion. Assume that the coefficient of sliding friction, \( \mu_{\text{kin}} \), is 0.20.

1. Is work done by the external force positive or negative? Show your calculations.
\[ F_{\text{external}} = +10 \text{ N} \]
\[ W_{\text{external}} = F_{\text{external}} \times \Delta x \]
\[ W_{\text{external}} = (+10 \text{ N})(0.40 \text{ m}) \]
\[ W_{\text{external}} = +4.0 \text{ J} \]

2. Does the friction force do positive work or negative work? Show your calculations.
\[ F_{\text{friction}} = -\mu_{\text{kin}} mg \]
\[ F_{\text{friction}} = -(0.20)(0.30 \text{ kg})(9.8 \frac{\text{m}}{s^2}) \]
\[ F_{\text{friction}} = -0.588 \text{ N} \]
\[ W_{\text{friction}} = F_{\text{friction}} \times \Delta x \]
\[ W_{\text{friction}} = (-0.588 \text{ N})(0.40 \text{ m}) \]
\[ W_{\text{friction}} = -0.235 \text{ J} \]

(d) Suppose you lift a 0.3 kg object through a distance of 1.0 m.

1. What is the work associated with the force that the earth exerts on the object? Is the work positive or negative? Show your calculations.
(2) What is the work associated with the external force you apply to the object? Is the work from the force you apply positive or negative? Show your calculations.

(assume applied force is just enough to counter gravity)

\[ \vec{F}_{\text{applied}} = +2.94 \, N \]
\[ W_{\text{gravity}} = \vec{F}_{\text{gravity}} \times \Delta x \]
\[ W_{\text{gravity}} = (-2.94 \, N)(1.0 \, m) \]
\[ W_{\text{gravity}} = -2.94 \, J \]

Pulling at an Angle – What Happens When the Force and the Displacement Are Not Along the Same Line?

Let's be more quantitative about measuring force and distance and calculating the work. How should work be calculated when the external force and the displacement of an object are not in the same direction? For this project you'll need:

- A 0-5 N spring scale
- A fairly smooth horizontal surface
- A wood block with a hook on it (or you can use an upside-down cart.
- Some weights

Before you make your simple force measurements, you should put some weights on your block so that it slides along a smooth surface at a constant velocity even when it is being pulled with a force that is 45 degrees from the horizontal.
Activity 10-4: Calculating Work

(a) Hold the spring scale horizontal to the table and use it to pull the block a distance of 0.5 metres along the horizontal surface in such a way that the block moves at a constant speed. Record the force in newtons and the distance in metres in the space below and calculate the work done on the block in joules. (Note that there is a special unit for work – the joule or J for short. One joule is equal to one newton times one metre, i.e., J = N·m)

Your values will be a bit different, but …

$$F = 1.6 \text{ N}$$
$$W = F \times \Delta x$$
$$\Delta x = 0.5 \text{ m}$$
$$W = (1.6 \text{ N})(0.5 \text{ m})$$
$$W = 0.8 \text{ J}$$

(b) Repeat the measurement, only this time pull on the block at a 45° angle with respect to the horizontal. Pull the block at the same speed. Is the force needed larger or smaller than you measured in part (a)?

You should see that the force required is less than in part (a), so I would expect a force of about 1.3 N this time if it was 1.6 N in part (a). This is because part of your force is now reducing the normal force, thus reducing the friction.

(c) Assuming that the actual physical work done in part (b) is the same as the physical work done in part (a) above, how could you enhance the mathematical definition of work so that the forces measured in part (b) could be used to calculate work? In other words, use your data to postulate a mathematical equation that relates the physical work, $W$, to the magnitude of the applied force, $F$, the magnitude of the displacement, $\Delta s$, and the angle, $\theta$, between $F$ and $\Delta s$. Explain your reasoning. Hint: $\sin 30^\circ = 0.500$, $\sin 45^\circ = 0.707$, $\cos 30^\circ = 0.865$, $\cos 45^\circ = 0.707$.

$$W = F \cos(\theta) \Delta s$$

(only the component of the force that is in the direction of motion causes work)

Work as a Dot Product

Review the definition of dot (or scalar) product as a special product of two vectors in your textbook, and convince yourself that the dot product can be used to define physical work in general cases when the force is constant but not necessarily in the direction of the displacement resulting from it.

$$W = \vec{F} \cdot \vec{s} \quad [\text{Equation 10-3}]$$
Activity 10-5: How Much Work Goes with Each Job?

(a) Re-examine the descriptions of the jobs shown in Figure 10-1. How much physical work is done in job 1?

\[ W_{box} = \Delta W \]
\[ W_{box} = F \Delta s \]
\[ W_{box} = (mg)(h) \]
\[ W = 100 \times W_{box} \]
\[ W_{box} = (10 \text{ kg}) \left( 9.8 \frac{m}{s^2} \right) (4 \text{ m}) \]
\[ W = (100)(392 \text{ J}) \]
\[ W_{box} = 392 \text{ J} \]
\[ W = 39200 \text{ J} \]

(b) How much physical work is done in job 2?

\[ W_{box} = \Delta W \]
\[ W_{box} = (mg \sin(\theta))(\Delta s) \]
\[ W_{box} = (10 \text{ kg}) \left( 9.8 \frac{m}{s^2} \right) \sin(30^\circ) (8 \text{ m}) \]
\[ W = (100)(392 \text{ J}) \]
\[ W_{box} = 392 \text{ J} \]
\[ W = 39200 \text{ J} \]

(c) How much physical work is done in job 3?

\[ W_{box} = \Delta W \]
\[ W_{box} = (mg)(\Delta h) \]
\[ W_{box} = (20 \text{ kg}) \left( 9.8 \frac{m}{s^2} \right) (2 \text{ m}) \]
\[ W = (50)(392 \text{ J}) \]
\[ W_{box} = 392 \text{ J} \]
\[ W = 19600 \text{ J} \]

(d) Was your original intuition about which job to take correct? Which job should Load 'n' Go try to land?

Yes, my original intuition was correct, the third job requires only half as much work as the first two, so it would be the easier job to do. Load 'n' Go should try to get job 3.

The Concept of Power

People are interested in more than physical work. They are also interested in the rate at which physical work can be done. Average power, \( <P> \), is defined as the ratio of the amount of work done, \( \Delta W \), to the time interval, \( \Delta t \), it takes to do the work, so that

\[ P = \frac{\Delta W}{\Delta t} \]
Instantaneous power is given by the derivative of work with respect to time, or

\[ P = \frac{dW}{dt} \]  

[Equation 10-4]

If work is measured in joules and time in seconds then the fundamental unit of power is in joules/second where 1 joule/second equals one watt. However, a more traditional unit of power is the horsepower, which represents the rate at which a typical work horse can do physical work. It turns out that

1 horsepower (or hp) = 746 watts = 746 joules/second.

Those of you who are car buffs know that horsepower is a big deal in rating high performance cars. The hp in a souped-up car is in the hundreds. How does your lifting ability stack up? Let's see how long it takes you or one of your classmates to lift a heavy object like a bowling ball a distance of one metre. For this observation you'll need to work with a partner and use the following items:

- A metre stick
- something heavy, like a bowling ball
- a stopwatch

Activity 10-6: Rate the Horsepower in Your Arms

(a) Lift or time someone who is lifting a bowling ball through a known height as fast as possible. Measure the time and height of the lift and compute the work done against the force of gravity.

Assume a height of 1 m, average person could lift a bowling ball that high in about 0.5 s. Bowling ball is 13 lbs.

\[ m_{ball} = 13 \text{ lbs} = 5.9 \text{ kg} \]

\[ h = 1.0 \text{ m} \]

\[ t = 0.5 \text{ s} \]

\[ W = F \times \Delta s \]

\[ W = mg \]

\[ W = (5.9 \text{ kg})(9.8 \frac{m}{s^2})(1.0 \text{ m}) \]

\[ W = 57.82 \text{ J} \]

How does this compare to the horsepower of your favourite car?
automobile? If you're not into cars, how do you stack up against a horse?

\[ \langle P \rangle = \frac{\Delta W}{\Delta t} \]

\[ \langle P \rangle = \frac{57.82 J}{0.5 s} \]

\[ \langle P \rangle = 115.64 W = 0.155 \text{ hp} \]

I am about as powerful as one-sixth of a horse (or one-sixth as powerful as a whole horse, if you want to look at it that way)

As far as cars go, considering both the Pinto and the Geo, which both get about 55 hp out of their engines, I am more than 300 times weaker than those vehicles. I guess I'd better hit the gym ...
SESSION TWO: WORK AND KINETIC ENERGY

Review of Homework Problems
Come prepared to ask and answer questions about your homework assignment.

The Force Exerted on a Mass by an Extended Spring
So far we have pushed and pulled on an object with a constant force and calculated the work needed to displace that object. In most real situations the force on an object can change as it moves.

What happens to the average force needed to stretch a spring from 0 to 1 cm compared to the average force needed to extend the same spring from 10 to 11 cm? How does the applied force on a spring affect the amount by which it stretches, i.e. its displacement. To complete these observations you will need:

• a spring such as the Pasco harmonic spring
• a support rod for the spring

Activity 10-7: Are Spring Forces Constant?
Hang the spring from a support rod. Extend the spring from 0 to 1 cm. Feel the force needed to extend the spring. Extend the spring from 10 to 11 cm. Feel the force needed to extend the spring again. How do the two forces compare? Are they the same?

It is definitely harder to stretch the string from 11 to 11 cm than it is from 0 to 1 cm. The further you try to stretch the spring, the harder it is to stretch (the more force is required).
The Force and Work Needed to Stretch a Spring

Now we would like to be able to quantify the force and work needed to extend a spring as a function of its displacement from an equilibrium position (i.e., when it is "unstretched").

Let's start by carefully measuring the forces needed to stretch a spring. To do this you can use a force and motion system. You will need:

- A computer system with a force probe and motion detector
- A block with a hook (such as the friction block)
- A spring scale to calibrate the force probe for a pull
- A spring
- A track

If you attach a spring to a calibrated force probe as shown in the figure below, you will be able to measure the spring's distance from a motion detector and the force you are applying to it at the same time.

![Figure 10-4: Schematic of set up for measuring spring forces. $x_0$ is the equilibrium position of the block at the end of the spring when it is not stretched.](image)

You will have to figure out how to calculate the displacement of the spring from its equilibrium (unstretched) position.

Activity 10-8: Force vs. Displacement for a Spring

(a) Record 10 values of $x$ and $F$. Stretch the spring a distance of $X = (x + 0.5) = 5.0$ cm, then $X = 10$ cm, and so on. Create and save a spreadsheet file with the data. Summarize the data in the table below. (Don't worry about the...
Next, display a plot of $<F_{\text{ext}}>$ vs. $X$. 

**Note:** Let $\Delta X$ represent the additional stretch each time. For example, if the spring is stretched from 20 cm to 25 cm, $\Delta X = 5.0$ cm while the total displacement $X = 25$ cm after the pull.

<table>
<thead>
<tr>
<th>$X=x_0-x$ (m)</th>
<th>$\Delta X$ (m)</th>
<th>$F_{\text{ext}}$ (N)</th>
<th>$\Delta W$ (J)</th>
<th>$W_{\text{total}}$ (J)</th>
</tr>
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<td>0</td>
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</table>

(b) Is the graph linear? If the force, $F_{\text{ext}}$, increases with the displacement in a proportional way, use a linear fit to find the slope of the line. Use the symbol $k$ to represent the slope of the line. What is the value of $k$? What are its units? **Note:** $k$ is known as the spring constant.

The graph should be pretty linear, and you should see a nice proportional relation between the force and the displacement. The slope for the above data is

$$k = 3.0 \frac{N}{m}$$

(c) Write the equation describing the relationship between the external force, $F_{\text{ext}}$, and the total displacement, $X$, of the spring from its equilibrium using the symbols $F_{\text{ext}}$, $k$, and $X$.  

Any restoring force on an object which is proportional to its displacement is known as a Hooke's Law Force. There was an erratic, contentious genius named Robert Hooke who was born in 1635. He played with springs and argued with Newton.

Calculating Work when the Force is not Constant

We would like to expand the definition of work so it can be used to calculate the work associated with stretching a spring and the work associated with other forces that are not constant. A helpful approach is to plot the average force needed to move an object for each successive displacement \( \Delta x \) as a bar graph like that shown in Figure 10-5 below.

Note: Any restoring force on an object which is proportional to its displacement is known as a Hooke's Law Force. There was an erratic, contentious genius named Robert Hooke who was born in 1635. He played with springs and argued with Newton.

Because the force is directly proportional to the displacement, and the constant of proportionality is \( k \). Although, since the force is always in the opposite direction of the displacement, it’s more accurate to write

\[
\bar{F}_{ext} = -k \bar{x}
\]

Figure 10-5: A graph representing the average applied force causing each unit of displacement of an object. This graph represents force that is not constant but not the force vs. displacement of a typical spring.

Activity 10-9: Force vs. Distance in a Bar Graph

(a) Replot your spring data in a bar graph format in the next grid.
How can we calculate the work done in stretching the spring? We can use several equivalent techniques:

1. adding up little pieces of $F_{\text{ext}} \Delta x$,
2. finding the area under the "curve" you created,
3. using mathematical integration.

All three methods should yield about the same result. If you have not yet encountered integrals in a calculus course, you can compare the results of using the first two methods. If you have studied integrals in calculus you may want to consult your instructor or the textbook about how to set up the appropriate definite integral to calculate the work needed to stretch the spring.

Activity 10-10: Calculation of Work

(a) Calculate the work needed to stretch the spring to a distance $d$ specified by you by adding up small increments of $<F_{\text{ext}}> \Delta X$ in your spreadsheet calculations. Place the running sum in the table in Activity 10-8 above and summarize the result below. Don't forget to specify units.

\[ W_{\text{total}} = 0.338 \text{ J} \]

(b) Calculate the work needed to stretch the spring to a distance $\Delta x$ specified by you by computing the area under the curve in the graph of $F$ vs. $X$ that you just created. Using the graph from part 10-8(a), the area under the curve from 0 to 0.45 m is

\[ W = 0.5(0.45 \text{ m})(1.35 \text{ N}) = 0.304 \text{ J} \]
(c) How does adding up the little rectangles in part (a) compare to finding the area under the curve in part (b)?

The two answers were very close, but the "area under the curve" method got me a smaller result. I would trust the "area under the curve" method more, though, because adding up discrete Force x Displacement chunks from the table assumes that the force was constant over those displacements, which it was not.

Note that in the limit where the ΔX values are very small the sum of \(<F_{ext}\>\Delta X\), known by mathematicians as the Riemann sum, converges to the mathematical integral and to the area under the curve.

**Defining Kinetic Energy and Its Relationship to Work**

What happens when you apply an external force to an object that is free to move and has no friction forces on it? Obviously it should experience an acceleration and end up being in a different state of motion. Can we relate the change in motion of the object to the amount of work that is done on it?

Let's consider a fairly simple situation. Suppose an object is lifted through a distance \(s\) near the surface of the earth and then allowed to fall. During the time it is falling it will experience a constant force as a result of the attraction between the object and the earth – glibly called the force of gravity. You can use the theory we have already developed for the gravitational force to compare the velocity of the object to the work done on it by the gravitational field as it falls through a distance \(h\). This should lead naturally to the definition of a new quantity called kinetic energy, which is a measure of the amount of "motion" gained as a result of the work done on the mass.
Activity 10-11: Equations for Falling $v$ vs. $\Delta y$

(a) An object of mass $m$ is dropped near the surface of the earth. What is the magnitude and direction of its acceleration?

$$\vec{a} = -g = -9.8 \frac{m}{s^2} \text{ [down]}$$

(b) If the object has no initial velocity and is allowed to fall for a time $t$ under the influence of the gravitational force, what kinematic equation describes the relationship between $\Delta y$, and its time of fall, $t$? Assume $g = +9.8 \text{ m/s}^2$.

$$\Delta y = vt + \frac{1}{2}at^2$$

$$\Delta y = (0)t + \frac{1}{2}gt^2$$

$$\Delta y = \frac{1}{2}gt^2$$

(c) Do you expect the magnitude of the velocity to increase, decrease or remain the same as the distance increases? 

**Note:** This answer is obvious!!

I would expect the magnitude of the velocity to increase, of course.

(d) Differentiate the equation you wrote down in part (b) to find a relationship between $v$, $g$, and time $t$.

$$\frac{d}{dt} (\Delta y) = \frac{d}{dt} \left( \frac{1}{2}gt^2 \right)$$

$$v = gt$$
(e) Combine the equations you obtained in parts (b) and 
(d) by eliminating $t$ to describe how the velocity, $v$, of the 
falling object depends on the displacement, $\Delta y$, through 
which it has fallen.

\[
\begin{align*}
\Delta y &= \frac{1}{2} gt^2 \\
t &= \sqrt{\frac{2 \Delta y}{g}} \\
\frac{v}{g} &= \sqrt{\frac{2 \Delta y}{g}} \\
v &= \sqrt{2 g \Delta y}
\end{align*}
\]

You can use the kinematic equations to derive the functional relationship you hopefully discovered experimentally in the last activity. If we define the kinetic energy ($K$) of a moving object as the quantity

\[K = \frac{1}{2} mv^2\]

then we can relate the change in kinetic energy as an object falls to the work done on it. Note that for an object initially at rest the initial kinetic energy is $K_i = 0$, so the change in kinetic energy is given by the difference between the initial and final kinetic energies.

\[\Delta K = K_f - K_i = \frac{1}{2} mv^2.\]
Activity 10-12: Computing Work and Kinetic Energy of a Falling Mass

(a) Suppose the mass of your falling object is 0.35 kg. What is the value of the work done by the gravitational force when the mass is dropped through a displacement of \(\Delta y = -1.2 \text{ m}\)?

\[
W_g = F_g \Delta y = -mg \Delta y = -(0.35 \text{ kg})(9.8 \frac{m}{s^2})(-1.2 \text{ m}) = 4.12 \text{ J}
\]

(b) Use the kinematic equation you derived in Activity 10-11 (e) that relates \(v\) and \(\Delta y\) to find the velocity of the falling object after it has fallen 1.2 m.

\[
v = \sqrt{2g\Delta y} = \sqrt{2(9.8 \frac{m}{s^2})(1.2 \text{ m})} = 4.85 \frac{m}{s}
\]

(c) What is the kinetic energy of the object before it is dropped? After it has fallen 1.2 m? What is the change in kinetic energy, \(\Delta K\), as a result of the fall?

\[
K_i = 0 \quad K_f = \frac{1}{2}mv_f^2 \quad \Delta K = K_f - K_i
\]

\[
K_f = \frac{1}{2}(0.35 \text{ kg})(4.85 \frac{m}{s})^2 = 4.12 \text{ J} \quad \Delta K = 4.12 \text{ J} - 0 \text{ J}
\]

(c) How does the work done by the gravitational force compare to the kinetic energy change, \(\Delta K\), of the object?

The work done by the gravitational force is exactly the same as the change in the object’s kinetic energy.

Activity 10-13: The Mathematical Relationship between Work and Kinetic Energy Change During a Fall

(a) Since our simplified case involves a constant acceleration, write down the equation you derived in Activity 10-11 (e) to describe the speed, \(v\), of a falling object as a function of \(\Delta y\).

\[
v = \sqrt{2g\Delta y}
\]
(b) Using the definition of work, show that $W = mg|\Delta y|$ when the object is dropped through a distance $|\Delta y|$.

\[
W = F \Delta s \\
W = F_g |\Delta y| \\
W = mg |\Delta y|
\]

(c) By combining the equations in parts (a) and (b) above, show that in theory the work done on a mass falling under the influence of the gravitational attraction exerted on it by the earth is given by the equation $W = \Delta K$.

\[
\Delta y = \frac{v^2}{2g} \\
W = mg \frac{v^2}{2} \\
W = \frac{1}{2}mv^2 \\
W = \Delta K
\]

You have just proven an example of the work-energy theorem which states that the change in kinetic energy of an object is equal to the net work done on it for all the forces acting on it.

\[
\Delta K = W \quad \text{[Work-Energy Theorem]}
\]

Although you have only shown the work-energy theorem for a special case where no friction is present, it can be applied to any situation in which the net force can be calculated. For example, the net force on an object might be calculated as a combination of applied, spring, gravitational, and friction forces.
SESSION THREE: KARATE AND PHYSICS (OPTIONAL)

Can you Break a Pine Board with Your Bare Hand?
The Japanese style of Karate which is currently popular in the United States both as a sport and as a method of self-defense was developed in the 17th century on the Island of Okinawa. It is claimed that even a beginner at Karate with sufficient athletic prowess and confidence can learn to break a substantial wood plank.

As a focal point for the application of the concepts of work, kinetic energy and momentum, we are going to explore whether you can break a pine board with your bare hand. As a result of the activities that follow you may be motivated to attempt to break a pine board (28 cm x 15 cm x 1.9 cm) along the grain with your bare hand. You should understand that regardless of the outcome of any tests you might conduct to gauge your ability to perform this karate movement, your attempts are entirely voluntary. You will be proceeding at your own risk and are not expected to do this as part of this course.

Before proceeding you should have read the article referenced in the footnote below. For this activity you may request to use any equipment we already have on hand in the laboratory. However, the use of some of the following equipment may be of value to you:

- 2 clear pine boards (28cm x 15cm x 1.9cm)
- 1 small chain
- Up to thirty 2 kg masses (or a collection of bricks)
- 4 large mass pans with bottom hooks (or a platform)
- A set of Vernier calipers
- A photogate or motion detection setup to measure hand speed
- A 20 N spring scale
- A balance

Figure 10-7: The pine board cut with proper grain alignment and no knots.


There are a series of questions you will want to answer to make sure you can break the pine board without injuring yourself. These include the following:

**Energy Considerations – Can you Break the Board?**

1. How much work is required to break a typical pine board cut to the specified dimensions? If you know how much work it takes to break the board, then how much kinetic energy would you have to transfer to the board to break it?

2. How much kinetic energy can you hit the board with? Assume that as you break the board your hand makes a collision that is approximately inelastic. If you hit the board with the calculated amount of kinetic energy, how much of that energy will be transferred to the board in the collision? Is there enough energy transfer to break the board in this case? **Hint:** Consider the Work-Energy Theorem.

**Momentum Considerations – Could you break a bone?**

1. Suppose as you break the board your hand makes a more or less inelastic collision with it. What is the momentum change of your hand? Suppose you have a failure of nerve because you are afraid of hurting yourself so your hand slows down to a speed just below that which you need to break the board. What is the momentum change of your hand in that case?

2. Suppose the total time that your hand is in contact with the board during the "hit" is the same as the time it takes a clay blob to make an inelastic collision. What is the maximum force your hand can feel in an elastic collision; in an inelastic collision? How many g's is this in each case?

3. If the injury you sustain is a function of the maximum force on your hand and if you know theoretically that you can break the board, what is the consequence of having a failure of nerve and slowing down your hand in mid-hit? Are you more likely to be injured or less likely to be injured?

**Note:** Research has shown that an impulse leading to a 900 N force for 0.006 s is enough to break a typical cheekbone.
Activity 10-14: Does Your Hand Have Enough Kinetic Energy to Break a Board?

(a) In discussions with your classmates and/or partners develop and implement a method for measuring the amount of work needed to break the pine board directly. Describe your procedures and results in the space below.

(b) In discussions with your classmates and/or partners develop and implement a method for determining the amount of kinetic energy your hand will have just before you hit the board. Describe your procedures and results in the space below.

(c) If the collision between your hand and the board is inelastic and no momentum is transferred to the board supports, use the law of conservation of momentum to find the velocity of your hand and the board fragments as they move down together.

(d) Next, calculate the amount of kinetic energy the board can acquire as a result of your hit. Is this enough energy
to break the board? **Note:** If this collision is totally inelastic mechanical energy will be not be conserved.

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## Activity 10-15: What is the Potential For Injury?

(a) If as you break the board your hand makes an inelastic collision with it, what is the momentum change of your hand?

Explain your assumptions and show your calculations.

(b) Suppose you have a failure of nerve because you are afraid of hurting yourself so your hand slows down to a speed just below that which you need to break the board. Calculate the momentum change of your hand in that case. Show all your assumptions and equations.

(c) Suppose the total time that your hand is in contact with the board during the "hit" is the same as the time it takes a clay blob to make an inelastic collision. Using the results of (a) and (b) above, what is the maximum force your hand can feel if you break the board? If you just barely fail to break the board?
(d) If the injury you sustain is a function of the maximum force on your hand and if you know theoretically that you can break the board, what is the consequence of having a failure of nerve and slowing down your hand in mid hit? Are you more likely to be injured or less likely to be injured? Why?

★ Activity 10-16: OPTIONAL–Breaking a Board
(a) After duly considering the situation, I have convinced myself and my instructor that I can do it, and I want to try it in class.

_____________________________            _________________
Your Signature                                                               Date

______________________________       _________________
Your Instructor's Signature                                                  Date

(b) Describe what happened:

"There's no challenge in breaking a board. Boards don't hit back."

Bruce Lee