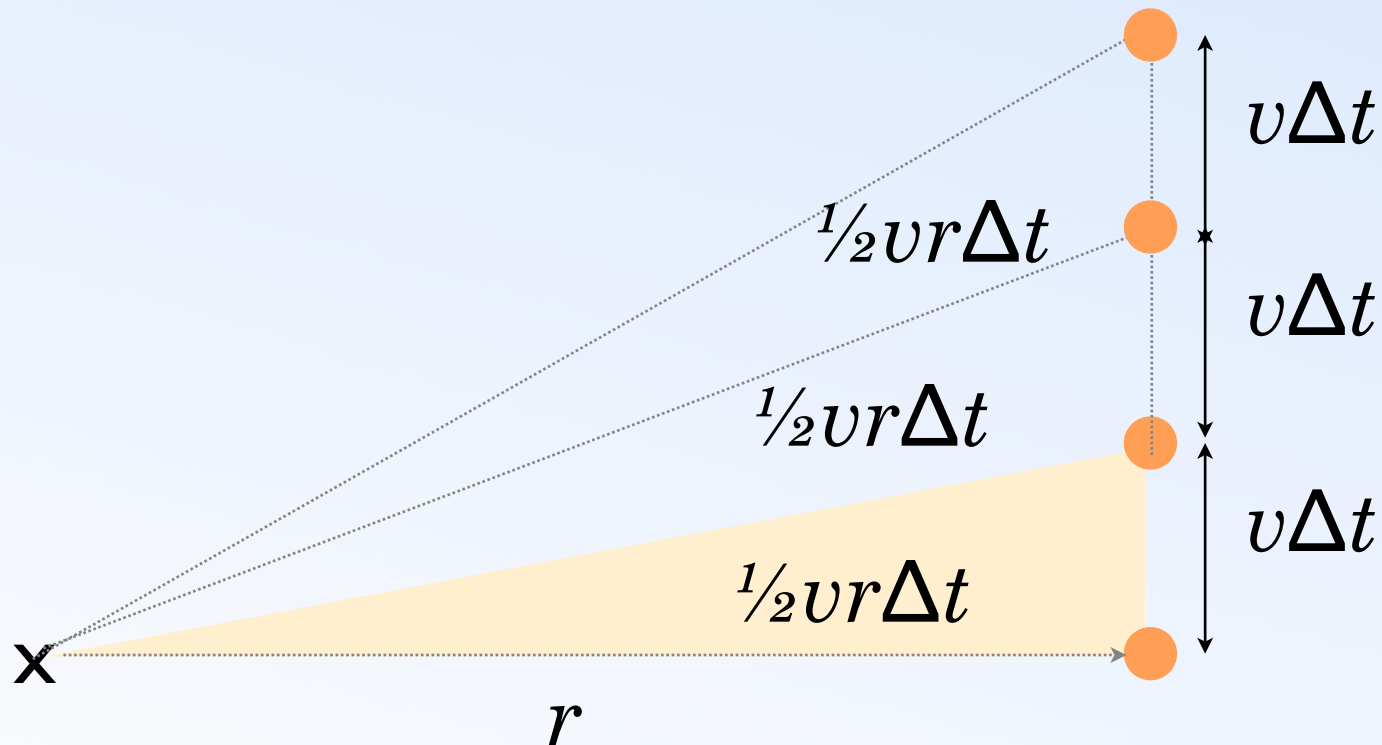


Rotations

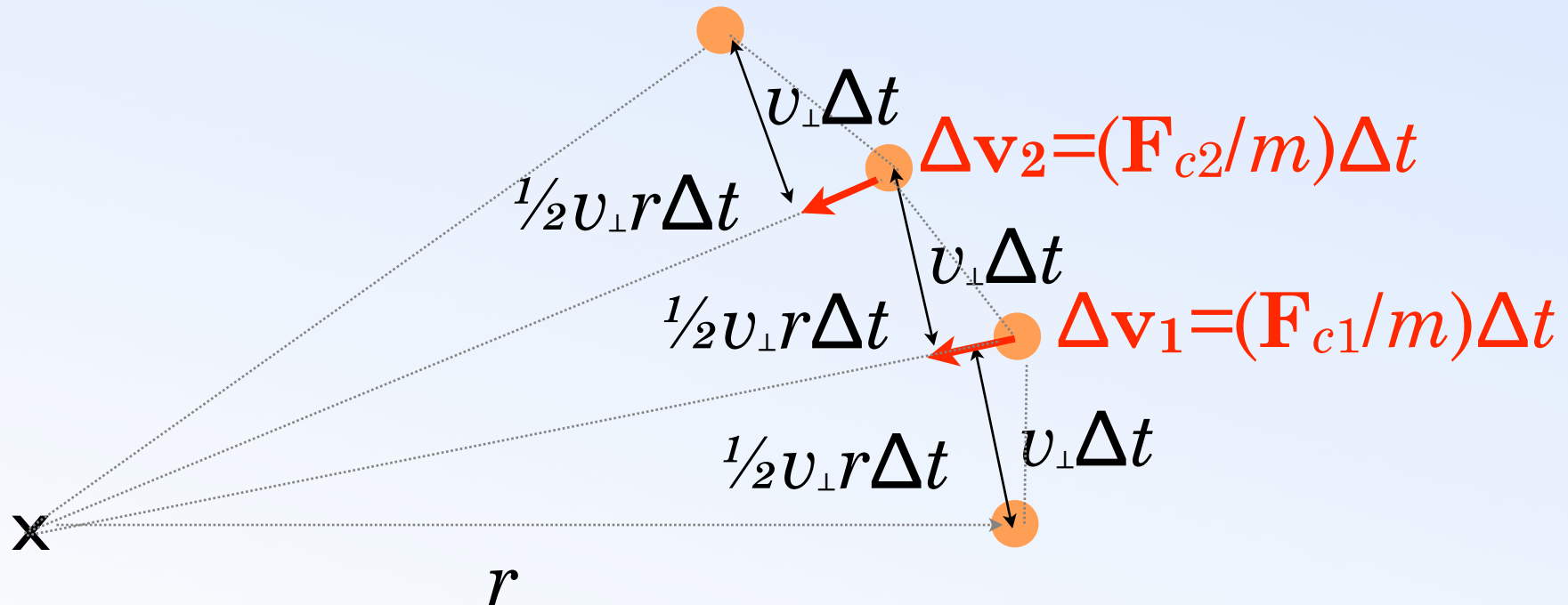
Linear Motion

- Kepler's 2nd Law
- Equal areas are swept out in equal times.



Central Force

- Kepler's 2nd Law
- Equal areas are swept out in equal times.



$v_{\perp}$ is the component of velocity perpendicular to $\mathbf{r}$.

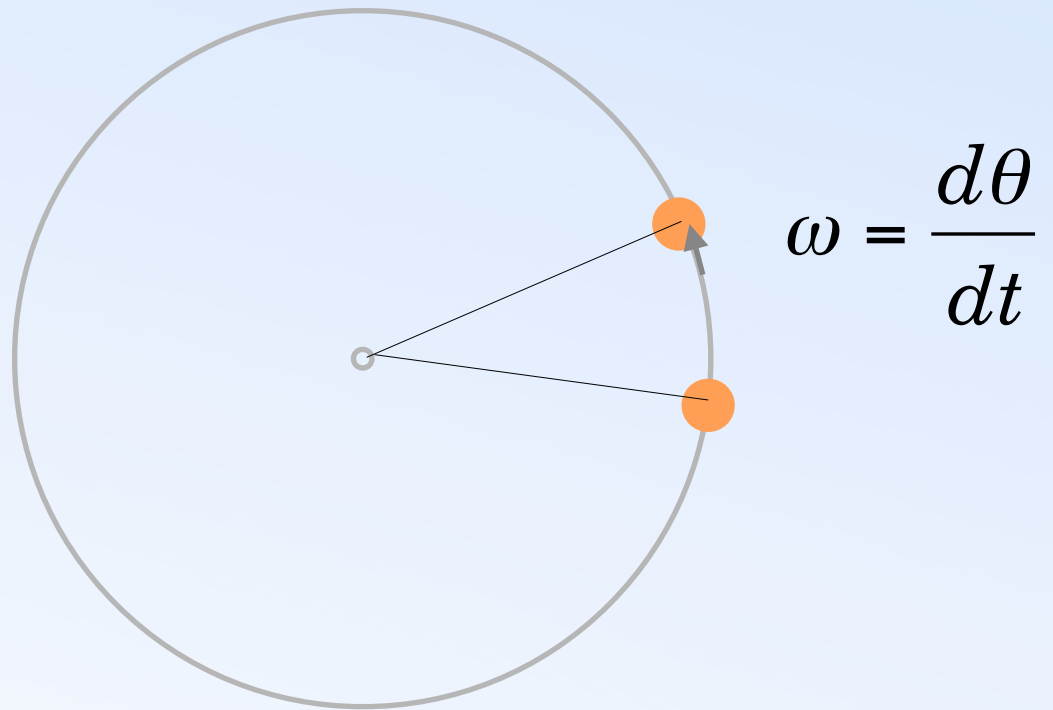
Angular Momentum

$$L = mv_{\perp}r$$

- When there are only central forces, L doesn't change.
- $L = 2m(\text{Area})$ swept out.
- Angular momentum is conserved

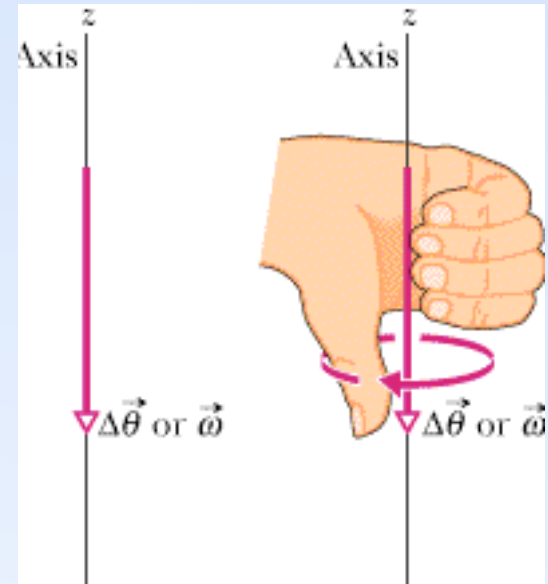
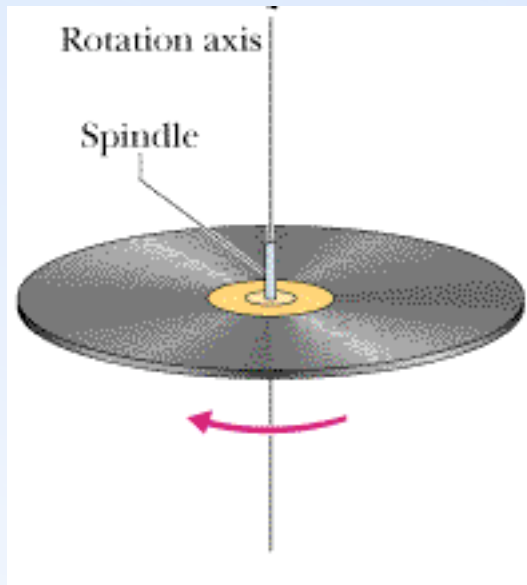
Angular momentum

- In terms of angular variables.



$$L = m\omega r^2$$

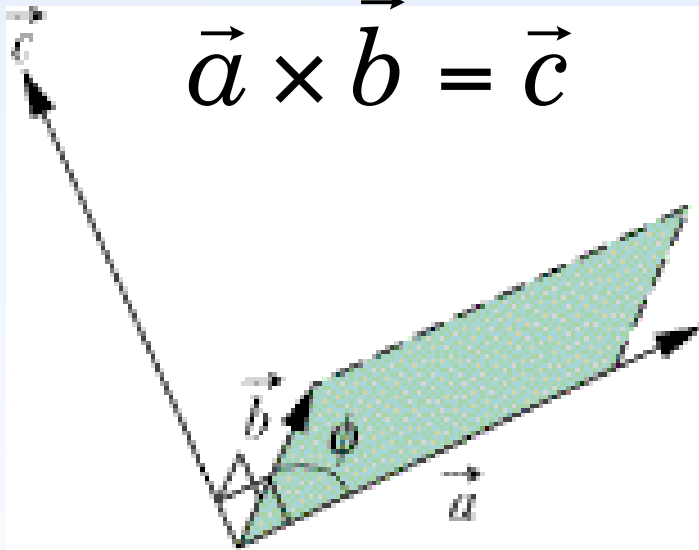
Vector nature of angular variables



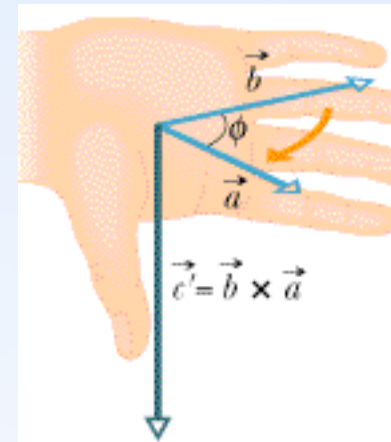
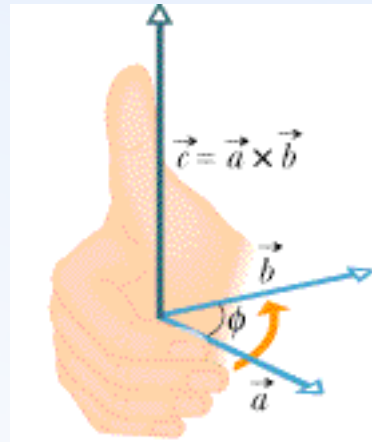
- use the “right hand rule” to assign a direction to angular rotations.

Cross Product

- Direction is given by “right hand rule”
- Imagine turning a screw from **a** to **b**.
- The direction a normal r.h. screw moves is the direction of **c**.



area of parallelogram = $ab \sin \phi$

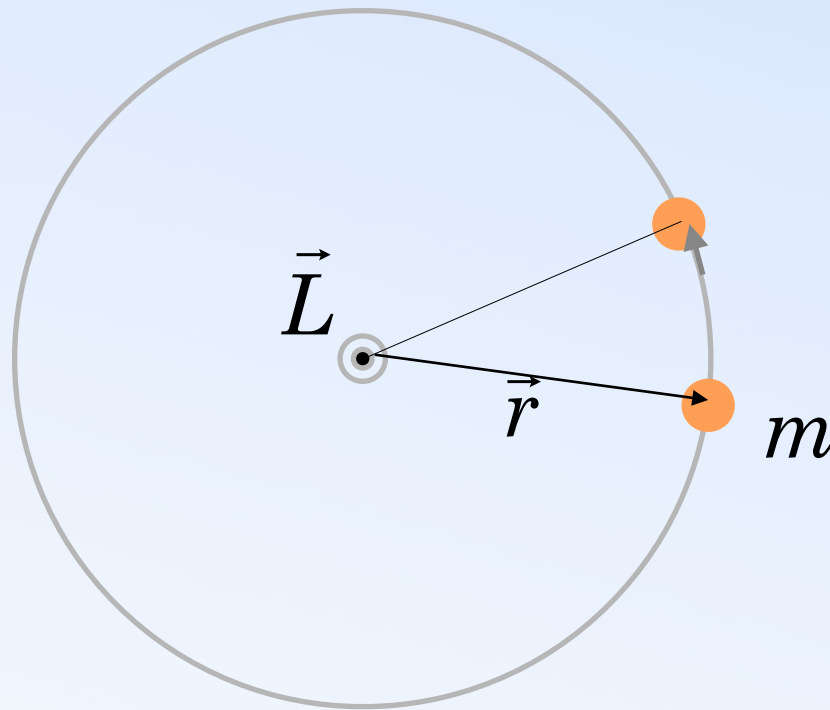


Angular Momentum

- Defines as a vector (or cross) product of $\mathbf{r}$ and $\mathbf{p}$.

$$\vec{p} = m\vec{v}$$

$$\vec{L} = \vec{r} \times \vec{p}$$



out of page



in to page

Kinetic Energy

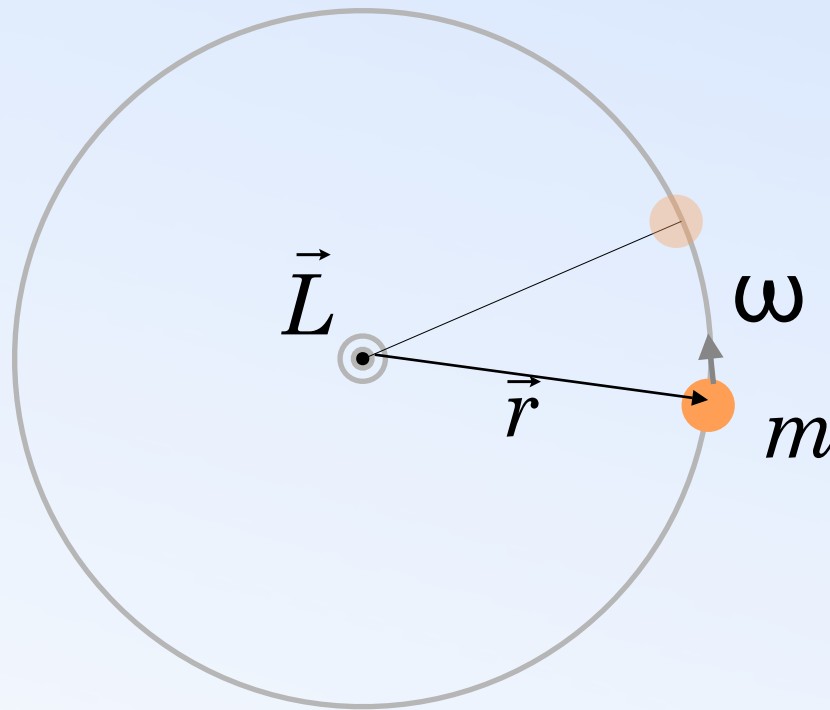
$$K = \frac{1}{2}mv^2 = \left(\frac{1}{2}mr^2\right)\omega^2$$

“Moment of Inertia” or
“Rotational Inertia”

$$I = mr^2$$

$$K = \frac{1}{2}I\omega^2$$

Analogous to $K = \frac{1}{2}mv^2$



Rigid Body

Everything's all stuck together.

ω is the same for all the m 's

$$K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2$$

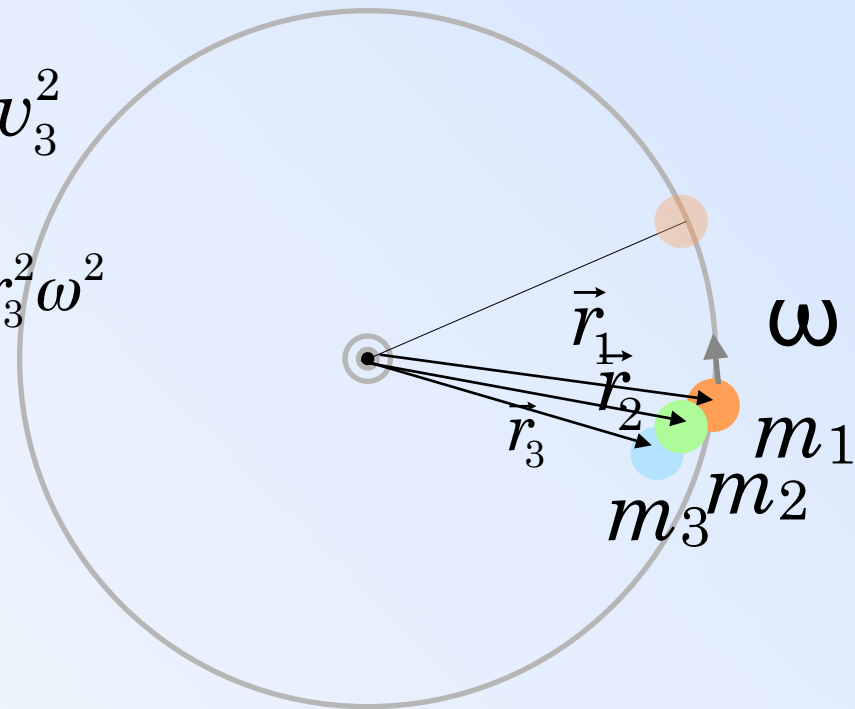
$$K = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \frac{1}{2}m_3r_3^2\omega^2$$

$$K = \frac{1}{2}(m_1r_1^2 + m_2r_2^2 + m_3r_3^2)\omega^2$$

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$$

$$K = \frac{1}{2}I\omega^2$$

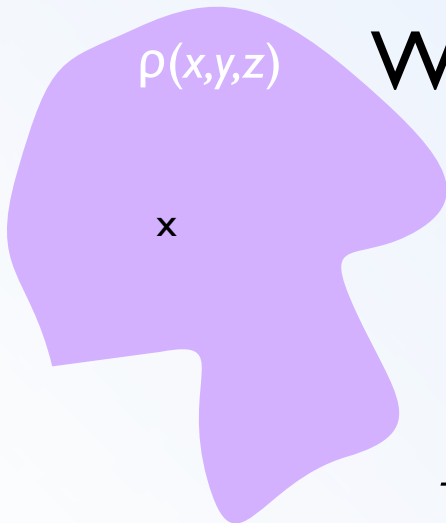
$$\vec{L} = I\vec{\omega}$$



Moment of Inertia

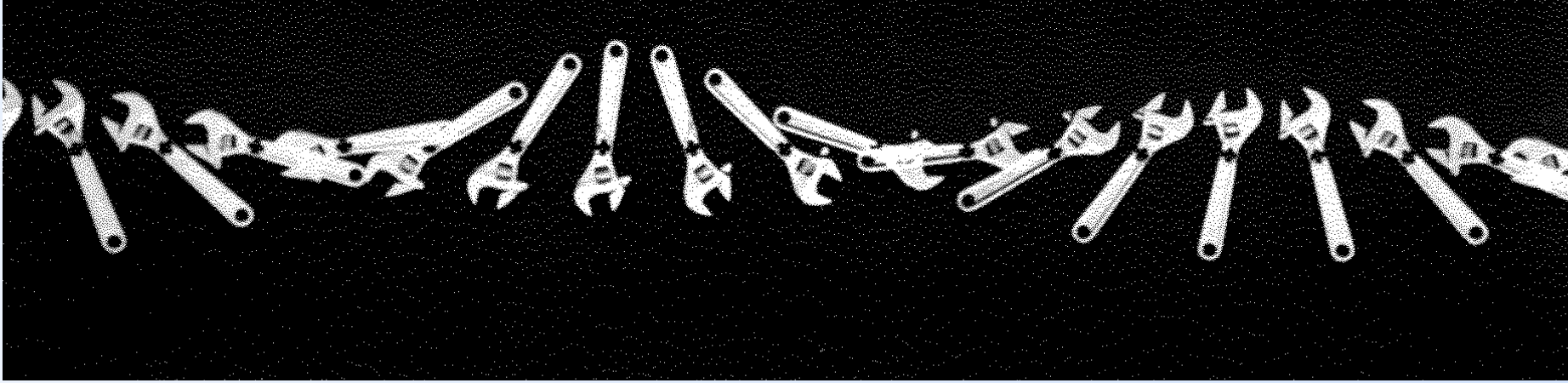
In general

$$I = \int r^2 dm = \iiint r^2 \rho(x, y, z) dx dy dz$$

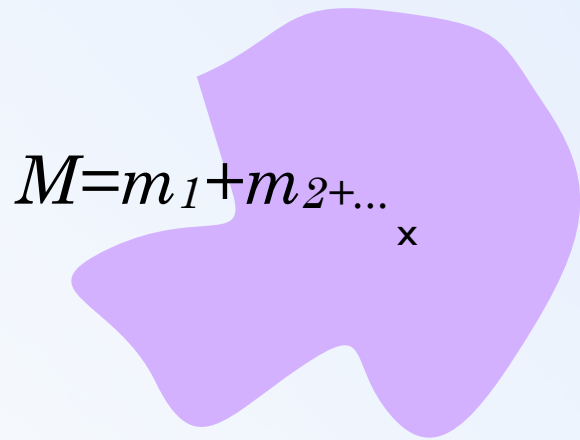


Where $\rho(x,y,z)$ is the mass density.

I depends on the reference point x .



Centre of Mass



$$x_{\text{cm}} = \frac{1}{M} \sum_i m_i x_i$$

$$y_{\text{cm}} = \frac{1}{M} \sum_i m_i y_i$$

$$z_{\text{cm}} = \frac{1}{M} \sum_i m_i z_i$$

A natural reference point for I is the centre of mass.
That's the points the object would rotate about in free space.

Parallel Axis Theorem

$$I = I_{\text{cm}} + Md^2$$

