Sample Formal Report

A Comparison of the Experimental Properties of a Compound Pendulum with Simple Theory

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Abstract

The period of oscillation P of a rigid bar 1 metre long has been measured as a function of distance l of the point of suspension from the centre of gravity.

The observed curve of P versus l agrees within experimental error with a curve calculated from simple theory though in the region of the minimum values of P the theoretical curve gives values which are systematically about 2% too low. The deviation is not consistent with viscous effects proportional to the angular velocity of the bar and probably arises because of a more complex damping mechanism.

The graph of P^2l versus l^2 is linear as expected from theory and leads to a value of the radius of gyration of the bar which agrees wiithin 1% with the value of 0.289 m calculated from the geometry of the bar. The acceleration due to gravity was found to be

$$g = 9.71 \pm 0.08 \text{ m/s}^2.$$

The various disagreements between the theoretical and experimental results do not exceed about ±1% and are thought to be due to damping mechanisms not considered in the simple theory.

Introduction

A compound pendulum is a rigid body swinging in a vertical plane about any horizontal axis passing through the body. The present article gives the results of an experimental investigation of the dependence of *P* on *l* for a particular compound pendulum where *P* is the period for small oscillations of the pendulum when the distance between the centre of gravity and the axis of rotation is *l*.

Theory

According to Newman and Searle[1] the period *P* of a compound pendulum is

$$P = 2\pi \left(\frac{k^2 + l^2}{lg} \right)^{1/2} \tag{1}$$

where *g* is the acceleration of gravity and *k* is the radius of gyration. The radius of gyration is defined by

$$k^2 = \frac{\int \rho r^2 dV}{\int r dV} \tag{2}$$

where ρ is the density of the material of the pendulum at a distance *r* from the centre of gravity.

Equation 1 applies if the damping is negligible and if α, the angular amplitude of oscillation, is infinitely small. When α is finite the period is given by

$$P' = P \left(1 + \frac{1}{4} \sin^2 \frac{\alpha}{2} \right) \tag{3}$$

If it is assumed that the damping force is of the form $MR(k^2 + l^2)\omega$ where ω is the angular velocity of the pendulum, *M* its mass and *R* a constant then the period of the damped motion is[1]

$$P_D = 2\pi \left[\frac{lg}{(k^2 + l^2)} - \frac{R^2}{4} \right]^{1/2} \tag{4}$$

When the damping is small $R^2/4 << lg/(k^2 + l^2)$ and in this case equations 4 and 1 may be used to obtain the relation

$$P_D - P = P_D^3 R^2 / 32\pi^2. \quad (5)$$

Experimental Procedure

The compound pendulum used in the present experiments consisted of a rectangular iron bar 1.00 m long x 3.80 cm wide x 0.95 cm thick in which a number of holes each 0.47 cm in diameter had been drilled (see figure 1) the distance between the centres of adjacent holes being 5 cm. The axes of the holes were perpendicular to the face of largest area of the bar and the axis of one of the holes (A) passed through the centre of gravity of the bar. In a given experiment the bar was suspended by means of an axle which passed through one of the holes and which was supported on a ball bearing mount so that the rod was able to oscillate in a vertical plane with a minimum of friction at the bearing.

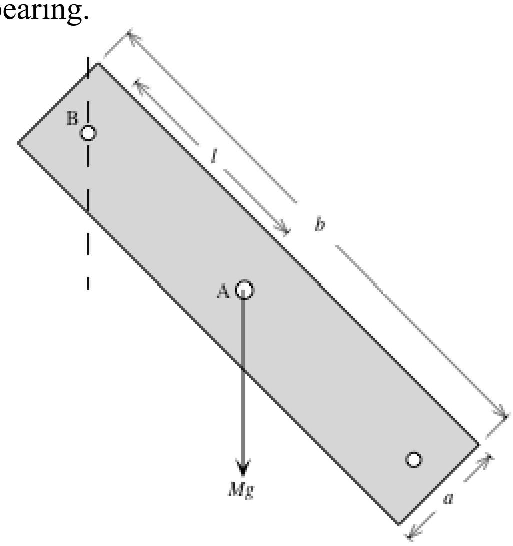

Figure 1: The Compound Pendulum

The period of oscillation was obtained by timing twenty swings with a stop watch. In order to obtain information about errors several such sets of readings were obtained and the period P was calculated by averaging. The amplitude of oscillation was kept below 10° so as to ensure that the period of oscillation was within 0.2% of the period for infinitely small oscillations (see equation 3). The distance l between the axes of the central hole B was measured with a metre ruler which enabled l to be obtained to within 1% accuracy.

Results and Discussion

Figure 2 shows a graph of P versus l.

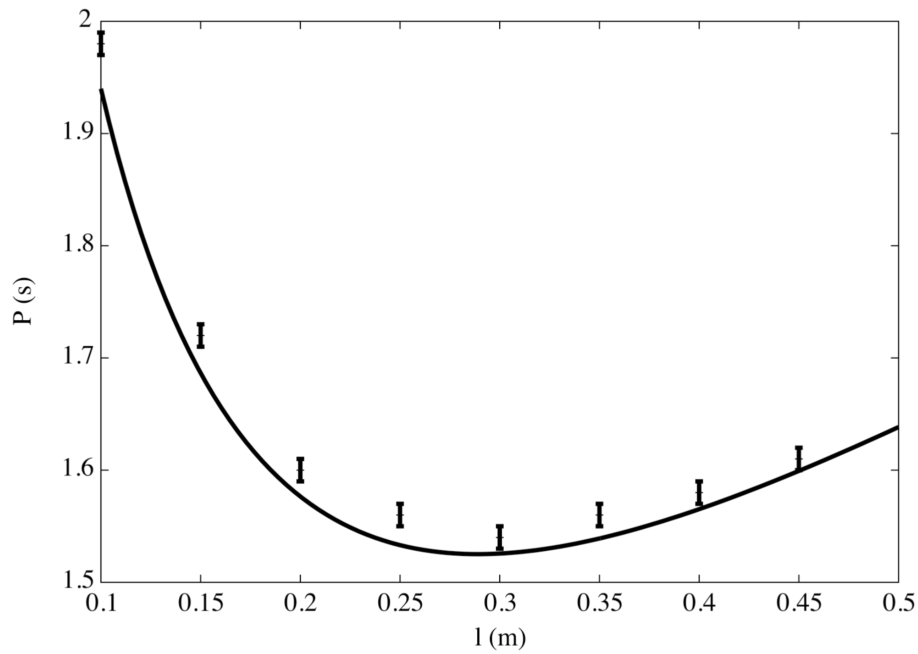

Figure 2: Graph of P versus l for a compound pendulum where P is the period in seconds and l is the distance in cm between the centre of gravity of the pendulum and the point of suspension. (Solid line shows calculated values; points with error bars show experimental results.)

By differentiation of equation 1 it is easily shown that the minimum value of P is

$$P_{min} = 2\pi\sqrt{k/g} \qquad (6)$$

From figure 2

$P_{\text{min}} = 1.54 \pm 0.01$ s hence

$$k = 0.295 \pm 0.005 \text{ m} \qquad (7)$$

where we have used the value of $g = 9.81$ m/s^2 for the acceleration of gravity. A non-linear least-squares fit using Marquhart-Levinsen algorithm in gnuplot gives

$$k = 0.2961 \pm 0.0004 \text{ m} \quad (\pm 0.12\%) \qquad (8)$$

which is consistent with (7).

Equation 1 has two solutions l_1 and l_2 for a fixed value of P. Algebraic manipulations of the equation show that

$$\sqrt{l_1 l_2} = k \tag{9}$$

When P = 1.58 s the observed values for l were $l_1 = 0.425 \pm 0.001$ m and $l_2 = 0.200 \pm 0.001$ m. These values give the result $k = 0.292 \pm 0.001$ m which is in reasonable agreement with the value of k obtained from $P_{\min}$ (equation 6).

When equation 2 is applied to a uniform bar of width a and length b (see figure 1) the result is

$$k^2 = a^2/12 + b^2/12. \tag{10}$$

In the present experiments the values of a and b were 3.80 ± 0.01 cm and 100.0 ± 0.1 cm respectively, hence the calculated value of k is

$$k = 0.289 \pm 0.001 \text{ m} \tag{11}$$

In this calculation the presence of holes has been neglected for simplicity. When the holes are considered the calculated value of k is reduced but is within 1% of that just given above.

The curve shown in figure 2 was calculated using equations 1, 7 and 8. The calculated and measured values of P disagree by about 2%. This discrepancy cannot be explained in terms of viscous damping proportional to the angular velocity of the bar because as equation (5) shows, the discrepancy between the observed damped period P_D and the undamped period P should decrease as P_D decreases. I think that the difference in the calculated and observed results may be due to damping in the roller bearing. [In a scientific paper it would be necessary to produce evidence for this conjecture, for example, the experiment should have been repeated after the bearing had been lubricated. However, in an elementary laboratory there is no time to follow up all possible explanations. Despite this limitation it is essential to try to think of plausible explanations of discrepancies.]

As a further check Equation 1 may be rewritten in the form

$$P^2 l = \frac{4\pi^2}{g}(k^2 + l^2) \tag{12}$$

This shows that the graph of $P^2 l$ versus l^2 should be a straight line of slope $4\pi^2/g$ and intercept $4\pi^2 k^2/g$ on the $P^2 l$ axis.

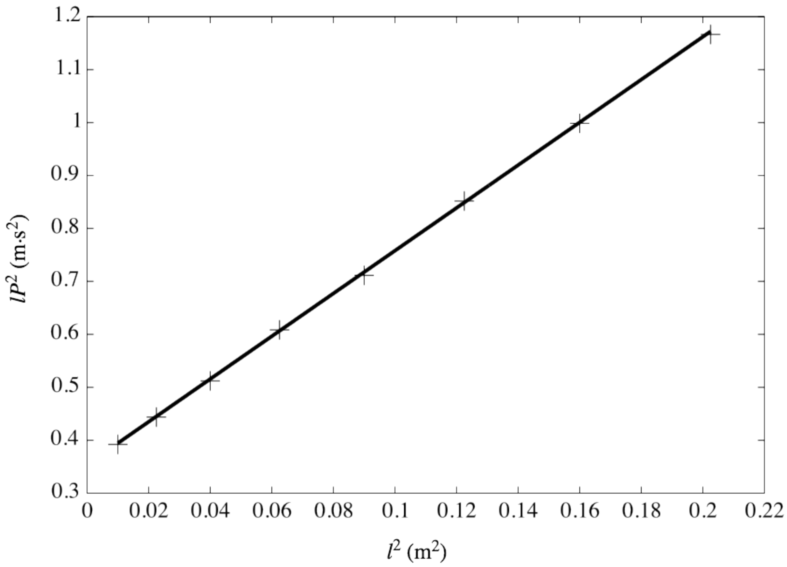

Figure 3: Graph of P^2l versus l^2 for a compound pendulum.

Slope = 4.04 ± 0.004 s^2/m.

Intercept on the lP^2 axis = 0.354 ± 0.0004 m^2/s.

Figure 3 shows that the graph of P^2l versus l^2 is in fact a straight line whose slope and intercept on the P^2l axis lead to the values

$$k = 0.289 \pm 0.0004 \text{ m} \tag{13}$$

$$g = 9.71 \pm .08 \text{ m/s}^2. \tag{14}$$

The value of k agrees well with the calculated result (see equation 11) and g is close to the accepted value of 9.81 m/s^2. Reference to figure 3 shows that the majority of the length of the straight line has points with P not close to P_{min} (because P^2l is plotted), hence k is largely determined by points having P in excess of P_{min}. Figure 2 shows that the calculated values of P agree best with the experimental values for P in excess of P_{min}. It is this fact which explains why the straight line graph gives a value of k which agrees well with the calculated value (see equations 11 and 13). It follows by a similar argument that the difference between k values given by relations 7 and 11 occurs because the value given in equation 7 depends on the observed P_{min} which is not the same as the result calculated from simple theory.

The random errors in the times and distances measured in the present experiments were about ±1%. In figure 3 the range of possible straight lines which could be drawn to fit the data was used to obtain the errors given in relations 13 and 14. In other cases the standard rules for combining errors were used to obtain the errors quoted.

It has already been pointed out that the measured values of P are systematically about 2% too high. This is may be due to friction at the bearing and not to any fundamental limitation of the theory. This being so it is reasonable to assert that the above experiments show that the behaviour of a compound pendulum is in accord with Newton's laws of rotation.

References

1. F. H. Newman and V. H. L. Searle, 1951, *The General Properties of Matter*, Edward Arnold and Co., London, pp. 22-23.
2. R. W. B. Stephens and A. E. Bate, 1950, *Wave Motion and Sound*, Edward Arnold and Co., London, p. 358.